Information Disclosure and Regret Returns in a Social Enterprise

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We study the problem faced by a social enterprise that is a distributor of new life-improving technologies in a developing market. This distributor seeks to be profitable while maximizing the adoption of the products it sells and must induce a (small) retailer to carry and sell a new product that it wants to distribute. The retailer sells the product to consumers that are heterogeneous, have uncertain valuations, and are risk averse. The distributor considers a mix of two possible strategies: (i) improve the accuracy of the information provided to the consumers through marketing and education, and (ii) invest in a reverse logistics channel that allows for higher refunds for consumers' regret-returns. We propose a model that captures this set-up, incorporating consumer regret-returns, information control, and the present value of growing a base of satisfied customers. We confirm that, in equilibrium, these two strategies are substitutes. We also show that if the distributor highly values customer satisfaction compared to immediate profits, it will likely prefer to invest more in reverse logistics than in information to the consumers. This suggests that reverse logistics are a better strategic option to increase customer satisfaction, for a social enterprise selling a product with uncertain valuation to risk-averse consumers, through a for-profit distribution channel. We find this insight to be robust to different model specifications, even if these lead to qualitatively different retailer behavior.

Key words: Social entrepreneurship, developing countries, bottom of the pyramid, reverse logistics, game theory, supply chain management.
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1. Introduction

Since the early 2000s, there has been an increasing interest in new, innovative technologies for low-income users in emerging markets around the world. Newly developed products like affordable solar lanterns, non-electric water purifiers, and smoke-reducing cooking stoves have the potential to address the unmet needs of millions of people (IEA (2015) and Purvis (2015)). Strong interest in these technologies is demonstrated by the growth of academic programs in which students design products for international development, and global initiatives like the Global Alliance for Clean Cookstoves, which plans to distribute 100 million clean-burning stoves by 2020.

The idea of designing technologies for use at the Bottom of the Pyramid (BOP)[1] is not new. After World War II, economists began considering a new form of technology to create non-agricultural jobs in rural areas of newly independent countries, as discussed by Schumacher (1970). Today, the idea of "design for the other 90%" is a growing movement that has inspired life-improving technologies like non-electric baby infant warmers, bicycle-powered mobile phone chargers, and drip irrigation systems for small-plot farmers, see Smith (2007).

There are hundreds of life-improving technologies that meet the critical needs of populations around the world (for a sample see Essmart (2016)). However, there is a very distinctive problem with these life-improving technologies for development. No matter how well-designed or well-intentioned they may be, there is no guarantee that they will reach the people for whom they are made, as discussed in Polak (2008) and Polak (2010).

The failure in distribution of life-improving technologies is critical and is the main motivation for this paper. Failure in distribution occurs because of low consumer awareness, affordability, lack of availability, risk aversion, and lack of confidence in the performance of these products (for the case of solar lanterns, see Brine et al. (2015)). Although nonprofit organizations, government programs, large multinational companies, and small social enterprises[2] have attempted to move these technologies out of the lab and into the land, no strategy has been completely successful (Jue (2012)). In particular, nonprofit organizations that have traditionally distributed technologies on a project-by-project basis are limited in funding and scale. Additionally, inappropriate design and lack of long-term maintenance and proper incentives have led to failures, such as Play Pumps in Mozambique (Costello (2010)). Another well-utilized dissemination strategy consists of increasing consumer awareness and access through massive door-to-door campaigns that combine education and subsidized direct sales to consumers, see for example (Vidal (2013)). However, managing the operations of these massive campaigns is expensive and labor-intensive, and scaling these operations is, in many cases, difficult.

Motivated by this context and by our collaboration with Essmart, a distributor of life-improving technologies[3] in India, we introduce a model that incorporates some of these distribution challenges,

[1] The "Bottom of the Pyramid" (also known as the "Base of the Pyramid") has been relabeled and redefined by multiple authors. Despite the semantics, the shared concept of the global poor as a market for new technologies remains constant.

[2] For the purposes of this research, social enterprises are organizations that use market-based methods to address social issues (Miller et al. (2012)).

[3] Examples of life-improving technologies that Essmart sells are are clean cooking stoves, solar lamps, water purifiers, and motorcycle helmets.

and we study an alternative strategy to tackle this problem. This strategy, which was developed by Essmart, uses small local "mom-and-pop" retail shops as points of sale of new life-improving technologies, and also as warranty providers and collectors of returned products. Essmart perceives this process innovation as crucial for scaling the distribution and sales of life-improving products, since millions of consumers in the developing world already buy most of their household products in local shops from shopkeepers whom they know and trust. In particular, in India, 90% of the annual retail spending occurs through more than 14 million small local retailers, see Kohli and Bhagwati (2011).

Essmart's strategy has 3 main components: (i) distribute, (ii) demonstrate and (iii) guarantee. To *distribute* products, Essmart partners with local retail shops and offers expedited product delivery to these retailers. Specifically, they give them a catalogue with all the products and a few sample items. When the retailer has a sale opportunity, Essmart delivers the product within a few days. This "deliver-to-order" strategy effectively removes the inventory risk from the retailer. This is important since the cost of many of the products sold by Essmart is equivalent to 2-3 weeks of the average salary in rural India. To *demonstrate* products Essmart representatives educate consumers and the retailer at local shops. These demonstrations are significantly less labor-intensive than door-to-door campaigns, but have a more limited reach. To *guarantee* the quality of their products, Essmart offers consumers the return of products that they did not like, and also services faulty products under warranty.

To achieve all three components of its strategy, Essmart has invested in facilities and employee training that allows them to service manufacturer warranties, and to have a significantly higher salvage value for consumer returns, e.g. by using refurbished items in their demonstrations. This last aspect is particularly novel for life-improving technologies being sold to the BOP. *Given Essmart's distribution strategy, the main objective of this paper is to compare the effectiveness of providing better information to the consumer (demonstrate), versus investing in a reverse logistics channel that allows to offer competitive refunds for regret returns (guarantee).* This is important in order to implement Essmart's strategy efficiently. Moreover, the success of this strategy can have a significant impact on increasing sustainable and long-lasting adoptions of life-improving technologies in developing countries.

1.1. Main contributions

The main contributions of this paper are:

- **Developing a modeling framework for information disclosure and regret returns.** We introduce a modeling framework to study the problem faced by a social enterprise (Essmart) that distributes life-improving technologies in a developing market and seeks profitability and the adoption of the products it sells. This distributor must induce (small) retailers to carry and sell a new product that it wants to distribute. Retailers sell this product to consumers that are heterogeneous, have uncertain valuations, and are risk averse. The distributor considers a mix of two possible strategies: (i) improve information provided to the consumers, and (ii) invest in a reverse logistics channel that allows for higher refunds for consumers' regret-returns. Our modeling framework captures this set-up and incorporates consumer regret-returns, information control, and the present value of growing a base of satisfied customers. In particular, we study two models: (i) a two types model, where consumers' valuation is either "high or low" (see Section 3), and (ii) a model where consumer valuation is continuously distributed (see Section 4).

- **Establish that consumers' risk aversion can lead to market collapse.** The first common insight from both models is that, without intervention from the distributor, it may unprofitable for the retailer to carry a product with relatively low margin if consumers are very risk averse, *even if the retailer faces no inventory risk*. This insight is consistent with the observations made by Essmart in India. It suggests that additional market interventions that reduce the risk faced by the consumers may be required to support the "deliver-to-order" distribution strategy.
- **Identify reverse logistics as a main driver of consumer satisfaction.** Both models confirm that, for the distributor, (*i*) investing in improving the accuracy of the information available to the consumers, and (*ii*) investing in reverse logistics, are *strategic substitutes*, see Bulow et al. (1985). Thus, the marginal benefit of increasing the salvage value of a returned product decreases with an increase in the information accuracy given to consumers (and vice-versa). In addition, we find that a distributor that highly values customers' satisfaction compared to immediate profits (with the goal of expanding its customer base for example) is more likely to prefer to invest in reverse logistics rather than to invest in improving consumer information accuracy. This is the main insight in this paper, and we find it to be robust to the two different model specifications we consider.

1.2. Model Overview

Our model has three players: the distributor (Essmart), the retailer, and the consumers. We assume that the market has a constant number of consumers, each with an individual valuation for the new product. To simplify the notation, we normalize the total demand to 1, and we work on a per-unit accounting basis. All results carry to any fixed pool of consumers.

The set of possible valuations is $\mathcal{V} \subseteq \mathbb{R}$. A consumer's true valuation, V, is only revealed after she purchases the product and uses it extensively. The distribution of types f_V among the population is common knowledge. Prior to the purchasing, the information that a consumer has about the product is a random marketing signal S received from the distributor (the signal is representative of a marketing, or educational, campaign). For each individual consumer of type $v \in \mathcal{V}$, this signal is drawn from a distribution $f_S(s|v,\theta)$, defined by a parameter θ. In our strategic analysis, we will assume that the distributor can invest in marketing and consumer education, increasing the accuracy of the marketing signal by changing the value of θ. The joint distribution of signals and types is $f_{S,V}(s,v|\theta)$.

The retailer sells each product for a price p, and offers a refund r for consumers that decide to return the product after purchasing it. We assume that a consumer returns a product if, after purchasing and using it, she finds that her true valuation is less than the refund r. Additionally, we assume that all consumers have the same *risk averse* utility function U and, for a signal s, will buy the product if her expected utility, denoted by $U_0(p,r,s)$, is nonnegative, i.e. if

$$U_0(p,r,s) := \mathbb{E}_V[U(\max(V,r)-p)|S=s,\theta] \geq 0.$$

The assumption that consumers are risk averse is representative of the challenging environment where organizations working on distributing new life-improving technologies in the developing world operate. The relationship between risk aversion and poverty is discussed by Haushofer and Fehr (2014). Moreover, this assumption captures the insight that eliminating the retailer's inventory risk may not be sufficient to activate the market.

To simplify notation, and summarize the consumer behavior in our model, we define

$$\mathbb{P}(\mathrm{B}(p,r)|\theta) = \mathbb{P}(U_0(p,r,S) \geq 0|\theta), \qquad \text{(Probability of buying the product)}$$

$$\mathbb{P}(\mathrm{R}(p,r)|\theta) = \mathbb{P}(U_0(p,r,S) \geq 0, V < r|\theta), \qquad \text{(Probability of returning the product)}$$

$$\mathbb{P}(\mathrm{A}(p,r)|\theta) = \mathbb{P}(U_0(p,r,S) \geq 0, V \geq r|\theta), \qquad \text{(Probability of adopting the product)}$$

$$\mathbb{P}(\mathrm{S}(p,r)|\theta) = \mathbb{P}(U_0(p,r,S) \geq 0, V \geq p|\theta). \qquad \text{(Probability of a satisfied customer)}$$

Note that a customer is considered satisfied if she adopted the product, and has no regret about the purchasing decision, equivalently she derives a non-negative consumer surplus.

The retailer buys the product from the distributor for a price c, and salvages returned items through the distributor for a value u. We assume that all sales are deliver-to-order. Whenever a sale occurs, the item is delivered to the retailer by the distributor, and then the consumer picks the product up. This assumption is representative of the operations of Essmart in India, which motivated this paper.

Hence, the retailer's decision variables are p and r, and its profit function, $\Pi_R(r,p)$, is

$$\Pi_R(r,p) = (p-c)\cdot\mathbb{P}(\mathrm{B}(p,r)|\theta) - (r-u)\cdot\mathbb{P}(\mathrm{R}(p,r)|\theta). \tag{1}$$

We assume that the retailer carries the product only if he derives a profit larger than some baseline profit $\pi_r \geq 0$. The profit π_r can be interpreted as the retailer's opportunity cost.

The distributor purchases items from the OEM at a unit price w, and can salvage returned products for y. Furthermore, the distributor decides the price it will charge the retailer c, as well as the amount it will pay the retailer for each return u.

We assume that the retailer lacks the adequate facilities to refurbish returned items and, in general, can extract very little value from returned products on its own. This is consistent with the realities of poorer regions in developing countries. In our strategic analysis we assume that the distributor, on the other hand, can invest in processing capabilities that allows it to extract value from consumer returns, increasing the salvage value y. For example, the distributor could refurbish returned products and use them in demonstrations.

Since we assume a fixed demand pool, the distributor always has enough inventory to satisfy the demand. Finally, we assume that the distributor values satisfied customers. This is in part due to the distributor's mission as a social enterprise. More generally, we model the fact that many companies value their market share, even when improving it may potentially reduce its short term profits. The distributor's profit function is

$$\Pi_D(c,u) = (c-w)\cdot \mathbb{P}(\mathrm{B}(p,r)|\theta) - (u-y)\cdot \mathbb{P}(\mathrm{R}(p,r)|\theta) + \gamma \cdot \mathbb{P}(\mathrm{S}(p,r)|\theta), \tag{2}$$

where $\gamma \geq 0$ models the relative value, for the distributor, of satisfied customers with respect to short term profits.

We assume that the distributor is a Stackelberg leader. That is, given the strategic decisions of the distributor (θ and y), the dynamics of our modeling framework are as follows:

1. Given a unit salvage value y, the informational level θ, and the distribution of types $f_{S,V}(s,v|\theta)$, the distributor chooses c, u anticipating the reaction from the retailer and consumers;
2. The retailer chooses p and r after observing c, u, and $f_{S,V}(\cdot|\theta, S=s)$, and starts selling the product if $\Pi_R(r,p) \geq \pi_R$;
3. Each consumer observes p, r, and their individual signal s. Based on the distribution $f_{S,V}$, the consumer estimates her utility and purchases the product if $U_0(p,r,s) \geq 0$;
4. If a purchase occurs, the product is delivered to the retailer and is picked by the consumer at the store;
5. The consumer uses the product, learns her true valuation $v \in \mathcal{V}$, and returns the product if $v < r$, obtaining a refund r;
6. The retailer returns the products that were returned by consumers to the distributor for a refund u per unit. The distributor, in turn, salvages the return for a salvage value y per unit.

In our strategic analysis, we will study the sensitivity of the equilibrium with respect to changes in: (i) the informational level θ, that corresponds to campaigns that increase the accuracy of the signal received by the consumers; and (ii) the salvage value y, that corresponds to investments in reverse-logistics that increase the value extracted from returned products. Additionally, we will analyze the effect of the weight γ that the distributor gives to customer satisfaction versus its short term profit, and the role of the retailer in the distribution channel.

Importantly, we do not explicitly model the cost of marketing and of reverse-logistics. The reason for this is that these costs depend on the realities of each specific firm. For example, Essmart's marketing costs are a convex increasing function of its marketing effort, since many of their customers are located in isolated regions. On the other hand, firms that rely on mass communication strategies for their marketing campaigns could experience economies of scale on their marketing efforts. Salvage values also depend on contracts with suppliers, and ease of recycling and refurbishment, and are product specific. Instead of modeling these costs, we will analyze the sensitivity of the equilibrium behaviors of each player, the distributor's optimal objective value, and the retailer's profits, to changes in the salvage value y, and the signal distribution (defined by the parameter θ). With this sensitivity analysis, we can take into account the costs of marketing and reverse-logistics in order to find the optimal mix between both strategies.

We will analyze two specifications of this model. In the first version of the model we assume that the consumer valuations, and the distributor's marketing signal, are discrete and can be of two types each. This model allows us to write the distributor's objective value, and the retailer's profits, in closed form at equilibrium, and gives us simple expressions for the amount of sales and consumer returns. In the second version of the model, we assume that the consumer valuations, and the distributor's marketing signal, are continuous and normally distributed. This set-up is considerably more difficult to study analytically, therefore we explore it mostly through numerical simulations.

The rest of this paper is structured as follows. In Section 2 we present a brief literature review. Section 3 will be dedicated to analyzing the discrete version of this problem, while in Section 4 we analyze the case where both Essmart's signal and consumer valuations are continuous. Finally, in Section 5 we discuss our conclusions and future research directions.

2. Literature Review

This paper draws from the literature on social entrepreneurship, reverse logistics, and the intersection of marketing and operations management. We position our paper with respect to this literature below, and also present an overview of previous research on these topics.

Our model is applied to the context of social entrepreneurship, which is a form of entrepreneurship that combines social and economic value creation, differentiating itself not only from traditional entrepreneurship but also from charities and philanthropy (Miller et al. (2012)). Mair and Marti (2006) describe social entrepreneurship as "a process involving the innovative use and combination of resources to pursue opportunities to catalyze social change and/or address social needs." Other researchers highlight social entrepreneurs' extensive search of different types of funding sources (Austin et al. (2006)), the cross-sector partnerships of commercial businesses that foray into the realm of creating social value based on common interests with nonprofit organizations (Sagawa and Segal (2000)), and the identification of an opportunity at forging a new, stable equilibrium from a previously unjust equilibrium (Martin and Osberg (2007)).

Through our research, we can particularly address problems confronted by social enterprises because of the contexts in which they are implemented. Social entrepreneurship emerges in contexts where goods and services are not being adequately provided by public agencies or private markets (Dees (1994)), where market and government failures are perceived (McMullen (2011)), and where there are institutional voids (Austin et al. (2006)). These markets have been described as "challenging" (Mair and Marti (2006)), and consumers have been described as "underserved, neglected, or highly disadvantaged" (Martin and Osberg (2007)).

Historically, traditional entrepreneurs have underestimated the financial returns of serving low-income markets. They lacked awareness of consumer wants and needs, as well as the obstacles that prevent their fulfillment (Webb et al. (2010)). Very often, entrepreneurs have assumed that poor consumers would not or could not repay loans (Yunus (2007)) and would not pay for brand name consumer products that are popular wealthier areas (Prahalad (2006)). However, relatively simple innovations like extending credit to groups as microfinance and selling single-serving-sized goods, have proven that there is indeed a market in low-income markets. The model that we present in this research is also an innovative strategy that further enables social enterprises to reach their markets with new, durable, life-improving products.

Operations management problems in developing countries that target BOP suppliers and consumers has been an area of increasing interest in the supply chain management community. Sodhi and Tang (2016) present an overview of this literature, together with business cases and a discussion of research opportunities. Also, Sodhi and Tang (2017) challenge the supply chain community to develop models and frameworks for supply chains that seek to have social impact and be financially sustainable. The model presented in this paper is one answer to this challenge, since we consider a distributor that values not only profits, but also the dissemination of life-improving products, leading to new insights.

One of the main features of our model, and of Essmart's operations strategy, is the possibility for consumers to return products. Thus, we build upon the literature on operations of reverse logistics systems that support warranties and customer returns. An overview of this literature is presented in Guide and Van Wassenhove (2009). In particular, Su (2009) proposes a model where consumers face valuation uncertainty and realize their valuation only after purchase. Su (2009) proposes variations on well known supply contracts (e.g. buy-back contracts) such that they coordinate the supply chain even when taking into account consumer returns. In contrast, we incorporate information control to the consumer, while we ignore aggregate demand uncertainty and the related inventory management component of the problem. More recently, managing reverse logistics systems when prices and demand is uncertain was investigated in Calmon and Graves (2016), while Pinçe et al. (2016) study the relationship between pricing and re-manufacturing decisions made by a company attempting to maximize the value it extracts from returned products.

