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Public information and uninformed trading: Implications for market liquidity and price efficiency ☆

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Abstract

We develop a rational expectations equilibrium model in which noise trading comes from discretionary liquidity traders. The equilibrium quantity of aggregate noise trading is endogenously determined by the population size of liquidity traders active in the financial market. By improving market liquidity, public information reduces the expected trading loss of liquidity traders and thus attracts more such traders to the market, which negatively affects information aggregation. Analyzing an alternative setting that models noise trading as coming from hedgers yields similar insights. In a setting with endogenous information, public information can harm information aggregation both through crowding out private information and through attracting noise trading.

JEL classification: D61; G14; G30; M41

Keywords: Discretionary liquidity trading; Market liquidity; Information aggregation; Information production; Hedging

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1. Introduction

Rational expectations equilibrium (REE) models have been the workbench for analyzing financial markets by providing a machinery of Hayek's (1945) idea that prices aggregate information dispersed among market participants. These models typically introduce "noise trading" or "liquidity trading" to prevent the market price from fully revealing private information and to circumvent the "no trade" problem (Milgrom and Stokey, 1982). The essential feature of noise trading is that it has no informational content; that is, in a statistical sense, it is independent of the fundamental value of the traded asset.[1] The theoretical literature has so far focused on studying the behavior of investors who trade on private information and it largely ignores how the quantity of noise trading is determined.[2]

In the modern financial market, much of uninformed trading is engaged by financial institutions. For example, fund managers need to rebalance their portfolios for non-informational reasons when receiving large inflows or redemptions from clients.[3] The resulting trading can be viewed as "discretionary liquidity trading," which has been studied in the microstructure literature (e.g., Admati and Pfleiderer, 1988; Foster and Viswanathan, 1990). Another example of uninformed trading is algorithmic trading, which has become increasingly dominant in the stock market. Skjeltorp et al. (2016) document that algorithmic trading originating from large institutional investors is likely to be uninformed. Uninformed trading may also result from hedging activities of financial institutions. For instance, investment banks may invest in commodity futures to hedge their issuance of commodity-linked notes (CLNs) whose payoffs are linked to the price of commodity futures. Henderson et al. (2015) provide evidence that futures investments of CLN issuers do not convey information about fundamentals but nonetheless significantly impact commodity futures prices.

What determines the size of noise trading in financial markets? What are the implications of this endogenous noise trading for market outcomes? In this paper, we provide theoretical models to answer these important questions. The baseline model in Section 3 generates uninformed trading using the notion of discretionary liquidity traders. These traders are uninformed and may experience future liquidity shocks. Anecdotal evidence suggests that transaction cost is an important factor in determining the behavior of discretionary liquidity traders.[4] Our mechanism of determining noise trading makes an effort to capture this feature.

Formally, we develop a model with one risky asset. Differentially privately informed speculators and uninformed discretionary noise traders exist. Speculators trade on their private information to maximize expected utility. Noise traders are "discretionary" in the sense that each chooses whether to participate in the market by optimally balancing the expected loss from trading against informed speculators versus a liquidity benefit of market participation. The expected

[1] Throughout the paper, we follow the literature and use the terms "noise trading"/"liquidity trading"/"uninformed trading" interchangeably. Similarly, we use "noise traders"/"liquidity traders" interchangeably to refer to those investors whose trading behavior generates the uninformed trading.

[2] For example, the classical studies by Grossman and Stiglitz (1980), Hellwig (1980), and Verrecchia (1982). More recent references include Kondor (2012), García and Urošević (2013), Kovalenkov and Vives (2014), and Cespa and Vives (2015), among many others.

[3] Da et al. (2015) find that pension fund companies in Chile often face redemption requests amounting to 10% of their domestic equity and 20% of their bond portfolios within a few days.

[4] For instance, "(m)inimizing trading costs is a priority for DFA's strategy and its managers spend much time working out ways to trade optimally," where DFA refers to Dimensional Fund Advisors, one of the top U.S. mutual fund companies (*The Wall Street Journal*, November 6, 2006, "The Dimensions of A Pioneering Strategy").

loss is endogenously determined by market illiquidity (price impact) while the constant benefit represents the exogenous liquidity needs (and hence the "liquidity" part in the term "liquidity traders"). This trade-off is central to the transaction-cost minimization behavior of real world investors. The optimal mass of discretionary noise traders participating in the market determines the equilibrium quantity of noise trading.

We use our model with endogenous noise trading to investigate the implications of public information for market liquidity and price efficiency, two key variables that represent market quality and are of central importance to regulators.[5] Market liquidity refers to a market's ability to facilitate the purchase or sale of an asset without drastically affecting the asset's price. Price efficiency, also called "market efficiency" or "informational efficiency," concerns how well the price transmits or aggregates the information that is relevant to the asset's fundamental value. We use public information as a way to change market environment, because public-information disclosure has been proposed as the foundation of financial regulations.[6]

We show that disclosing payoff-relevant public information attracts noise trading, improves market liquidity, and harms price efficiency. The intuition is as follows. More public information reduces information asymmetry and adverse selection; thus, for a given amount of noise trading, it improves market liquidity. In turn, better liquidity lowers the expected loss of discretionary noise traders thereby attracting more such traders to the market, leading to more non-informational trading in the market. Hence, the information asymmetry problem weakens, which further improves market liquidity. As a result, both the equilibrium amount of aggregate noise trading and market liquidity increase with the precision of the public signal. Since noise traders are uninformed, increased noise trading negatively impacts the effectiveness of asset price in aggregating speculators' private information, which implies that disclosure negatively affects price efficiency.

In Section 4, we extend our analysis to endogenize speculators' private-information acquisition decisions. The effect of public information on information production is ambiguous, as there are two competing forces. First, a negative crowding out effect has been documented in the literature (e.g., Diamond, 1985): more disclosure can crowd out speculators' trading gains from private information thereby discouraging information production. The second effect is a positive effect highlighted by our analysis. That is, as we show in the baseline model with exogenous information, disclosure attracts noise trading, which in turn can encourage information production. We characterize conditions under which the crowding out effect dominates. For instance, when the public information is sufficiently precise, disclosure harms private information production. When this happens, public information negatively affects price efficiency through two reinforcing channels—i.e., by attracting noise trading and by discouraging information production.

In Section 5, we study an alternative model in which uninformed trading is provided by hedgers. Hedgers can incur a cost to develop a private technology whose return is correlated with the risky asset payoff. So, hedgers can invest in the risky asset to hedge their investment in the developed technology. We endogenize the mass of active hedgers, which in turn determines

[5] O'Hara (2003, p. 1335) stated that "(m)arkets have two important functions—liquidity and price discovery—and these functions are important for asset pricing." Relatedly, when describing short sales, the Securities and Exchange Commission (SEC, 1999) also highlighted that "short selling provides the market with two important benefits: market liquidity and pricing efficiency."

[6] For instance, Greenstone et al. (2006, p. 399) state: "(s)ince the passage of the Securities Act of 1933 and the Securities Exchange Act of 1934, the federal government has actively regulated U.S. equity markets. The centerpiece of these efforts is the mandated disclosure of financial information."

the size of noise trading in the risky asset market. This model well describes the issuance of CLNs in reality: the tradable asset is commodity futures, while CLNs represent the private technology accessible to investment banks that determine whether to issue CLNs and use commodity futures to hedge issuance. We show that our main insight continues to hold in this alternative model. That is, the equilibrium size of noise trading depends negatively on the transaction cost incurred by hedgers, which is in turn negatively affected by endogenous market liquidity. Thus, public information improves market liquidity but can harm private information aggregation through attracting noise trading.

2. Relation to the literature

2.1. The literature on public information

There is a voluminous literature examining the implications of public information for firm value, market liquidity, efficiency, prices, and investor welfare (for excellent surveys, see Verrecchia, 2001 and Leuz and Wysocki, 2007). Previous studies have used REE models to explore the implications of public information for the cost of capital (Hughes et al., 2007; Lambert et al., 2007), for private information acquisition and price informativeness (Lundholm, 1991; Demski and Feltham, 1994), and for disagreement and trading volume (Kim and Verrecchia, 1991; Kondor, 2012). None of these studies has examined the channel of liquidity-chasing uninformed trading and the resulting negative price-efficiency consequences of public information, which we focus upon in our paper.

The crux of our analysis is the endogenous determination of the size of aggregate noise trading. Through affecting the size of noise trading, public information enhances market liquidity but harms price efficiency. Such contrasting implications of public information for market liquidity and price efficiency differ from what the existing literature suggests. For instance, Diamond and Verrecchia (1991) show that releasing public information helps to improve market liquidity and Gao (2008) argues that disclosing accounting information improves price efficiency. The difference in results lies in the fact that the size of noise trading is endogenous in our models while it is exogenous in previous studies on public information.

Two closely related studies, Diamond (1985) and Gao and Liang (2013), also show that disclosure can harm price efficiency. In Diamond (1985), although noisy trading arises as an outcome of investors' utility maximization, the size of total noise trading *is still fixed and does not respond to public information*, and thus the noise trading channel highlighted by our analysis is absent. In Diamond (1985), releasing public information harms market efficiency by crowding out private information production, while in our model, disclosing public information harms price efficiency through attracting noise trading. In both models, releasing public information has two effects on market liquidity—one positive direct effect (through weakening information asymmetry) and one indirect effect. In our model, the indirect effect works through attracting uninformed trading therefore strengthening the positive direct effect. By contrast, in Diamond (1985), the indirect effect works through crowding out private information, which harms liquidity and weakens the positive direct effect by making the price more responsive to uninformed trading (i.e., a higher price impact).

Gao and Liang (2013) explicitly model real decisions and formally bridge the link from price efficiency to real efficiency. However, similar to Diamond (1985), the negative efficiency effect in their model still occurs through crowding out private information production. Therefore, our paper complements Diamond (1985) and Gao and Liang (2013).

Our paper is also related to studies that examine the dark side of public information. Hirshleifer (1971) points out that public information destroys risk-sharing opportunities and thereby impairs the social welfare. Our results are not driven by this so-called Hirshleifer effect. Several papers rely on payoff externality and coordination failures across economic agents to show that public information release may harm welfare (Morris and Shin, 2002; Angeletos and Pavan, 2007; and Colombo et al., 2014). In contrast with this line of work, our results are not driven by any kind of payoff externality—speculators or liquidity traders do not care what other investors do in our setting. More recently, Amador and Weill (2010) and Goldstein and Yang (2014) show that disclosing public information can make investors trade less aggressively on their private information, which in turn harms price informativeness. In our model, speculators' trading aggressiveness is not affected by public information. Instead, the negative implication for price efficiency arises from increased uninformed trading when more public information is available.

2.2. *The literature on endogenous noise trading*

Virtually all of the previous studies on discretionary noise trading have adopted a Kyle (1985) framework in which informed, risk-neutral speculators trade strategically by taking into account their price impact, and discretionary liquidity traders decide when and/or where to trade.[7] By contrast, in our model, risk-averse speculators trade competitively, and discretionary liquidity traders decide whether to participate in the financial market. This modeling difference generates dramatically different implications for market liquidity and price efficiency. For example, in Admati and Pfleiderer's (1988) dynamic model with endogenous information, price informativeness does not depend directly on the level of noise trading and is only positively determined by the amount of private information. More noise trading will cluster in periods with higher market liquidity, which in turn stimulates more private information production and makes the price more efficient. In contrast, in our setting, when market liquidity attracts more noise trading, the effectiveness of information aggregation in financial markets is directly reduced.

Using a Kyle (1985) model with risk-averse speculators, Subrahmanyam (1991) also finds that increased liquidity trading can improve market liquidity but reduce price efficiency, which is consistent with our finding. Subrahmanyam (1991) treats the variance of noise trading as an exogenous parameter. In contrast, we endogenize the variance of noise trading by solving the optimal decisions of discretionary liquidity traders. Our paper can be viewed as providing a micro-foundation for the comparative statics with respect to the variance of noise trading in Subrahmanyam (1991). More importantly, we study the implications of public information, which cannot be conducted in Subrahmanyam (1991).

Recently, García and Urošević (2013) and Kovalenkov and Vives (2014) explore the relation between noise trading and information aggregation in financial markets with a large number of speculators. Both papers show that as long as the size of *exogenous* noise trading increases with the number of speculators, the limiting equilibrium is well-defined and leads to non-trivial information acquisition. Our analysis provides a micro-foundation for these studies by endogenizing the size of aggregate noise trading in financial markets. On top of this technical contribution, our paper also yields new and important economic insights on the implications of public information.

[7] A partial list includes Admati and Pfleiderer (1988), Foster and Viswanathan (1990), Chowdhry and Nanda (1991), Subrahmanyam (1994), and Foucault and Gehrig (2008). For other approaches that endogenize noise trading, see the survey article by Dow and Gorton (2008).

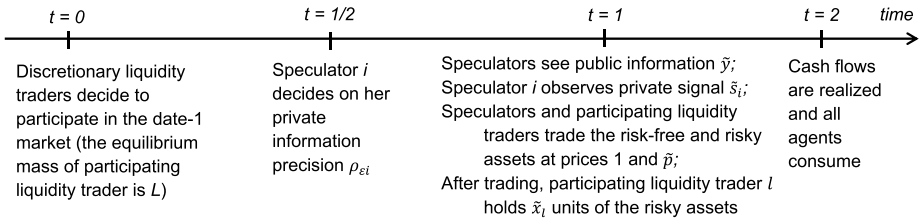

Fig. 1. *Timeline*. This figure plots the timeline. The baseline model with exogenous private information includes three dates 0, 1, and 2. The extended model with endogenous information acquisition includes four dates 0, 1/2, 1, and 2.

3. A model of financial markets with discretionary liquidity trading

3.1. The setup

Time is discrete, and there are three dates: $t = 0, 1$, and 2. The timeline of the economy is described in Fig. 1. At date 1, two assets are traded in a competitive market: a risk-free asset and a risky asset, which can be understood as a firm's stock or an index on the aggregate stock market. The risk-free asset has a constant value of 1 and is in unlimited supply. The risky asset is traded at an endogenous price $\tilde{p}$ and has a fixed supply, which is normalized as one share. It pays an uncertain cash flow at the final date $t = 2$, denoted $\tilde{v}$. We assume that $\tilde{v}$ is normally distributed with a mean of 0 and a precision (reciprocal of variance) of ρ_v—that is, $\tilde{v} \sim N(0, 1/\rho_v)$, with $\rho_v > 0$.

The economy is populated by two types of traders. The first type is a [0, 1] continuum of speculators who have constant absolute risk aversion (CARA) utility with a risk aversion coefficient of $\gamma > 0$. Speculators trade assets to speculate on their superior information. Because there is a continuum of speculators, they behave competitively and take the price as given although they still infer information from the price (which is standard in REE models). Speculators have access to both public and private information. Specifically, prior to trading, each speculator observes a public signal $\tilde{y}$ which communicates information regarding fundamental value $\tilde{v}$ of the risky asset in the following form:

$$\tilde{y} = \tilde{v} + \tilde{\eta}, \text{ with } \tilde{\eta} \sim N\left(0, 1/\rho_\eta\right) \text{ and } \rho_\eta > 0. \tag{1}$$

For example, $\tilde{y}$ can be an earnings announcement by the firm. The precision ρ_η controls the quality of the public signal $\tilde{y}$, with a high value of ρ_η signifying that $\tilde{y}$ is more informative about the asset cash flow $\tilde{v}$. In addition to public signal $\tilde{y}$, speculator i also observes a private signal $\tilde{s}_i$, which contains information about $\tilde{v}$ in the following form:

$$\tilde{s}_i = \tilde{v} + \tilde{\varepsilon}_i, \text{ with } \tilde{\varepsilon}_i \sim N(0, 1/\rho_\varepsilon) \text{ and } \rho_\varepsilon > 0. \tag{2}$$

In our baseline model, we assume that speculators are endowed with $\tilde{s}_i$ of an exogenous precision ρ_ε. Later, in Section 4, speculators endogenously choose an optimal precision.

The second type of traders are noise traders who consume liquidity in the financial market. Unlike the standard noisy-REE models (e.g., Grossman and Stiglitz, 1980; Kovalenkov and Vives, 2014), in which liquidity traders are purely exogenous and their only role is to provide the necessary "noise" in the market to prevent a fully revealing price, we explicitly model their decision on whether to participate in trading. In our model, the *size* (*variance*) of the aggregate noise trading is endogenously determined in the same spirit as discretionary liquidity trading in the microstructure literature (e.g., Admati and Pfleiderer, 1988; Foster and Viswanathan, 1990).

Specifically, there exist a large mass of potential noise traders who are risk neutral and ex-ante identical. These traders are uninformed and may demand liquidity from the markets when they experience liquidity shocks. They can represent institutional traders—e.g., index funds or ETFs—who need to rebalance portfolios around index recompositions or when receiving flow shocks.

We follow Admati and Pfleiderer (1988) and assume that if liquidity trader l decides to trade, she has to trade $\tilde{x}_l$ units of risky asset. At date 0, each liquidity trader decides whether to trade in the future date-1 financial market, and we normalize the utility of not participating in the market to be zero. Trading the assets necessitates the following trade-off. First, trading generates an exogenous benefit of $B > 0$, which represents the exogenous liquidity needs net of any participation cost. Second, because liquidity traders are trading against informed speculators, they will suffer losses on average, which represents an endogenous trading cost. Each potential liquidity trader therefore balances the costs and benefits in deciding whether to participate in the financial market, and in this sense, they are labeled *discretionary liquidity traders*. We refer to those liquidity traders who decide to participate in the market as *participating liquidity traders*. We use L to denote the endogenous mass of participating liquidity traders. As we show below, this endogenous variable L in turn determines the size of noise trading in the financial market.

We assume that $\tilde{x}_l$ consists of two components: (1) an idiosyncratic component $\tilde{z}_l \sim N\left(0, \sigma_z^2\right)$ (with $\sigma_z > 0$), which captures idiosyncratic liquidity motives that are specific to each noise trader; and (2) a systematic component $\tilde{u} \sim N(0, 1)$, which represents liquidity demands that are correlated across noise traders because of some common driving factors (such as correlated flow shocks to pension funds studied by Da et al., 2015). That is,

$$\tilde{x}_l = \tilde{u} + \tilde{z}_l, \tag{3}$$

where

$$\tilde{u} \sim N(0, 1) \text{ and } \tilde{z}_l \sim N\left(0, \sigma_z^2\right) \text{ (with } 0 < \sigma_z < \infty).$$

We have normalized the variance of $\tilde{u}$ as 1, but this normalization does not affect our results. The size of the idiosyncratic variance σ_z^2 does not affect our analysis because all of the idiosyncratic noise trading washes out in the aggregate.