The option value of product returns to consumers has been investigated empirically in Anderson et al. (2009). The authors use a data-set from an online fashion retail company and estimate that having the possibility to return the purchased product increases purchase rates by more than 50%. The impact of re-manufacturing and refurbished products on firm profitability and market share is analyzed in Atasu et al. (2008). In this paper, the authors consider re-manufacturing in a competitive environment, and show that re-manufacturing and selling refurbished products can be an effective marketing strategy, allowing firms to protect their market share. An analysis of different after-sales service contracts is presented in Bakshi et al. (2015), where the authors compare performance-based contracting and resource-based contracting, and investigate how they affect a vendor's inventory investments.

Shulman et al. (2009) consider an analytical model with risk neutral consumers and two horizontally differentiated products. They identify conditions under which it is optimal to provide product fit information to consumers. However, they consider a binary decision of providing either full information, or no information, to the consumers. They do identify information provision and reverse logistics as strategic substitutes, as we do. In contrast, we consider a richer information control model, and additionally identify reverse logistics as a main driver of consumer satisfaction. Shulman et al. (2011) extend these insights into a competitive environment, by studying a duopoly. They find that, surprisingly, restocking fees not only can be sustained in a competitive environment, but they can be higher than the ones that a monopolist would charge.

The structure of the return channel is considered in Shulman et al. (2010). There, the authors compare supply chain performance when the retailer accepts and salvages customer returns, with the case when the manufacturer receives and process returns. They show that it might be more effective for the manufacturer to salvage returns even if it is more efficient for the retailer to do so. This is because if a retailer handles returns, it might penalize consumers that are returning products more heavily than the profit maximizing penalty that the manufacturer would have chosen.

On a different topic, Taylor and Xiao (2016) compare the options of distributing socially-desirable products through non-commercial and commercial channels, where the latter include a for-profit intermediary, in a model that incorporates consumer awareness. The authors study the optimal subsidy by an international donor, and show that the subsidy level can be higher or lower in the commercial channel, depending on the level of consumer awareness. Additionally, the donor can be hurt by increased awareness in the presence of a for-profit intermediary. Our model is different in several aspects, including the absence of subsidies, and the presence of consumer returns. However, in our continuous model in Section 4, we also find that the social enterprise can be hurt by increasing accuracy of the information provided to the consumers, due to the presence of a for-profit retailer in the supply chain.

Finally, the information control framework we consider, particularly in our continuous model specification in Section 4, was introduced in Johnson and Myatt (2006). Recently, Chu and Zhang (2011) used this framework to analyze the effect of information release in the prices of pre-orders, before a product is officially launched, in the context of hi-tech products. They find that a small change in the amount of information provided to the consumers can have a dramatic effect in the optimal price of pre-orders.

3. Discrete Model

In this section, we will assume that consumers can have either a high or low valuation for the product. Thus, $\mathcal{V} = \{v_h, v_l\}$, where $v_h > v_l$. Furthermore, we assume that $\mathbb{P}(V = v_l) = \beta$.

As a result of the distributor's marketing and educational efforts, the consumers receive either a high or a low signal, $s \in \{s_h, s_l\}$, of the value of the product. We assume that θ is the *accuracy* of the signal and that this accuracy is symmetric, i.e. the same for both signal types. Specifically, we assume $\mathbb{P}(s_i|v_i) = \theta$ for $i \in \{l, h\}$, and $\theta \in [.5, 1]$. The case $\theta = 0.5$ corresponds to an uninformative signal, while $\theta = 1$ is a perfectly informative signal. We have

$$\mathbb{P}(s_h, v_h|\theta) = \theta(1-\beta),\ \mathbb{P}(s_l, v_l|\theta) = \theta\beta,\ \mathbb{P}(s_h, v_l|\theta) = (1-\theta)\beta, \text{and } \mathbb{P}(s_l, v_h|\theta) = (1-\theta)(1-\beta),$$

Our goal in this section is to characterize how the equilibrium behaviour of the retailer and the distributor, as well as their profits and the proportion of satisfied customers, change with different values of the information accuracy to the consumers θ, and the salvage value y.

All the proofs of this section are provided in the Appendix 6.1.

3.1. Consumer behavior

Once a consumer receives a signal, she will estimate her valuation. If a consumer receives a low (respectively high) signal, the inferred probability of having a low (respetively high) valuation is

$$\mathbb{P}(v_l|s_l, \theta) = \frac{\theta\beta}{\theta\beta + (1-\theta)(1-\beta)}, \quad \mathbb{P}(v_h|s_h, \theta) = \frac{\theta(1-\beta)}{\theta(1-\beta) + (1-\theta)\beta}.$$

Consumers are risk averse and have a CARA utility function with constant absolute risk parameter $\alpha > 0$. They anticipate that they will return the product if, after the product is purchased and used, they learn that their valuation is lower than the refund r. Thus, given a signal s, their expected utility $U_0(p,r,s)$ is

$$U_0(p,r,s) = E[U(\max(V,r)-p)|s,\theta] = 1 - \mathbb{P}(v_l|s,\theta)e^{-\alpha(\max(v_l,r)-p)} - \mathbb{P}(v_h|s,\theta)e^{-\alpha(\max(v_h,r)-p)}. \quad (3)$$

Hence, a consumer that received a signal $s \in \{s_h, s_l\}$ will purchase if and only if $U_0(p,r,s) \geq 0$.

For some refund r, and information accuracy θ, let $p_h(r,\theta)$ and $p_l(r,\theta)$ be the highest price that consumers that received signal s_h and s_l, respectively, are willing to pay for the product. Then,

$$\begin{aligned} p_h(r,\theta) &= -\frac{1}{\alpha}\ln\left(\mathbb{P}(v_l|s_h,\theta)e^{-\alpha\max(v_l,r)} + \mathbb{P}(v_h|s_h,\theta)e^{-\alpha\max(v_h,r)}\right), \\ p_l(r,\theta) &= -\frac{1}{\alpha}\ln\left(\mathbb{P}(v_l|s_l,\theta)e^{-\alpha\max(v_l,r)} + \mathbb{P}(v_h|s_l,\theta)e^{-\alpha\max(v_h,r)}\right). \end{aligned} \quad (4)$$

Clearly, $p_h(r,\theta) \geq p_l(r,\theta)$, for any refund r, and information accuracy θ.

3.2. Retailer's behavior

The retailer will choose to target either only consumers that received a high signal s_h, by charging the price $p_h(r,\theta)$, or the whole market, by charging $p_l(r,\theta)$. Thus, the retailer's optimal profit if carrying the product can be written as

$$\Pi_R^* = \max_{r,p\in\{p_h(r,\theta),p_l(r,\theta)\}} (p-c)\cdot\mathbb{P}(\mathrm{B}(p,r)|\theta) - (r-u)\cdot\mathbb{P}(\mathrm{R}(p,r)|\theta).$$

Note that the probabilities also depend on the distribution of types defined by β.

The retailer will carry the product if his profit is higher than some outside option π_R, i.e., if $\Pi_R^* \geq \pi_R$. To ensure that the setup is non-trivial, we make the following assumption.

Assumption 1. *Assume* $v_l - w < \pi_R < (1-\beta)(v_h - w)$.

Note that if the consumers are fully informed about their valuations, then the highest price that the retailer can charge and still target the whole market is v_l. It follows that the first inequality in Assumption 1 states that the wholesale price w is large enough, such that it is not profitable for the retailer to carry the product and serve the whole population of consumers if they are fully informed. Similarly, the second inequality in Assumption 1 states that, if the consumers are fully informed, then it is profitable for the retailer to target high valuation consumers. This assumption generates tension in the model. Specifically, if consumers are very risk averse, and are aware that the signal they receive is imperfect, then consumers that received a high signal s_h will require a price very close to v_l in order to buy the product, since they will fear that v_l is their real valuation.

The retailer's optimal response, for any given distributor's decisions, is characterized next.

Theorem 1. *Given* $\theta \in \left(\frac{1}{2}, 1\right]$, *if the retailer chooses to carry the distributor's product, his optimal profit is*

$$\Pi_R^* = \max\left\{\Pi_R^a, \Pi_R^b, \Pi_R^c, \Pi_R^d\right\}. \quad (5)$$

Where each component in the max *corresponds to a different non-dominated strategy by the retailer. Strategies* $(a),(b),(c)$, *and* (d) *are:*

Strategy	p	r	$\mathbb{P}(\mathrm{B}(p,r)\vert\theta)$	$\mathbb{P}(\mathrm{A}(p,r)\vert\theta)$	$\mathbb{P}(\mathrm{R}(p,r)\vert\theta)$	$\mathbb{P}(\mathrm{S}(p,r)\vert\theta)$
(a)	$\frac{-1}{\alpha}\ln\Big(\mathbb{P}(v_l\vert s_l,\theta)e^{-\alpha v_l} + \mathbb{P}(v_h\vert s_l,\theta)e^{-\alpha v_h}\Big)$	0	1	1	0	$1-\beta$
(b)	v_h	v_h	1	$1-\beta$	β	$1-\beta$
(c)	$\frac{-1}{\alpha}\ln\Big(\mathbb{P}(v_l\vert s_h,\theta)e^{-\alpha v_l} + \mathbb{P}(v_h\vert s_h,\theta)e^{-\alpha v_h}\Big)$	0	$\mathbb{P}(s_h\vert\theta)$	$\mathbb{P}(s_h\vert\theta)$	0	$\mathbb{P}(s_h,v_h\vert\theta)$
(d)	v_h	v_h^-	$\mathbb{P}(s_h\vert\theta)$	$\mathbb{P}(s_h,v_h\vert\theta)$	$\mathbb{P}(s_h,v_l\vert\theta)$	$\mathbb{P}(s_h,v_h\vert\theta)$

Table 1 Summary of retailer's strategies

(a) *Target full market adoptions, with no refunds. In this case,* $p^a = p_l(0,\theta)$*,* $r^a = 0$*, and*

$$\Pi_R^a = p_l(0,\theta) - c = \frac{-1}{\alpha}\ln(\mathbb{P}(v_l|s_l,\theta)e^{-\alpha v_l} + \mathbb{P}(v_h|s_l,\theta)e^{-\alpha v_h}) - c.$$

(b) *Target product adoptions by consumers with high valuation* v_h*, with full refunds. In this case,* $p^b = r^b = v_h$*, and*

$$\Pi_R^b = (1-\beta)(v_h - c) + \beta(u - c).$$

(c) *Target product adoptions by consumers that received a high signal* s_h*, with no refunds. In this case,* $p^c = p_h(0,\theta)$*,* $r^c = 0$*, and*

$$\Pi_R^c = \mathbb{P}(s_h|\theta)(p_h(0,\theta) - c) = \mathbb{P}(s_h|\theta)\left(\frac{-1}{\alpha}\ln(\mathbb{P}(v_l|s_h,\theta)e^{-\alpha v_l} + \mathbb{P}(v_h|s_h,\theta)e^{-\alpha v_h}) - c\right).$$

(d) *Target adoptions from consumers that received a high signal* s_h*,* and *have a high valuation* v_h*, with (essentially) full refunds. In this case,* $p^d = p_h(v_h^-,\theta) \approx v_h$*,* $r^d = v_h^-$*, and*

$$\Pi_R^d \approx \mathbb{P}(s_h|\theta)(\mathbb{P}(v_h|s_h,\theta)(v_h - c) + \mathbb{P}(v_l|s_h,\theta)(u - c)),$$

where $r^d = v_h^-$ *denotes* $r^d < v_h$*, and* r^d *arbitrarily close to* v_h*. Note that* $r^d = v_h^- < p_h(v_h^-,\theta) = p^d < v_h$*, hence this strategy implements essentially full refunds. Specifically, the approximations can be made arbitrarily accurate.*

On the other hand, if $\theta = \frac{1}{2}$*, i.e. the signal to the consumers is uninformative, then only strategies* (a) *and* (b) *are non-dominated.*

Table 1 provides a summary of the retailer's strategies and the consumer behavior they induce.

Table 1 displays the retailer's strategies in Theorem 1 in decreasing probability of satisfied customers, $\mathbb{P}(\mathrm{S}(p,r)|\theta)$. Moreover, Theorem 1 captures the *qualitative behavior* of the retailer in our discrete model. Specifically, Theorem 1 shows that the optimal refund strategy for the retailer is always one of two possible options: either (i) full refunds ($r^* = p^*$), or (ii) no refunds ($r^* = 0$). There are also two possible market targets: (i) inducing the whole market to buy the product

($\mathbb{P}(\mathrm{B}(p,r)|\theta)=1$), or (ii) inducing only the consumers that received a high signal to buy the product ($\mathbb{P}(\mathrm{B}(p,r)|\theta)=\mathbb{P}(s_h|\theta)$). The combination of the refund decision, and market target, leads to the four possible retailer strategies summarized in Table 1. We will see later, in Section 4, that this retailer's qualitative behavior is the result of the model specification in this section, as opposed to being a feature of the modeling framework described in Section 1.2.

In order to understand the approximations made in strategy (d) in Theorem 1, note that if the retailer sets the price $p=p_h(r,\theta)$, then only the consumers that receive a high signal s_h buy the product. From them, the consumers that realize they have a low valuation v_l return it. In the proof of Theorem 1 we show that the retailer's profit is increasing in $r\in[v_l,v_h)$ in this case. However, the consumers' behavior is *non-continuous* at $r=v_h$. Namely, if $r=v_h$ then there is no downside for consumers buying the product, and it is no longer possible to price out the consumers that received a low signal s_l. This results in the solution described in strategy (d) in Theorem 1.

3.2.1. Market collapse. Many social enterprises, particularly those in developing countries, operate in an environment that is characterized by low levels of recognition for life-improving products and brands, as well as by consumers that are highly risk-averse, and unwilling to spend a significant fraction of their income on new technologies. In this environment, it might be unprofitable for the retailer to carry a new life-improving technology in the absence of some proactive intervention by the distributor, such as setting-up a reverse logistics channel and investing in marketing and consumer education. This scenario is captured by our model, as it is described next.

Define the threshold level $\underline{\alpha}$ for the consumers' absolute risk aversion parameter α as follows.

Definition 1. *Let $\underline{\alpha}$ be such that $\frac{-1}{\underline{\alpha}}\ln(\beta e^{-\underline{\alpha}v_l}+(1-\beta)e^{-\underline{\alpha}v_h})-w=\pi_r$ if $(1-\beta)v_h+\beta v_l-w>\pi_r$, and $\underline{\alpha}=0$ otherwise.*

If consumers are highly risk averse, then the strategy of targeting full market adoptions (strategy (a) in Theorem 1) becomes unprofitable for the retailer, regardless of the accuracy of the information provided to the consumer. This is summarized in the following corollary.

Corollary 1. *For any accuracy level of the information provided to the consumers $\theta\in\left[\frac{1}{2},1\right]$, if the consumers are highly risk averse ($\alpha>\underline{\alpha}$), then it is unprofitable for the retailer to carry the distributor's product and target full market adoptions. Namely, it is unprofitable for the retailer to implement strategy (a) in Theorem 1.*

In order to focus our analysis in the challenging environment in which our partner social enterprise operates in, we will rule out strategy (a) in Theorem 1 by assuming that the market parameters satisfy the assumptions in Corollary 1. Thus, for any accuracy level θ, it will be unprofitable for the retailer to charge the price $p_l(0,\theta)$ and target the whole market.

Assumption 2. *Assume that consumers are highly risk averse, specifically assume $\alpha>\underline{\alpha}$.*

There are situations where unless the distributor increases the information disclosure to the consumers, or give larger refunds to the retailer, or both, the *market collapses* and it is always unprofitable for the retailer to carry the product, i.e., there are no sales. This scenario is described in the next corollary

Corollary 2. *Consider a market with the following challenging environment.*

- *Consumers are highly risk averse ($\alpha > \underline{\alpha}$).*
- *Consumers have no information about their type ($\theta = \frac{1}{2}$).*
- *Either the population of high valuation consumers, or the high valuation itself, is not large enough. Specifically $(1-\beta)v_h < \pi_r + w$.*

Then, if the distributor provides no additional information (i.e. maintains $\theta = \frac{1}{2}$), and it does not provide a salvage value for returns to the retailer ($u = 0$), then it is unprofitable for the retailer to carry the product, even if he faces no inventory risk.

3.3. Distributor's behavior

We assume that the conditions defined in Assumption 2 hold, such that it is always unprofitable for the retailer to implement strategy (a) in Theorem 1 (see Corollary 1). Then, the distributor will choose c, and u, that induces the retailer to either pursue strategy (b), (c) or (d), depending on which attains the largest objective value for the distributor.

The distributor's possible optimal strategies are given in the Theorem below. Depending on the information accuracy to the consumers θ, and the value of carrying an alternative product for the retailer π_R, some strategies might not be available to the distributor. In order to simplify the statement of the Theorem, we define the following functions. Let,

$$f_I(\theta,\pi_R) := \min\left\{\frac{p_h(0,\theta)}{\beta} - v_h\frac{1-\beta}{\beta} - \pi_r\frac{\mathbb{P}(s_l|\theta)}{\mathbb{P}(s_h|\theta)\beta}, \frac{p_h(0,\theta)}{\mathbb{P}(v_l|s_h,\theta)} - v_h\frac{\mathbb{P}(v_h|s_h,\theta)}{\mathbb{P}(v_l|s_h,\theta)}\right\}, \tag{6}$$

$$f_M(\theta,\pi_R) := v_h - \frac{\pi_r}{\mathbb{P}(s_h|\theta)(\mathbb{P}(v_h|s_h,\theta) - \mathbb{P}(v_h|s_l,\theta))} - \max\left\{\frac{p_h(0,\theta)}{\mathbb{P}(v_l|s_h,\theta)} - v_h\frac{\mathbb{P}(v_h|s_h,\theta)}{\mathbb{P}(v_l|s_h,\theta)}, 0\right\}. \tag{7}$$

Theorem 2. *The optimal profit for the distributor is*

$$\Pi_D^* = \max\left\{\Pi_D^{Log}, \Pi_D^{Info}\mathbb{1}_{\{f_I(\theta,\pi_R)\geq 0\}}, \Pi_D^{Mix}\mathbb{1}_{\{f_M(\theta,\pi_R)\geq 0\}}\right\}. \tag{8}$$

Where each component in the max *corresponds to a different non-dominated strategy by the distributor. The strategies Log, Info, and Mix are:*

- *Pure logistics strategy (Log), with full refunds. In this case, $c^{Log} = u^{Log} = v_h - \frac{\pi_r}{1-\beta}$, and*

$$\Pi_D^{Log} = \mathbb{P}(v_h)v_h + \mathbb{P}(v_l)y - w + \gamma\mathbb{P}(v_h) - \pi_r.$$

- *Pure information strategy (Info), with no refunds. In this case, $c^{Info} = p_h(0,\theta) - \frac{\pi_r}{\mathbb{P}(s_h|\theta)}$, $u^{Info} = 0$, and*

$$\Pi_D^{Info} = \mathbb{P}(s_h|\theta)(p_h(0,\theta) - w) + \gamma\mathbb{P}(v_h, s_h|\theta) - \pi_r.$$

- *Mixed logistics and information strategy (Mix), with partial refunds. In this case,* $c^{Mix} = v_h - \frac{\pi_r}{\mathbb{P}(s_h|\theta)\mathbb{P}(v_h|s_h,\theta)}$, $u^{Mix} = v_h - \frac{\pi_r}{\mathbb{P}(s_h|\theta)(\mathbb{P}(v_h|s_h,\theta)-\mathbb{P}(v_h|s_l,\theta))}$, *and*

$$\Pi_D^{Mix} = \mathbb{P}(s_h|\theta)\big(\mathbb{P}(v_h|s_h,\theta)v_h + \mathbb{P}(v_l|s_h,\theta)y - w\big) + \gamma\mathbb{P}(v_h, s_h|\theta) - \pi_r.$$

The proof of Theorem 2 is long and it requires the analysis of many cases. For the sake of clarity, we split it into Propositions 3-5 in the Appendix.

Proposition 3 shows that it is *always feasible* (although not necessarily profitable) for the distributor to induce the retailer to implement strategy (b) in Theorem 1. Furthermore, the optimal policy for the distributor in this case is to offer full refunds to the retailer. This induces the retailer to give full refunds to the consumers, effectively eliminating the need for more accurate information. Since improving the information accuracy is likely to be costly, it is optimal for the distributor *not* to invest in improving the current information accuracy when following this strategy. Therefore, we refer to this strategy as a *pure logistics strategy.*

Proposition 4 shows that the distributor can induce the retailer to implement strategy (c) in Theorem 1 if and only if $f_I(\theta, \pi_R) \geq 0$, see (6). If it is feasible for the distributor to induce strategy (c), then the optimal policy to do so is to offer no refunds to the retailer. This induces the retailer to give no refunds to the consumers, effectively eliminating the need for a reverse logistics channel to process returns. Since investing in reverse logistics is likely to be costly, if the distributor decides to induce strategy (c), no investments in reverse logistics should be made. Therefore, we refer to this strategy as a *pure information strategy.*

Finally, Proposition 5 shows that the distributor can induce the retailer to implement strategy (d) in Theorem 1 if and only if $f_M(\theta, \pi_R) \geq 0$, see (7). If this strategy can be induced, then it is optimal for the distributor to do so by offering *partial refunds* to the retailer. This induces the retailer to target consumers that received a high signal s_h, and implement (essentially) full refunds (see strategy (d) in Theorem 1). As a result, both the information accuracy to the consumers, and the capability of offering a high refund, remain relevant to the customers' purchasing decision. Therefore, we refer to this strategy as a *mixed logistics and information strategy.*

3.4. Strategic analysis

In this section, we study the relative effectiveness of two strategies that the distributor can use to tackle the consumers' risk aversion. Namely, (i) investing in increasing the accuracy θ of the information that consumers receive, and (ii) investing in a reverse logistics channel that allows to extract a larger salvage value y from returned products. The latter strategy supports a larger refund to the retailer for returns u, which induces a larger refund to the consumer r, and ultimately reduces the downside in the consumers' purchasing decision.

We leverage the fact that, for any fixed pair (θ, y), we have characterized the optimal behavior in equilibrium of the consumers, the retailer, and the distributor. Our goal is to investigate how these behaviors, and in particular the distributor's optimal objective value, change for different values of θ and y. This will provide insight into the relative effectiveness of reverse logistics, and marketing campaigns, for a social enterprise that distributes a product with uncertain valuation to risk averse consumers through a for-profit distribution channel.

Additionally, in Sections 3.4.1 and 3.4.2 we will study the sensitivity of the equilibrium with respect to π_R, the opportunity cost for the retailer, and γ, the relative value for the distributor of satisfied customers with respect to short profits, respectively.

Our first insight is summarized in the following corollary.

Corollary 3. *Investing in increasing the accuracy of the information provided to the consumers θ, and investing in increasing the distributor's salvage value y, are strategic substitutes. More specifically,* $\frac{\partial \Pi_D^{Log}}{\partial \theta} = 0$, $\frac{\partial \Pi_D^{Info}}{\partial y} = 0$, *and* $\frac{\partial^2 \Pi_D^{Mix}}{\partial \theta \partial y} = -\beta$.

Corollary 3 shows that the marginal benefit of one strategy in equilibrium decreases with an increase in the level of the alternative strategy. The managerial insight provided by Corollary 3 is that the distributor should emphasize either investing in improving the information accuracy to the consumers, or investing in reverse logistics to obtain a higher salvage value. A natural question that follows is: *Are there general conditions that favor one option over the other?*

In order to answer this question, we first characterize for which levels of accuracy of the information provided to the consumers θ, and salvage value y, the distributor will prefer to implement the pure logistics strategy *Log*, the pure information strategy *Info*, or the mixed logistics and information strategy *Mix*. In other words, we characterize the distributor's strategy that attains the maximum in equation (8), for any pair (θ, y). The results are summarized in Proposition 1. In order to simplify its statement, we define the following functions. Let,

$$g_{LM}(\theta,\gamma) := w - \frac{\mathbb{P}(v_h|s_l,\theta)}{\mathbb{P}(v_l|s_l,\theta)}(v_h + \gamma - w), \tag{9}$$

$$g_{MI}(\theta) := p_h(0,\theta) - \frac{\mathbb{P}(v_h|s_h,\theta)}{\mathbb{P}(v_l|s_h,\theta)}(v_h - p_h(0,\theta)), \tag{10}$$

$$g_{LI}(\theta,\gamma) := \theta g_{LM}(\theta,\gamma) + (1-\theta) g_{MI}(\theta). \tag{11}$$

Proposition 1. *Let* $\Theta := \left[\frac{1}{2}, 1\right]$, $Y := [0, w]$. *Additionally, let* $LOG(\gamma,\pi_R)$, $INFO(\gamma,\pi_R)$, *and* $MIX(\gamma,\pi_R)$, *be the partition of* $\Theta \times Y$ *given by*

$$LOG(\gamma,\pi_R) := \left\{(\theta,y) \in \Theta \times Y : y \geq \max\{g_{LI}(\theta,\gamma)\mathbb{1}_{\{f_I(\theta,\pi_R)\geq 0\}}, g_{LM}(\theta,\gamma)\mathbb{1}_{\{f_M(\theta,\pi_R)\geq 0\}}\}\right\}, \tag{12}$$

$$INFO(\gamma,\pi_R) := \left\{(\theta,y) \in \Theta \times Y : f_I(\theta,\pi_R) \geq 0, y \leq \min\{g_{LI}(\theta,\gamma), g_{MI}(\theta)\mathbb{1}_{\{f_M(\theta,\pi_R)\geq 0\}} + w\mathbb{1}_{\{f_M(\theta,\pi_R)<0\}}\}\right\}, \tag{13}$$

$$MIX(\gamma,\pi_R) := \left\{(\theta,y) \in \Theta \times Y : f_M(\theta,\pi_R) \geq 0, g_{LM}(\theta,\gamma) \geq y \geq g_{MI}(\theta)\mathbb{1}_{\{f_I(\theta,\pi_R)\geq 0\}}\right\}. \tag{14}$$

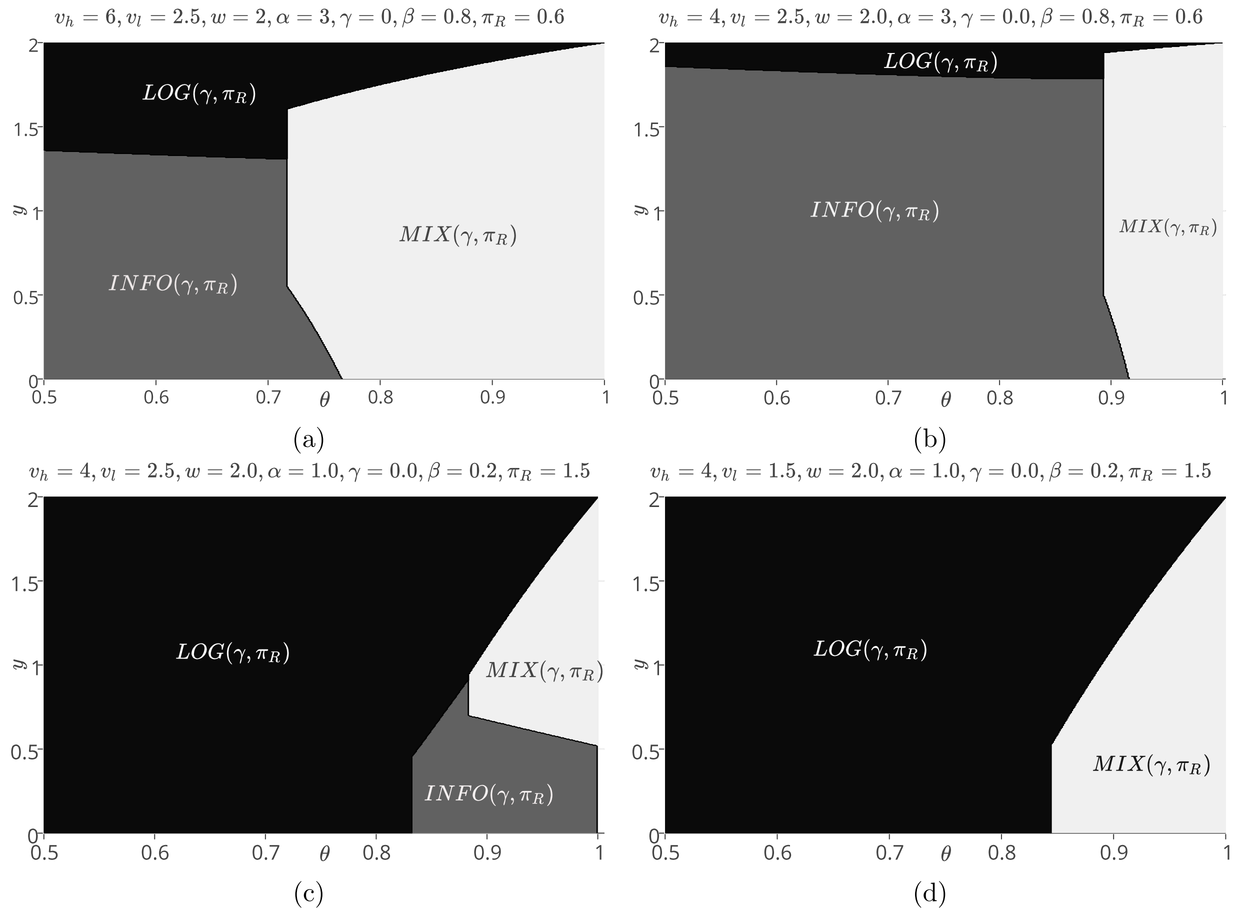
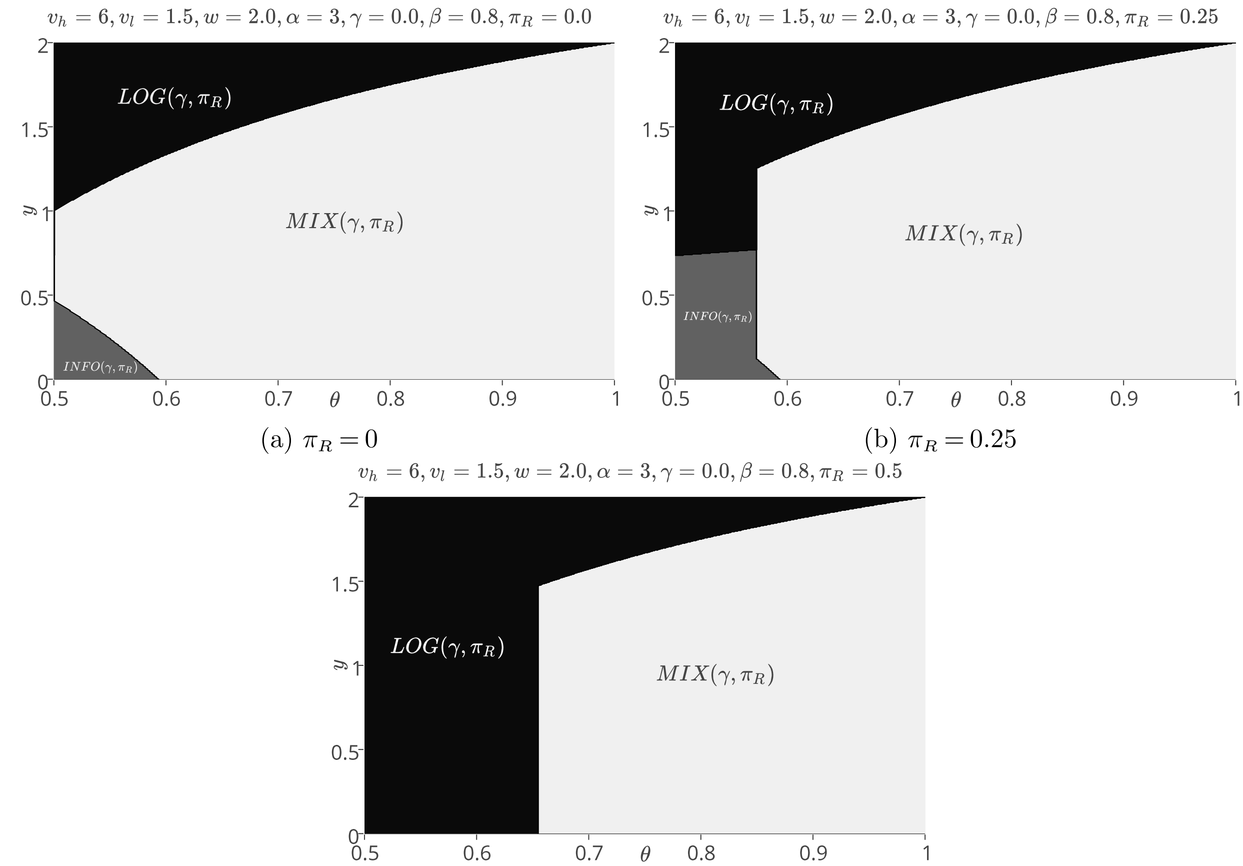


Figure 1 **Examples of optimal decision diagrams for the distributor.**

Then, if $(\theta, y) \in LOG(\gamma, \pi_R)$, the dominant strategy for the distributor is the pure logistics strategy Log; if $(\theta, y) \in INFO(\gamma, \pi_R)$, the dominant strategy for the distributor is the pure information strategy Info; and if $(\theta, y) \in MIX(\gamma, \pi_R)$, the dominant strategy for the distributor is the mixed strategy Mix. Moreover, the sets $LOG(\gamma, \pi_R)$ and $MIX(\gamma, \pi_R)$ are guaranteed to be non-empty.

The definition of the set $LOG(\gamma, \pi_R)$ in Equation (12) imposes a lower bound on the distributor's salvage value y, for any information accuracy θ, in order for strategy *Log* to be selected. As illustrated in Figures 1a and 1b, this lower bound may be non-monotonic with respect to θ. Similarly, the definition of the set $INFO(\gamma, \pi_R)$ in Equation (13) imposes an upper bound on the distributor's salvage value y, in order for strategy *Info* to be selected. However, it additionally requires that the distributor can actually induce the retailer to select strategy *Info*, which is only guaranteed if $f_I(\theta, \pi_R) \geq 0$. Finally, the definition of the set $MIX(\gamma, \pi_R)$ in Equation (14) states that the salvage value y must have an intermediate value, and the relevant incentive compatibility constraints for the retailer must be satisfied, in order for strategy *Mix* to be selected.



(a) $\pi_R = 0$

(b) $\pi_R = 0.25$

(c) $\pi_R = 0.5$

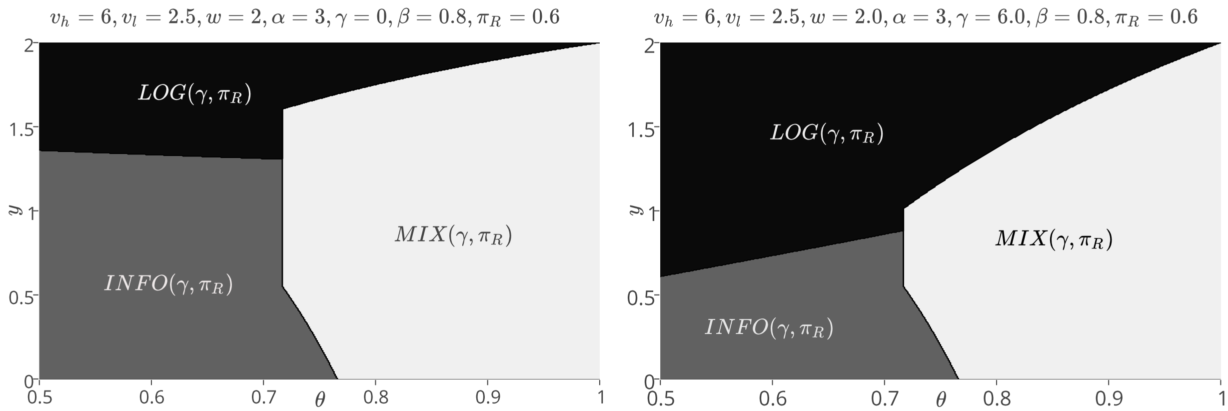
Figure 2 **Impact of π_R on optimal decision diagram.**

Different examples of the distributor's optimal decision diagram, defined by the sets $LOG(\gamma, \pi_R)$, $INFO(\gamma, \pi_R)$, and $MIX(\gamma, \pi_R)$, are provided in Figure 1. In particular, they confirm that the set $INFO(\gamma, \pi_R)$ may be empty.

3.4.1. The effect of a larger value of the retailer's outside option π_R.

Corollary 4. *Let π_R^1, π_R^2, be such that $\pi_R^1 > \pi_R^2 \geq 0$. Then $LOG(\gamma, \pi_R^2) \subseteq LOG(\gamma, \pi_R^1)$, for any $\gamma \geq 0$. Namely, a more profitable retailer's outside option makes the distributor more likely to select the pure logistics strategy Log.*

Corollary 4 suggests that, when dealing with a retailer that has a high opportunity cost, improving reverse logistics is more likely to be a better strategic option to induce a him to carry a product with valuation uncertainty for sale to risk averse consumers, compared to improving the information accuracy to the consumers. This is illustrated in Figure 2, where the distributor's decision diagram is depicted for various values of the outside option π_R. In particular, as π_R increases, the area in the distributor's decision diagram for which the pure logistics strategy *Log* is optimal also increases. Interestingly, the effect over the set $INFO(\gamma, \pi_R)$, where the pure information strategy



(a) $\gamma = 0$ (b) $\gamma = 6$

Figure 3 Impact of γ on the distributor's optimal decision diagram.