Finally, we assume that the random variables $\left(\tilde{v}, \tilde{\eta}, \{\tilde{\varepsilon}_i\}_{i\in[0,1]}, \tilde{u}, \{\tilde{z}_l\}_{l\in[0,L]}\right)$ are mutually independent. As a result, the total amount of liquidity trading in the market is[8]

$$\tilde{X} \equiv \int_0^L \tilde{x}_l dl = L\tilde{u}. \tag{4}$$

We define the size of the aggregate noise trading in the financial market to be $Var(X)$, and use ρ_X to denote its inverse. That is,

$$\rho_X \equiv \frac{1}{Var\left(\tilde{X}\right)} = \frac{1}{L^2}. \tag{5}$$

Thus, parameter L endogenously determines the size of the aggregate noise trading in the financial market.

[8] We here adopt the convention that a law of large numbers holds for a continuum of independent random variables (see the technical appendix in Vives, 2008).

We make two remarks regarding the setup. First, we have assumed that trading benefit B of discretionary liquidity traders is exogenous. In Section 5, we endogenize B using a different approach that generates uninformed trading from informed hedgers and show that our results are robust.

Second, our baseline model has assumed that liquidity traders do not receive any information. In particular, they do not observe public information, because the timeline in Fig. 1 has specified that liquidity traders make their participation decisions before the release of public information. Again, this feature is not necessary in deriving our results for the following two reasons. First, in our baseline model, even if we allow liquidity traders to observe public signal $\tilde{y}$ before they make participation decisions, our results do not change.[9] Second, in the alternative model analyzed in Section 5 in which noise trading arises from hedging-motivated trades, hedgers not only see $\tilde{y}$ but also actively infer information from the price, which makes them the most informed traders in that economy. Although the analysis in that alternative model is far more complex, our main insight still carries through. However, we caution that in our modeling framework, trade and liquidity before and after the release of the public signal cannot be analyzed. Instead, trades and information occur simultaneously in our economy.

3.2. The equilibrium

The equilibrium concept that we use is the rational expectations equilibrium (REE), as in Grossman and Stiglitz (1980), which involves the optimal decisions of agents and the statistical behavior of aggregate variables ($\tilde{p}$ and $\tilde{X}$). Specifically, at date 1, in the financial market, (1) taking the total liquidity trading $\tilde{X}$ as given, speculators choose investments in assets to maximize their expected utility conditioning on their private information $\tilde{s}_i$, the public information $\tilde{y}$, and the market-clearing asset price $\tilde{p}$; (2) the markets clear; and (3) speculators have rational expectations in the sense that their beliefs about all random variables are consistent with the true underlying distributions. At date 0, discretionary liquidity traders make market-participation decisions to maximize their participation benefit net of expected trading loss, taking the equilibrium price function and other discretionary liquidity traders' decisions as given. Their participation decisions determine the total liquidity trading $\tilde{X} = L\tilde{u}$ in the financial market. In the subsequent two subsections, we first solve the financial market equilibrium—taking as given a fixed mass L of participating liquidity traders—and we then endogenize the equilibrium mass L^* by solving the decision problem of discretionary liquidity traders.

3.2.1. Financial market equilibrium

As is standard in the literature, we consider a linear REE, in which traders conjecture about the following price function:

$$\tilde{p} = \alpha_0 + \alpha_y \tilde{y} + \alpha_v \tilde{v} + \alpha_X \tilde{X}, \tag{6}$$

where the coefficients will be endogenously determined. In particular, coefficient α_X is related to the market depth: a smaller α_X means that aggregate liquidity trading $\tilde{X}$ has a smaller price impact, and thus the market is deeper. Thus, we measure market liquidity by

$$LIQ \equiv \frac{1}{\alpha_X}. \tag{7}$$

[9] Formally, we can recompute the expected utility of market participation conditional on $\tilde{y}$ as $B + E\left[(\tilde{v} - \tilde{p})\,\tilde{x}_l \,|\, \tilde{y}\right] = B - \alpha_X\left(L; \rho_\eta\right) L$, which is the same as $W\left(L; \rho_\eta\right)$ defined in equation (12).

This measure of market liquidity is in the same spirit as Kyle's (1985) lambda.

We now derive the demand functions of speculators for given price $\tilde{p}$. Consider typical speculator i, who has information set $\{\tilde{p}, \tilde{y}, \tilde{s}_i\}$. The CARA-normal feature of the model implies that the speculator's demand function for the risky asset is

$$D(\tilde{p}, \tilde{y}, \tilde{s}_i) = \frac{E(\tilde{v}|\tilde{p}, \tilde{y}, \tilde{s}_i) - \tilde{p}}{\gamma Var(\tilde{v}|\tilde{p}, \tilde{y}, \tilde{s}_i)},$$

where $E(\tilde{v}|\tilde{p}, \tilde{y}, \tilde{s}_i)$ and $Var(\tilde{v}|\tilde{p}, \tilde{y}, \tilde{s}_i)$ represent speculator i's estimates of the mean and variance of random payoff $\tilde{v}$ conditional on her information set.

Given public signal $\tilde{y}$, the information in the price is equivalent to the following signal:

$$\tilde{s}_p \equiv \frac{\tilde{p} - \alpha_0 - \alpha_y \tilde{y}}{\alpha_v} = \tilde{v} + \frac{\alpha_X}{\alpha_v}\tilde{X}, \tag{8}$$

which, conditional on $\tilde{v}$, is normally distributed with mean $\tilde{v}$ and endogenous precision

$$\rho_p = (\alpha_v/\alpha_X)^2 \rho_X. \tag{9}$$

The endogenous precision ρ_p captures how much extra information the price conveys regarding the asset fundamental $\tilde{v}$ to the speculators in addition to the public signal and speculators' private information. In addition, in equilibrium, ρ_p also measures how effectively the price aggregates speculators' private information and is therefore a measure of price informativeness. As a result, we refer to ρ_p as price efficiency or price informativeness.

Using equations (1), (2), and (8), we apply Bayes' rule and compute

$$E(\tilde{v}|\tilde{p}, \tilde{y}, \tilde{s}_i) = \frac{\rho_\eta \tilde{y} + \rho_p \tilde{s}_p + \rho_\varepsilon \tilde{s}_i}{\rho_v + \rho_\eta + \rho_p + \rho_\varepsilon} \text{ and } Var(\tilde{v}|\tilde{p}, \tilde{y}, \tilde{s}_i) = \frac{1}{\rho_v + \rho_\eta + \rho_p + \rho_\varepsilon}.$$

Plugging these expressions into demand function $D(\tilde{p}, \tilde{y}, \tilde{s}_i)$ yields the demand function,

$$D(\tilde{p}, \tilde{y}, \tilde{s}_i) = \frac{\rho_\eta \tilde{y} + \rho_p \tilde{s}_p + \rho_\varepsilon \tilde{s}_i - \left(\rho_v + \rho_\eta + \rho_p + \rho_\varepsilon\right)\tilde{p}}{\gamma}. \tag{10}$$

In the aggregate, speculators and liquidity traders purchase $\int_0^1 D(\tilde{p}, \tilde{y}, \tilde{s}_i)\,di$ and $\tilde{X} = \int_0^L \tilde{x}_l dl$ units of the risky asset, respectively. Thus, the market-clearing condition is:

$$\int_0^1 D(\tilde{p}, \tilde{y}, \tilde{s}_i)\,di + \tilde{X} = 1, \tag{11}$$

where the left-hand side is the aggregate demand from speculators and liquidity traders and the right-hand side is the aggregate supply of the risky asset. Substituting demand function (10) and the expression (8) for $\tilde{s}_p$ in the market-clearing condition (11), we solve for $\tilde{p}$, and verify the conjectured form of the price function, which yields the following lemma.

Lemma 1. *For a given size L^2 of aggregate noise trading, there exists a unique, linear, partially revealing REE with the price function*

$$\tilde{p} = \alpha_0 + \alpha_y \tilde{y} + \alpha_v \tilde{v} + \alpha_X \tilde{X},$$

where

$$\alpha_0 = -\frac{\gamma}{\rho_v + \rho_\eta + \left(\frac{\rho_\varepsilon}{\gamma}\right)^2 \rho_X + \rho_\varepsilon}, \alpha_y = \frac{\rho_\eta}{\rho_v + \rho_\eta + \left(\frac{\rho_\varepsilon}{\gamma}\right)^2 \rho_X + \rho_\varepsilon},$$

$$\alpha_v = \frac{\left(\frac{\rho_\varepsilon}{\gamma}\right)^2 \rho_X + \rho_\varepsilon}{\rho_v + \rho_\eta + \left(\frac{\rho_\varepsilon}{\gamma}\right)^2 \rho_X + \rho_\varepsilon}, \text{ and } \alpha_X = \frac{\gamma + \frac{\rho_\varepsilon}{\gamma}\rho_X}{\rho_v + \rho_\eta + \left(\frac{\rho_\varepsilon}{\gamma}\right)^2 \rho_X + \rho_\varepsilon}.$$

3.2.2. Decision of discretionary liquidity traders

We now go back to date 0 to analyze discretionary liquidity traders' decision of whether to participate in the date-1 financial market. Consider a given discretionary liquidity trader l. She takes the price function and the decisions of other discretionary liquidity traders as given and compares her own utility of participating in the market versus not participating. Since her reservation value of not participating in the market has been normalized at 0, she chooses to participate in the market if and only if she expects that she will have positive utility from trading the assets.

By participating in the financial market, liquidity trader l can enjoy the exogenous liquidity benefit $B > 0$. Meanwhile, she will have to hold $\tilde{x}_l$ units of the risky asset, which yields an expected revenue of $E\left[(\tilde{v} - \tilde{p})\,\tilde{x}_l\right]$: if $\tilde{x}_l > 0$, she then buys each unit for $\tilde{p}$ at date 1 and sells it at date 2, receiving a payoff of $\tilde{v}$; if $\tilde{x}_l < 0$, she then shorts the risky asset, which generates a payoff of $\tilde{p}$ per unit at date 1 and an obligation of $\tilde{v}$ per unit at date 2. Since liquidity traders are trading against speculators who have superior information, the expected revenue $E\left[(\tilde{v} - \tilde{p})\,\tilde{x}_l\right]$ is negative and represents the expected costs for liquidity trader l to meet her liquidity demand $\tilde{x}_l$.

Therefore, for price function (6) and a given mass L of participating liquidity traders, the expected utility of participating in the market for a potential liquidity trader l is

$$\begin{aligned} W\left(L; \rho_\eta\right) &\equiv B + E\left[(\tilde{v} - \tilde{p})\,\tilde{x}_l\right] \\ &= B - \underbrace{\alpha_X\left(L; \rho_\eta\right)}_{\textit{illiquidity}} \times \underbrace{L}_{\textit{size}}, \end{aligned} \tag{12}$$

where the second equality follows from price function (6) and the definition of $\tilde{x}$ in (3), and

$$\alpha_X\left(L; \rho_\eta\right) = \frac{\gamma + \frac{\rho_\varepsilon}{\gamma}\frac{1}{L^2}}{\rho_v + \rho_\eta + \left(\frac{\rho_\varepsilon}{\gamma}\right)^2 \frac{1}{L^2} + \rho_\varepsilon}, \tag{13}$$

which follows from the expression of α_X in Lemma 1 and the definition of ρ_X in (5).

We have explicitly written the expressions $W\left(L; \rho_\eta\right)$ and $\alpha_X\left(L; \rho_\eta\right)$ as functions of L and ρ_η. Variable L is endogenous and will be solved as part of the full equilibrium. Parameter ρ_η is exogenous and it controls the precision of public information. Later, in Section 3.3, we will conduct comparative statics with respect to ρ_η to derive implications of public information.

To better understand expression (12), recall that liquidity traders bear transaction costs precisely because their trading can potentially move prices against their trades. When they buy the risky asset they can move the price up and when they sell the asset the price can go down. By (6), this price impact is the market illiquidity (i.e., the inverse of market liquidity): $\frac{1}{LIQ} = \frac{\partial \tilde{p}}{\partial \tilde{X}} = \alpha_X\left(L; \rho_\eta\right)$. This variable captures the cost a liquidity trader expects to bear by submitting an order to trade one unit of the risky asset. However, not all orders submitted by a liquidity trader will move prices. By (3), the orders $\tilde{x}_l$ of given liquidity trader l consist of

an idiosyncratic component $\tilde{z}_l$ and a systematic component $\tilde{u}$. After aggregation in (4), the idiosyncratic components become washed out because these components represent trading among liquidity traders. As a result, only the systematic component affects the aggregate noise trading (with speculators as the counterparty) and has the potential to move prices. Because of the common component $\tilde{u}$ in the orders of all liquidity traders, liquidity trader l expects the total size of orders that is correlated with her own trading to be $Cov\left(\tilde{X}, \tilde{x}_l\right) = Cov\left(L\tilde{u}, \tilde{u}\right) = L \times Var\left(\tilde{u}\right) = L$ (by $Var\left(\tilde{u}\right) = 1$).

The expression $W\left(L; \rho_\eta\right)$ in (12) clearly illustrates that in our economy, discretionary noise traders have incentives to chase market liquidity. That is, if there is an exogenous change in the trading environment so that market liquidity improves (i.e., $\alpha_X\left(L; \rho_\eta\right)$ decreases), then, other things being equal, discretionary liquidity traders are more likely to participate in the financial market. This observation precisely produces the intuition highlighted by Admati and Pfleiderer (1988, p. 5): "It is intuitive that, to the extent that liquidity traders have discretion over when they trade, they prefer to trade when the market is 'thick'—that is, when their trading has little effect on prices."

In equilibrium, each discretionary liquidity trader must be indifferent to participation and nonparticipation in the market. Thus, the equilibrium mass L^* of participating liquidity traders is determined by

$$W\left(L^*; \rho_\eta\right) = B - \alpha_X\left(L^*; \rho_\eta\right) L^* = 0. \tag{14}$$

To establish the existence and uniqueness of such an equilibrium, we examine the property of $W\left(L; \rho_\eta\right)$. First, by (12) and (13), we can show that $W\left(0; \rho_\eta\right) > 0$ and $\lim_{L\to\infty} W\left(L; \rho_\eta\right) = -\infty$. By the intermediate value theorem, there always exists an equilibrium mass L^* that satisfies (14). Second, the equilibrium may not be unique because L affects $W\left(L; \rho_\eta\right)$ in two opposite ways through its effect on the "illiquidity" and "size" components in (12), which means that the function $W\left(\cdot; \rho_\eta\right)$ may not always be downward sloping.

On the one hand, increasing L will improve market liquidity: an increase in L will increase the variance $Var\left(\tilde{X}\right) = L^2$ of aggregate noise trading, which in turn reduces the adverse selection problem and thus improves market liquidity. Formally, we have

$$\frac{\partial \alpha_X\left(L; \rho_\eta\right)}{\partial L} = -\frac{2\rho_\varepsilon\left(\rho_v + \rho_\eta\right)}{\gamma L^3\left[\rho_v + \rho_\eta + \left(\frac{\rho_\varepsilon}{\gamma L}\right)^2 + \rho_\varepsilon\right]^2} < 0. \tag{15}$$

This effect tends to increase $W\left(L; \rho_\eta\right)$ and strengthen the incentive of discretionary liquidity traders to participate in the market because they chase market liquidity. On the other hand, increasing L also has a second negative effect—it directly increases the size component L in (12), which tends to lower $W\left(L; \rho_\eta\right)$. It is easy to check that when L is large this negative effect dominates so that the overall effect of increasing L is to decrease $W\left(L; \rho_\eta\right)$.

Proposition 1. *(a) For any given parameter configuration $\left(\gamma, \rho_v, \rho_\eta, B, \rho_\varepsilon\right) \in \mathbb{R}^5_{++}$, there exists an equilibrium mass L^* of participating liquidity traders, where L^* is determined by*

$$W\left(L^*; \rho_\eta\right) = B - \frac{\gamma + \frac{\rho_\varepsilon}{\gamma}\frac{1}{L^{*2}}}{\rho_v + \rho_\eta + \left(\frac{\rho_\varepsilon}{\gamma}\right)^2 \frac{1}{L^{*2}} + \rho_\varepsilon} L^* = 0. \tag{16}$$

(b) If $\frac{3\rho_\varepsilon}{(\rho_v+\rho_\eta+\rho_\varepsilon)^2} < B^2 < \frac{1}{3(\rho_v+\rho_\eta+\rho_\varepsilon)}$ *and* $4\left(\rho_v+\rho_\eta+\rho_\varepsilon\right)^3 B^4 - \left(\rho_v+\rho_\eta\right)\left(\rho_v+\rho_\eta+20\rho_\varepsilon\right)B^2 + 8\rho_\varepsilon^2 B^2 + 4\rho_\varepsilon < 0$*, then there are three equilibria* L^**; otherwise, the equilibrium is unique except for a parameter set of Lebesgue measure zero.*

Proof. See the appendix. □

3.2.3. Multiplicity and stability of equilibrium

Proposition 1 shows that our economy admits either a unique equilibrium or three equilibria. The conditions determining uniqueness versus multiplicity are characterized in terms of $\left(B, \rho_v+\rho_\eta, \rho_\varepsilon\right) \in \mathbb{R}^3_{++}$. The condition for multiple equilibria is rather restrictive. In particular, the Lebesgue measure of the uniqueness region is infinite, while that of the multiplicity region is finite. Thus, for most parameter values, our model admits a unique equilibrium.

In the case of multiple equilibria, we follow Cespa and Vives (2015) to perform a stability analysis in order to select an equilibrium. An equilibrium is *stable* if and only if the expected utility function $W\left(L;\rho_\eta\right)$ of participating in the market is a downward-sloping function of L at the equilibrium mass L^* of participating liquidity traders, i.e., $\frac{\partial W\left(L^*;\rho_\eta\right)}{\partial L} < 0$. Similarly, an equilibrium is *unstable* if and only if $\frac{\partial W\left(L^*;\rho_\eta\right)}{\partial L} > 0$.