Info is optimal, is non-monotonic in π_R. Namely, the region $INFO(\gamma, \pi_R)$ in Figure 2 first increases with π_R (from 2a to 2b), and then it decreases to the point where it vanishes (from 2b to 2c).

This result could be valuable for social enterprises that are disseminating new clean technologies that seek to replace polluting products (for example, solar lamps that aim to replace kerosene lamps). If these social enterprises seek to distribute their products through small retail shops that are accustomed to distributing a traditional alternative product that is relatively profitable, investing in a reverse logistics system, and offering refunds through the retailer, can be an effective strategy for the distributor to increase product adoption.

3.4.2. The effect of increasing the value of consumer satisfaction γ.

Corollary 5. *Let γ_1, γ_2, be such that $\gamma_1 > \gamma_2 \geq 0$. Then $LOG(\gamma_2, \pi_R) \subset LOG(\gamma_1, \pi_R)$, for any $\pi_R \geq 0$. Namely, a larger weight on consumer satisfaction makes the distributor more likely to select the pure logistics strategy Log.*

Corollary 5 states the main result in this paper, for the discrete model we study in this section. It shows that, everything else being the same, a distributor that puts a larger weight on consumer satisfaction (compared to short term profits), is more likely to choose the pure logistics strategy *Log*. This result is demonstrated in Figure 3 where, as the value of γ increases, the set of pairs (θ, y) for which the distributor selects the pure logistics strategy becomes larger. This result follows from the observation that the pure logistics strategy attains the upper bound on satisfied customers $\mathbb{P}(v_h) = (1-\beta)$, while the information based strategies attain an intermediate level of satisfied customers $\mathbb{P}(v_h, s_h|\theta) = (1-\beta)\theta$, for any information accuracy less than perfect information. Interestingly, increasing the information accuracy θ only increases the probability of satisfied customers. In Section 4 we will see that this is a characteristic of the model specification we study in this section, as opposed to being a feature of the modeling framework described in Section 1.2.

In summary, the equilibrium behavior of the distributor, retailer, and consumers in this model suggests that, *when dealing with risk averse consumers and valuation uncertainty, improving reverse logistics is a better strategic option to increase the fraction of satisfied customers, than improving the accuracy of the information provided through extensive marketing campaigns.* A natural question that arises is: how robust is this result to different specifications of the modeling framework described in Section 1.2? More precisely, is this insight driven by the retailer's optimal behavior in this specification of the model, where only full refunds, or no refunds, can be optimal? We explore this question by analyzing a different model specification in Section 4.

4. Continuous Model

The discrete model from Section 3 provides closed-form results regarding optimal prices, refunds, and information disclosure by the distributor. However, since we only considered two consumer types, the retailer's behavior is non-smooth and the model does not allow for a detailed sensitivity analysis on how changes in various parameters affect profits and satisfied customers. We address this issue in this section by assuming that the consumer valuation types, and the signal sent by the distributor, are continuous. Although the model we obtain is analytically intractable, we perform a comparative statics analysis through numerical simulations, and investigate the sensitivity of the equilibrium to changes in the information disclosed to consumers θ, and on the salvage value y.

In this section, we assume that $\mathcal{V}=\mathbb{R}$, and that consumer valuations are normally distributed, i.e., $V \sim N(\mu, \sigma^2)$. As a result of the distributor's marketing and educational efforts, the consumers receive a signal S, of the form $S = V + \eta$, where $\eta \sim N(0, \sigma_\eta^2)$ corresponds to a random noise, and it is independent of V. Hence, each customer receives a noisy signal that is correlated with her valuation for the product. The distributor can improve the quality of the signals by decreasing σ_η^2. In particular, when $\sigma_\eta = 0$ the consumers receive perfect information, i.e. the signal is her exact valuation. Conversely, when $\sigma_\eta \to \infty$, consumers have no information about their valuation.

Our analysis in this section has a similar outline to the discrete model in Section 3. All proofs are provided in Appendix 6.2.

4.1. Consumer behavior

Consumers form an estimate of their valuation based on the signal they receive. Let V_0 be the valuation estimate of a consumer that received signal S, i.e. $V_0 = E[V|S]$. Then,

$$V_0 = E[V|S] = \mu + \rho \frac{\sigma}{\sqrt{\sigma^2 + \sigma_\eta^2}}(S - \mu) = \rho^2 S + (1 - \rho^2)\mu,$$

where $\rho = \frac{\sigma}{\sqrt{\sigma^2+\sigma_\eta^2}} \in [0,1]$, is the correlation coefficient of V and S, see for example Bertsekas and Tsitsiklis (2002).

In order to simplify the notation and analysis, let $\theta := \rho^2 \in [0,1]$. It follows that the consumers' valuation estimate V_0 is also normally distributed, and has the same mean and a fraction of the variance of the consumers' valuations. Specifically $V_0 \sim N(\mu, \theta\sigma^2)$. Moreover, let ϵ be the consumer's valuation estimation error, then $\epsilon = V - V_0 \sim N(0, (1-\theta)\sigma^2)$, see Bertsekas and Tsitsiklis (2002). Namely, we can write the consumer valuations V as $V = V_0 + \epsilon$.

As before, $\theta \in [0,1]$ represents the *accuracy* of the signal sent to the customers (although with a different definition from the discrete model in the previous section). In particular, when $\theta = 0$ (equivalently $\sigma_\eta \to \infty$), the consumers have no information about their valuation before buying the product (they only know the average valuation μ), while when $\theta = 1$ (equivalently $\sigma_\eta = 0$), the consumers have perfect information and their valuation is equal to their estimate, i.e. $V = V_0$.

We assume, once again, that the consumers are risk averse and have a CARA utility function. However, we leverage our simplified notation and express the consumer's utility as a function of V_0 instead of S. Recall that consumers anticipate that they will return the product if, after the product is purchased, they find out that their true valuation is less than the refund r. Then, for a valuation estimate $V_0 = v_0$, we have that the consumer expected utility is

$$U_0(p,r,v_0) = \mathbb{E}_V[U(\max(V,r)-p)|V_0 = v_0, \theta] = 1 - \mathbb{E}_\epsilon[e^{-\alpha(\max(v_0+\epsilon,r)-p)}|\theta]. \tag{15}$$

A consumer purchases the product if $U_0(p,r,v_0) \geq 0$, and we say the consumer is *satisfied* if she purchases the product and her valuation v is such that $v \geq p$, i.e. if she has no regret of having purchased the product (derives positive surplus).

4.2. Retailer's behavior

There is a one to one mapping between the price p and v_0, where v_0 is the lowest valuation estimate of a customer still willing to buy the product, i.e.,

$$U_0(p,r,v_0) \geq 0 \iff p \leq -\frac{1}{\alpha}\ln\left(\mathbb{E}_\epsilon[e^{-\alpha\max(v_0+\epsilon,r)}|\theta]\right).$$

Therefore, instead of choosing the price p, we can assume that the retailer directly targets the fraction of consumers that buy the product $\mathbb{P}(V_0 \geq v_0|\theta)$. We use this observation to define the retailer's pricing function

$$p(r,v_0) = -\frac{1}{\alpha}\ln\left(\mathbb{E}_\epsilon[e^{-\alpha\max(v_0+\epsilon,r)}|\theta]\right). \tag{16}$$

This is the highest price that the retailer can charge when targeting consumers with a valuation estimate of at least v_0. It follows that we can write the retailer's profit function as

$$\Pi_R(r,v_0) = (p(r,v_0)-c)\mathbb{P}(V_0 \geq v_0|\theta) - (r-u)\mathbb{P}(V_0 \geq v_0, V \leq r|\theta). \tag{17}$$

This model is challenging to study analytically. Specifically, although the retailer's objective function is quasi-concave, there are no closed form expressions for the retailer's optimal response. The following proposition states the quasi-concavity of $\Pi_R(r,v_0)$ with respect to the refund r.

Proposition 2. *The retailer's profit $\Pi_R(r, v_0)$ is quasi-concave in the refund r, for any given estimated valuation v_0. Moreover, for any v_0, let r^* be the unique maximizer of $\Pi_R(r, v_0)$. Then, $0 \le u < r^* < p(r^*, v_0)$. Namely, partial refunds are optimal for the retailer.*

Proposition 2 shows that the retailer's behavior is *qualitatively different* in this continuous model, compared to the discrete model in Section 3. Specifically, Theorem 1 shows that in the discrete model it is always optimal for the retailer to implement either full refunds or no refunds. In contrast, Proposition 2 shows that partial refunds are optimal for the retailer in the continuous model. In particular, implementing full refunds, or no refunds, is *never optimal.* Moreover, the retailer's optimal response changes *smoothly* with the product cost c, and the salvage value u, offered by the distributor. This is a fundamental difference between the two model specifications we consider in this paper.

From a technical perspective, Proposition 2 reduces the retailer's problem to a one variable optimization problem in terms of the valuation estimation v_0, since the optimal refund r^* is a function of v_0. This problem can be solved numerically for all practical purposes. For the special case with no information, i.e. $\theta = 0$, we have that $V_0 = \mu$ with probability one. Therefore, Proposition 2 fully characterizes the retailer's optimal response in this special case.

In the numerical simulations in Section 4.4 we will assume for simplicity that the retailer does not have a profitable alternative product to carry, i.e. $\pi_R = 0$. This does not affect our main insights. This follows because, in contrast to the discrete model in Section 3, the individual rationality constraint for the retailer can be loose in the equilibrium. Namely, the retailer can always derive positive profits even if $\pi_R = 0$. This is the case since we assume that $\mathcal{V} = \mathbb{R}$. Specifically, for any fixed distributor's price c and refund u, the retailer can always charge a price $p > c$, set $r = u$, and obtain a positive probability of a consumer buying the product, leading to a positive profit.

4.2.1. Market collapse. Similarly to Corollary 2, the next corollary shows that there are non-trivial setups where the distributor needs to be proactive in either increasing the information disclosure to the consumers, or giving larger refunds to the retailer, or both, in order to incentivize the retailer to carry its product.

Corollary 6. *Consider a market where consumers have no information about their type ($\theta = 0$), i.e. $V_0 = \mu$ with probability one. Let $z = \frac{\mu - c}{\sigma}$ denote the normalized margin of the product. Then, for any customers' absolute risk aversion parameter $\alpha > 0$, and refund to the retailer u, there exists a unique value $\bar{z}\left(\alpha, u, \frac{\mu}{\sigma}, \pi_r\right)$ such that $\max_r \Pi(r, \mu) < \pi_R$ if and only if $z < \bar{z}\left(\alpha, u, \frac{\mu}{\sigma}, \pi_r\right)$.*

Namely, if the distributor provides no additional information ($\theta = 0$), the product has a small enough normalized margin and the consumers are risk averse enough, ($z < \bar{z}\left(\alpha, u, \frac{\mu}{\sigma}, \pi_r\right)$), it is unprofitable for the retailer to carry the product.

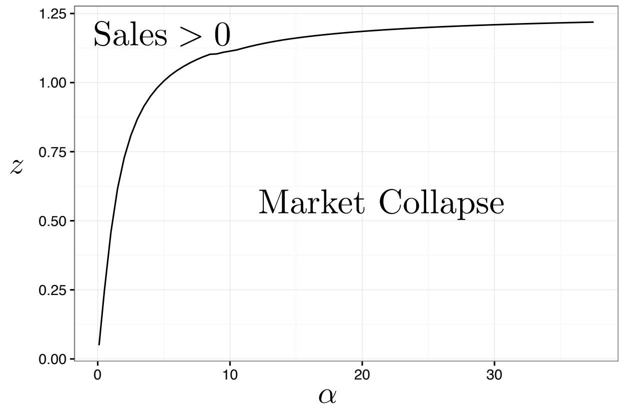


Figure 4 **Minimum normalized margin for the retailer to carry the product. The line represents the threshold** $\bar{z}(\alpha, 0, 3, 0)$**.**

Corollary 6 shows that for any consumers' risk aversion parameter α, and refund to the retailer u, if the standardized margin of the product z is smaller than a threshold $\bar{z}\left(\alpha, u, \frac{\mu}{\sigma}, \pi_r\right)$, then the retailer has no incentive to carry the product.

Figure 4 depicts the threshold $\bar{z}(\alpha, 0, 3, 0)$. In the figure, we assume that the distributor provides no information to the consumers ($\theta = 0$), and also does not provide a refund to the retailer ($u = 0$). Additionally, we assume that the probability of consumers deriving a negative value from the object is negligible, namely $\mu = 3\sigma$. A smaller value for the ratio $\frac{\mu}{\sigma}$ implies that some consumers may get a negative valuation, and it has the effect of shifting the threshold from Figure 4 upwards, making the risk of market collapse larger. Finally, we assume that the retailer does not have a profitable alternative product to carry ($\pi_R = 0$). This makes Figure 4 consistent with the numerical simulations in Section 4.4. A larger value for π_R has the effect of shifting the threshold from Figure 4 upwards, once again making the risk of market collapse larger.

In summary, Figure 4 shows that if the product has a small enough normalized margin, and the consumers are risk averse enough, then it is unprofitable for the retailer to carry the product, unless the distributor gives a refund ($u > 0$), or more accurate information is provided to the consumers ($\theta > 0$). To make sense of the scale for the normalized margin z in Figure 4, note that $z \in [-1, 1.5]$ has been considered as a reasonable interval in the context of pre-orders of hi-tech products in developed countries, see Chu and Zhang (2011). Life-improving technologies in the developing world are likely to have a normalized margin well below 1.25, while their markets are characterized by highly risk averse consumers, which is consistent with the need for the intervention of a proactive distributor in our model. To conclude, let us point out that an analogous result can be shown for $\theta > 0$.

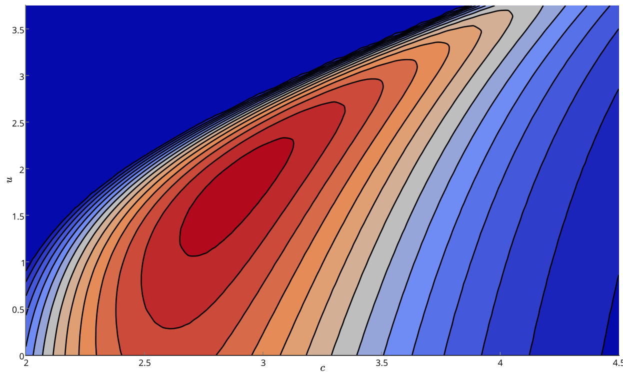


Figure 5 **Depiction of the objective function of the distributor as a function of c and u. We assume $\theta = 0.5$, $\alpha = 3.0$, $\mu = 3.0$, $\sigma = 1.5$, $w = 3.0$, $y = 1.0$, and $\gamma = 0$**

4.3. Distributor's problem

We assume that the distributor acts as a Stackelberg leader, anticipating the retailer's response, and the consumers' purchasing behavior. As in Section 3.3, the distributor can choose the price c, and the refund u, offered to the retailer. The distributor's objective function can be written as

$$\Pi_D(c,u) = (c-w)\mathbb{P}(V_0 \geq v_0|\theta) - (u-y)\mathbb{P}(V_0 \geq v_0, V < r|\theta) + \gamma\mathbb{P}(V_0 \geq v_0, V > p(v_0,r)|\theta). \tag{18}$$

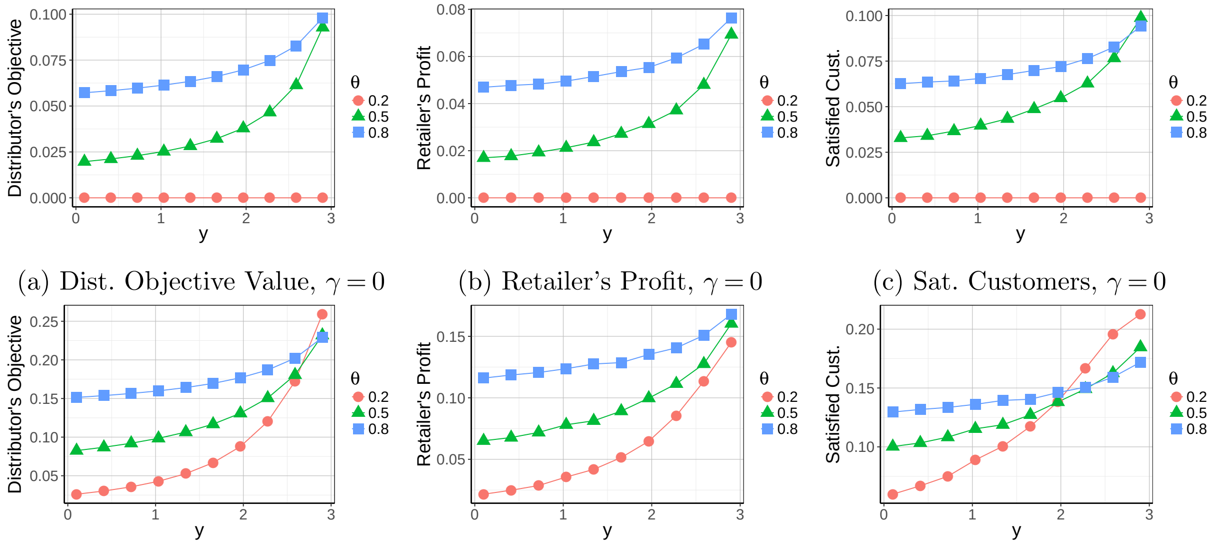
Where $\mathbb{P}(V_0 \geq v_0|\theta)$ is the probability that a consumer chosen at random will buy the product. Similarly, $\mathbb{P}(V_0 \geq v_0, V < r|\theta)$ is the probability of a consumer returning the product. Finally, the last term in the distributor's objective corresponds to the value associated with satisfied customers, i.e., customers that purchase the product and extract positive surplus from it. If $\gamma = 0$, the distributor's objective is the same as its profits.

Once again, note that v_0 and r implicitly depend on c and u, since the retailer responds to the decisions made by the distributor. This makes the analytical characterization of the optimal distributor's behavior challenging in the continuous model. For this reason, in this section we resort to numerical simulations in order to study the distributor's response. All the simulations were carried out using the Julia programming language, see Bezanson et al. (2012).

Although the distributor's objective function is analytically intractable, it is numerically stable. In particular, Figure 5 depicts the distributor's objective for $\theta = 0.5$, $\alpha = 3.0$, $\mu = 3.0$, $\sigma = 1.5$, $w = 2.0$, $y = 1.0$, and $\gamma = 0$. The darkest (blue in the colored figure) area corresponds to values of u and c where the profit of the distributor is negative. The qualitative dependence of the distributor's objective value with respect to c and u, depicted in Figure 5, is representative of the results obtained in extensive simulations.

4.4. Strategic analysis

In the same spirit as in Section 3.4, we study the relative effectiveness of (i) investing in increasing the accuracy of the information provided to the consumers θ, and (ii) investing in a reverse logistics



(a) Dist. Objective Value, $\gamma = 0$ (b) Retailer's Profit, $\gamma = 0$ (c) Sat. Customers, $\gamma = 0$

(d) Dist. Objective Value, $\gamma = 1$ (e) Retailer's Profit, $\gamma = 1$ (f) Sat. Customers, $\gamma = 1$

Figure 6 Simulation results for a low margin product, where $w = 3$, $\mu = 3$, $\sigma = 1.5$, and $\alpha = 3$. Plots of the distributor's objective value, retailer's profit, and fraction of satisfied customers as a function of the salvage value y, and the accuracy of the information to the consumers θ. Includes the case when the distributor does not value satisfied customers (top row, $\gamma = 0$), and when it does (bottom row, $\gamma = 1$).

channel that allows to extract a larger salvage value y. Specifically, we study how the distributor's objective value, the retailer's profit, and the fraction of satisfied customers, change for different values of θ and y, in equilibrium. In Section 4.4.1, we additionally study the effect of changing γ, the relative value for the distributor of satisfied customers with respect to short term profits.

Since the retailer's behavior changes smoothly with the price c and refund u, the difference between a strategy that emphasizes logistics, and one that emphasizes information may not be as sharp when compared to the discrete model from Section 3. However, we will see that the continuous model in this section has an additional fundamental difference with respect to the discrete model, which allows for the interpretation of a change in the distributor's strategy.

Specifically, in the discrete model, improving the accuracy of the information provided to the consumers always increases the fraction of satisfied customers, as well as the retailer's profit and the distributor's objective value. In contrast, in the continuous model, increasing the information accuracy θ may actually *decrease* the fraction of satisfied customers, and even the distributor's objective value, for a product with a high enough salvage value y. In this case, as the salvage value increases, the distributor's strategy switches, from favoring an improvement in the accuracy of the information provided to the consumers, to preferring to keep the status quo in the information accuracy level.

We calibrated our simulations using data from a water purifier sold by Essmart. This product's wholesale price is ₹1100 (around US$ 15) and is a low margin product for both the distributor and the retailer. It is also a product with a high adoption potential, since a significant fraction of households targeted by Essmart don't have access to non-murky bore well water, municipal water lines, or pre-filtered water cans. However, given the wholesale cost of the water filter, Essmart estimates that less than 30% of their consumers would be willing to purchase this product. Essmart also perceives consumers to be risk averse towards purchasing the water purifier, since it requires changing their water consumption behavior. Both returns and warranties are available for this product.

Figure 6 depicts a simulation based on this setup. Specifically, we assume $w = \mu = 3$, i.e. the normalized margin of the product is $z \leq 0$, and only a small fraction of the consumers can potentially benefit from adopting the product. The figure depicts the distributor's objective value, the retailer's profit, and the fraction of satisfied customers, as a function of the salvage value y, for different accuracy levels of the information provided to the consumers θ. The first row in Figure 6 considers the case where $\gamma = 0$, i.e. the distributor only values its short terms profits, while the second row depicts the case where $\gamma = 1$, i.e. the distributor values an increase in the fraction of satisfied customers as much as an increase in its short term profit.

First, from the first row in Figure 6, note that when $\theta = 0.2$ the market collapses since consumers are risk averse and will not purchase the product, even if there is a high refund available. Intuitively, for a small information accuracy θ, the estimated valuation for all the consumers is similar and close to μ. If $w = \mu$, then there is no room for the distributor to incentivize the retailer to offer a competitive price to the consumers. When there is more information available (e.g. $\theta = 0.5$, $\theta = 0.8$), the market is active. This suggests that, for products with a small normalized margin, at least *some level of information is actually required* in order for the distributor to push the product. This is consistent with the observations made by Essmart in their operations in India.

All the effects we discuss next are even stronger for products with a moderate and large normalized margin, see the Appendix 6.2 for a similar discussion for the case of a product with a moderate normalized margin.

In particular, from Figures 6a and 6d, note that the distributor's objective value is increasing with a higher salvage value y, for all the accuracy levels of the information provided to the consumers θ. However, the *marginal* effect of increasing y (respectively, θ), on the distributor's objective value, is decreasing with a larger θ (respectively, y). Visually, the curves associated with lower information accuracy (e.g. $\theta = 0.2$ or $\theta = 0.5$) are catching up, and even surpassing, the curves associated with high information accuracy (e.g. $\theta = 0.8$). Namely, similarly to Corollary 3, increasing the information accuracy θ, and increasing the salvage value y, are *strategic substitutes* for the distributor.

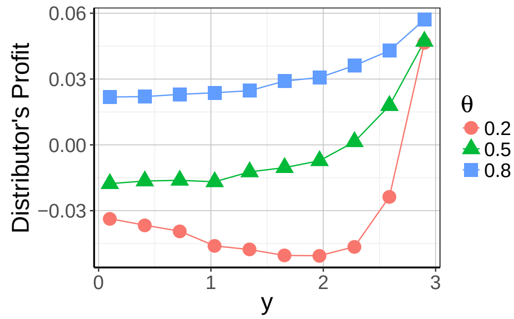


Figure 7 **Distributor's profit when product adoptions are valued ($\gamma = 1$), for a low margin product, where $\mu = 3$, $\sigma = 1.5$, $w = 3$ and $\alpha = 3$.**

Moreover, from Figures 6c and 6f note that, for large enough salvage values y, increasing θ, i.e., disclosing more information, actually *reduces* the fraction of satisfied customers. This is the case because, when the salvage value is high, the distributor will "pass along" the high salvage value to the retailer, who in turn will offer a high refund to the consumers. In this case, the impact that increasing the information accuracy θ has on reducing the downside risk for the consumers is already accounted for by the large refund for returns. In other words, when the salvage value is already high, the main effect of increasing θ is a reduction in the perceived *upside* for the consumers. Since the consumers are risk averse, a smaller upside leads to less consumers trying out the product, therefore to a smaller number of satisfied consumers.

Furthermore, when y is very large, increasing θ does not only reduce the fraction of satisfied customers, but it also reduces the distributor's objective value in Figure 6d. Namely, as the salvage value y increases, the distributor switches from a strategy that favors more accuracy in the information provided to the consumers to a strategy that prefers less accurate information.

4.4.1. The effect of increasing the value of consumer satisfaction γ. As noted in the previous subsections, both the retailer's behavior and the effect of improving the accuracy of the information provided to the consumers are qualitatively different in the continuous model in this section compared to the discrete model in Section 3. Nonetheless, we now corroborate that our main insight is preserved in both models.

Namely, we find evidence in the results of the numerical simulations in this section that, everything else being the same, a distributor that puts a larger weight on consumer satisfaction, compared to its short term profits, is more likely to pursue a strategy that emphasizes investing in reverse logistics over improving the accuracy of the information provided to the consumers.

Specifically, comparing Figure Figure 6a to Figure 6d, suggests that the pairs of information accuracy θ, and salvage value y, for which the distributor follows a strategy that favors *less* accuracy in the information provided to the consumers *increases* with the value of satisfied consumers for the distributor γ. In other words, for a larger γ, the distributor is willing to switch to a strategy with low accuracy in the information provided to the consumers for smaller values of the salvage value y. We find this observation to be consistent across the extensive simulations we ran for the continuous model. In particular, see the Appendix 6.2 for the case of a product with a moderate normalized margin. Similarly to Corollary 5, this suggests that reverse logistics are a better strategic option to increase customer satisfaction, for a social enterprise pushing a product with uncertain valuation to risk-averse consumers, through a for-profit distribution channel.

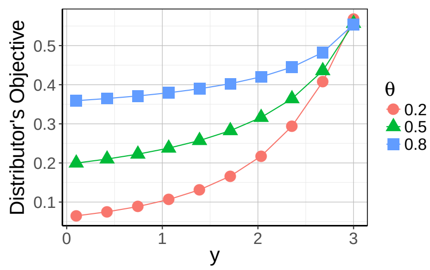
One additional effect of having a larger distributor's value for consumer satisfaction γ, is that it can capture the willingness of a social enterprise to distribute products in a challenging environment, where a traditional for-profit company may not be interested in doing so.

In particular, consider Figure 6a and note that, since $\gamma = 0$, the distributor's objective coincides with its short term profits. Moreover, as already noted, when $\theta = 0.2$ the market collapses since the distributor has no room to incentivize the retailer to carry a low margin product profitably. In contrast, in Figure 6d, and for the same product and information accuracy $\theta = 0.2$, we find that a social enterprise that values consumer satisfaction as much as its own short term profits, i.e. with $\gamma = 1$, is willing to distribute the product, and target reasonable product adoption levels. In particular, in this numerical example, the expected fraction of satisfied customers ranges between 8% and 15% for moderate salvage values y.

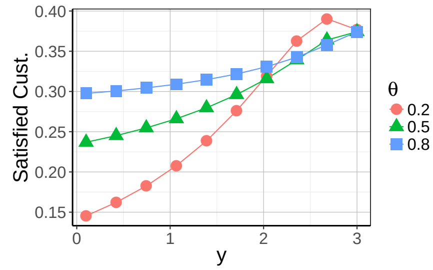
Not surprisingly, in order to induce positive sales in this challenging environment, the distributor must be willing to accept a negative short term profit (otherwise the market would not collapse for $\theta = 0.2$ in Figure 6a). This is depicted in Figure 7, where the distributor's profit is negative when $\theta = 0.2$, for almost all salvage values below the product wholesale cost. Note that a negative distributor's profit is also attained in other cases in Figure 7. This behavior has been observed in practice, where social enterprises, like Essmart, are willing to operate at a profit loss in the short run, in order to fulfill their social mission, as well as to build up a consumer base that sustains their operations in the long run.

4.4.2. The consequences of distributing through a for profit channel. In this section we study the role of the retailer in our continuous model. In contrast to the analysis for the discrete model in Section 3.4.1, the retailer has an important impact on the equilibrium outcome even if he does not have an attractive alternative product to carry, i.e. even if $\pi_R = 0$. This is depicted in Figure 8, where the distributor's objective, and the fraction of satisfied customers, are shown assuming that the distributor sells directly to the consumers, i.e. without the retailer.

Note that the effect of removing the retailer in the continuous model is dramatic, both qualitatively and quantitatively. First, from Figure 8a, the distributor does prefer more accuracy of the



(a) Dist. Objective Value (no retailer)



(b) Sat. Customers (no retailer)

Figure 8 **Distributor's objective value, and fraction of satisfied customers, when the distributor sells directly to the consumers. For a low margin product, where $w = 3$, $\gamma = 1$, $\mu = 3$, $\sigma = 1.5$, and $\alpha = 3$.**

information provided to the consumers, even for a very high salvage value. From Figure 6, note that this was also the case for the retailer before. This suggests that the presence of a for-profit retailer in the supply chain is the main driver behind the distributor's change of strategy discussed in Section 4.4.1. Namely, with a very high salvage value, the problem of double marginalization gets exacerbated as the accuracy of the information provided to the consumers θ increases, to the point where the distributor prefers to have a lower θ, as discussed in Section 4.4.1.

Second, Essmart's strategy of using local retailers as a point of sale to the consumers is associated to a cost that can be significant, both in terms of reducing the distributor's optimal objective value, and the fraction of satisfied consumers. This cost was limited in the discrete model in Section 3, since the retailer's individual rationality constraint is always tight at the equilibrium. Namely, the distributor could always extract all the retailer's profits beyond π_R, due to having a finite support for the consumers' valuations. Having said that, the cost associated with directly selling life-improving technologies to millions of consumers in the developing world is likely to be prohibitive. In this context, an interesting direction of future research is to evaluate the performance of alternative distribution channels, in terms of supply chain profits and the fraction of satisfied consumers, and compare them with Essmart's current strategy of distributing through local retailers.

5. Conclusions

We propose a model that indicates that (i) investing in improving the information to the consumers, and (ii) investing in reverse logistics, are strategic substitutes, i.e. the marginal benefit of one strategy decreases with an increase in the value of the alternative strategy. The model additionally shows that a distributor that highly values customer satisfaction, compared to immediate profits, is likely to prefer to invest more in reverse logistics versus in information improvement.

We find this insight to be robust to different model specifications. Namely, a two-type discrete model, and a continuous model. Interestingly, the retailer's behavior is qualitatively different in each model. Specifically, in the discrete model either full refunds or no refunds are optimal, while in the continuous model partial refunds are always optimal. Even if this is the case, in both model specifications we find evidence that a distributor that highly values customer satisfaction is more likely to follow a strategy that emphasizes reverse logistics.

This issue is important for social enterprises that are trying to distribute life-improving products in developing countries. From the authors' experience, there is little thought given to the value of reverse logistics and after sales warranties for these products. Most investments, especially in small social start-ups, are directed towards marketing and education. Our analysis suggests that this is not necessarily the optimal strategy. Moreover, investing in reverse logistics is an effective strategy when the goal is to increase the proportion of satisfied customers. This observation is exacerbated when consumers are very risk averse, which is the case in developing countries where consumers are budget constrained, and have limited protection to the downside risk of poor purchasing decisions.

Identifying and developing effective reverse logistics and marketing strategies for social enterprises in the real-world can have a profound impact in the way new technologies are disseminated in these challenging environments. Furthermore, we believe that social enterprises that are trying to scale and be financially sustainable in the developing world can be a rich source of interesting Operations Management research questions. Many of these companies are attempting to align profit, technology, and positive social and environmental impact. The expansion, analysis, and optimization of the operations of these companies can be a valuable source of new operational models.

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6. Appendix:

6.1. Proofs of Section 3

Theorem 1. *Given $\theta \in \left(\frac{1}{2}, 1\right]$, if the retailer chooses to carry the distributor's product, his optimal profit is*

$$\Pi_R^* = \max\left\{\Pi_R^a, \Pi_R^b, \Pi_R^c, \Pi_R^d\right\}. \tag{5}$$

Where each component in the max *corresponds to a different non-dominated strategy by the retailer. Strategies $(a), (b), (c)$, and (d) are:*

(a) Target full market adoptions, with no refunds. In this case, $p^a = p_l(0,\theta)$, $r^a = 0$, and

$$\Pi_R^a = p_l(0,\theta) - c = \frac{-1}{\alpha}\ln(\mathbb{P}(v_l|s_l,\theta)e^{-\alpha v_l} + \mathbb{P}(v_h|s_l,\theta)e^{-\alpha v_h}) - c.$$

(b) Target product adoptions by consumers with high valuation v_h, with full refunds. In this case, $p^b = r^b = v_h$, and

$$\Pi_R^b = (1-\beta)(v_h - c) + \beta(u - c).$$

(c) Target product adoptions by consumers that received a high signal s_h, with no refunds. In this case, $p^c = p_h(0,\theta)$, $r^c = 0$, and

$$\Pi_R^c = \mathbb{P}(s_h|\theta)(p_h(0,\theta) - c) = \mathbb{P}(s_h|\theta)\left(\frac{-1}{\alpha}\ln(\mathbb{P}(v_l|s_h,\theta)e^{-\alpha v_l} + \mathbb{P}(v_h|s_h,\theta)e^{-\alpha v_h}) - c\right).$$

(d) Target adoptions from consumers that received a high signal s_h, and *have a high valuation v_h, with (essentially) full refunds. In this case, $p^d = p_h(v_h^-,\theta) \approx v_h$, $r^d = v_h^-$, and*

$$\Pi_R^d \approx \mathbb{P}(s_h|\theta)(\mathbb{P}(v_h|s_h,\theta)(v_h - c) + \mathbb{P}(v_l|s_h,\theta)(u - c)),$$

where $r^d = v_h^-$ denotes $r^d < v_h$, and r^d arbitrarily close to v_h. Note that $r^d = v_h^- < p_h(v_h^-,\theta) = p^d < v_h$, hence this strategy implements essentially full refunds. Specifically, the approximations can be made arbitrarily accurate.

On the other hand, if $\theta = \frac{1}{2}$, i.e. the signal to the consumers is uninformative, then only strategies (a) and (b) are non-dominated.

Table 1 provides a summary of the retailer's strategies and the consumer behavior they induce.

Proof. The retailer's decision variable is the refund level r. We analyze the outcome of all possible values that r can take. Note that $p_l(r,\theta) < p_h(r,\theta)$ if and only if $r < v_h$ and $\theta \in \left(\frac{1}{2}, 1\right]$. Otherwise, $p_l(r,\theta) = p_h(r,\theta)$.

In particular, if $r > v_h$ then $p_l(r,\theta) = p_h(r,\theta) = r > v_h$, and $\mathbb{P}(\mathrm{B}(r,r)|\theta) = \mathbb{P}(\mathrm{R}(r,r)|\theta) = 1$. Namely, all the consumers buy and return the product, for any information accuracy $\theta \in \left[\frac{1}{2}, 1\right]$. Then, the retailer's profit in this case is $\Pi(r,\theta) = c - u \leq 0 \leq \pi_r$. Namely, this strategy is never profitable for the retailer, for any reasonable product cost c, and refund u, from the distributor.

If $r = v_h$ then $p_l(r,\theta) = p_h(r,\theta) = v_h$, $\mathbb{P}(\mathrm{B}(v_h,v_h)|\theta) = 1$, and $\mathbb{P}(\mathrm{R}(v_h,v_h)|\theta) = \beta$, for any information accuracy level $\theta \in \left[\frac{1}{2},1\right]$. Namely, with full refunds there is no downside for the consumers, and everyone buys the product. Only the consumers with low valuation v_l return the product. Then, the retailer's profit in this case is $\Pi(v_h,\theta) = (1-\beta)(v_h - c) + \beta(u-c)$, for any $\theta \in \left[\frac{1}{2},1\right]$.