The concepts of stable and unstable equilibria can be understood via the following thought experiment. Assume the economy is at an equilibrium level L^* so that $W\left(L^*;\rho_\eta\right) = 0$. Suppose a small perturbation to L^* occurs so that the mass of participating liquidity traders deviates to L'. If $W\left(\cdot;\rho_\eta\right)$ is downward sloping at L^*, then: (1) If $L' > L^*$, we would have $W\left(L';\rho_\eta\right) < 0$, so in response, some liquidity traders would withdraw from the market, thus decreasing L' towards L^*; and (2) if $L' < L^*$, then $W\left(L';\rho_\eta\right) > 0$, which means that more liquidity traders would like to participate in the market, thus increasing L' towards L^*. In either case, the economy goes back to the original L^*, and in this sense, the equilibrium is stable. A similar argument explains the concept of unstable equilibrium.

Given $W\left(0;\rho_\eta\right) > 0$ and $\lim_{L\to\infty} W\left(L;\rho_\eta\right) = -\infty$, we know that if a unique equilibrium exists, function $W\left(\cdot;\rho_\eta\right)$ must cross zero from above so that the unique equilibrium is stable. When multiplicity occurs, there must be three equilibria by Proposition 1. We label these three equilibrium values of L as L_1^*, L_2^*, and L_3^*, where $0 < L_1^* < L_2^* < L_3^* < \infty$. Again, by $W\left(0;\rho_\eta\right) > 0$ and $\lim_{L\to\infty} W\left(L;\rho_\eta\right) = -\infty$, it must be the case that $W\left(\cdot;\rho_\eta\right)$ first crosses zero from above at L_1^*, then from below at L_2^*, and finally from above at L_3^*. That is, L_1^* and L_3^* are stable equilibria, while L_2^* is unstable. Formally, we have the following corollary.

Corollary 1. *When there is a unique equilibrium mass* L^* *of participating liquidity traders,* L^* *is stable. When there are three equilibrium masses* L_1^**,* L_2^**, and* L_3^* *of participating liquidity traders (with* $L_1^* < L_2^* < L_3^*$*),* L_1^* *and* L_3^* *are stable and* L_2^* *is unstable.*

3.3. Implications of public information

In this subsection, we examine the implications of public information by conducting comparative statics with respect to parameter ρ_η that controls the quality of public signal $\tilde{y}$. Our analysis focuses on stable equilibria in which function $W\left(\cdot;\rho_\eta\right)$ is downward sloping at L^*.

First, releasing public information induces more discretionary liquidity traders to choose to participate in the market. That is, $\frac{\partial L^*}{\partial \rho_\eta} > 0$. With more public information, speculators face less

information asymmetry and trade more aggressively so that for any given level L of aggregate noise trading, the price is more responsive to the firm's fundamental than to the noise trading. This causes the market to be more liquid and thereby strengthens the ex-ante incentive of discretionary liquidity traders to participate in the market. Formally, it follows from (13) that $\frac{\partial \alpha_X(L;\rho_\eta)}{\partial \rho_\eta} < 0$, and therefore by applying the implicit function theorem to (16) we obtain $\frac{\partial L^*}{\partial \rho_\eta} = \frac{\frac{\partial \alpha_X(L^*;\rho_\eta)}{\partial \rho_\eta} L^*}{\frac{\partial W(L^*;\rho_\eta)}{\partial L}} > 0$, where we have used the fact that $W(\cdot;\rho_\eta)$ is downward sloping at L^* for stable equilibrium (and hence $\frac{\partial W(L^*;\rho_\eta)}{\partial L} < 0$).

Second, in equilibrium, market liquidity increases with public information precision as well, that is, $\frac{\partial LIQ^*}{\partial \rho_\eta} > 0$. This is due to two effects. The direct effect is that public information improves market liquidity holding the amount of noise trading L fixed. There is also an indirect effect of public information—that is, the increased L^* implies that more non-informational trading exists in the market and hence the information asymmetry problem weakens, which in turn further improves market liquidity. Formally, by (13) and the definition $LIQ = \frac{1}{\alpha_X}$, we obtain

$$\frac{\partial LIQ^*}{\partial \rho_\eta} = \underbrace{\frac{\partial}{\partial \rho_\eta} \frac{1}{\alpha_X\left(L^*;\rho_\eta\right)}}_{\textit{direct effect}} + \underbrace{\frac{\partial}{\partial L} \frac{1}{\alpha_X\left(L^*;\rho_\eta\right)} \frac{\partial L^*}{\partial \rho_\eta}}_{\textit{indirect effect}} > 0,$$

where both the direct and indirect effects are positive.

Finally, more public information reduces price efficiency because more public information attracts additional liquidity trading and, as a result, the price reveals less fundamental information. That is, $\frac{\partial \rho_p^*}{\partial \rho_\eta} < 0$. Formally, by equations (5) and (9) and Lemma 1, we have

$$\text{Price Efficiency } \rho_p^* = \left(\frac{\rho_\varepsilon}{\gamma L^*}\right)^2, \tag{17}$$

and thus $\frac{\partial \rho_p^*}{\partial \rho_\eta} < 0$ follows directly from $\frac{\partial L^*}{\partial \rho_\eta} > 0$.

Proposition 2. *When information is exogenous, at stable equilibria, public information attracts discretionary liquidity trading, improves market liquidity, and harms market efficiency. That is, for a fixed $\rho_\varepsilon > 0$, we have $\frac{\partial L^*}{\partial \rho_\eta} > 0$, $\frac{\partial LIQ^*}{\partial \rho_\eta} > 0$, and $\frac{\partial \rho_p^*}{\partial \rho_\eta} < 0$.*

The price efficiency result in Proposition 2 also has real efficiency implications when some real decision maker learns information from the asset price $\tilde{p}$. For instance, the real decision maker can be a firm manager, while the public signal $\tilde{y}$ is an earnings announcement made by the firm. Since $\tilde{p}$ aggregates private information of speculators (such as hedge funds), the firm cares to learn from the price to guide real investments, where ρ_p^* captures how much information that the firm can glean from the price. Given that the firm is also the agent who makes the disclosure, the firm manager has nothing to directly learn from $\tilde{y}$, and in this case variable ρ_p can readily work as a measure for real efficiency. So, by (17), public information can harm real efficiency through attracting noise trading.

In other cases, the real decision maker may also directly learn from public information $\tilde{y}$, and a more relevant real efficiency measure is the total precision obtained from $\tilde{y}$ and $\tilde{p}$, i.e., $\frac{1}{Var(\tilde{v}|\tilde{y},\tilde{p})} = \rho_v + \rho_\eta + \rho_p$. An example for this case is the various economic statistics on aggregate

outcomes such as GDP, unemployment, and inflation. These statistics, typically disclosed by the government or other regulation agencies, are closely monitored by public companies and affect their real investment decisions.

In this case, disclosing public information has two effects on $\frac{1}{Var(\tilde{v}|\tilde{y},\tilde{p})}$:

$$\frac{\partial}{\partial \rho_\eta}\frac{1}{Var\left(\tilde{v}|\tilde{y},\,\tilde{p}\right)} = \underbrace{1}_{\text{direct effect}} + \underbrace{\frac{\partial \rho_p}{\partial \rho_\eta}}_{\text{indirect effect}}. \tag{18}$$

First, the real decision maker benefits directly from the disclosed public information, which is captured by the term "1" (i.e., $\frac{\partial \rho_\eta}{\partial \rho_\eta}$) in (18). The second effect is an indirect effect as highlighted by our analysis: public information harms price informativeness ρ_p, which in turn negatively affects how the real decision maker learns from the price, as captured by the term $\frac{\partial \rho_p}{\partial \rho_\eta}$ in (18). The overall effect of public information on $\frac{1}{Var(\tilde{v}|\tilde{y},\tilde{p})}$ is determined by the trade-off of these two effects. In the Online Appendix, we show that releasing public information lowers $\frac{1}{Var(\tilde{v}|\tilde{y},\tilde{p})}$ if and only if $\rho_p^* > \rho_\varepsilon$. Intuitively, because the negative indirect effect works through price informativeness, when the additional information content ρ_p^* in the price is sufficiently high, this negative effect is particularly strong.

Proposition 2 has important policy implications. Many recent regulation rules, such as the Sarbanes–Oxley Act (2002) and the Dodd–Frank Act (2010), are concerned with requiring increased transparency and providing more accurate information to the public with the intent of improving the functioning of the financial market. For example, in its final ruling on shortening the reporting deadline for insider trading to two business days, the SEC (2002) argued that "making this information available to all investors on a more timely basis should increase market transparency, which will likely enhance *market efficiency* and *liquidity*." Proposition 2 suggests that providing public information may not achieve both goals simultaneously when discretionary noise traders chase liquidity. Thus, transparency measures might work against regulators' original intentions if efficiency is their top priority, and thus they need to take the effect of discretionary liquidity trading into account when designing disclosure policies.

4. Endogenous information acquisition

4.1. Characterization of the overall equilibrium

We now extend our baseline model presented in Section 3 by adding an information acquisition date, $t = \frac{1}{2}$, which is between dates 0 and 1, again as shown in Fig. 1. Our analysis of information acquisition closely follows Verrecchia (1982) and Holmström and Tirole (1993). Specifically, at date $\frac{1}{2}$, speculator i can acquire a private signal with precision $\rho_{\varepsilon i}$ according to an increasing and convex cost function $C\left(\rho_{\varepsilon i}\right) \geq 0$, with $C(0) = C'\left(0\right) = 0$. For example, the cost function can take the quadratic form of $C\left(\rho_{\varepsilon i}\right) = \frac{c}{2}\rho_{\varepsilon i}^2$ where c is a positive constant. Given that we have a continuum of speculators, each speculator will take the equilibrium choice ρ_ε^* of other speculators as given when making her own information acquisition decision on $\rho_{\varepsilon i}$. Of course, she will also take as given the equilibrium mass L^* of participating liquidity traders and the price function characterized by Lemma 1.

Because speculators are ex-ante identical, we look for a symmetric equilibrium of information acquisition in which all speculators choose the same precision ρ_ε^*. Following an argument similar

to that of Grossman and Stiglitz (1980), we can compute that the expected net benefit of a private signal with precision $\rho_{\varepsilon i}$ to a speculator i is

$$\pi\left(\rho_{\varepsilon i};\rho_{\varepsilon}^{*},L^{*}\right)=\frac{1}{2\gamma}\log\left[\frac{Var\left(\tilde{v}|\tilde{p},\tilde{y}\right)}{Var\left(\tilde{v}|\tilde{p},\tilde{y},\tilde{s}_{i}\right)}\right]-C\left(\rho_{\varepsilon i}\right)$$
$$=\frac{1}{2\gamma}\log\left[\frac{\rho_{v}+\rho_{\eta}+\left(\frac{\rho_{\varepsilon}^{*}}{\gamma L^{*}}\right)^{2}+\rho_{\varepsilon i}}{\rho_{v}+\rho_{\eta}+\left(\frac{\rho_{\varepsilon}^{*}}{\gamma L^{*}}\right)^{2}}\right]-C\left(\rho_{\varepsilon i}\right),$$

where we have explicitly incorporated equilibrium precision ρ_{ε}^{*} of other speculators' private information and equilibrium mass L^{*} of participating liquidity traders as arguments in the benefit function defined above.

We label the equilibrium in this extended economy with both endogenous liquidity trading and endogenous private information acquisition as an *overall equilibrium*, which is composed of three sub-equilibriums. As previously, at date 1, the financial market reaches an REE that determines the price function for given L^{*} and ρ_{ε}^{*}. At date 0, discretionary liquidity traders make market participation decisions to maximize their preferences, taking as given the equilibrium price function and private information precision ρ_{ε}^{*}. Their behavior jointly determines L^{*}. We also now require that at date $\frac{1}{2}$, speculators make information acquisition decisions on $\rho_{\varepsilon i}^{*}$ to maximize their expected indirect utility in future financial markets, taking L^{*} and the price function as given. The equilibrium price function, the equilibrium mass L^{*} of participating liquidity traders, and the equilibrium level ρ_{ε}^{*} of private information precision are mutually consistent across the three dates.

Given $\left(\rho_{\varepsilon}^{*},L^{*}\right)$, each speculator i optimally chooses her own signal precision $\rho_{\varepsilon i}$ by solving the first-order condition of $\pi\left(\rho_{\varepsilon i};\rho_{\varepsilon}^{*},L^{*}\right)$ as follows[10]:

$$\frac{\partial\pi\left(\rho_{\varepsilon i};\rho_{\varepsilon}^{*},L^{*}\right)}{\partial\rho_{\varepsilon i}}=0\Rightarrow\frac{1}{2\gamma\left[\rho_{v}+\rho_{\eta}+\left(\frac{\rho_{\varepsilon}^{*}}{\gamma L^{*}}\right)^{2}+\rho_{\varepsilon i}\right]}=C'\left(\rho_{\varepsilon i}\right).$$

Since the above equation holds for any speculator i, we have $\rho_{\varepsilon i}=\rho_{\varepsilon}^{*}$ for any $i\in[0,1]$ (i.e., the equilibrium is indeed symmetric). Thus,

$$\frac{1}{2\gamma\left[\rho_{v}+\rho_{\eta}+\left(\frac{\rho_{\varepsilon}^{*}}{\gamma L^{*}}\right)^{2}+\rho_{\varepsilon}^{*}\right]}=C'\left(\rho_{\varepsilon}^{*}\right). \tag{19}$$

This equation implicitly determines the optimal response of ρ_{ε}^{*} to a given value of L^{*} (and disclosure precision ρ_{η}):

$$\rho_{\varepsilon}^{*}=\rho_{\varepsilon}\left(L^{*};\rho_{\eta}\right). \tag{20}$$

By Proposition 1, at date 0, the equilibrium condition for determining L^{*} is

$$B-\frac{\gamma+\frac{\rho_{\varepsilon}^{*}}{\gamma}\frac{1}{L^{*2}}}{\rho_{v}+\rho_{\eta}+\left(\frac{\rho_{\varepsilon}^{*}}{\gamma}\right)^{2}\frac{1}{L^{*2}}+\rho_{\varepsilon}^{*}}L^{*}=0, \tag{21}$$

[10] Because $\pi\left(\rho_{\varepsilon i};\rho_{\varepsilon}^{*},L^{*}\right)$ is concave in $\rho_{\varepsilon i}$, the first-order condition is both necessary and sufficient.

which determines the optimal response of L^* to a given value of ρ_ε^* (and ρ_η):

$$L^* = L\left(\rho_\varepsilon^*; \rho_\eta\right). \tag{22}$$

The two unknowns, ρ_ε^* and L^*, which define the overall equilibrium, are pinned down by the two best response functions, (20) and (22) (or (19) and (21)).

Proposition 3. *There exists a symmetric overall equilibrium in which equilibrium precision $\rho_\varepsilon^* \in (0, \bar{\rho}_\varepsilon)$ of speculators' private information is determined by*

$$B^2\left[\frac{1}{2\gamma C'\left(\rho_\varepsilon^*\right)} - \left(\rho_v + \rho_\eta + \rho_\varepsilon^*\right)\right] = \left[1 - 2\gamma C'\left(\rho_\varepsilon^*\right)\left(\rho_v + \rho_\eta\right)\right]^2, \tag{23}$$

where $\bar{\rho}_\varepsilon$ is uniquely determined by

$$\frac{1}{2\gamma C'\left(\bar{\rho}_\varepsilon\right)} = \rho_v + \rho_\eta + \bar{\rho}_\varepsilon; \tag{24}$$

the equilibrium mass L^* of liquidity traders choosing to participate in the financial market is

$$L^* = \frac{\rho_\varepsilon^*}{\gamma\sqrt{\frac{1}{2\gamma C'\left(\rho_\varepsilon^*\right)} - \left(\rho_v + \rho_\eta + \rho_\varepsilon^*\right)}}; \tag{25}$$

and the equilibrium price function is given by Lemma 1 with $\rho_X = \frac{1}{L^{*2}}$. In addition, if $B > \sqrt{\frac{8}{27(\rho_v+\rho_\eta)}}$, then the overall equilibrium is unique.

Proof. See the appendix. □

4.2. The effect of public information

4.2.1. Information production

Since the overall equilibrium is given by conditions on ρ_ε^* in Proposition 3, we first examine the implications of disclosure for this variable. Inserting (22) into (20), we have

$$\rho_\varepsilon^* = \rho_\varepsilon\left(L\left(\rho_\varepsilon^*; \rho_\eta\right); \rho_\eta\right).$$

Taking total differentiation with respect to ρ_η on the above equation yields

$$\frac{d\rho_\varepsilon^*}{d\rho_\eta} = \frac{\overbrace{\frac{\partial\rho_\varepsilon\left(L^*;\rho_\eta\right)}{\partial L}\frac{\partial L\left(\rho_\varepsilon^*;\rho_\eta\right)}{\partial\rho_\eta}}^{\text{liquidity trading effect (+)}} + \overbrace{\frac{\partial\rho_\varepsilon\left(L^*;\rho_\eta\right)}{\partial\rho_\eta}}^{\text{crowding out effect (-)}}}{1 - \frac{\partial\rho_\varepsilon\left(L^*;\rho_\eta\right)}{\partial L}\frac{\partial L\left(\rho_\varepsilon^*;\rho_\eta\right)}{\partial\rho_\varepsilon}}, \tag{26}$$

where the total differentiation operator $\frac{d}{d\rho_\eta}$ indicates that both L^* and ρ_ε^* are now endogenous.

In equation (26), the term $\frac{\partial\rho_\varepsilon\left(L^*;\rho_\eta\right)}{\partial\rho_\eta}$ captures the crowding out effect of disclosure, as analyzed by Diamond (1985). That is, when more public information exists in financial markets, speculators expect lower trading gains and thus collect less private information. Formally, by examining equation (19), we can show that $\frac{\partial\rho_\varepsilon\left(L^*;\rho_\eta\right)}{\partial\rho_\eta} < 0$.