For any refund $r \in [v_l, v_h)$, and any information accuracy $\theta \in \left[\frac{1}{2},1\right]$. Consider the option of setting the price $p = p_l(r,\theta)$. Then, $\mathbb{P}(\mathrm{B}(p_l(r,\theta),r)|\theta) = 1$, and $\mathbb{P}(\mathrm{R}(p_l(r,\theta),r)|\theta) = \beta$. Namely, all the consumers buy the product, and only the consumers with low valuation v_l return it. Therefore, the retailer's profit in this case is $\Pi(r,\theta) = p_l(r,\theta) - c - (r-u)\beta$. Note that $\partial_r \Pi(r,\theta) = \partial_r p_l(r,\theta) - \beta > 0$, for any $r < v_h$. Specifically, $\partial_r p_l(r,\theta) = \frac{\mathbb{P}(v_l|s_l,\theta)}{\mathbb{P}(v_l|s_l,\theta)+\mathbb{P}(v_h|s_l,\theta)e^{-\alpha(v_h - r)}} > \mathbb{P}(v_l|s_l,\theta) \geq \beta$, for any $r < v_h$, where the last inequality follows from $\theta \geq \frac{1}{2}$. Moreover, the consumers' behavior is continuous at $r = v_h$. Hence, it follows that the optimal refund in this setup is $r^* = v_h$. This implies $p^* = p_l(v_h,\theta) = v_h$, i.e. full refunds are optimal. Finally, the retailer's maximum profit in this case is $\Pi(v_h,\theta) = (1-\beta)(v_h - c) + \beta(u-c)$. This completes the proof for strategy (b) in the Theorem.

For any refund $r \in [v_l, v_h)$, and any information accuracy $\theta \in \left(\frac{1}{2},1\right]$. Now consider the option of setting the price $p = p_h(r,\theta)$. Then, $\mathbb{P}(\mathrm{B}(p,r)|\theta) = \mathbb{P}(s_h|\theta)$, and $\mathbb{P}(\mathrm{R}(p,r)|\theta) = \mathbb{P}(s_h, v_l|\theta)$. Namely, only the consumers that receive a high signal s_h buy the product. From them, the consumers that realize they have a low valuation v_l return the product. Therefore, the retailer's profit in this case is $\Pi(r,\theta) = \mathbb{P}(s_h|\theta)(p_h(r,\theta) - c - (r-u)\mathbb{P}(v_l|s_h,\theta))$. Note that $\partial_r \Pi(r,\theta) = \mathbb{P}(s_h|\theta)(\partial_r p_h(r,\theta) - \mathbb{P}(v_l|s_h,\theta)) > 0$, for any $r < v_h$. Specifically, $\partial_r p_h(r,\theta) = \frac{\mathbb{P}(v_l|s_h,\theta)}{\mathbb{P}(v_l|s_h,\theta)+\mathbb{P}(v_h|s_h,\theta)e^{-\alpha(v_h - r)}} > \mathbb{P}(v_l|s_l,\theta)$, for any $r < v_h$. However, the consumers' behavior is *not continuous* at $r = v_h$. Namely, if $r = v_h$ then there is no downside for consumers buying the product, and it is no longer possible to price out the low signal consumers. Hence, it follows that the optimal refund in this setup is $r^* = v_h^-$, where $r^* = v_h^-$ denotes $r^* < v_h$, and r^* arbitrarily close to v_h. This implies $p^* = p_h(v_h^-,\theta) \approx v_h$, where the approximation can be made arbitrarily accurate. Therefore, the retailer's maximum profit in this case is

$$
\begin{aligned}
\Pi_R(v_h^-, p_h(v_h^-,\theta)) &= \mathbb{P}(s_h|\theta)(\mathbb{P}(v_h|s_h,\theta)(p_h(v_h^-,\theta) - c) + \mathbb{P}(v_l|s_h,\theta)(u-c)), \\
&= \mathbb{P}(s_h|\theta)\left(\mathbb{P}(v_h|s_h,\theta)\left(\frac{-1}{\alpha}\ln(\mathbb{P}(v_l|s_h,\theta)e^{-\alpha v_h^-} + \mathbb{P}(v_h|s_h,\theta)e^{-\alpha v_h}) - c\right) + \mathbb{P}(v_l|s_h,\theta)(u-c)\right), \\
&\approx \mathbb{P}(s_h|\theta)(\mathbb{P}(v_h|s_h,\theta)(v_h - c) + \mathbb{P}(v_l|s_h,\theta)(u-c)).
\end{aligned}
$$

Again, the approximation can be made arbitrarily accurate. This completes the proof for strategy (d) in the Theorem.

For any refund $r < v_l$, no consumer returns the product, i.e. $\mathbb{P}(\mathrm{R}(p,r)|\theta) = 0$ for any p. Moreover, $p_l(r,\theta) = p_l(0,\theta)$, $p_h(r,\theta) = p_h(0,\theta)$, and

$$
\mathbb{P}(\mathrm{B}(p,r)|\theta) = \begin{cases} 1 & \text{if } p = p_l(0,\theta) \\ \mathbb{P}(s_h|\theta) & \text{if } p = p_h(0,\theta). \end{cases}
$$

Hence, in particular we can set $r^* = 0$ in this case. If the price is set to $p^* = p_l(0,\theta)$, then the retailer's maximum profit is $\Pi(0,\theta) = p_l(0,\theta) - c$. Similarly, if the price is set to $p^* = p_h(0,\theta)$, then the retailer's maximum profit is $\Pi(0,\theta) = \mathbb{P}(s_h|\theta)(p_h(0,\theta) - c)$. This completes the proof for strategies (a) and (c) in the Theorem, respectively.

Finally, if $\theta = \frac{1}{2}$, then $\mathbb{P}\left(s_h|\theta = \frac{1}{2}\right) = \mathbb{P}\left(s_l|\theta = \frac{1}{2}\right) = \frac{1}{2}$, $\mathbb{P}\left(v_l|s, q = \frac{1}{2}\right) = \mathbb{P}(v_l) = \beta$, and $\mathbb{P}\left(v_h|s, q = \frac{1}{2}\right) = \mathbb{P}(v_h) = 1 - \beta$, for any signal $s \in \{s_h, s_l\}$. Therefore, we conclude $\Pi_{\mathrm{R}}^c = \frac{1}{2}\Pi_{\mathrm{R}}^a$, and $\Pi_{\mathrm{R}}^d = \frac{1}{2}\Pi_{\mathrm{R}}^b$. Namely, strategies (c) and (d) are dominated by strategies (a) and (b), respectively. This concludes the proof. □

Corollary 1. *For any accuracy level of the information provided to the consumers $\theta \in \left[\frac{1}{2}, 1\right]$, if the consumers are highly risk averse ($\alpha > \underline{\alpha}$), then it is unprofitable for the retailer to carry the distributor's product and target full market adoptions. Namely, it is unprofitable for the retailer to implement strategy (a) in Theorem 1.*

Proof. If $(1-\beta)v_h + \beta v_l \leq \pi_r + w$, then it would be unprofitable for the retailer to target the whole market to adopt the product *even with risk neutral consumers.* Risk aversion simply makes it even less profitable. Specifically, we get $\Pi_{\mathrm{R}}^a = p_l(0,\theta) - c \leq p_l(0,\theta) - w < \mathbb{P}(v_l|s_l,\theta)v_l + \mathbb{P}(v_h|s_l,\theta)v_h - w \leq (1-\beta)v_h + \beta v_l - w \leq \pi_r$. Where the first inequality follows from $c \geq w$, the second inequality follows from noticing that $p_l(0,\theta)$ is decreasing in α and taking the limit $\alpha \to 0$, the third inequality follows from $\mathbb{P}(v_l|s_l,\theta) \geq \mathbb{P}(v_l) = \beta$ for any $\theta \in [.5, 1]$, and the last inequality is the assumption.

Alternatively, if $(1-\beta)v_h + \beta v_l > \pi_r + w$, then by assumption consumers are so risk averse that it is unprofitable for the retailer to carry the product. Specifically, we have $\Pi_{\mathrm{R}}^a = p_l(0,\theta) - c \leq p_l(0,\theta) - w = \frac{-1}{\alpha}\ln(\mathbb{P}(v_l|s_l,\theta)e^{-\alpha v_l} + \mathbb{P}(v_h|s_l,\theta)e^{-\alpha v_h}) - w \leq \frac{-1}{\alpha}\ln(\beta e^{-\alpha v_l} + (1-\beta)e^{-\alpha v_h}) - w < \frac{-1}{\underline{\alpha}}\ln(\beta e^{-\underline{\alpha} v_l} + (1-\beta)e^{-\underline{\alpha} v_h}) - w = \pi_r$. Where the first inequality follows from $c \geq w$, the second inequality follows from $\mathbb{P}(v_l|s_l,\theta) \geq \mathbb{P}(v_l) = \beta$ for any $\theta \in [.5, 1]$, the last inequality follows from the assumption $\alpha > \underline{\alpha}$, and the last equality is the definition $\underline{\alpha}$ in the statement of the corollary. □

Corollary 2. *Consider a market with the following challenging environment.*

- *Consumers are highly risk averse ($\alpha > \underline{\alpha}$).*
- *Consumers have no information about their type ($\theta = \frac{1}{2}$).*
- *Either the population of high valuation consumers, or the high valuation itself, is not large enough. Specifically $(1-\beta)v_h < \pi_r + w$.*

Then, if the distributor provides no additional information (i.e. maintains $\theta = \frac{1}{2}$), and it does not provide a salvage value for returns to the retailer ($u = 0$), then it is unprofitable for the retailer to carry the product, even if he faces no inventory risk.

Proof. From $\theta = \frac{1}{2}$ and Theorem 1 it follows that we can restrict our attention to the retailer's strategies (a) and (b) in Theorem 1. Moreover, from Assumption 2 it follows that the retailer follows strategy (b) in Theorem 1. Finally, from $u = 0$ it follows that the retailer's profit from carrying the product is $\Pi_{\mathrm{R}}^{*} = (1-\beta)v_h - c \leq (1-\beta)v_h - w < \pi_r$. The first inequality follows from $c \geq w$, and the second inequality corresponds to the third assumption in the statement of the corollary. □

Proof of Theorem 2 For the sake of clarity, we split the proof of Theorem 2 into Propositions 3, 4, and 5 below.

Proposition 3. *It is always possible for the distributor to induce the retailer to implement strategy* (b) *from Theorem 1. Given that the distributor is interested in inducing the retailer to choose strategy (b), she maximizes her profits by giving full refunds to the retailer,* $u^b = c^b = v_h - \frac{\pi_r}{1-\beta}$. *In this case, the distributor's profit is*

$$\Pi_D^{Info} = (1-\beta)v_h + \beta y - w + \gamma(1-\beta) - \pi_r.$$

Proof. In strategy (b), the retailer sets $p = r = v_h$. Then, $\mathbb{P}(\mathrm{B}(v_h, v_h)|\theta) = 1$, $\mathbb{P}(\mathrm{R}(v_h, v_h)|\theta) = \beta$, and $\mathbb{P}(\mathrm{S}(v_h, v_h)|\theta) = 1-\beta$, for any information quality $\theta \in \left[\frac{1}{2}, 1\right]$. Namely, all consumers purchase the product, and a consumer chosen at random returns the product with probability $\mathbb{P}(v_l) = \beta$. The distributor's profit in this case is

$$\Pi_D(c, u) = c - w - (u - y)\beta + \gamma(1-\beta).$$

From Assumption 2 it follows that the retailer will never choose strategy (a) in Theorem 1. Moreover, the retailer will choose strategy (b) over (c) if $\Pi_R^b \geq \Pi_R^c$, i.e., if

$$(1-\beta)v_h + \beta u \geq \mathbb{P}(s_h|\theta)p_h(0, \theta) + \mathbb{P}(s_l|\theta)c$$

This is the first incentive compatibility constraint, (IC_1), that the distributor must satisfy to induce the retailer to implement strategy (b).

Similarly, the retailer will choose strategy (b) over (d) if $\Pi_R^b \geq \Pi_R^d$, i.e., if

$$\mathbb{P}(s_l|\theta)(\mathbb{P}(v_h|s_l, \theta)(v_h - c) + \mathbb{P}(v_l|s_l, \theta)(u - c)) \geq 0$$

This is the second incentive compatibility constraint (IC_2). It states that targeting the consumers that received a low signal s_l must lead to a non-negative expected profit.

Furthermore, the retailer needs to prefer strategy (b) to the status-quo product that offers a profit of π_r. Thus, in order for the retailer to use strategy (b), we must have

$$(1-\beta)(v_h - c) + \beta(u - c) \geq \pi_r.$$

This is the individual rationality, (IR), constraint. With these constraints in hand, the distributor can determine the optimal price c, and refund u, by solving

$$\begin{aligned}
\max_{c,u} \quad & c - w + (y-u)\beta + \gamma(1-\beta) \\
\text{s.t.} \quad & (1-\beta)v_h + \beta u \geq \mathbb{P}(s_h|\theta)p_h(0,\theta) + \mathbb{P}(s_l|\theta)c && (\text{IC}_1) \\
& \mathbb{P}(s_l|\theta)(\mathbb{P}(v_h|s_l,\theta)(v_h - c) + \mathbb{P}(v_l|s_l,\theta)(u-c)) \geq 0 && (\text{IC}_2) \\
& (1-\beta)(v_h - c) + \beta(u-c) \geq \pi_r && (\text{IR}) \\
& 0 \leq u \leq c.
\end{aligned} \tag{19}$$

From Lemma 1 below we have that, without loss of generality, the (IR) constraint can be assumed to be tight at optimality in problem (19).

Assume that the (IR) constraint is tight. Then, by replacing $c^b = (1-\beta)v_h + \beta u - \pi_r$, and recognizing terms, we can re-write problem (19) as

$$\begin{aligned}
\max_{u} \quad & (1-\beta)v_h + \beta y - w + \gamma(1-\beta) - \pi_r \\
\text{s.t.} \quad & u \geq \frac{p_h(0,\theta)}{\beta} - v_h\frac{1-\beta}{\beta} - \pi_r\frac{\mathbb{P}(s_l|\theta)}{\mathbb{P}(s_h|\theta)\beta} && (\text{IC}_1) \\
& u \geq v_h - \frac{\pi_r}{\mathbb{P}(v_l|s_l,\theta) - \beta} && (\text{IC}_2) \\
& 0 \leq u \leq v_h - \frac{\pi_r}{1-\beta}.
\end{aligned} \tag{20}$$

Note that the distributor's objective function is independent of of the actual value of the refund offered to the retailer u, as long as it is feasible. This is due to the fact that any feasible refund u that is chosen is counterbalanced by an appropriate price to the retailer c^b, such that the (IR) constraint is tight, i.e. the distributor extracts all the retailer's utility beyond π_r.

We now verify that the feasible set of problem (20) is always non-empty. First note that Assumption 1 implies that the interval $\left[0, v_h - \frac{\pi_r}{1-\beta}\right]$ is non-empty. Specifically, we have $(1-\beta)v_h > (1-\beta)(v_h - w) > \pi_r$.

To conclude we need to verify that the intersection of $\left[0, v_h - \frac{\pi_r}{1-\beta}\right]$ with the lower bounds on u given by constraints (IC_1) and (IC_2) in problem (20) is non-empty. A sufficient condition is that the upper bound of the interval is feasible for (IC_1) and (IC_2) in problem (20). In particular, for constraint (IC_2) we have $u^b = v_h - \frac{\pi_r}{1-\beta} \geq v_h - \frac{\pi_r}{\mathbb{P}(v_l|s_l,\theta)-\beta}$, since $\mathbb{P}(v_l|s_l,\theta) \leq 1$. Moreover, note that the inequality is strict as long as the information to the consumers is not perfect, i.e as long as $\theta < 1$. On the other hand, replacing $u^b = v_h - \frac{\pi_r}{1-\beta}$ in constraint (IC_2) leads to the inequality

$$v_h \geq p_h(0,\theta) + \pi_r\frac{\beta - \mathbb{P}(s_l|\theta)}{(1-\beta)\mathbb{P}(s_h|\theta)}. \tag{21}$$

We now show that under Assumption 1 inequality (21) always hold. This implies that the feasible set of problem (20) is always non-empty.

First, note that the inequality (21) holds for any $\beta \leq \frac{1}{2}$. Specifically, we have that $v_h \geq p_h(0,\theta)$, and $\mathbb{P}(s_l|\theta) \geq \beta$ for any $\beta \leq \frac{1}{2}$. Again, note that the inequality is strict for any $\theta < 1$. Now assume $\beta > \frac{1}{2}$, then we have

$$\begin{aligned} p_h(0,\theta) + \pi_r \frac{\beta - \mathbb{P}(s_l|\theta)}{(1-\beta)\mathbb{P}(s_h|\theta)} &\leq \mathbb{P}(v_h|s_h,\theta)v_h + \mathbb{P}(v_l|s_h,\theta)v_l + \pi_r \frac{\beta - \mathbb{P}(s_l|\theta)}{(1-\beta)\mathbb{P}(s_h|\theta)} \\ &= v_h + \pi_r \frac{\beta - \mathbb{P}(s_l|\theta)}{(1-\beta)\mathbb{P}(s_h|\theta)} - \mathbb{P}(v_l|s_h,\theta)(v_h - v_l) \\ &= v_h + \frac{(1-\theta)}{\mathbb{P}(s_h|\theta)} \left(\pi_r \frac{2\beta - 1}{1-\beta} - \beta(v_h - v_l) \right) \\ &\leq v_h + \frac{(1-\theta)}{\mathbb{P}(s_h|\theta)} \left((2\beta-1)(v_h - w) - \beta(v_h - v_l) \right) \\ &= v_h + \frac{(1-\theta)}{\mathbb{P}(s_h|\theta)} \left(\beta(v_l - w) - (1-\beta)(v_h - w) \right) \\ &\leq v_h. \end{aligned}$$

The first inequality follows from noticing that $p_h(0,\theta)$ is decreasing in $\alpha > 0$, and taking the limit $\alpha \to 0$. The second equality follows from noticing that $\beta - \mathbb{P}(s_l|\theta) = (1-\theta)(2\beta - 1)$. The second inequality follows from Assumption 1, specifically from $\pi_r < (1-\beta)(v_h - w)$. For the last inequality, note that it trivially holds if $v_l \leq w$; alternatively we have $\beta(v_l - w) < v_l - w < (1-\beta)(v_h - w)$, where the last inequality follows from Assumption 1. Again, all inequalities are strict for any $\theta < 1$.

This completes the proof that inequality (21) always hold. Moreover, the inequality is strict for any $\theta < 1$. Therefore, the feasible set of problem (20) is always non-empty. In particular, the existence of the optimal solution $c^b = u^b = v_h - \frac{\pi_r}{1-\beta}$ is guaranteed. Moreover, for any $\theta < 1$ we have an interval of optimal solutions that satisfy $u^b < c^b$.

Hence, we conclude that the distributor's optimal profits in this case are

$$\Pi_D^{Info} = (1-\beta)v_h + \beta y - w + \gamma(1-\beta) - \pi_r.$$

□

Lemma 1. *Without loss of generality, the (IR) constraint can be assumed to be tight at optimality in problem (19).*

Proof. Note that at least one of the constraints (IC_1), (IC_2) and (IR) must be tight at optimality in problem (19), otherwise we could increase c, or decrease u, strictly increasing the distributor's profits.

Assume first that the (IC_2) constraint is tight. Then, by replacing $c^* = \mathbb{P}(v_h|s_l,\theta)v_h + \mathbb{P}(v_l|s_l,\theta)u$, and recognizing terms, we can re-write problem (19) as

$$\begin{aligned} \max_{u,q} \quad & \mathbb{P}(v_h|s_l,\theta)v_h + (\mathbb{P}(v_l|s_l,\theta) - \beta)u - w + \beta y - g(\theta - \underline{q}) + \gamma(1-\beta) \\ \text{s.t.} \quad & \mathbb{P}(v_h|s_h,\theta)v_h + \mathbb{P}(v_l|s_h,\theta)u \geq p_h(0,\theta) && \text{(IC}_1\text{)} \\ & u \leq v_h - \frac{\pi_r}{\mathbb{P}(v_l|s_l,\theta) - \beta} && \text{(IR)} \\ & 0 \leq u \leq v_h. \end{aligned} \tag{22}$$

From $\mathbb{P}(v_l|s_l,\theta) \geq \beta$, for any $\theta \in \left[\frac{1}{2}, 1\right]$, it follows that without loss of generality we can increase the value of u up until it attains its upper bound, i.e. until the (IR) constraint is tight.

Now assume that the (IC_1) constraint is tight. Then, by replacing $c^* = \frac{(1-\beta)}{\mathbb{P}(s_l|\theta)} v_h + \frac{\beta}{\mathbb{P}(s_l|\theta)} u - \frac{\mathbb{P}(s_h|\theta)}{\mathbb{P}(s_l|\theta)} p_h(0,\theta)$, and recognizing terms, we can re-write problem (19) as

$$\begin{aligned}
\max_{u,q} \quad & \frac{(1-\beta)}{\mathbb{P}(s_l|\theta)} v_h + \beta \frac{\mathbb{P}(s_h|\theta)}{\mathbb{P}(s_l|\theta)} u - \frac{\mathbb{P}(s_h|\theta)}{\mathbb{P}(s_l|\theta)} p_h(0,\theta) - w + \beta y - g(\theta - \underline{q}) + \gamma(1-\beta) \\
\text{s.t.} \quad & \mathbb{P}(v_h|s_h,\theta) v_h + \mathbb{P}(v_l|s_h,\theta) u \leq p_h(0,\theta) \qquad (\text{IC}_2) \\
& (1-\beta) v_h + \beta u + \frac{\mathbb{P}(s_l|\theta)}{\mathbb{P}(s_h|\theta)} \pi_r \leq p_h(0,\theta) \qquad (\text{IR}) \\
& 0 \leq u \\
& (\mathbb{P}(s_l|\theta) - \beta) u \leq (1-\beta) v_h - \mathbb{P}(s_h|\theta) p_h(0,\theta).
\end{aligned} \tag{23}$$

From $\beta \frac{\mathbb{P}(s_h|\theta)}{\mathbb{P}(s_l|\theta)} > 0$, for any $\theta \in \left[\frac{1}{2}, 1\right]$, it follows that u must attain its upper bound at optimality. We now show that the fourth constraint is not tight at optimality. We have three possible cases.

- If $\beta \geq \frac{1}{2}$, then $\mathbb{P}(s_l|\theta) \leq \beta$, therefore u must be such that either the (IR) constraint is tight, or the (IC_2) constraint is tight. In the latter case, we have already shown that it implies that, without loss of generality, we can assume that the (IR) constraint is also tight, and we are done in this case.
- If $\theta = 1$, then the fourth constraint in problem (23) reduces to $0 \leq 0$, i.e. it is vacuous.
- If $\beta < \frac{1}{2}$ and $\theta < 1$, then we have $(\mathbb{P}(s_l|\theta) - \beta)u = (2\beta - 1)(1-\theta)u \leq (2\beta - 1)(1-\theta)v_h = ((1-\beta) - \mathbb{P}(s_h|\theta))v_h < (1-\beta)v_h - \mathbb{P}(s_h|\theta)p_h(0,\theta)$. Namely, the fourth constraint in problem (23) is redundant. Hence, the logic of the first bullet point applies.

This completes the proof. □

Proposition 4. *The distributor can induce the retailer to implement strategy* (c) *from Theorem 1 if and only if*

$$\min\left\{ \frac{p_h(0,\theta)}{\beta} - v_h \frac{1-\beta}{\beta} - \pi_r \frac{\mathbb{P}(s_l|\theta)}{\mathbb{P}(s_h|\theta)\beta}, \frac{p_h(0,\theta)}{\mathbb{P}(v_l|s_h,\theta)} - v_h \frac{\mathbb{P}(v_h|s_h,\theta)}{\mathbb{P}(v_l|s_h,\theta)} \right\} \geq 0.$$

Given that the distributor is interested in inducing the retailer to choose strategy (c)*, she maximizes her profits by charging* $c^c = p_h(0,\theta) - \frac{\pi_r}{\mathbb{P}(s_h|\theta)}$*, and not giving any refunds to the retailer, i.e setting* $u^c = 0$*.*

In this case, the distributor's profit is

$$\Pi_D^{Info} = \mathbb{P}(s_h|\theta)(p_h(0,\theta) - w) + \gamma \mathbb{P}(s_h, v_h|\theta) - \pi_r.$$

Proof. In strategy (c), the retailer sets $r = 0$, $p = p_h(0,\theta)$. Then, $\mathbb{P}(\mathrm{B}(v_h, v_h)|\theta) = \mathbb{P}(s_h|\theta)$, $\mathbb{P}(\mathrm{R}(v_h, v_h)|\theta) = 0$, and $\mathbb{P}(\mathrm{S}(v_h, v_h)|\theta) = \mathbb{P}(s_h, v_h|\theta)$. Namely, consumers that received a high signal

s_h purchase the product, and no one returns it because the retailer does not offer a refund. Therefore, the satisfied customers are the ones that received a high signal s_h *and* have a high valuation for the product v_h. The distributor's profit in this case is

$$\Pi_D(c,u) = \mathbb{P}(s_h|\theta)(c-w) + \gamma\mathbb{P}(s_h, v_h|\theta).$$

From Assumption 2 it follows that the retailer will never choose strategy (a) in Theorem 1. Moreover, the retailer will choose strategy (c) over (b) if $\Pi_R^c \geq \Pi_R^b$, i.e., if

$$(1-\beta)v_h + \beta u \leq \mathbb{P}(s_h|\theta)p_h(0,\theta) + \mathbb{P}(s_l|\theta)c$$

This is the first incentive compatibility constraint, (IC_1), that the distributor must satisfy to induce the retailer to implement strategy (c).

Similarly, the retailer will choose strategy (c) over (d) if $\Pi_R^c \geq \Pi_R^d$, i.e., if

$$p_h(0,\theta) \geq \mathbb{P}(v_h|s_h,\theta)v_h + \mathbb{P}(v_l|s_h,\theta)u.$$

This is the second incentive compatibility constraint (IC_2).

Furthermore, the retailer needs to prefer strategy (c) to the status-quo product that offers a profit of π_r. Thus, in order for the retailer to use strategy (c), we must have

$$\mathbb{P}(s_h|\theta)(p_h(0,\theta) - c) \geq \pi_r.$$

This is the individual rationality, (IR), constraint. With these constraints in hand, the distributor can determine the optimal price c, and refund u, by solving

$$\begin{aligned}
\max_{c,u} \quad & \mathbb{P}(s_h|\theta)(c-w) + \gamma\mathbb{P}(s_h, v_h|\theta) \\
\text{s.t.} \quad & (1-\beta)v_h + \beta u \leq \mathbb{P}(s_h|\theta)p_h(0,\theta) + \mathbb{P}(s_l|\theta)c \quad (\text{IC}_1) \\
& \mathbb{P}(v_h|s_h,\theta)v_h + \mathbb{P}(v_l|s_h,\theta)u \leq p_h(0,\theta) \quad (\text{IC}_2) \\
& \mathbb{P}(s_h|\theta)(p_h(0,\theta) - c) \geq \pi_r \quad (\text{IR}) \\
& 0 \leq u \leq c.
\end{aligned} \tag{24}$$

Assume that the feasible set of problem (24) is non-empty. Then, note that the (IR) constraint must be tight at optimality, otherwise we could increase c without bound, strictly increasing the distributor's profits.

Assume that the (IR) constraint is tight. Then, by replacing $c^c = p_h(0,\theta) - \frac{\pi_r}{\mathbb{P}(s_h|\theta)}$, and recognizing terms, we can re-write problem (24) as

$$\begin{aligned}
\max_{u} \quad & \mathbb{P}(s_h|\theta)(p_h(0,\theta) - w) + \gamma\mathbb{P}(s_h, v_h|\theta) - \pi_r \\
\text{s.t.} \quad & u \leq \frac{p_h(0,\theta)}{\beta} - v_h\frac{1-\beta}{\beta} - \pi_r\frac{\mathbb{P}(s_l|\theta)}{\mathbb{P}(s_h|\theta)\beta} \quad (\text{IC}_1) \\
& u \leq \frac{p_h(0,\theta)}{\mathbb{P}(v_l|s_h,\theta)} - v_h\frac{\mathbb{P}(v_h|s_h,\theta)}{\mathbb{P}(v_l|s_h,\theta)} \quad (\text{IC}_2) \\
& 0 \leq u \leq p_h(0,\theta) - \frac{\pi_r}{\mathbb{P}(s_h|\theta)}.
\end{aligned} \tag{25}$$

Note that the distributor's objective function is independent of the actual value of the refund offered to the retailer u, as long as it is feasible. This is due to the fact that any feasible refund u that is chosen is counterbalanced by an appropriate price to the retailer c^c, such that the (IR) constraint is tight, i.e. the distributor extracts all the retailer's utility beyond the reservation profit π_r. In particular, assuming that the feasible set of problem (24) is non-empty, then giving no refund to the retailer, i.e. setting the refund equal to its lower bound $u^c = 0$, will be feasible.

Moreover, the feasible set of problem (24) is non-empty if and only if

$$\min\left\{\frac{p_h(0,\theta)}{\beta} - v_h\frac{1-\beta}{\beta} - \pi_r\frac{\mathbb{P}(s_l|\theta)}{\mathbb{P}(s_h|\theta)\beta}, \frac{p_h(0,\theta)}{\mathbb{P}(v_l|s_h,\theta)} - v_h\frac{\mathbb{P}(v_h|s_h,\theta)}{\mathbb{P}(v_l|s_h,\theta)}\right\} \geq 0.$$

Namely, if both upper bounds for u defined by the constraints (IC_1) and (IC_2) in problem (25) are non-negative. This follows from the observation that the third upper bound on u in problem (25) is redundant with (IC_1). Specifically, if $\beta \leq \frac{1}{2}$ then we have that

$$\begin{aligned}&\left(p_h(0,\theta) - \frac{\pi_r}{\mathbb{P}(s_h|\theta)}\right) - \left(\frac{p_h(0,\theta)}{\beta} - v_h\frac{1-\beta}{\beta} - \pi_r\frac{\mathbb{P}(s_l|\theta)}{\mathbb{P}(s_h|\theta)\beta}\right)\\ &= \frac{\mathbb{P}(s_h|\theta)(1-\beta)(v_h - p_h(0,\theta)) + \pi_r(\mathbb{P}(s_l|\theta)-\beta)}{\mathbb{P}(s_h|\theta)\beta} \geq 0.\end{aligned}$$

The inequality follows from $v_h \geq p_h(0,\theta)$, and $\mathbb{P}(s_l|\theta) \geq \beta$ for any $\beta \leq \frac{1}{2}$.

On the other hand, if $\beta > \frac{1}{2}$ then we have that

$$\begin{aligned}&\left(p_h(0,\theta) - \frac{\pi_r}{\mathbb{P}(s_h|\theta)}\right) - \left(\frac{p_h(0,\theta)}{\beta} - v_h\frac{1-\beta}{\beta} - \pi_r\frac{\mathbb{P}(s_l|\theta)}{\mathbb{P}(s_h|\theta)\beta}\right)\\ &= \frac{\mathbb{P}(s_h|\theta)(1-\beta)(v_h - p_h(0,\theta)) - \pi_r(\beta - \mathbb{P}(s_l|\theta))}{\mathbb{P}(s_h|\theta)\beta}\\ &\geq \frac{\mathbb{P}(s_h|\theta)(1-\beta)(v_h - p_h(0,\theta)) - (1-\beta)(v_h - w)(\beta - \mathbb{P}(s_l|\theta))}{\mathbb{P}(s_h|\theta)\beta}\\ &= \frac{(1-\beta)((\mathbb{P}(v_h,s_h|\theta)v_h + \mathbb{P}(v_l,s_h|\theta)w - \mathbb{P}(s_h|\theta)p_h(0,\theta)) + (v_h - w)\mathbb{P}(v_h,s_l|\theta))}{\mathbb{P}(s_h|\theta)\beta}\\ &\geq 0.\end{aligned}$$

The first inequality follows from $\mathbb{P}(s_l|\theta) \leq \beta$ for any $\beta > \frac{1}{2}$, and taking the upper bound on π_r from Assumption 1 (i.e. $\pi_r < (1-\beta)(v_h - w)$). The second equality follows from $(\beta - \mathbb{P}(s_l|\theta)) = (1-2\beta)(1-\theta) = \mathbb{P}(v_h,s_l|\theta) - \mathbb{P}(v_l,s_h|\theta)$, and rearranging terms. The last inequality follows from $v_h > w$, and Lemma 2. $\square$

Lemma 2. *For any information accuracy θ, we have that*

$$\mathbb{P}(v_h|s_h,\theta)v_h + \mathbb{P}(v_l|s_h,\theta)w \geq p_h(0,\theta).$$

Moreover, for any $\theta < 1$ the inequality is strict.

Proof. From the definition of $p_h(r,\theta)$ in (4), the statement in the lemma is equivalent to

$$h_1(\theta) := \mathbb{P}(v_l|s_h,\theta)e^{-\alpha v_l} + \mathbb{P}(v_h|s_h,\theta)e^{-\alpha v_h} - e^{-\alpha(\mathbb{P}(v_h|s_h,\theta)v_h + \mathbb{P}(v_l|s_h,\theta)w)} \geq 0.$$

We now show that $h_1(\theta)$ is quasiconcave for any $\theta \in \left[\frac{1}{2},1\right]$. Specifically, note that $h_1'(\theta) > 0$ if and only if $h_2(\theta) := e^{-\alpha v_h} - e^{-\alpha v_l} + e^{-\alpha(\mathbb{P}(v_h|s_h,\theta)v_h + \mathbb{P}(v_l|s_h,\theta)w)}\alpha(v_h - w) > 0$. Moreover, $h_2(\theta)$ is clearly decreasing for any $\theta \in \left[\frac{1}{2},1\right]$, hence we conclude that $h_1(\theta)$ is quasiconcave.

It follows that we only need to check the statement of the lemma at the extremes values of $\theta \in \left[\frac{1}{2},1\right]$. In particular, if $\theta = \frac{1}{2}$, then the statement in the lemma becomes $(1-\beta)v_h + \beta w \geq p_h\left(0,\frac{1}{2}\right) = p_l\left(0,\frac{1}{2}\right)$. This inequality strictly holds, since from Assumptions 1 and 2 we equivalently have $p_l\left(0,\frac{1}{2}\right) - w < \pi_r < (1-\beta)(v_h - w)$. Finally, if $\theta = 1$ then the statement in the lemma becomes $v_h \geq v_h$. This completes the proof. □

Proposition 5. *The distributor can induce the retailer to implement strategy* (d) *from Theorem 1 if and only if*

$$v_h - \frac{\pi_r}{\mathbb{P}(s_h|\theta)(\mathbb{P}(v_h|s_h,\theta) - \mathbb{P}(v_h|s_l,\theta))} \geq \max\left\{\frac{p_h(0,\theta)}{\mathbb{P}(v_l|s_h,\theta)} - \frac{\mathbb{P}(v_h|s_h,\theta)}{\mathbb{P}(v_h|s_h,\theta)v_h}v_h, 0\right\}.$$

Given that the distributor is interested in inducing the retailer to choose strategy (d)*, she maximizes her profits by charging* $c^d = v_h - \frac{\pi_r}{\mathbb{P}(s_h|\theta)\mathbb{P}(v_h|s_h,\theta)}$*, and giving the partial refund to the retailer* $u^d = v_h - \frac{\pi_r}{\mathbb{P}(s_h|\theta)(\mathbb{P}(v_h|s_h,\theta) - \mathbb{P}(v_h|s_l,\theta))} < c^d$.

In this case, the distributor's profit is

$$\Pi_D^{Mix} = \mathbb{P}(s_h|\theta)(\mathbb{P}(v_h|s_h,\theta)v_h + \mathbb{P}(v_l|s_h,\theta)y - w) + \gamma(1-\beta)q - \pi_r.$$

Proof. In strategy (d), the retailer sets $r = v_h^-$, $p = p_h(v_h^-,\theta) \approx v_h$. Then, $\mathbb{P}(\mathrm{B}(v_h,v_h)|\theta) = \mathbb{P}(s_h|\theta)$, $\mathbb{P}(\mathrm{R}(v_h,v_h)|\theta) = \mathbb{P}(s_h,v_l|\theta)$, and $\mathbb{P}(\mathrm{S}(v_h,v_h)|\theta) = \mathbb{P}(s_h,v_h|\theta)$. Namely, consumers that received a high signal s_h purchase the product, and the consumers among them that realize they had a low valuation v_l return the product. The distributor's profit in this case is

$$\Pi_D(c,u,\theta) = \mathbb{P}(s_h|\theta)(c-w) - \mathbb{P}(s_h,v_l|\theta)(u-y) + \gamma\mathbb{P}(s_h,v_h|\theta).$$

From Assumption 2 it follows that the retailer will never choose strategy (a) in Theorem 1. Moreover, the retailer will choose strategy (d) over (b) if $\Pi_R^d \geq \Pi_R^b$, i.e., if

$$\mathbb{P}(s_l|\theta)(\mathbb{P}(v_h|s_l,\theta)(v_h - c) + \mathbb{P}(v_l|s_l,\theta)(u-c)) \leq 0.$$

This is the first incentive compatibility constraint, (IC_1), that the distributor must satisfy to induce the retailer to implement strategy (d).

Similarly, the retailer will choose strategy (d) over (c) if $\Pi_R^b \geq \Pi_R^d$, i.e., if

$$\mathbb{P}(v_h|s_h,\theta)v_h + \mathbb{P}(v_l|s_h,\theta)u \geq p_h(0,\theta).$$

This is the second incentive compatibility constraint (IC_2).