The term $\frac{\partial \rho_\varepsilon(L^*;\rho_\eta)}{\partial L}\frac{\partial L(\rho_\varepsilon^*;\rho_\eta)}{\partial \rho_\eta}$ represents the new channel emphasized by our analysis. Specifically, public information attracts more noise trading through improving market liquidity (i.e., $\frac{\partial L(\rho_\varepsilon^*;\rho_\eta)}{\partial \rho_\eta} > 0$ as stated in Proposition 2). This increased noise trading in turn improves the trading gains of speculators, which strengthens their ex-ante information acquisition incentives (i.e., $\frac{\partial \rho_\varepsilon(L^*;\rho_\eta)}{\partial L} > 0$). Therefore, public information tends to promote information production through this noise trading channel (i.e., $\frac{\partial \rho_\varepsilon(L^*;\rho_\eta)}{\partial L}\frac{\partial L(\rho_\varepsilon^*;\rho_\eta)}{\partial \rho_\eta} > 0$ in equation (26)). As a result, this new channel attenuates the crowding out channel in terms of information production. The total effect of disclosure on private information production is determined by the relative strength of the liquidity trading effect and the crowding out effect.

Proposition 4. *(a) Suppose that discretionary liquidity traders coordinate on a stable equilibrium at date 0. The liquidity trading effect dominates the crowding out effect in information production if and only if price efficiency is sufficiently high. That is,*

$$\frac{d\rho_\varepsilon^*}{\partial \rho_\eta} > 0 \Longleftrightarrow \rho_p^* > \rho_\varepsilon^* \Longleftrightarrow L^* < \frac{\sqrt{\rho_\varepsilon^*}}{\gamma}. \tag{27}$$

(b) When $B > \sqrt{\frac{1}{2(\rho_v+\rho_\eta)}}$, the crowding out effect dominates, so that $\frac{d\rho_\varepsilon^*}{d\rho_\eta} < 0$.

Proof. See the appendix. □

To understand condition (27), note that the liquidity trading effect works through attracting noise trading and affecting price informativeness, and thus when price efficiency ρ_p^* is sufficiently high (or equivalently, when there is little noise trading L^* in the market), this effect is particularly strong. In Part (b) of Proposition 4, we further provide a sufficient condition under which condition (27) is violated so that the crowding out effect dominates. Intuitively, when the liquidity benefit B is high, many discretionary liquidity traders choose to participate in the market, so that the endogenous value of L^* is high, which makes condition (27) difficult to hold. In addition, note that when ρ_η is sufficiently large, the sufficient condition in Part (b) of Proposition 4 is satisfied. This implies that the crowding out effect always dominates when the public information is sufficiently precise.

Corollary 2. *For sufficiently large values of ρ_η, the crowding out effect dominates, so that ρ_ε^* gradually decreases to* 0. *That is,*

$$\frac{d\rho_\varepsilon^*}{\partial \rho_\eta} < 0 \textit{ for sufficiently large values of } \rho_\eta,$$

and

$$\lim_{\rho_\eta\to\infty} \rho_\varepsilon^* = 0 \textit{ and } \lim_{\rho_\eta\to\infty} \frac{d\rho_\varepsilon^*}{\partial \rho_\eta} = 0.$$

Proof. See the appendix. □

Corollary 2 suggests that although ρ_ε^* may initially increase with ρ_η, it will eventually switch and decrease. Thus, we expect that ρ_ε^* either monotonically decreases with ρ_η or is hump-shaped.

According to Proposition 4, the hump shape is more likely to occur when the liquidity benefit B of participating in the market is small and/or when the cost $C'(\rho_\varepsilon)$ of acquiring fundamental information is small. We use Fig. 2 to verify these intuitions.

In Fig. 2, we choose a quadratic cost function for information acquisition, $C(\rho_\varepsilon) = \frac{c}{2}\rho_\varepsilon^2$ with $c > 0$. In this case, equation (23) of determining ρ_ε^* becomes a cubic equation. Thus, we can easily solve all the roots of this cubic equation and check whether each root falls in the range of $(0, \bar{\rho}_\varepsilon)$ to pin down all the possible equilibria. We plot the equilibrium values of ρ_ε^* against disclosure precision ρ_η for various combinations of (B, c), where $B \in \{0.2, 0.5, 1, 5\}$ and $c \in \{0.2, 0.5, 1, 5\}$. Other parameters are set as $\rho_v = 1$ and $\gamma = 2$. Consistent with Corollary 2, we see that for all panels of Fig. 2, ρ_ε^* decreases with ρ_η for large values of ρ_η. In addition, for small (B, c), there is an increasing segment of ρ_ε^* at small values of ρ_η. In contrast, when (B, c) becomes large, ρ_ε^* always decreases with ρ_η. All of these observations are consistent with Proposition 4.

In the top four panels, there can exist multiple (three) equilibria. When this multiplicity arises, we follow Section 3.2.3 and focus on equilibria featuring stable date-0 stable equilibria (i.e., $\frac{\partial W(L^*;\rho_\eta)}{\partial L} < 0$). Unstable ones are labeled by dashed segments in the figure. Multiplicity suggests jumps in ρ_ε^* in response to small changes in ρ_η, when agents coordinate on a particular stable equilibrium. Take the first panel as an example. When $\rho_\eta < 3.5$, a unique equilibrium exists, and when $\rho_\eta \in (3.5, 5.5)$, three equilibria exist. Now suppose that agents coordinate on a stable equilibrium with a higher value of ρ_ε^*. As ρ_η slightly increases around 3.5, the equilibrium value of ρ_ε^* can then jump from 0.05 to 0.25. If agents coordinate on a stable equilibrium with a smaller value of ρ_ε^*, then this jump occurs at $\rho_\eta = 5.5$. Nonetheless, the overall pattern is still that ρ_ε^* first increases and then decreases with ρ_η no matter which stable equilibrium that agents coordinate upon. What multiplicity adds is a discontinuity of ρ_ε^* as a function of ρ_η. Taken together, for small (B, c), there is an inverted U-shape between ρ_ε^* and ρ_η, while for large (B, c), ρ_ε^* monotonically decreases with ρ_η.

4.2.2. *Liquidity trading, market liquidity, and price efficiency*

After we derive the effect of ρ_η on ρ_ε^*, we can use equation (25) to determine the effect on L^*. We employ equations (13) and (17) to pin down the effect of ρ_η on market liquidity LIQ^* and price efficiency ρ_p^*, respectively. The complexity of the overall equilibrium precludes a full analytical characterization of these outcomes, and so we use Fig. 3 to conduct a numerical analysis. Specifically, Fig. 2 suggests two patterns of ρ_ε^*, and thus in Fig. 3, we report two economies.[11] In the top panels, we set $B = c = 0.5$ so that ρ_ε^* first increases and then decreases with ρ_η; in the bottom panels, we set $B = c = 1$, in which case ρ_ε^* monotonically decreases with ρ_η. The other parameters are the same as in Fig. 2. We see that in both economies, the conclusion in Proposition 2 of our baseline model continues to hold. That is, public information attracts discretionary liquidity trading L^*, improves market liquidity LIQ^*, and harms price efficiency ρ_p^*. This suggests that the liquidity channel highlighted by our analysis is quite robust.

In particular, in terms of price efficiency ρ_p^*, the crowding out effect and the liquidity trading effect can strengthen each other. Specifically, by (17), we have

$$\rho_p^* = \left(\frac{\rho_\varepsilon^*}{\gamma L^*}\right)^2 \Rightarrow \frac{1}{2\rho_p^*}\frac{d\rho_p^*}{d\rho_\eta} = \frac{1}{\rho_\varepsilon^*}\frac{d\rho_\varepsilon^*}{d\rho_\eta} - \frac{1}{L^*}\frac{dL^*}{d\rho_\eta}.$$

[11] In the Online Appendix, we report the results for all 16 economies studied in Fig. 2 and find that our results are robust.

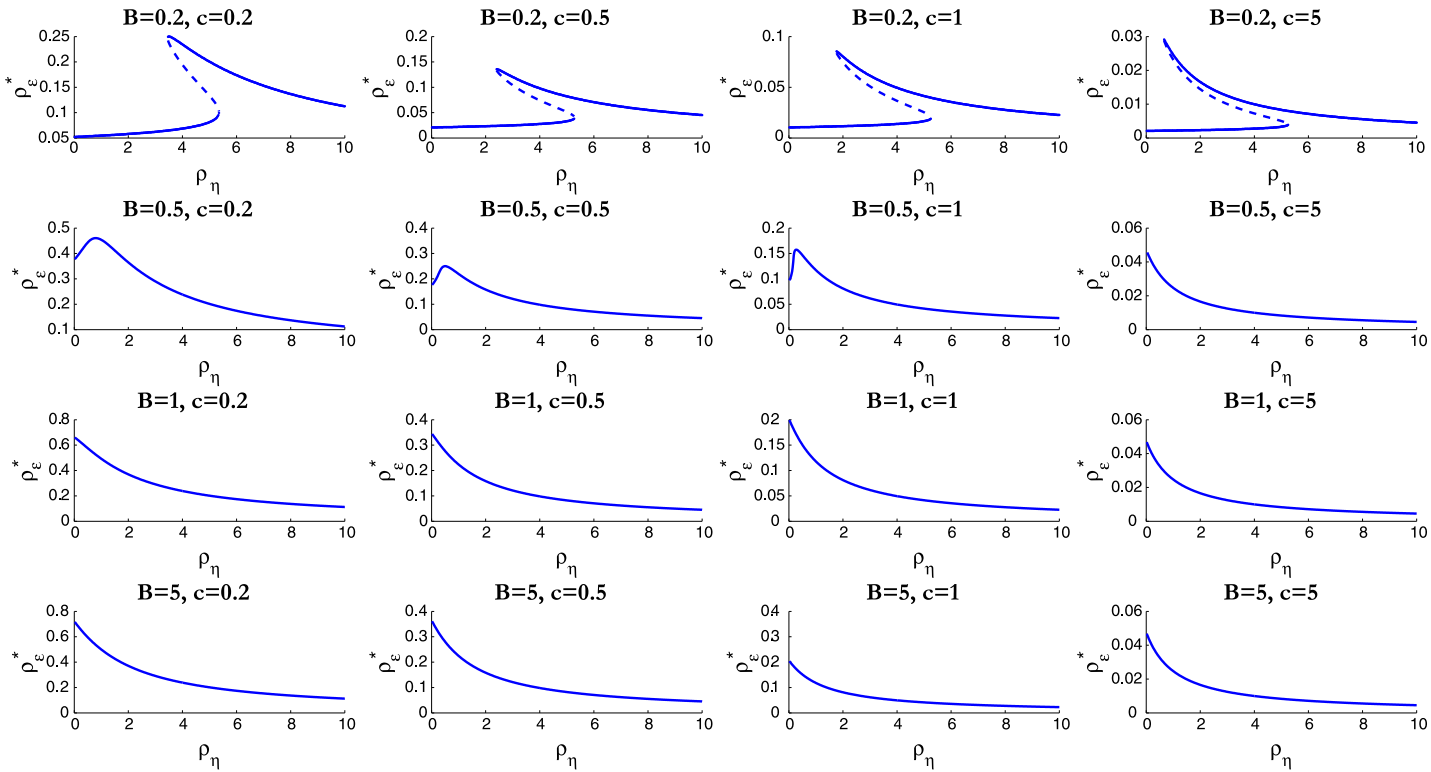

Fig. 2. *The Effect of Public Information on Information Production*. This figure plots the implications of public information for information production for various combinations of parameters B and c. The information acquisition cost takes a quadratic functional form of $C(\rho_\varepsilon) = \frac{c}{2}\rho_\varepsilon^2$. Parameter ρ_η controls the precision of public information. The other parameter values are: $\rho_v = 1$ and $\gamma = 2$. The dashed segments indicate unstable equilibria.

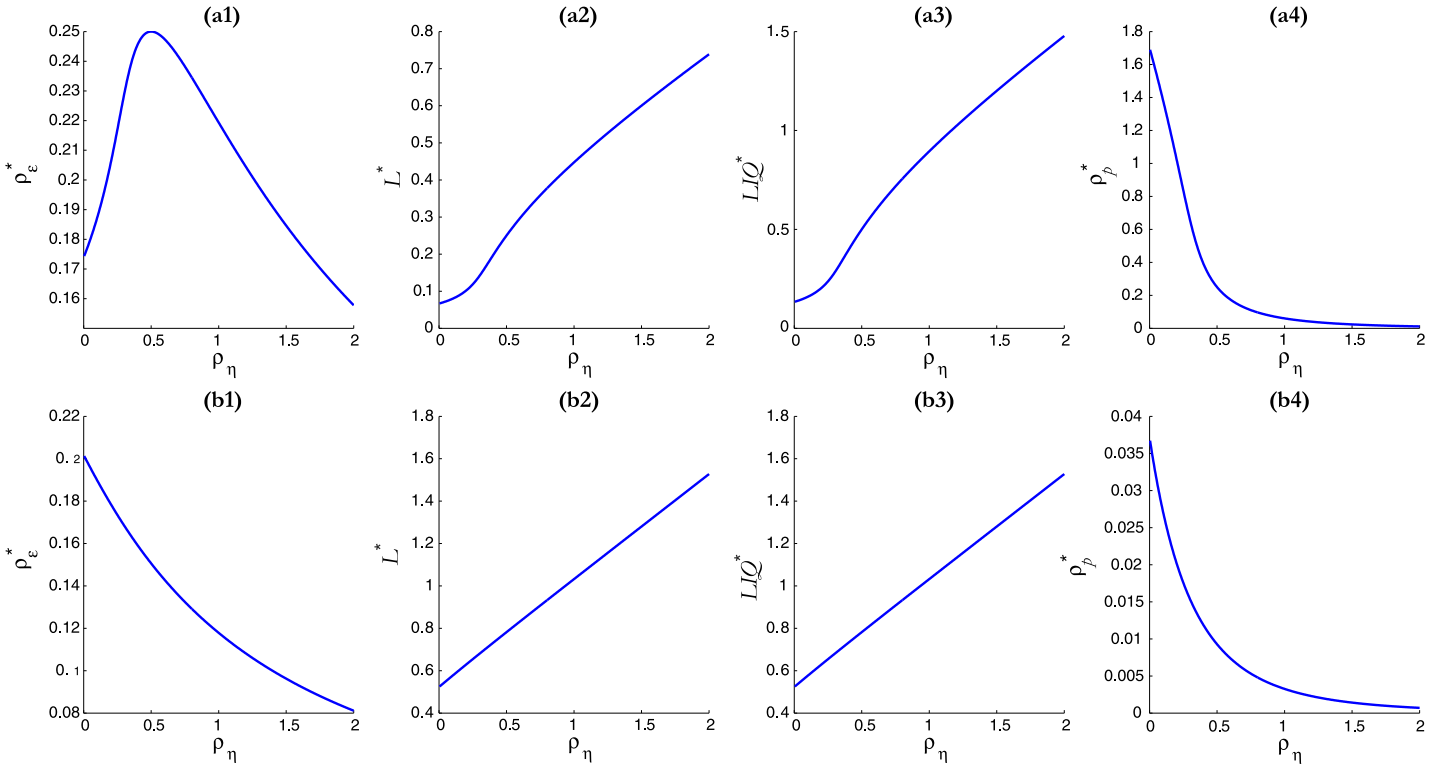

Fig. 3. *The Effect of Public Information in Economies with Endogenous Information Acquisition*. This figure plots the implications of public information when both information acquisition and liquidity trading are endogenous. The information acquisition cost takes a quadratic functional form of $C(\rho_\varepsilon) = \frac{c}{2}\rho_\varepsilon^2$. Parameter ρ_η controls the precision of public information. In the top four panels, we set $B = c = 0.5$, while in the bottom four panels, we set $B = c = 1$. The other parameter values are: $\rho_v =$ and $\gamma = 2$.

Thus, when $\frac{d\rho_\varepsilon^*}{d\rho_\eta} < 0$ (disclosure crowds out private information production) and $\frac{dL^*}{d\rho_\eta} > 0$ (public disclosure attracts noise trading), we must have $\frac{d\rho_p^*}{d\rho_\eta} > 0$. The following proposition provides a sufficient condition under which the price-efficiency effect of disclosure is negative.

Proposition 5. *Suppose $B > \sqrt{\frac{8}{27(\rho_v+\rho_\eta)}}$. In the unique, symmetric, overall equilibrium with both endogenous information acquisition and endogenous liquidity trading, public information harms price efficiency. That is, $\frac{d\rho_p^*}{d\rho_\eta} < 0$ in the overall equilibrium.*

Proof. See the appendix. □

5. Informed hedgers as "noise" providers

5.1. The setup

The model setup is almost identical to the baseline model except for the behavior of liquidity traders, who are now modeled as rational hedgers. Specifically, there are still three dates, $t = 0, 1$, and 2. At date 1, a risk-free asset (with a zero net rate of return) and a risky asset are traded in the financial market. The risky asset has an endogenous price $\tilde{p}$ and it pays off $\tilde{v} \sim N(0, 1/\rho_v)$ (with $\rho_v > 0$) at date 2. There are three groups of traders in the financial market: informed speculators (with mass one), hedgers (with an endogenous mass L), and a market maker sector. Both speculators and hedgers have CARA preferences, and we use $\gamma_s > 0$ and $\gamma_h > 0$ to denote their risk aversion, respectively. We follow Vives (1995) and introduce a competitive, uninformed, risk-neutral market maker who sees the aggregate orders $\tilde{\omega}$ from speculators and hedgers and sets the price according to the weak-efficiency form. Similar to our baseline model, prior to trading, all traders observe a public signal $\tilde{y}$ as specified by equation (1), where parameter ρ_η controls the quality of public information $\tilde{y}$. To simplify the analysis, we assume that all speculators observe a common private signal $\tilde{s}$: $\tilde{s} = \tilde{v} + \tilde{e}$, with $\tilde{e} \sim N(0, 1/\rho_e)$ and $\rho_e > 0$. In the Online Appendix, we allow speculators to see diverse signals and show that our results continue to hold.

We borrow from Wang (1994) and Easley et al. (2014) to model hedgers' behavior. At date 0, there is a fixed mass $\bar{L} > 0$ of identical potential hedgers, each of whom can spend a fixed cost $\bar{C} > 0$ to develop a private investment technology. We call those hedgers who decide to develop the technology *active hedgers* and those who do not *inactive hedgers*. We use $L \in \left[0, \bar{L}\right]$ to denote the endogenous mass of active hedgers. At date 1, in addition to the risk-free and risky assets, an active hedger can invest in the developed private technology, which has a net return of $\tilde{\theta} + \phi\tilde{v} + \tilde{\xi}$, where $\tilde{\theta} \sim N(0, 1/\rho_\theta)$ (with $\rho_\theta > 0$), $\tilde{\xi} \sim N\left(0, 1/\rho_\xi\right)$ (with $\rho_\xi > 0$), and ϕ is a non-zero constant. All underlying random variables $\left\{\tilde{v}, \tilde{e}, \tilde{\eta}, \tilde{\theta}, \tilde{\xi}\right\}$ are mutually independent. The variable $\tilde{\theta}$ represents the forecastable part of the return on the private technology and it is privately observable to the active hedger at date 1. Active hedgers can hedge their private technology investment by the risky asset because the unforecastable part $\phi\tilde{v} + \tilde{\xi}$ of the technology return and the risky asset payoff $\tilde{v}$ are correlated, with the correlation being controlled by parameter ϕ (a higher value of $|\phi|$ corresponds to a higher correlation). Such hedging trades by the active hedgers generate "noise" in the risky asset market (see equation (31)), which relates to the variable $\tilde{\theta}$ that is privately observable to active hedgers.

The timeline is as follows. At date 0, each potential hedger decides whether to develop the technology and L mass of hedgers decide to do so. At date 1, the public signal $\tilde{y}$ is announced; active hedgers observe information set $\left\{\tilde{\theta}, \tilde{p}, \tilde{y}\right\}$, invest in the technology, and trade the risk-free and risky assets; inactive hedgers observe $\{\tilde{p}, \tilde{y}\}$ and make investments in the risk-free and risky assets (as we will show shortly, they optimally choose to only invest in the risk-free asset); speculators see $\{\tilde{s}, \tilde{p}, \tilde{y}\}$ and trade both assets in the financial market; the market maker sees the public signal $\tilde{y}$ and the aggregate order flow $\tilde{\omega}$ from hedgers and speculators, and sets the price $\tilde{p}$ according to the weak-efficiency rule: $\tilde{p} = E(\tilde{v}|\tilde{\omega}, \tilde{y})$. All traders behave competitively. At date 2, the assets pay off and all agents consume. The equilibrium in this economy still consists of two sub-equilibriums: (1) the date-1 financial market equilibrium in which hedgers and speculators respectively maximize their expected utility given their information sets and the market maker sets the price according to the weak-efficiency rule, and (2) the date-0 equilibrium in which hedgers optimally decide whether to develop the private technology. Again, hedgers' date-0 behavior determines the equilibrium amount of noise trading in the date-1 financial market.

5.2. *The financial market equilibrium*

5.2.1. *The price function*

The aggregate order flow $\tilde{\omega}$ depends on speculators' private information $\tilde{s}$ and active hedgers' private information $\tilde{\theta}$ and thus we consider the following linear pricing rule:

$$\tilde{p} = E(\tilde{v}|\tilde{\omega}, \tilde{y}) = \alpha_y \tilde{y} + \alpha_s \tilde{s} + \alpha_X \tilde{X}, \tag{28}$$

where α's are endogenous coefficients and where

$$\tilde{u} \equiv -\frac{\phi\rho_\xi}{\gamma_h}\tilde{\theta} \text{ and } \tilde{X} \equiv L \times \tilde{u} \tag{29}$$

respectively represent the uninformed component in active hedgers' demand for the risky asset and the resulting aggregate uninformed trading in the financial market (which will be formally shown in (31)). We now derive this price function.

Similar to (10) in the baseline model, speculators' demand function for the risky asset is

$$D_s(\tilde{p}, \tilde{y}, \tilde{s}) = \frac{E(\tilde{v} - \tilde{p}|\tilde{p}, \tilde{y}, \tilde{s})}{\gamma_s Var(\tilde{v}|\tilde{p}, \tilde{y}, \tilde{s})} = \frac{\rho_e \tilde{s} + \rho_\eta \tilde{y} - \left(\rho_v + \rho_e + \rho_\eta\right)\tilde{p}}{\gamma_s}. \tag{30}$$

Inactive hedgers observe $\{\tilde{p}, \tilde{y}\}$ and can only invest in the two tradable assets. Their demand for the risky asset is $D_h(\tilde{p}, \tilde{y}) = \frac{E(\tilde{v}|\tilde{p},\tilde{y}) - \tilde{p}}{\gamma_h Var(\tilde{v}|\tilde{p},\tilde{y})}$. By equation (28), we have $E(\tilde{v}|\tilde{p}, \tilde{y}) - \tilde{p} = 0$, and thus $D_h(\tilde{p}, \tilde{y}) = 0$. That is, inactive hedgers optimally choose not to participate in the risky asset market.

Active hedgers observe $\left\{\tilde{p}, \tilde{y}, \tilde{\theta}\right\}$ and choose investment D_h in the risky asset and investment K in the private technology to maximize

$$E\left(-\exp\left[-\gamma_h\left(D_h(\tilde{v} - \tilde{p}) + K(\tilde{\theta} + \phi\tilde{v} + \tilde{\xi}) - \bar{C}\right)\right]\middle|\, \tilde{p}, \tilde{y}, \tilde{\theta}\right),$$

where we have normalized their initial wealth at 0. By price function (28), active hedgers can use the price $\tilde{p}$, the public signal $\tilde{y}$, and their own signal $\tilde{\theta}$ to back out speculators' signal $\tilde{s}$. Thus, their demand for the risky asset consists of two components: an informed component due

to speculation on $\tilde{s}$ and an uninformed component due to hedging the investment in the private technology. Formally, we can compute

$$D_h\left(\tilde{p},\tilde{y},\tilde{\theta}\right)=\underbrace{\frac{\rho_e\tilde{s}+\rho_\eta\tilde{y}-\left(\phi^2\rho_\xi+\rho_v+\rho_e+\rho_\eta\right)\tilde{p}}{\gamma_h}}_{\text{informed trades}}+\underbrace{\left(-\frac{\phi\rho_\xi}{\gamma_h}\tilde{\theta}\right)}_{\text{uninformed trades}\equiv\tilde{u}}. \tag{31}$$

In particular, the second term generates the endogenous uninformed trading and, to be consistent with the baseline model, we label it as $\tilde{u}$ (also see equation (29)).

The aggregate order flow is $\tilde{\omega}=D_s\left(\tilde{p},\tilde{y},\tilde{s}\right)+L\times D_h\left(\tilde{p},\tilde{y},\tilde{\theta}\right)$ (note that inactive hedgers optimally choose $D_h\left(\tilde{p};\tilde{y}\right)=0$). By equations (30) and (31), we have

$$\begin{aligned}\tilde{\omega}=&\frac{\rho_e\tilde{s}+\rho_\eta\tilde{y}-\left(\rho_v+\rho_e+\rho_\eta\right)\tilde{p}}{\gamma_s}\\&+L\frac{\rho_e\tilde{s}+\rho_\eta\tilde{y}-\left(\phi^2\rho_\xi+\rho_v+\rho_e+\rho_\eta\right)\tilde{p}}{\gamma_h}+\tilde{X},\end{aligned} \tag{32}$$

where we have used (29) to replace $\tilde{\theta}$ with $\tilde{X}$ in $L\times D_h\left(\tilde{p},\tilde{y},\tilde{\theta}\right)$. The market maker uses $\{\tilde{\omega},\tilde{y}\}$ to predict $\tilde{v}$ and determine the price $\tilde{p}$. Equation (32) indicates that, when combined with $\tilde{y}$, $\tilde{\omega}$ is equivalent to the following in predicting $\tilde{v}$:

$$\tilde{s}_\omega\equiv\tilde{s}+\frac{1}{\frac{\rho_e}{\gamma_s}+L\frac{\rho_e}{\gamma_h}}\tilde{X}. \tag{33}$$

That is,

$$\tilde{p}=E\left(\tilde{v}|\tilde{\omega},\tilde{y}\right)=E\left(\tilde{v}|\tilde{s}_\omega,\tilde{y}\right). \tag{34}$$

Applying the Bayes' rule, we can express $E\left(\tilde{v}|\tilde{s}_\omega,\tilde{y}\right)$ to compute the price function, which is summarized in the following proposition.

Proposition 6. *There exists a unique linear financial market equilibrium in which*

$$\tilde{p}=\alpha_y\tilde{y}+\alpha_s\tilde{s}+\alpha_X\tilde{X},$$

where

$$\alpha_y=\frac{\rho_\eta}{\rho_v+\rho_\eta+\rho_{pv}},\ \alpha_s=\frac{\rho_{pv}}{\rho_v+\rho_\eta+\rho_{pv}},\ \textit{and}\ \alpha_X=\frac{\rho_{pv}\left(\frac{\rho_e}{\gamma_s}+L\frac{\rho_e}{\gamma_h}\right)^{-1}}{\rho_v+\rho_\eta+\rho_{pv}} \tag{35}$$

with

$$\rho_{pv}=\left(\rho_e^{-1}+\frac{\phi^2\rho_\xi^2}{\rho_e^2\rho_\theta}\left(1+\frac{\gamma_h}{\gamma_sL}\right)^{-2}\right)^{-1}. \tag{36}$$

5.2.2. Price efficiency and market liquidity

By (28), the price $\tilde{p}$ is equivalent to the following signal in predicting $\tilde{s}$ (and hence $\tilde{v}$):

$$\tilde{s}_p\equiv\frac{\tilde{p}-\alpha_y\tilde{y}}{\alpha_s}=\tilde{s}+\frac{\alpha_X}{\alpha_s}\tilde{X}=\tilde{v}+\tilde{e}+\frac{\alpha_X}{\alpha_s}\tilde{X}. \tag{37}$$

This signal $\tilde{s}_p$ has a precision of

$$\rho_p \equiv \left(\frac{\alpha_s}{\alpha_X}\right)^2 \frac{1}{Var\left(\tilde{X}\right)} \tag{38}$$

in predicting $\tilde{s}$, and a precision of

$$\rho_{pv} \equiv \left(\rho_e^{-1} + \rho_p^{-1}\right)^{-1} \tag{39}$$

in predicting the asset fundamental $\tilde{v}$. That is, ρ_{pv} is a monotonic transformation of ρ_p (recall that ρ_e is an exogenous parameter). Therefore, either ρ_p or ρ_{pv} is a proxy for how the market price transmits fundamental information about $\tilde{v}$. As in the baseline model, we use ρ_p to measure price efficiency.

Note that by Proposition 6, we have

$$\frac{\alpha_s}{\alpha_X} = \frac{\rho_e}{\gamma_s} + L\frac{\rho_e}{\gamma_h}. \tag{40}$$

Thus, the signal $\tilde{s}_p$ given by (37) is the same as the signal $\tilde{s}_\omega$ given by (33), which makes sense since the price transmits information through order flows. This observation also helps in understanding how the mass L of active hedgers affects price efficiency ρ_p.

By (29), (38), and (40), we have

$$\rho_p = \underbrace{\left(\frac{\rho_e}{\gamma_s} + L\frac{\rho_e}{\gamma_h}\right)^2}_{\text{speculation on } \tilde{s}} \times \underbrace{\left(\frac{\gamma_h}{\phi\rho_\xi L}\right)^2 \rho_\theta}_{\text{noise injected by hedgers}} = \frac{\rho_e^2\rho_\theta}{\phi^2\rho_\xi^2}\left(1 + \frac{\gamma_h}{\gamma_s L}\right)^2. \tag{41}$$

As their demand function (31) shows, active hedgers play two roles in affecting price efficiency through their demands. First, because active hedgers trade on $\tilde{s}$, the higher the number of active hedgers, the higher the amount of fundamental information is brought into the price. This effect tends to make price efficiency ρ_p increase with L. Second, active hedgers hedge their investment in the private technology, which injects noise $\tilde{\theta}$ into the price, thus reducing price efficiency. Overall, the negative effect dominates, so that similar to our baseline model, price informativeness ρ_p decreases with an increase in the mass L of active hedgers.

As in our baseline model, $LIQ \equiv \frac{1}{\alpha_X}$ is still the metric for market liquidity (i.e., equation (7)). By Proposition 6,

$$LIQ = \underbrace{\left(\frac{\rho_e}{\gamma_s} + L\frac{\rho_e}{\gamma_h}\right)}_{\text{direct effect}} \times \underbrace{\left(1 + \frac{\rho_v + \rho_\eta}{\rho_{pv}}\right)}_{\text{indirect effect}}. \tag{42}$$

Active hedgers affect market liquidity also through two channels. First, by trading on $\tilde{s}$, more active hedgers make the order-flow signal $\tilde{s}_\omega$ in (33) more responsive to fundamentals $\tilde{s}$ than to aggregate noise trading $\tilde{X}$. Other things being equal, this effect directly makes the price less sensitive to $\tilde{X}$, thereby improving market liquidity. Second, an indirect effect exists. By (41), increasing L reduces ρ_p (and hence ρ_{pv} by (39)), and so the market maker faces a harder time in reading order flow information, which makes the price less sensitive to $\tilde{s}_\omega$. This further reduces the price impact of $\tilde{X}$ and hence market liquidity further improves. So, overall, LIQ increases with L.

5.3. The decision of hedgers: equilibrium L^*

Let vector $\tilde{\mathbf{V}}$ denote the net returns on the two risky investment opportunities, that is, $\tilde{\mathbf{V}} \equiv \left(\tilde{v} - \tilde{p}, \tilde{\theta} + \phi\tilde{v} + \tilde{\xi}\right)'$. The date-1 certainty equivalent $\widetilde{CE}_{h,1}$ of an active hedger as

$$\widetilde{CE}_{h,1} = \frac{1}{2\gamma_h} E\left(\tilde{\mathbf{V}}|\tilde{p}, \tilde{y}, \tilde{\theta}\right)' Var^{-1}\left(\tilde{\mathbf{V}}|\tilde{p}, \tilde{y}, \tilde{\theta}\right) E\left(\tilde{\mathbf{V}}|\tilde{p}, \tilde{y}, \tilde{\theta}\right) - \bar{C}.$$

Thus, back to date 0, we can compute the ex-ante certainty equivalent as

$$\begin{aligned} CE_{h,0} &\equiv -\frac{1}{\gamma_h} \log E\left(e^{-\gamma_h \widetilde{CE}_{h,1}}\right) \\ &= \frac{1}{2\gamma_h} \log\left[\left|Var\left(\tilde{\mathbf{V}}\right)\right|\right] - \frac{1}{2\gamma_h} \log\left[\left|Var\left(\tilde{\mathbf{V}}|\tilde{p}, \tilde{y}, \tilde{\theta}\right)\right|\right] - \bar{C}, \end{aligned} \tag{43}$$

where the second equality follows from the moment generating function of a Chi-square distribution. Equation (43) is intuitive. The first term $\left|Var\left(\tilde{\mathbf{V}}\right)\right|$ is a measure for the size of the ex-ante investment opportunities available to active hedgers: the higher the volatility of $\tilde{\mathbf{V}}$, the more chances that active hedgers see to explore their investment opportunities. The second term $\left|Var\left(\tilde{\mathbf{V}}|\tilde{p}, \tilde{y}, \tilde{\theta}\right)\right|$ in (43) captures the risk faced by active hedgers when they invest in the risky returns $\tilde{\mathbf{V}}$. Being risk averse, active hedgers dislike this risk ex ante and thus $\left|Var\left(\tilde{\mathbf{V}}|\tilde{p}, \tilde{y}, \tilde{\theta}\right)\right|$ negatively affects $CE_{h,0}$ in equation (43).

Direct computation shows

$$\left|Var\left(\tilde{\mathbf{V}}\right)\right| = \underbrace{\frac{1}{\rho_v + \rho_\eta + \rho_{pv}}\left(\frac{1}{\rho_\theta} + \frac{\phi^2}{\rho_v} + \frac{1}{\rho_\xi}\right)}_{Var(\tilde{v}-\tilde{p}) \times Var\left(\tilde{\theta}+\phi\tilde{v}+\tilde{\xi}\right)} - \underbrace{\left(\frac{\phi}{\rho_v + \rho_\eta + \rho_{pv}} + \alpha_X L \frac{\phi\rho_\xi}{\gamma_h \rho_\theta}\right)^2}_{\left[Cov\left(\tilde{v}-\tilde{p}, \tilde{\theta}+\phi\tilde{v}+\tilde{\xi}\right)\right]^2 \text{(liquidity channel)}}. \tag{44}$$

In particular, the term $\alpha_X L \frac{\phi\rho_\xi}{\gamma_h \rho_\theta}$ captures the liquidity channel that we analyzed in our baseline model (i.e., the term $\alpha_X L$ in equation (12)). The liquidity channel thus works through $Cov\left(\tilde{v} - \tilde{p}, \tilde{\theta} + \phi\tilde{v} + \tilde{\xi}\right)$ in this alternative setup, which makes sense given that the uninformed trading in the risky asset is generated by the hedging motive related to the private technology. We can also compute

$$\left|Var\left(\tilde{\mathbf{V}}|\tilde{p}, \tilde{y}, \tilde{\theta}\right)\right| = \frac{1}{\rho_\xi\left(\rho_e + \rho_v + \rho_\eta\right)}. \tag{45}$$

Inserting (44) and (45) into (43) yields the following expression:

$$\begin{aligned} &CE_{h,0}\left(L; \rho_\eta\right) \\ &= \frac{1}{2\gamma_h} \log\left[\frac{\frac{1}{\rho_\theta} + \frac{\phi^2}{\rho_v} + \frac{1}{\rho_\xi}}{\rho_v + \rho_\eta + \rho_{pv}} - \left(\frac{\phi}{\rho_v + \rho_\eta + \rho_{pv}} + \alpha_X L \frac{\phi\rho_\xi}{\gamma_h \rho_\theta}\right)^2\right] \\ &\quad + \frac{1}{2\gamma_h} \log\left[\rho_\xi\left(\rho_e + \rho_v + \rho_\eta\right)\right] - \bar{C}, \end{aligned} \tag{46}$$

where we explicitly express $CE_{h,0}$ as a function of L and ρ_η.

Function $CE_{h,0}\left(L;\rho_\eta\right)$—which is the counterpart of $W\left(L;\rho_\eta\right)$ in (12) in our baseline model—determines the equilibrium mass L^* of active hedgers. Specifically, if $CE_{h,0}\left(0;\rho_\eta\right)\le 0$, then a potential hedger does not benefit from developing the private technology when all hedgers do not develop, and thus $L^*=0$ constitutes an equilibrium. If $CE_{h,0}\left(\bar{L};\rho_\eta\right)\ge 0$—a potential hedger is weakly better off by becoming active when all other hedgers are also active—then an equilibrium exists in which the entire mass $\bar{L}$ of hedgers are active, that is, $L^*=\bar{L}$. Given an interior mass of active hedgers ($0<L^*<\bar{L}$), if every potential hedger is indifferent to becoming active versus remaining inactive—i.e., $CE_{h,0}\left(L^*;\rho_\eta\right)=0$—then that mass L^* is an equilibrium at date 0.

5.4. The effects of public information

As in the baseline model, how the equilibrium mass L^* of active hedgers responds to public information precision ρ_η depends on how $CE_{h,0}\left(L;\rho_\eta\right)$ changes with L and ρ_η. Analyzing (46), we obtain the following proposition that summarizes the behavior of $CE_{h,0}$.

Proposition 7. *(a) The sign of $\frac{\partial CE_{h,0}(L;\rho_\eta)}{\partial L}$ is determined by a 4th order polynomial of $\frac{\gamma_s L}{\gamma_h}$. That is,*

$$\frac{\partial CE_{h,0}\left(L;\rho_\eta\right)}{\partial L}\propto F_L\left(L;\rho_\eta\right)\equiv -1-\left[3-\frac{\rho_\xi\left(\rho_e+\rho_\eta-\rho_v\right)}{\rho_v\left(\rho_e+\rho_\eta+\rho_v\right)}\phi^2\right]\frac{\gamma_s L}{\gamma_h}-o_{L,4}\left(\frac{\gamma_s L}{\gamma_h}\right), \tag{47}$$

with

$$o_{L,4}\left(\frac{\gamma_s L}{\gamma_h}\right)\equiv A_{L,2}\left(\frac{\gamma_s L}{\gamma_h}\right)^2+A_{L,3}\left(\frac{\gamma_s L}{\gamma_h}\right)^3+A_{L,4}\left(\frac{\gamma_s L}{\gamma_h}\right)^4,$$

where the A coefficients are polynomials of ϕ, whose exact expressions are provided in the Online Appendix.

(b) The sign of $\frac{\partial CE_{h,0}(L;\rho_\eta)}{\partial \rho_\eta}$ is determined by a 6th order polynomial of $\frac{\gamma_s L}{\gamma_h}$. That is,

$$\frac{\partial CE_{h,0}\left(L;\rho_\eta\right)}{\partial \rho_\eta}\propto F_{\rho_\eta}\left(L;\rho_\eta\right)\equiv 1+\frac{2\left(3\rho_\theta+\rho_\xi\right)}{\rho_\theta}\frac{\gamma_s L}{\gamma_h}+o_{\rho_\eta,6}\left(\frac{\gamma_s L}{\gamma_h}\right), \tag{48}$$

with

$$\begin{aligned}o_{\rho_\eta,6}\left(\frac{\gamma_s L}{\gamma_h}\right)\equiv{}& A_{\rho_\eta,2}\left(\frac{\gamma_s L}{\gamma_h}\right)^2+A_{\rho_\eta,3}\left(\frac{\gamma_s L}{\gamma_h}\right)^3+A_{\rho_\eta,4}\left(\frac{\gamma_s L}{\gamma_h}\right)^4\\&+A_{\rho_\eta,5}\left(\frac{\gamma_s L}{\gamma_h}\right)^5+A_{\rho_\eta,6}\left(\frac{\gamma_s L}{\gamma_h}\right)^6,\end{aligned}$$

where the A coefficients are polynomials of ϕ, whose exact expressions are provided in the Online Appendix.

Proof. Plug the expressions of ρ_{pv} and α_X in Proposition 6 into (46) and then compute the derivatives directly. □

Proposition 7 suggests the following sufficient condition for the robustness of Proposition 2 in our baseline model.

Corollary 3. *Suppose that*

$$F_L\left(L;\rho_\eta\right) < 0 \text{ and } F_{\rho_\eta}\left(L;\rho_\eta\right) > 0 \text{ for } L \in \left[0,\bar{L}\right], \tag{49}$$

where $F_L\left(L;\rho_\eta\right)$ and $F_{\rho_\eta}\left(L;\rho_\eta\right)$ are given by equations (47) and (48), respectively. Then:

(a) there exists a unique equilibrium mass L^* of active hedgers;

(b) public information attracts more active hedgers, improves market liquidity, and harms price efficiency (i.e., $\frac{\partial L^*}{\partial \rho_\eta} \geq 0$, $\frac{\partial LIQ^*}{\partial \rho_\eta} \geq 0$, and $\frac{\partial \rho_p^*}{\partial \rho_\eta} \leq 0$).

Proof. See the appendix. □

Under condition (49), we have $\frac{\partial CE_{h,0}(L;\rho_\eta)}{\partial L} < 0$ and $\frac{\partial CE_{h,0}(L;\rho_\eta)}{\partial \rho_\eta} > 0$ for $L \in \left[0,\bar{L}\right]$. Thus, increasing ρ_η will increase L^* by the implicit function theorem. Since price efficiency ρ_p^* decreases with L^* and ρ_η affects ρ_p^* only through L^* (see equation (41)), ρ_p^* decreases with ρ_η. The intuition is still the same as in the baseline model. That is, public information improves market liquidity and reduces the trading cost of uninformed trading, which therefore encourages more hedgers to develop the private technology. This in turn brings more uninformed trading as noise into the asset price and thus price informativeness is reduced. We can analytically show that condition (49) holds when $\bar{L}$ or ϕ is small. When $\bar{L}$ or ϕ becomes large, condition (49) can be violated. In this case, we use numerical analysis to show the robustness of our main results.

We first use Fig. 4 to explore the possible shapes of $CE_{h,0}\left(L;\rho_\eta\right)$. Specifically, we plot $CE_{h,0}$ as functions of L for various values of ϕ and ρ_η, while other parameters are fixed at $\rho_v = \rho_e = \rho_\xi = \rho_\theta = 1$, $\gamma_s = \gamma_h = 2$, and $\bar{C} = 0$. Consistent with Part (a) of Proposition 7, $CE_{h,0}\left(\cdot;\rho_\eta\right)$ is downward sloping at small values of L for all economies. When ϕ is small, for instance, in Panels (a) and (b), $CE_{h,0}\left(\cdot;\rho_\eta\right)$ is downward sloping, independent of the value of ρ_η. By contrast, when both ϕ and ρ_η become larger in Panels (c) and (d), $CE_{h,0}\left(\cdot;\rho_\eta\right)$ can be upward sloping at large values of L, so that $CE_{h,0}\left(\cdot;\rho_\eta\right)$ is U-shaped. Finally, consistent with Part (b) of Proposition 7, increasing ρ_η shifts the entire curve of $CE_{h,0}\left(\cdot;\rho_\eta\right)$ upward for all economies, which suggests that $\frac{\partial CE_{h,0}(L;\rho_\eta)}{\partial \rho_\eta} > 0$ is a more robust feature.

The U-shape of $CE_{h,0}\left(\cdot;\rho_\eta\right)$ as a function of L does not change our main result qualitatively. The only complication is the possibility of multiple equilibrium L^* due to the increasing part of $CE_{h,0}\left(\cdot;\rho_\eta\right)$. Nonetheless, as in Section 3.2.3, if we stick to a particular stable equilibrium in case of multiplicity, then our result in Proposition 2 continues to hold.

We use Fig. 5 to illustrate this point. In Panel (a1), we choose a *large* value of $\phi = 5$ and plot $CE_{h,0}\left(\cdot;\rho_\eta\right)$ for two possible values of ρ_η: $\rho_\eta = 0.4$ and 0.45. We also choose a *large* value of $\bar{L} = 2$, which means that the entire mass of potential hedgers is twice as large as the mass of speculators. The other parameters are fixed at $\rho_v = \rho_e = \rho_\xi = \rho_\theta = 1$, $\gamma_s = \gamma_h = 2$, and $\bar{C} = 0.66$. Given the large value of ϕ, we see that $CE_{h,0}\left(\cdot;\rho_\eta\right)$ exhibits a U-shape for both values of ρ_η in Panel (a1). When $\rho_\eta = 0.4$, there is a unique equilibrium, $L^* = 0.25$. When ρ_η increases to 0.45, the curve $CE_{h,0}\left(\cdot;\rho_\eta\right)$ moves upward, which achieves three equilibria: $L_1^* = 0.41$, $L_2^* = 1.42$, and $L_3^* = 2$. Equilibrium L_2^* is unstable, while L_1^* and L_3^* are stable.[12] Thus, increasing ρ_η will increase L^*.

[12] As in Section 3.2.3, L^* is a stable equilibrium if $CE_{h,0}\left(\cdot;\rho_\eta\right)$ is downward sloping at L^*, and L^* is an unstable equilibrium if $CE_{h,0}\left(\cdot;\rho_\eta\right)$ is upward sloping at L^*.

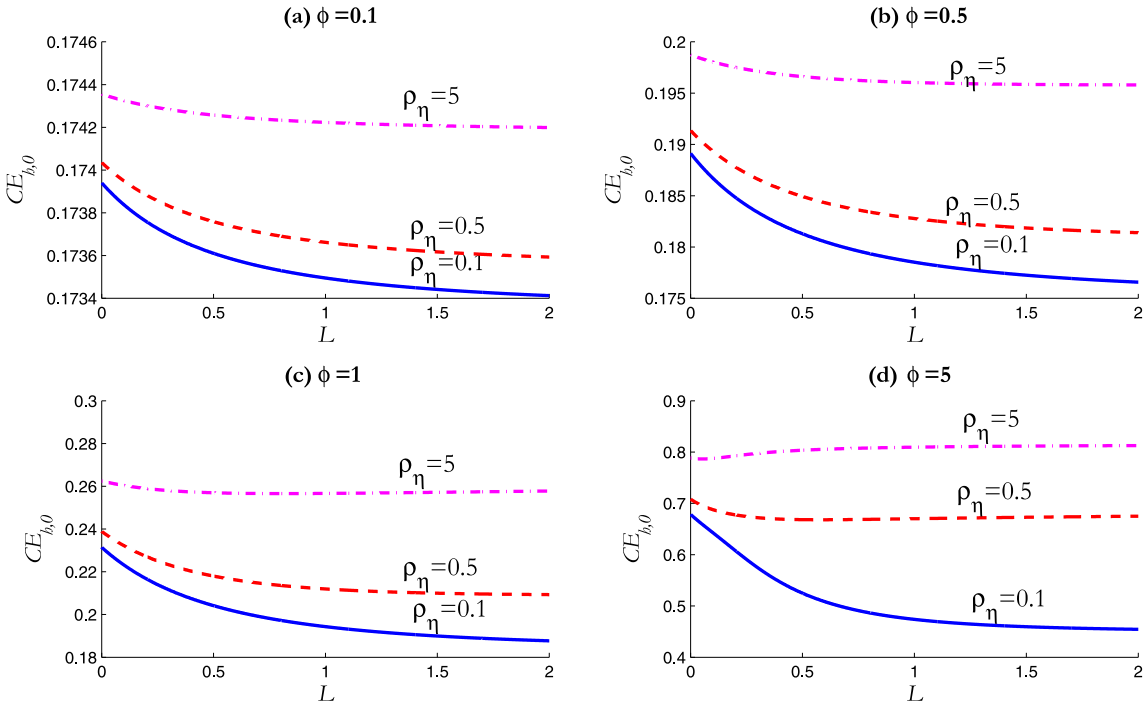

Fig. 4. *Ex-Ante Expected Utility of Active Hedgers in Economies with Hedgers as Noise Providers*. This figure plots the date-0 certainty equivalent $CE_{h,0}$ of active hedgers as a function of the mass L of the active hedger population for various values of ϕ and ρ_η (i.e., equation (46)). In all panels, the other parameter values are: $\rho_v = \rho_e = \rho_\theta = \rho_\xi = 1$, $\gamma_s = \gamma_h = 2$, and $\overline{C} = 0$.

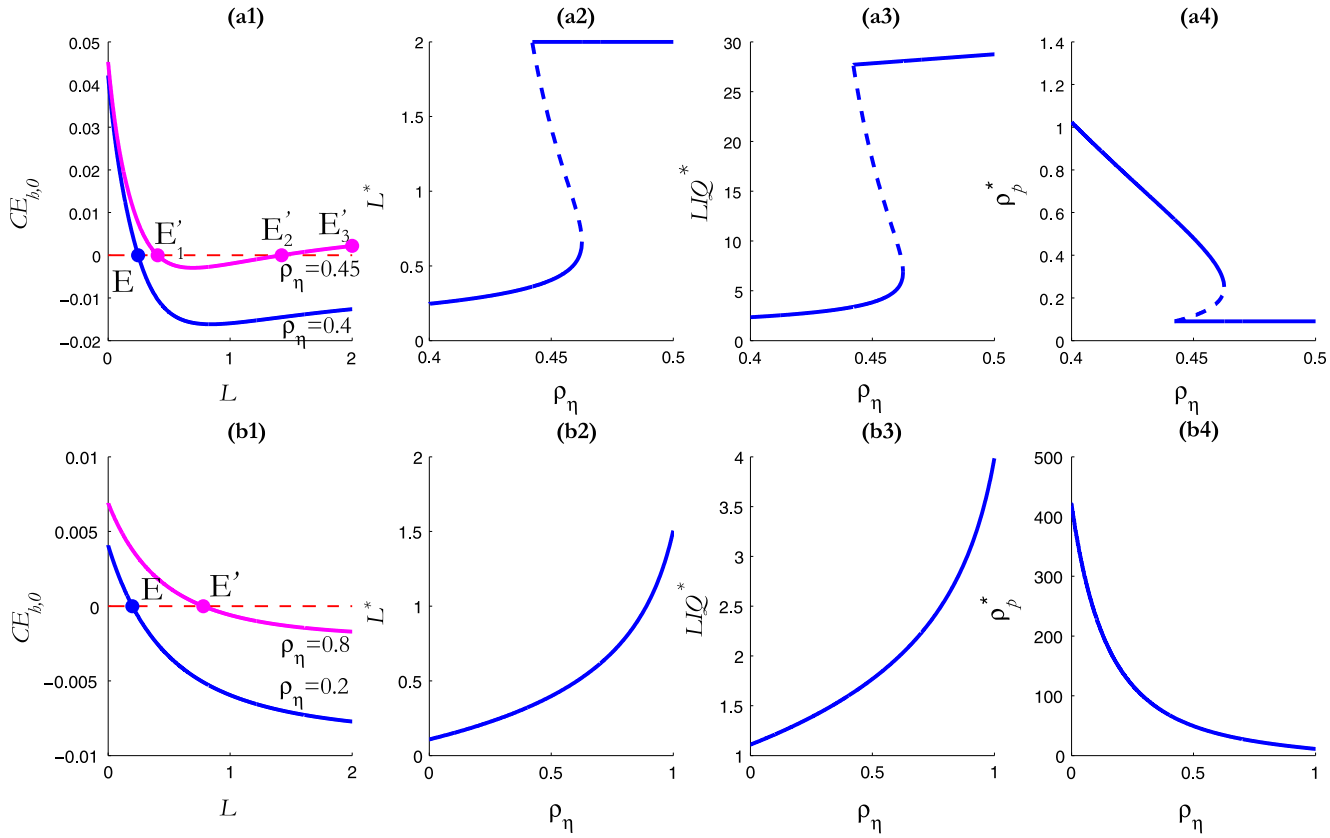

Fig. 5. *The Effect of Public Information in Economies with Hedgers as Noise Providers*. Panels (a1) and (b1) plot the date-0 certainty equivalent $CE_{h,0}$ of active hedgers as a function of the mass L of the active hedger population, which are useful for determining the equilibrium value of L. Other panels plot the implications of public information for equilibrium outcomes, where parameter ρ_η controls the precision of public information. The dashed segments in Panels (a2)–(a4) indicate unstable equilibria. In the top four panels, we set $\phi = 5$ and $\overline{C} = 0.66$, while in the bottom four panels, we set $\phi = 0.5$ and $\overline{C} = 0.1857$. In all panels, the other parameter values are: $\rho_v = \rho_e = \rho_\eta = \rho_\theta = \rho_\xi = 1$, $\gamma_s = \gamma_h = 2$, and $\overline{L} = 2$.

Panel (a2) of Fig. 5 plots the equilibrium L^* against a continuous range of ρ_η, while other parameter values are fixed at the same values as Panel (a1). Only when ρ_η entertains intermediate values do multiple equilibria exist. The dashed segment indicates the unstable equilibrium. If hedgers coordinate on a particular stable equilibrium, then L^* still increases with ρ_η. What multiplicity does is to generate a discontinuity in L^*, so that there can exist jumps in L^* in response to a small change in ρ_η. For instance, suppose that hedgers coordinate on the equilibrium with a smaller L^*. At $\rho_\eta = 0.46$, three equilibria then exist and the smaller stable L^* is around 0.5. When ρ_η slightly increases, there is a unique equilibrium, $L^* = 2$. Thus, a small change in ρ_η leads to a large jump in L^*. Note that this jump result exists no matter which stable equilibrium hedgers coordinate upon. If they coordinate on the larger L^*, then the jump occurs at a smaller value of ρ_η around 0.44.

Panels (a3) and (a4) of Fig. 5 plot the equilibrium values of market liquidity LIQ^* and price efficiency ρ_p^* against a continuous range of ρ_η. Again, we can see that if hedgers coordinate on a particular stable equilibrium, then LIQ^* increases with ρ_η while ρ_p^* decreases with ρ_η. Overall, Panels (a1)–(a4) suggest that when both $\bar{L}$ and ϕ take large values ($\bar{L} = 2$ and $\phi = 5$), our results continue to hold, which shows the robustness of our Proposition 2.

We also use Panels (b1)–(b4) to conduct a similar analysis for a smaller value of $\phi = 0.5$. The other parameters are the same as those in the top panels, except that we have set $\bar{C} = 0.1857$. We found that Panels (b1)–(b4) are consistent with Corollary 3: In Panels (b1) and (b2), there is a unique equilibrium L^*, which increases with ρ_η; market liquidity LIQ^* increases with ρ_η in Panel (b3), and price efficiency ρ_p^* decreases with ρ_η in Panel (b4).

6. Empirical relevance and model predictions

A leading example of discretionary liquidity trading that we seek to model is uninformed trades by institutional investors. Chordia et al. (2011) find that institutions are the driving forces behind the uptrend in stock market trading activities over the last two decades. Uninformed trading by institutional investors may arise due to their need to rebalance portfolios around index recompositions or liquidate their positions following shocks to their portfolios such as large losses in part of the portfolios, large inflows, or redemptions from clients. When executing these orders, fund managers often break them into small pieces spread over several days in an attempt to trade inside the spread and buy close to the bid price. The well-known Dimensional Fund Advisors (DFA) provides a good example of the discretionary executing strategy of passive investors and the importance of such trading discretion.[13]

Another example of uninformed trading by institutional investors is algorithmic trading, which has become increasingly dominant in the stock market. Skjeltorp et al. (2016) find that algorithmic trading originating from large institutional investors is likely to be uninformed. Chaboud et al. (2014) find evidence consistent with the strategies of algorithmic traders being highly correlated.

[13] Minimizing trading costs is a priority for DFA's strategy and its managers spend much time working out ways to trade optimally (see *The Wall Street Journal*, November 6, 2006, "The Dimensions of A Pioneering Strategy"). Although DFA is a passive portfolio, it pursues a strategy that sacrifices tracking accuracy by allowing actual weights to deviate from the underlying index thereby reducing the trading costs. See also Stein (2009) for an example that, surrounding a change by Morgan Stanley in the methodology of its indexes, how index funds employ some discretion to do the adjustment when market liquidity is more favorable.

There is a large literature on mutual funds herding (i.e., funds tend to buy into or out of the same stocks at the same time). Lou (2012) shows that persistence and the performance-chasing nature of investment flows from retail investors drive institutional investors to herd. Chordia et al. (2000) argue that correlated trading and herding among institutional funds can lead to commonality in liquidity. Da et al. (2015) document that coordinated noise trading by pension investors can exert a large price impact in Chile's financial markets.

As illustrated in Section 5, uninformed trading in one asset could also come from the hedging activities of investors who are informed about a correlated asset. One motivating real-world example is issuers of commodity-linked notes (CLNs) that are traded over the counter and have payoffs linked to the price of commodity or commodity futures. Henderson et al. (2015) argue that futures trades originating from CLN issuers are more likely to be motivated by hedging rather than based on information about subsequent futures prices, but that these trades have significant impacts on commodity prices.

Chae (2005) finds that liquidity trading decreases prior to scheduled firm announcements (such as earnings) and increase afterwards. His argument is consistent with our model of discretionary liquidity trading: discretionary liquidity traders who receive exogenous trade demands prior to announcements will postpone trading until the announcement is made and the information asymmetry is resolved. Schoenfeld (2014) finds that disclosure improves liquidity using trading of index funds around a company's inclusion to the S&P 500 Index. Balakrishnan et al. (2014) use an exogenous variation in the supply of public information to show that firms voluntarily disclose more information than regulation mandates and that such efforts improve liquidity. Their findings suggest a causal link between public information disclosure and market liquidity, consistent with our model's prediction.

Our model links public information to market liquidity and price efficiency in three steps, each of which has received supportive empirical evidence in the literature: (1) public information improves market liquidity (Blankespoor et al., 2014; Schoenfeld, 2014); (2) uninformed trades chase market liquidity (Chae, 2005; Menkveld et al., 2014); and (3) uninformed trades harm price efficiency (Brogaard et al., 2015; Da et al., 2015). Thus, our model predicts a negative relation between disclosure quality and price informativeness in settings in which discretionary liquidity trading plays an important role, such as those settings documented by Chae (2005). To test this new prediction, we can use the quality of a firm's financial reports to measure its disclosure quality (e.g., Hutton et al., 2009) and use price non-synchronicity measure in Morck et al. (2000) or the VAR-approach-based measure in Hasbrouck (1991) for price efficiency.

7. Conclusion

We construct a tractable REE model to endogenize noise trading and examine the implications of public information. The crux of our analysis is that noise trading endogenously chases market liquidity, which implies that the size of aggregate noise trading in the market responds to changes in the market environment. We use two approaches to implement our idea. In the baseline model, noise trading comes from uninformed, discretionary liquidity traders, while in the alternative model, it comes from informed hedgers. We identify that one key factor that determines the size of noise trading is the endogenous transaction cost that is negatively affected by market liquidity. We show that in our economy, more public information disclosure increases market liquidity but decreases price efficiency. Public information improves market liquidity through weakening adverse selection. However, the improved market liquidity attracts noise trading to the market, which harms information aggregation in the price system. When information is en-

dogenous, public information can negatively affect price efficiency both through crowding out private information and through attracting noise trading.

Appendix A. Proofs

A.1. Proof of Proposition 1

We have proved Part (a) in the main text. We now prove Part (b). We can rewrite equation (16), which determines equilibrium level L^*, as the following cubic equation:

$$Q\left(L^*\right) \equiv \gamma L^{*3} - B\left(\rho_v + \rho_\eta + \rho_\varepsilon\right) L^{*2} + \left(\frac{\rho_\varepsilon}{\gamma}\right) L^* - B\left(\frac{\rho_\varepsilon}{\gamma}\right)^2 = 0. \tag{A.1}$$

It is easy to compute $Q'(L) = 3\gamma L^2 - 2B\left(\rho_v + \rho_\eta + \rho_\varepsilon\right) L + \frac{\rho_\varepsilon}{\gamma}$, which is a quadratic. Its discriminant is $4\left[B^2\left(\rho_v + \rho_\eta + \rho_\varepsilon\right)^2 - 3\rho_\varepsilon\right]$, and so if

$$4\left[B^2\left(\rho_v + \rho_\eta + \rho_\varepsilon\right)^2 - 3\rho_\varepsilon\right] \le 0 \Longleftrightarrow B^2\left(\rho_v + \rho_\eta + \rho_\varepsilon\right)^2 \le 3\rho_\varepsilon,$$

then $Q'(L) \ge 0$ for all $L > 0$, and thus $Q(L)$ is monotonically increasing in L. Therefore, in this case, there exists a unique solution to $Q(L^*) = 0$.

Now, suppose that $B^2\left(\rho_v + \rho_\eta + \rho_\varepsilon\right)^2 > 3\rho_\varepsilon$. Then, there are two positive solutions to $Q'(L) = 0$, and we denote them by L_{small} and L_{large} as follows:

$$L_{small} \equiv \frac{\rho_\varepsilon}{\gamma\left[B\left(\rho_v + \rho_\eta + \rho_\varepsilon\right) + \sqrt{B^2\left(\rho_v + \rho_\eta + \rho_\varepsilon\right)^2 - 3\rho_\varepsilon}\right]}, \tag{A.2}$$

$$L_{large} \equiv \frac{\rho_\varepsilon}{\gamma\left[B\left(\rho_v + \rho_\eta + \rho_\varepsilon\right) - \sqrt{B^2\left(\rho_v + \rho_\eta + \rho_\varepsilon\right)^2 - 3\rho_\varepsilon}\right]}. \tag{A.3}$$

These two values correspond to a local maximum and a local minimum of $Q(L)$, respectively. That is, $Q(L)$ first increases with L on $(0, L_{small})$, then decreases with L on $\left(L_{small}, L_{large}\right)$, and finally increases with L on $\left(L_{large}, \infty\right)$. Only when $Q(L_{small}) > 0$ and $Q\left(L_{large}\right) < 0$ are there three equilibrium levels L^* that satisfy $Q(L^*) = 0$. If one of $Q(L_{small})$ and $Q\left(L_{large}\right)$ is zero, then there are two equilibria L^*. Otherwise, there will be a unique equilibrium.

Next, we find expressions for the conditions $Q(L_{small}) > 0$ and $Q\left(L_{large}\right) < 0$. Plugging (A.2) and (A.3) into (A.1), after massive algebra, we can show

$$\begin{aligned} Q(L_{small}) > 0 \Longleftrightarrow\ & 2\left[3B^2\left(\rho_v + \rho_\eta + \rho_\varepsilon\right) - 1\right]\sqrt{B^2\left(\rho_v + \rho_\eta + \rho_\varepsilon\right)^2 - 3\rho_\varepsilon} \\ & < B\left[9\rho_\varepsilon + \left(\rho_v + \rho_\eta + \rho_\varepsilon\right) - 6B^2\left(\rho_v + \rho_\eta + \rho_\varepsilon\right)^2\right], \end{aligned} \tag{A.4}$$

$$\begin{aligned} Q\left(L_{large}\right) < 0 \Longleftrightarrow\ & 2\left[3B^2\left(\rho_v + \rho_\eta + \rho_\varepsilon\right) - 1\right]\sqrt{B^2\left(\rho_v + \rho_\eta + \rho_\varepsilon\right)^2 - 3\rho_\varepsilon} \\ & < B\left[6B^2\left(\rho_v + \rho_\eta + \rho_\varepsilon\right)^2 - 9\rho_\varepsilon - \left(\rho_v + \rho_\eta + \rho_\varepsilon\right)\right]. \end{aligned} \tag{A.5}$$

These two conditions are equivalent to the following two conditions:

$$2\left[3B^2\left(\rho_v+\rho_\eta+\rho_\varepsilon\right)-1\right]\sqrt{B^2\left(\rho_v+\rho_\eta+\rho_\varepsilon\right)^2-3\rho_\varepsilon}<0\Rightarrow$$
$$B^2\left(\rho_v+\rho_\eta+\rho_\varepsilon\right)<\frac{1}{3};$$

and

$$\left\{B\left[6B^2\left(\rho_v+\rho_\eta+\rho_\varepsilon\right)^2-9\rho_\varepsilon-\left(\rho_v+\rho_\eta+\rho_\varepsilon\right)\right]\right\}^2$$
$$<\left\{2\left[3B^2\left(\rho_v+\rho_\eta+\rho_\varepsilon\right)-1\right]\sqrt{B^2\left(\rho_v+\rho_\eta+\rho_\varepsilon\right)^2-3\rho_\varepsilon}\right\}^2\Rightarrow$$
$$4\left(\rho_v+\rho_\varepsilon+\rho_\eta\right)^3B^4-\left(\rho_v+\rho_\eta\right)\left(\rho_v+20\rho_\varepsilon+\rho_\eta\right)B^2+8\rho_\varepsilon^2B^2+4\rho_\varepsilon<0.$$

So, if

$$\left(\rho_v+\rho_\eta,B,\rho_\varepsilon\right)\in\mathcal{S}_3\equiv\left\{\begin{array}{c}\left(\rho_v+\rho_\eta,B,\rho_\varepsilon\right)\in\mathbb{R}^3_{++}:\\ \frac{3\rho_\varepsilon}{\left(\rho_v+\rho_\eta+\rho_\varepsilon\right)^2}<B^2<\frac{1}{3\left(\rho_v+\rho_\eta+\rho_\varepsilon\right)},\\ 4\left(\rho_v+\rho_\eta+\rho_\varepsilon\right)^3B^4-\left(\rho_v+\rho_\eta\right)\left(\rho_v+\rho_\eta+20\rho_\varepsilon\right)B^2\\ +8\rho_\varepsilon^2B^2+4\rho_\varepsilon<0\end{array}\right\},$$

there will be three equilibria.

By a similar argument, we can show that if

$$\left(\rho_v+\rho_\eta,B,\rho_\varepsilon\right)\in\mathcal{S}_2\equiv\left\{\begin{array}{c}\left(\rho_v+\rho_\eta,B,\rho_\varepsilon\right)\in\mathbb{R}^3_{++}:\\ \frac{3\rho_\varepsilon}{\left(\rho_v+\rho_\eta+\rho_\varepsilon\right)^2}<B^2<\frac{1}{3\left(\rho_v+\rho_\eta+\rho_\varepsilon\right)},\\ 4\left(\rho_v+\rho_\eta+\rho_\varepsilon\right)^3B^4-\left(\rho_v+\rho_\eta\right)\left(\rho_v+\rho_\eta+20\rho_\varepsilon\right)B^2\\ +8\rho_\varepsilon^2B^2+4\rho_\varepsilon=0\end{array}\right\},$$

there will be two equilibria. Note that this $\mathcal{S}_2$ set has a zero Lebesgue measure in the $\mathbb{R}^3_{++}$ space. If $\left(\rho_v+\rho_\eta,B,\rho_\varepsilon\right)\in\mathbb{R}^3_{++}\backslash\mathcal{S}_3\cup\mathcal{S}_2$, the equilibrium is unique. □

A.2. *Proof of Proposition 3*

We prove this proposition in four steps. First, we derive the conditions that characterize the overall equilibrium in terms of a single unknown ρ_ε^*. Second, we characterize an upper bound for the equilibrium value of ρ_ε^*. Third, we show the existence of an equilibrium. Lastly, we provide a sufficient condition to ensure the uniqueness of the equilibrium.

Conditions characterizing the overall equilibrium. The equilibrium values of $\left(\rho_\varepsilon^*,L^*\right)$ are jointly determined by (19) and (21). Now we rewrite this system of two equations with two unknowns into a system of one equation and one inequality in terms of one unknown ρ_ε^*.

Specifically, by equation (19), we have

$$\left(\frac{\rho_\varepsilon^*}{\gamma L^*}\right)^2=\frac{1}{2\gamma C'\left(\rho_\varepsilon^*\right)}-\left(\rho_v+\rho_\eta+\rho_\varepsilon^*\right). \tag{A.6}$$

Since the left-hand side (LHS) of equation (A.6) is positive, then we must have

$$\frac{1}{2\gamma C'\left(\rho_\varepsilon^*\right)}-\left(\rho_v+\rho_\eta+\rho_\varepsilon^*\right)>0. \tag{A.7}$$

From (A.6), we can also expresses L^* as the following function of ρ_ε^*:

$$L^* = \frac{\rho_\varepsilon^*}{\gamma\sqrt{\frac{1}{2\gamma C'(\rho_\varepsilon^*)} - \left(\rho_v + \rho_\eta + \rho_\varepsilon^*\right)}},$$

which is equation (25) in Proposition 3.

Plugging (25) into (21), we have

$$B = \frac{1 - 2\gamma C'\left(\rho_\varepsilon^*\right)\left(\rho_v + \rho_\eta\right)}{\sqrt{\frac{1}{2\gamma C'(\rho_\varepsilon^*)} - \left(\rho_v + \rho_\varepsilon^* + \rho_\eta\right)}}. \tag{A.8}$$

Given $B > 0$, equation (A.8) requires

$$1 - 2\gamma C'\left(\rho_\varepsilon^*\right)\left(\rho_v + \rho_\eta\right) > 0 \Longleftrightarrow \frac{1}{2\gamma C'\left(\rho_\varepsilon^*\right)} - \left(\rho_v + \rho_\eta\right) > 0. \tag{A.9}$$

Nonetheless, (A.7) implies (A.9), and thus condition (A.9) is redundant.

We can rewrite equation (A.8) as the following condition in terms of ρ_ε^*:

$$B^2\left[\frac{1}{2\gamma C'\left(\rho_\varepsilon^*\right)} - \left(\rho_v + \rho_\eta + \rho_\varepsilon^*\right)\right] = \left[1 - 2\gamma C'\left(\rho_\varepsilon^*\right)\left(\rho_v + \rho_\eta\right)\right]^2,$$

which is equation (23) in Proposition 3. So, the overall equilibrium is jointly determined by condition (A.7) and equation (23), in terms of ρ_ε^*.

The upper bound of ρ_ε^*. We now use condition (A.7) to characterize an upper bound for ρ_ε^*. Define

$$H\left(\rho_\varepsilon\right) \equiv \frac{1}{2\gamma C'\left(\rho_\varepsilon\right)} - \left(\rho_v + \rho_\eta + \rho_\varepsilon\right). \tag{A.10}$$

Condition (A.7) is equivalent to $H\left(\rho_\varepsilon^*\right) > 0$. Thus, in order to obtain an upper bound for ρ_ε^*, let us examine the property of $H\left(\rho_\varepsilon\right)$.

First, $H(0) = \frac{1}{2\gamma C'(0)} - \left(\rho_v + \rho_\eta + 0\right) = \infty$ by $C'(0) = 0$. Second, $\lim_{\rho_\varepsilon \to \infty} H\left(\rho_\varepsilon\right) = -\infty$, by the convexity of $C\left(\rho_\varepsilon\right)$. Last, because $C\left(\rho_\varepsilon\right)$ is convex, $H\left(\rho_\varepsilon\right)$ is decreasing in ρ_ε. So, $H\left(\rho_\varepsilon\right)$ monotonically decreases from ∞ to $-\infty$. This implies that there exists a unique $\bar{\rho}_\varepsilon$, such that $H\left(\bar{\rho}_\varepsilon\right) = 0$ (i.e., equation (24) in Proposition 3).

Given $H\left(\rho_\varepsilon\right)$ is decreasing in ρ_ε, we know that $H\left(\bar{\rho}_\varepsilon\right) = 0$ and $H\left(\rho_\varepsilon^*\right) > 0$ jointly imply $\rho_\varepsilon^* < \bar{\rho}_\varepsilon$. That is, $\rho_\varepsilon^* \in \left(0, \bar{\rho}_\varepsilon\right)$.

Existence of equilibrium. We now use equation (23) to show the existence of an equilibrium. When $\rho_\varepsilon = 0$, the LHS of equation (23) takes the value of $B^2 H(0) = \infty$, while the right-hand side (RHS) is equal to 1. When $\rho_\varepsilon = \bar{\rho}_\varepsilon$, the LHS of (23) is 0, while the RHS is $\left[1 - 2\gamma C'\left(\bar{\rho}_\varepsilon\right)\left(\rho_v + \rho_\eta\right)\right]^2 = \left(\frac{\bar{\rho}_\varepsilon}{\rho_v + \rho_\eta + \bar{\rho}_\varepsilon}\right)^2 > 0$, where the equality follows from the definition of $\bar{\rho}_\varepsilon$ in (24). Therefore, by the intermediate value theorem, there exists an equilibrium ρ_ε^* satisfying (23), and hence there is an overall equilibrium.

Uniqueness of equilibrium. Finally, let us prove that the condition $B > \sqrt{\frac{8}{27\left(\rho_v + \rho_\eta\right)}}$ implies uniqueness. We can rewrite (23) as follows:

$$\frac{B^2}{2\gamma C'\left(\rho_\varepsilon^*\right)} - \left[1 - 2\gamma C'\left(\rho_\varepsilon^*\right)\left(\rho_v + \rho_\eta\right)\right]^2 = B^2\left(\rho_v + \rho_\eta + \rho_\varepsilon^*\right). \tag{A.11}$$

Note that the RHS of (A.11) is increasing in ρ_ε^*, and thus, as long as we can ensure that the LHS is decreasing on $[0, \bar{\rho}_\varepsilon]$, we will have a unique equilibrium.

Let us define the LHS of (A.11) as follows:

$$T(x) \equiv \frac{B^2}{x} - \left[1 - x\left(\rho_v + \rho_\eta\right)\right]^2, \tag{A.12}$$

where

$$x \equiv 2\gamma C'\left(\rho_\varepsilon^*\right) \in \left[0, 2\gamma C'\left(\bar{\rho}_\varepsilon\right)\right] \subset \left[0, \frac{1}{\rho_v + \rho_\eta}\right]. \tag{A.13}$$

Next, we show that the condition of $B > \sqrt{\frac{8}{27(\rho_v+\rho_\eta)}}$ implies that $T(x)$ is decreasing in x on $\left[0, \frac{1}{\rho_v+\rho_\eta}\right]$, which implies that the LHS of (A.11) is decreasing on $[0, \bar{\rho}_\varepsilon]$.

Taking the derivative of $T'(x)$ yields:

$$T'(x) = -\frac{B^2}{x^2} + 2\left[1 - x\left(\rho_v + \rho_\eta\right)\right]\left(\rho_v + \rho_\eta\right). \tag{A.14}$$

Thus,

$$T'(x) < 0 \Longleftrightarrow B^2 > 2x^2\left[1 - x\left(\rho_v + \rho_\eta\right)\right]\left(\rho_v + \rho_\eta\right).$$

It is easy to check that the maximum of $2x^2\left[1 - x\left(\rho_v + \rho_\eta\right)\right]\left(\rho_v + \rho_\eta\right)$ is achieved at $x = \frac{2}{3(\rho_v+\rho_\eta)}$ and that the maximum value is $\frac{8}{27(\rho_v+\rho_\eta)}$. Thus, if $B^2 > \frac{8}{27(\rho_v+\rho_\eta)}$, we will have $T'(x) < 0$. □

A.3. Proof of Proposition 4

Part (a) Applying the implicit function theorem to equation (19), we have

$$\frac{\partial \rho_\varepsilon\left(L^*; \rho_\eta\right)}{\partial \rho_\eta} = -\frac{1}{2\gamma C''\left(\rho_\varepsilon^*\right)\left[\rho_v + \rho_\eta + \left(\frac{\rho_\varepsilon^*}{\gamma L^*}\right)^2 + \rho_\varepsilon^*\right]^2 + \frac{2\rho_\varepsilon^*}{(\gamma L^*)^2} + 1}, \tag{A.15}$$

$$\frac{\partial \rho_\varepsilon\left(L^*; \rho_\eta\right)}{\partial L} = \frac{\frac{2}{L^*}\left(\frac{\rho_\varepsilon^*}{\gamma L^*}\right)^2}{2\gamma C''\left(\rho_\varepsilon^*\right)\left[\rho_v + \rho_\eta + \left(\frac{\rho_\varepsilon^*}{\gamma L^*}\right)^2 + \rho_\varepsilon^*\right]^2 + \frac{2\rho_\varepsilon^*}{(\gamma L^*)^2} + 1}. \tag{A.16}$$

Similarly, applying the implicit function theorem to equation (21), we have

$$\frac{\partial L\left(\rho_\varepsilon^*; \rho_\eta\right)}{\partial \rho_\eta} = \frac{\frac{\gamma + \left(\frac{\rho_\varepsilon^*}{\gamma}\right)\frac{1}{L^2}}{\left(\rho_v+\rho_\eta+\left(\frac{\rho_\varepsilon^*}{\gamma}\right)^2\frac{1}{L^2}+\rho_\varepsilon^*\right)^2}L^*}{\frac{\left(\frac{\rho_\varepsilon^*}{\gamma}\right)(\rho_v+\rho_\eta)}{\left(\rho_v+\rho_\eta+\left(\frac{\rho_\varepsilon^*}{\gamma}\right)^2\frac{1}{L^{*2}}+\rho_\varepsilon\right)^2}\left(-\frac{2}{L^{*3}}\right)L^* + \frac{\gamma + \left(\frac{\rho_\varepsilon^*}{\gamma}\right)\frac{1}{L^{*2}}}{\rho_v+\rho_\eta+\left(\frac{\rho_\varepsilon^*}{\gamma}\right)^2\frac{1}{L^{*2}}+\rho_\varepsilon^*}}, \tag{A.17}$$

$$\frac{\partial L\left(\rho_\varepsilon^*;\rho_\eta\right)}{\partial\rho_\varepsilon}=\frac{\frac{\gamma\left(\rho_\varepsilon^{*2}+2L^{*2}\gamma^2\rho_\varepsilon^*+L^{*4}\gamma^4-L^2\gamma^{*2}\rho_\eta-L^{*2}\gamma^2\rho_v\right)}{\left(\rho_\varepsilon^{*2}+L^{*2}\gamma^2\rho_v+L^{*2}\gamma^2\rho_\varepsilon^*+L^{*2}\gamma^2\rho_\eta\right)^2}L^*}{\frac{\left(\frac{\rho_\varepsilon^*}{\gamma}\right)(\rho_v+\rho_\eta)}{\left(\rho_v+\rho_\eta+\left(\frac{\rho_\varepsilon^*}{\gamma}\right)^2\frac{1}{L^{*2}}+\rho_\varepsilon\right)^2}\left(-\frac{2}{L^{*3}}\right)L^*+\frac{\gamma+\left(\frac{\rho_\varepsilon^*}{\gamma}\right)\frac{1}{L^{*2}}}{\rho_v+\rho_\eta+\left(\frac{\rho_\varepsilon^*}{\gamma}\right)^2\frac{1}{L^{*2}}+\rho_\varepsilon^*}}. \tag{A.18}$$

Note that equation (A.17) is essentially the expression of $\frac{\partial L^*}{\partial\rho_\eta}$ in Proposition 2.

We use the above four equations to examine the expression of $\frac{d\rho_\varepsilon^*}{d\rho_\eta}$ in equation (26). Inserting equations (A.15) and (A.16) into the numerator of (26), we find that

$$\begin{aligned}\text{Numerator} \quad &> \quad 0 \Longleftrightarrow 1-\frac{2}{L^*}\left(\frac{\rho_\varepsilon^*}{\gamma L^*}\right)^2\frac{\partial L\left(\rho_\varepsilon^*;\rho_\eta\right)}{\partial\rho_\eta}<0\\ &\Longleftrightarrow 1-\rho_p^*\frac{2}{L^*}\frac{\partial L^*}{\partial\rho_\eta}=1+\frac{\partial\rho_p^*}{\partial\rho_\eta}<0 \text{ (by (17))}\\ &\Longleftrightarrow \frac{\partial}{\partial\rho_\eta}\frac{1}{Var(v|y,p)}<0 \text{ (by (18))}\\ &\overset{\text{on stable equilibria}}{\Longleftrightarrow} \rho_p^* \quad > \quad \rho_\varepsilon^* \Leftrightarrow L^*<\frac{\sqrt{\rho_\varepsilon^*}}{\gamma} \text{ (by Proposition OA1).}\end{aligned}$$

Finally, we show the denominator of (26) is always positive, which completes the proof of the proposition. By equations (A.16) and (A.18), we have

$$\begin{aligned}&\text{Denominator } >0 \Longleftrightarrow\\ &\left(2\gamma C''\left(\rho_\varepsilon^*\right)\left[\rho_v+\rho_\eta+\left(\frac{\rho_\varepsilon^*}{\gamma L^*}\right)^2+\rho_\varepsilon^*\right]^2+\frac{2\rho_\varepsilon^*}{(\gamma L^*)^2}+1\right)\left(-\frac{\partial W\left(L^*;\rho_\eta\right)}{\partial L}\right)\\ &>\frac{2}{L^*}\left(\frac{\rho_\varepsilon^*}{\gamma L^*}\right)^2\frac{\gamma\left(\rho_\varepsilon^{*2}+2L^{*2}\gamma^2\rho_\varepsilon^*+L^{*4}\gamma^4-L^2\gamma^{*2}\rho_\eta-L^{*2}\gamma^2\rho_v\right)}{\left(\rho_\varepsilon^{*2}+L^{*2}\gamma^2\rho_v+L^{*2}\gamma^2\rho_\varepsilon^*+L^{*2}\gamma^2\rho_\eta\right)^2}L^*,\end{aligned} \tag{A.19}$$

where $-\frac{\partial W(L^*;\rho_\eta)}{\partial L}=\frac{\left(\frac{\rho_\varepsilon^*}{\gamma}\right)(\rho_v+\rho_\eta)}{\left(\rho_v+\rho_\eta+\left(\frac{\rho_\varepsilon^*}{\gamma}\right)^2\frac{1}{L^{*2}}+\rho_\varepsilon\right)^2}\left(-\frac{2}{L^{*3}}\right)L^*+\frac{\gamma+\left(\frac{\rho_\varepsilon^*}{\gamma}\right)\frac{1}{L^{*2}}}{\rho_v+\rho_\eta+\left(\frac{\rho_\varepsilon^*}{\gamma}\right)^2\frac{1}{L^{*2}}+\rho_\varepsilon^*}$. Note that at a stable equilibrium, we have $-\frac{\partial W(L^*;\rho_\eta)}{\partial L}>0$, and thus the LHS of (A.19) is greater than $\left(\frac{2\rho_\varepsilon^*}{(\gamma L^*)^2}+1\right)\left(-\frac{\partial W(L^*;\rho_\eta)}{\partial L}\right)$. We then compare this term with the RHS of (A.19). Specifically, direct computation shows

$$\begin{aligned}&\left(\frac{2\rho_\varepsilon^*}{(\gamma L^*)^2}+1\right)\left(-\frac{\partial W\left(L^*;\rho_\eta\right)}{\partial L}\right)\\ &-\frac{2}{L^*}\left(\frac{\rho_\varepsilon^*}{\gamma L^*}\right)^2\frac{\gamma\left(\rho_\varepsilon^{*2}+2L^{*2}\gamma^2\rho_\varepsilon^*+L^{*4}\gamma^4-L^2\gamma^{*2}\rho_\eta-L^{*2}\gamma^2\rho_v\right)}{\left(\rho_\varepsilon^{*2}+L^{*2}\gamma^2\rho_v+L^{*2}\gamma^2\rho_\varepsilon^*+L^{*2}\gamma^2\rho_\eta\right)^2}L^*\\ &=\frac{\gamma\left(\rho_\varepsilon^*+L^{*2}\gamma^2\right)}{\rho_\varepsilon^{*2}+L^{*2}\gamma^2\rho_v+L^{*2}\gamma^2\rho_\varepsilon+L^{*2}\gamma^2\rho_\eta}>0,\end{aligned}$$

and thus condition (A.19) always holds.

Part (b) Taking the derivative of both sides of (A.11) with respect to ρ_η and then using the notation of $x \equiv 2\gamma C'(\rho_\varepsilon^*)$ and the expression of $T'(x)$ in (A.14), we can compute

$$T'(x) \times 2\gamma C''(\rho_\varepsilon^*) \times \frac{d\rho_\varepsilon^*}{d\rho_\eta} + 2x[1 - x(\rho_v + \rho_\eta)] = B^2\left(1 + \frac{d\rho_\varepsilon^*}{d\rho_\eta}\right),$$

which implies

$$\frac{d\rho_\varepsilon^*}{d\rho_\eta} = \frac{2x[1 - x(\rho_v + \rho_\eta)] - B^2}{B^2 - T'(x)2\gamma C''(\rho_\varepsilon^*)}. \tag{A.20}$$

Note that as the proof of Proposition 3 shows, when $B > \sqrt{\frac{8}{27(\rho_v+\rho_\eta)}}$—which is true when $B > \sqrt{\frac{1}{2(\rho_v+\rho_\eta)}}$—we have $T'(x) < 0$. Thus, the denominator of (A.20) is positive (since $C''(\rho_\varepsilon^*) > 0$ by the convexity of $C(\cdot)$). Therefore, when $B > \sqrt{\frac{1}{2(\rho_v+\rho_\eta)}}$,

$$\frac{d\rho_\varepsilon^*}{d\rho_\eta} < 0 \Longleftrightarrow 2x[1 - x(\rho_v + \rho_\eta)] - B^2 < 0. \tag{A.21}$$

Note that the quadratic function $q(x) \equiv 2x[1 - x(\rho_v + \rho_\eta)]$ has a maximum value of $\frac{1}{2(\rho_v+\rho_\eta)}$. Thus, under the condition of $B > \sqrt{\frac{1}{2(\rho_v+\rho_\eta)}}$, we have $\frac{d\rho_\varepsilon^*}{d\rho_\eta} < 0$. □

A.4. *Proof of Corollary 2*

When ρ_η is large, the condition of $B > \sqrt{\frac{1}{2(\rho_v+\rho_\eta)}}$ in Part (b) of Proposition 4 is satisfied. So, for large values of ρ_η, we must have $\frac{d\rho_\varepsilon^*}{d\rho_\eta} < 0$.

Now let us consider the limit as $\rho_\eta \to \infty$. Equation (24) in Proposition 3 implies that $\frac{1}{2\gamma C'(\bar{\rho}_\varepsilon)} \to \infty$, which in turn implies $\bar{\rho}_\varepsilon \to 0$. Since $\rho_\varepsilon^* \in (0, \bar{\rho}_\varepsilon)$, we have $\lim_{\rho_\eta\to\infty} \rho_\varepsilon^* = 0$. Given that for large values of ρ_η, $\frac{d\rho_\varepsilon^*}{d\rho_\eta} < 0$, we must have $\lim_{\rho_\eta\to\infty} \frac{d\rho_\varepsilon^*}{d\rho_\eta} = 0$, because otherwise, $\frac{d\rho_\varepsilon^*}{d\rho_\eta}$ will be negatively bounded above, which means $\lim_{\rho_\eta\to\infty} \rho_\varepsilon^* < 0$, a contradiction. □

A.5. *Proof of Proposition 5*

Equation (A.6) and the expression of ρ_p in (17) imply that

$$\rho_p^* = \left(\frac{\rho_\varepsilon^*}{\gamma}\right)^2 \frac{1}{L^{*2}} = \frac{1}{2\gamma C'\left(\rho_\varepsilon^*\right)} - \left(\rho_v + \rho_\eta + \rho_\varepsilon^*\right). \tag{A.22}$$

Direct computation shows

$$\frac{d\rho_p^*}{d\rho_\eta} = -\left(\frac{C''(\rho_\varepsilon^*)}{2\gamma C'^2(\rho_\varepsilon^*)} + 1\right)\frac{d\rho_\varepsilon^*}{d\rho_\eta} - 1. \tag{A.23}$$

Plugging (A.20) into (A.23), we know that $\frac{d\rho_p^*}{d\rho_\eta} < 0$ if and only if

$$-\left(\frac{2\gamma C''(\rho_\varepsilon^*)}{x^2} + 1\right)\frac{2x[1 - x(\rho_v + \rho_\eta)] - B^2}{B^2 - T'(x)2\gamma C''(\rho_\varepsilon^*)} - 1 < 0 \Longleftrightarrow$$
$$\left[2\gamma C''(\rho_\varepsilon^*) + x^2\right]\left(2x[1 - x(\rho_v + \rho_\eta)] - B^2\right) + x^2\left[B^2 - T'(x)2\gamma C''(\rho_\varepsilon^*)\right] > 0,$$

where we have used the fact that $B^2 - T'(x)2\gamma C''(\rho_\varepsilon^*) > 0$, which is implied by $T'(x) < 0$ (when $B > \sqrt{\frac{8}{27(\rho_\upsilon+\rho_\eta)}}$). Inserting the expression $T'(x)$ in (A.14) into the above condition shows

$$\frac{d\rho_p^*}{d\rho_\eta} < 0 \Longleftrightarrow x > -\frac{1}{x}\left(1 - x\left(\rho_\upsilon + \rho_\eta\right)\right)2\gamma C''(\rho_\varepsilon^*),$$

which is true, because $1 - x\left(\rho_\upsilon + \rho_\eta\right) > 0$ by (A.9) and $x \equiv 2\gamma C'(\rho_\varepsilon^*)$. □

A.6. *Proof of Corollary 3*

Given that $F_L\left(L; \rho_\eta\right) < 0$ for $L \in \left[0, \bar{L}\right]$, we know that, by equation (47) in Proposition 7, $CE_{h,0}\left(L; \rho_\eta\right)$ is downward sloping as a function of L on $\left[0, \bar{L}\right]$. Thus, L^* is unique.

As stated in the main text, by the implicit function theorem, we have $\frac{\partial L^*}{\partial \rho_\eta} = -\frac{\partial CE_{h,0}\left(L^*;\rho_\eta\right)/\partial \rho_\eta}{\partial CE_{h,0}\left(L^*;\rho_\eta\right)/\partial L} \propto -\frac{F_{\rho_\eta}\left(L^*;\rho_\eta\right)}{F_L\left(L^*;\rho_\eta\right)} \geq 0$. By equation (41), we have $\frac{\partial \rho_p^*}{\partial \rho_\eta} = \frac{\partial \rho_p^*}{\partial L^*}\frac{\partial L^*}{\partial \rho_\eta} \leq 0$. By (42), ρ_η both directly improves liquidity, but also indirectly does through its effect on L^*. That is,

$$\frac{\partial LIQ^*}{\partial \rho_\eta} = \frac{\partial LIQ^*}{\partial L^*}\frac{\partial L^*}{\partial \rho_\eta} + \frac{1}{\rho_{pv}}\left(L^*\frac{\rho_e}{\gamma_h} + \frac{\rho_e}{\gamma_s}\right) > 0,$$

because $\frac{\partial LIQ^*}{\partial L^*} \geq 0$ and $\frac{\partial L^*}{\partial \rho_\eta} \geq 0$. □

Appendix B. Supplementary material

Supplementary material related to this article can be found online at http://dx.doi.org/10.1016/j.jet.2016.02.012.

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