Furthermore, the retailer needs to prefer strategy (d) to the status-quo product that offers a profit of π_r. Thus, in order for the retailer to use strategy (d), we must have

$$\mathbb{P}(s_h|\theta)(\mathbb{P}(v_h|s_h,\theta)(v_h - c) + \mathbb{P}(v_l|s_h,\theta)(u - c)) \geq \pi_r.$$

This is the individual rationality, (IR), constraint. With these constraints in hand, the distributor can determine the optimal price c, and refund u, by solving

$$\begin{aligned}
\max_{c,u} \quad & \mathbb{P}(s_h|\theta)(c-w) - \mathbb{P}(s_h,v_l|\theta)(u-y) + \gamma\mathbb{P}(s_h,v_h|\theta) \\
\text{s.t.} \quad & \mathbb{P}(s_l|\theta)(\mathbb{P}(v_h|s_l,\theta)(v_h - c) + \mathbb{P}(v_l|s_l,\theta)(u - c)) \leq 0 \qquad (\text{IC}_1) \\
& \mathbb{P}(v_h|s_h,\theta)v_h + \mathbb{P}(v_l|s_h,\theta)u \geq p_h(0,\theta) \qquad (\text{IC}_2) \\
& \mathbb{P}(s_h|\theta)(\mathbb{P}(v_h|s_h,\theta)(v_h - c) + \mathbb{P}(v_l|s_h,\theta)(u - c)) \geq \pi_r \qquad (\text{IR}) \\
& 0 \leq u \leq c.
\end{aligned} \tag{26}$$

Assume that the feasible set of problem (26) is non-empty. Then, note that the (IR) constraint must be tight at optimality, otherwise we could increase c without bound, strictly increasing the distributor's profits.

Assume that the (IR) constraint is tight. Then, by replacing $c^d = \mathbb{P}(v_h|s_h,\theta)v_h + \mathbb{P}(v_l|s_h,\theta)u - \frac{\pi_r}{\mathbb{P}(s_h|\theta)}$, and recognizing terms, we can re-write problem (26) as

$$\begin{aligned}
\max_{u} \quad & \mathbb{P}(s_h|\theta)(\mathbb{P}(v_h|s_h,\theta)v_h + \mathbb{P}(v_l|s_h,\theta)y - w) + \gamma(1-\beta)q - \pi_r \\
\text{s.t.} \quad & u \leq v_h - \frac{\pi_r}{\mathbb{P}(s_h|\theta)(\mathbb{P}(v_h|s_h,\theta) - \mathbb{P}(v_h|s_l,\theta))} \qquad (\text{IC}_1) \\
& u \geq \frac{p_h(0,\theta)}{\mathbb{P}(v_l|s_h,\theta)} - \frac{\mathbb{P}(v_h|s_h,\theta)}{\mathbb{P}(v_h|s_h,\theta)v_h} v_h \qquad (\text{IC}_2) \\
& 0 \leq u \leq v_h - \frac{\pi_r}{\mathbb{P}(s_h|\theta)\mathbb{P}(v_h|s_h,\theta)}.
\end{aligned} \tag{27}$$

Note that the distributor's objective function is independent of the actual value of the refund offered to the retailer u, as long as it is feasible. This is due to the fact that any feasible refund u that is chosen is counterbalanced by an appropriate price to the retailer c^d, such that the (IR) constraint is tight, i.e. the distributor extracts all the retailer's utility beyond the reservation profit π_r. In particular, assuming that the feasible set of problem (26) is non-empty, then setting the refund equal to its upper bound $u^d = v_h - \frac{\pi_r}{\mathbb{P}(s_h|\theta)(\mathbb{P}(v_h|s_h,\theta) - \mathbb{P}(v_h|s_l,\theta))}$, will be feasible.

Moreover, the feasible set of problem (26) is non-empty if and only if

$$v_h - \frac{\pi_r}{\mathbb{P}(s_h|\theta)(\mathbb{P}(v_h|s_h,\theta) - \mathbb{P}(v_h|s_l,\theta))} \geq \max\left\{\frac{p_h(0,\theta)}{\mathbb{P}(v_l|s_h,\theta)} - \frac{\mathbb{P}(v_h|s_h,\theta)}{\mathbb{P}(v_h|s_h,\theta)v_h} v_h, 0\right\}.$$

Namely, if the upper bound for u defined by constraint (IC_1) in problem (27), is larger than its lower bounds defined by constraint (IC_2) in problem (27) and zero. This follows from the observation that the second upper bound on u in problem (27) is redundant with (IC_1). □

Proposition 1. *Let $\Theta := \left[\frac{1}{2}, 1\right]$, $Y := [0, w]$. Additionally, let $LOG(\gamma, \pi_R)$, $INFO(\gamma, \pi_R)$, and $MIX(\gamma, \pi_R)$, be the partition of $\Theta \times Y$ given by*

$$LOG(\gamma,\pi_R) := \left\{(\theta,y) \in \Theta \times Y : y \geq \max\{g_{LI}(\theta,\gamma)\mathbb{1}_{\{f_I(\theta,\pi_R)\geq 0\}}, g_{LM}(\theta,\gamma)\mathbb{1}_{\{f_M(\theta,\pi_R)\geq 0\}}\}\right\}, \tag{12}$$

$$INFO(\gamma,\pi_R) := \left\{(\theta,y) \in \Theta \times Y : f_I(\theta,\pi_R) \geq 0, y \leq \min\{g_{LI}(\theta,\gamma), g_{MI}(\theta)\mathbb{1}_{\{f_M(\theta,\pi_R)\geq 0\}} + w\mathbb{1}_{\{f_M(\theta,\pi_R)<0\}}\}\right\}, \tag{13}$$

$$MIX(\gamma,\pi_R) := \left\{(\theta,y) \in \Theta \times Y : f_M(\theta,\pi_R) \geq 0, g_{LM}(\theta,\gamma) \geq y \geq g_{MI}(\theta)\mathbb{1}_{\{f_I(\theta,\pi_R)\geq 0\}}\right\}. \tag{14}$$

Then, if $(\theta, y) \in LOG(\gamma, \pi_R)$, the dominant strategy for the distributor is the pure logistics strategy Log; if $(\theta, y) \in INFO(\gamma, \pi_R)$, the dominant strategy for the distributor is the pure information strategy Info; and if $(\theta, y) \in MIX(\gamma, \pi_R)$, the dominant strategy for the distributor is the mixed strategy Mix. Moreover, the sets $LOG(\gamma, \pi_R)$ and $MIX(\gamma, \pi_R)$ are guaranteed to be non-empty.

Proof. From the definitions of Π_D^{Log}, Π_D^{Info}, and Π_D^{Mix}, in Theorem 2, as well as $g_{LM}(\theta, \gamma)$, and $g_{MI}(\theta)$ in equations (9) and (10), respectively, it follows directly that $\Pi_D^{Log} \geq \Pi_D^{Mix}$ if and only if $s \geq g_{LM}(\theta, \gamma)$, and $\Pi_D^{Mix} \geq \Pi_D^{Info}$ if and only if $y \geq g_{MI}(\theta)$.

Additionally, note that

$$\begin{aligned}
& \Pi_D^{Log} \geq \Pi_D^{Info} \\
\iff & \mathbb{P}(v_h)v_h + \mathbb{P}(v_l)y - w + \gamma\mathbb{P}(v_h) - \mathbb{P}(s_h|\theta)(p_h(0,\theta) - w) - \gamma\mathbb{P}(v_h, s_h|\theta) \geq 0 \\
\iff & \mathbb{P}(v_h)v_h + \mathbb{P}(v_l)y - \mathbb{P}(s_l|\theta)w + \gamma\mathbb{P}(v_h, s_l|\theta) - \mathbb{P}(s_h|\theta)p_h(0,\theta) \geq 0 \\
\iff & y \geq \theta\left(w - \frac{\mathbb{P}(v_h|s_l,\theta)}{\mathbb{P}(v_l|s_l,\theta)}(v_h + \gamma - w)\right) + (1-\theta)\left(p_h(0,\theta) - \frac{\mathbb{P}(v_h|s_h,\theta)}{\mathbb{P}(v_l|s_h,\theta)}(v_h - p_h(0,\theta))\right) \\
\iff & y \geq g_{LI}(\theta,\gamma).
\end{aligned}$$

Then, $\Pi_D^{Log} = \max\left\{\Pi_D^{Log}, \Pi_D^{Info}\mathbb{1}_{\{f_I(\theta)\geq 0\}}, \Pi_D^{Mix}\mathbb{1}_{\{f_M(\theta)\geq 0\}}\right\}$ if and only if $\Pi_D^{Log} \geq \Pi_D^{Info}\mathbb{1}_{\{f_I(\theta)\geq 0\}}$ and $\Pi_D^{Log} \geq \Pi_D^{Mix}\mathbb{1}_{\{f_M(\theta)\geq 0\}}$, or equivalently $y \geq g_{LI}(\theta,\gamma)\mathbb{1}_{\{f_I(\theta)\geq 0\}}$ and $y \geq g_{LM}(\theta,\gamma)\mathbb{1}_{\{f_M(\theta)\geq 0\}}$. This completes the characterization of the set $LOG(\gamma, \pi_R)$. The characterizations of the sets $INFO(\gamma, \pi_R)$ and $MIX(\gamma, \pi_R)$ are analogous, and are ommited for the sake of brevity.

Now we show that the set $LOG(\gamma, \pi_R)$ is guaranteed to be non-empty. Note that $g_{LM}(\theta, \gamma) < w$, for any $\theta < 1$. Moreover, from Lemma 2 it follows that $g_{MI}(\theta) < w$, for any $\theta < 1$. From the definition of $g_{LM}(\theta, \gamma)$ in equation (11), it then follows that $g_{LI}(\theta, \gamma) < w$, for any $\theta < 1$. Therefore, $\max\{g_{LI}(\theta,\gamma)\mathbb{1}_{\{f_c(\theta)\geq 0\}}, g_{LM}(\theta,\gamma)\mathbb{1}_{\{f_M(\theta)\geq 0\}}\} < w$, for any $\theta < 1$. Hence, $(\theta, w) \in LOG(\gamma, \pi_R)$, for any $\theta \in \left[\frac{1}{2}, 1\right]$.

To conclude, we show that the set $MIX(\gamma, \pi_R)$ is guaranteed to be non-empty. In particular, if $\theta = 1$ then $f_I(1) = 0$, $f_M(1) = v_h - \frac{\pi_r}{1-\beta} > 0$, $g_{LM}(1, \gamma) = w$, $g_{MI}(1) = 0$. Hence, $(1, s) \in MIX(\gamma, \pi_R)$, for any $s \in [0, w]$. □

Corollary 4. *Let π_R^1, π_R^2, be such that $\pi_R^1 > \pi_R^2 \geq 0$. Then $LOG(\gamma,\pi_R^2) \subseteq LOG(\gamma,\pi_R^1)$, for any $\gamma \geq 0$. Namely, a more profitable retailer's outside option makes the distributor more likely to select the pure logistics strategy Log.*

Proof. We define the following functions in order to simplify the notation and exposition. Let,

$$h_1(\theta,\pi_R) := \frac{p_h(0,\theta)}{\beta} - v_h\frac{1-\beta}{\beta} - \pi_r\frac{\mathbb{P}(s_l|\theta)}{\mathbb{P}(s_h|\theta)\beta}, \quad h_2(\theta) := \frac{p_h(0,\theta)}{\mathbb{P}(v_l|s_h,\theta)} - v_h\frac{\mathbb{P}(v_h|s_h,\theta)}{\mathbb{P}(v_l|s_h,\theta)}.$$

Note that, for any θ, $\frac{\partial f_I(\theta,\pi_R)}{\partial \pi_R} = -\frac{\mathbb{P}(s_l|\theta)}{\mathbb{P}(s_h|\theta)}\mathbb{1}_{\{h_1(\theta,\pi_R)\leq h_2(\theta)\}} \leq 0$, $\frac{\partial f_M(\theta)}{\partial \pi_R} = -\frac{1}{\mathbb{P}(s_h|\theta)(\mathbb{P}(v_h|s_h,\theta)-\mathbb{P}(v_h|s_l,\theta))} < 0$. Therefore, for any $\pi_R^1 > \pi_R^2 \geq 0$,

$$\max\left\{g_{LI}(\theta,\gamma)\mathbb{1}_{\left\{f_I(\theta,\pi_R^1)\geq 0\right\}}, g_{LM}(\theta,\gamma)\mathbb{1}_{\left\{f_M(\theta,\pi_R^1)\geq 0\right\}}\right\} \geq \max\left\{g_{LI}(\theta,\gamma)\mathbb{1}_{\left\{f_I(\theta,\pi_R^2)\geq 0\right\}}, g_{LM}(\theta,\gamma)\mathbb{1}_{\left\{f_M(\theta,\pi_R^2)\geq 0\right\}}\right\}.$$

Hence, we conclude $LOG(\gamma,\pi_R^2) \subseteq LOG(\gamma,\pi_R^2)$ for any $\pi_R^1 > \pi_R^2 \geq 0$. □

Corollary 5. *Let γ_1, γ_2, be such that $\gamma_1 > \gamma_2 \geq 0$. Then $LOG(\gamma_2,\pi_R) \subset LOG(\gamma_1,\pi_R)$, for any $\pi_R \geq 0$. Namely, a larger weight on consumer satisfaction makes the distributor more likely to select the pure logistics strategy Log.*

Proof. From Proposition 1 we have that $MIX(\gamma)$ is non-empty. It follows that there exists a $\theta \in \left[\frac{1}{2},1\right]$ such that $\max\{g_{LI}(\theta,\gamma)\mathbb{1}_{\{f_I(\theta)\geq 0\}}, g_{LM}(\theta,\gamma)\mathbb{1}_{\{f_M(\theta)\geq 0\}}\} > 0$.

Note that, for any θ, $\frac{\partial g_{LM}(\theta,\gamma)}{\partial \gamma} = -\frac{\mathbb{P}(v_h|s_l,\theta)}{\mathbb{P}(v_l|s_l,\theta)} < 0$. Therefore, $\frac{\partial g_{LI}(\theta,\gamma)}{\partial \gamma} = -q\frac{\mathbb{P}(v_h|s_l,\theta)}{\mathbb{P}(v_l|s_l,\theta)} < 0$. Hence, for any θ, either $\max\{g_{LI}(\theta,\gamma)\mathbb{1}_{\{f_I(\theta)\geq 0\}}, g_{LM}(\theta,\gamma)\mathbb{1}_{\{f_M(\theta)\geq 0\}}\} = 0$, or $\frac{\partial \max\{g_{LI}(\theta,\gamma)\mathbb{1}_{\{f_I(\theta)\geq 0\}}, g_{LM}(\theta,\gamma)\mathbb{1}_{\{f_M(\theta)\geq 0\}}\}}{\partial \gamma} < 0$. Hence, we conclude $LOG(\gamma_2) \subset LOG(\gamma_1)$ for any $\gamma_1 > \gamma_2 \geq 0$. □

6.2. Proofs of Section 4

6.2.1. Some properties of the failure rate of a standard normal distribution $\frac{\phi(x)}{1-\Phi(x)}$.

Lemma 3.

$$0 < \left(\frac{\phi(x)}{1-\Phi(x)}\right)' = \frac{\phi(x)}{1-\Phi(x)}\left(\frac{\phi(x)}{1-\Phi(x)} - x\right) < 1.$$

We omit the proof for the sake of brevity.

Lemma 4. *$\frac{\phi(x)}{1-\Phi(x)}$ is convex for any x.*

We omit the proof for the sake of brevity.

Lemma 5. *For any x we have that*

$$\left(x+\frac{\phi(x)}{\Phi(x)}\right)\frac{1-\Phi(x)}{\phi(x)}\Phi(x) + \left(x+\frac{\phi(x)}{\Phi(x)}\right)\left(\frac{\phi(x)}{1-\Phi(x)} - x\right)(1-\Phi(x)) \leq 1$$

Proof. We can rewrite the left hand side of the inequality in the lemma as

$$\left(x+\frac{\phi(x)}{\Phi(x)}\right)(1-\Phi(x))\left(\frac{\Phi(x)}{\phi(x)} + \frac{\phi(x)}{1-\Phi(x)} - x\right).$$

If $x \geq 0$, note that

$$\frac{\Phi(x)}{\phi(x)} = \frac{1}{\phi(x)} - \frac{1-\Phi(x)}{\phi(x)} \leq \frac{1}{\phi(x)} - \frac{\sqrt{4+x^2}-x}{2}.$$

Also,

$$\frac{\phi(x)}{1-\Phi(x)} \leq \frac{\sqrt{4+x^2}+x}{2}.$$

Both inequalities are due to Komatu (1955) Komatu (1955). Thus, we have

$$\frac{\Phi(x)}{\phi(x)} + \frac{\phi(x)}{1-\Phi(x)} - x \leq \frac{1}{\phi(x)} - \frac{\sqrt{4+x^2}-x}{2} + \frac{\sqrt{4+x^2}+x}{2} - x = \frac{1}{\phi(x)}$$

Since $1-\Phi(x) \leq \frac{2\phi(x)}{\sqrt{2+x^2+x}}$, we have

$$\left(x + \frac{\phi(x)}{\Phi(x)}\right)(1-\Phi(x)) \leq \phi(x)\left(\frac{2x}{\sqrt{2+x^2}+x} + \frac{1-\Phi(x)}{\Phi(x)}\right)$$

Putting everything together,

$$\left(x + \frac{\phi(x)}{\Phi(x)}\right)(1-\Phi(x))\left(\frac{\Phi(x)}{\phi(x)} + \frac{\phi(x)}{1-\Phi(x)} - x\right) \leq \frac{2x}{\sqrt{2+x^2}+x} + \frac{1-\Phi(x)}{\Phi(x)} = \frac{(1+\Phi(x))x + (1-\Phi(x))\sqrt{2+x^2}}{\Phi(x)(\sqrt{2+x^2}+x)}$$

Rewriting the right hand side of the equality above, we obtain

$$\frac{(1+\Phi(x))x + (1-\Phi(x))\sqrt{2+x^2}}{\Phi(x)(\sqrt{2+x^2}+x)} = 1 - \frac{(2\Phi(x)-1)\sqrt{2+x^2}-x}{\Phi(x)(\sqrt{2+x^2}+x)} \leq 1$$

The last inequality comes from the fact that, for $x \geq 0$, $\Phi(x) \geq 1/2$ and $\sqrt{2+x^2} \geq x$.

If $x < 0$, a similar analysis completes the proof. □

6.2.2. Some properties of the retailer's pricing function $p(r, v_0)$. Recall that

$$\begin{aligned} p(r,v_0) &= -\frac{1}{\alpha}\ln\left(\mathbb{E}_\epsilon[e^{-\alpha\max(v_0+\epsilon,r)}|\theta]\right). \\ &= \frac{-1}{\alpha}\ln\left(e^{-\alpha r}\Phi\left(\frac{r-v_0}{\sqrt{1-\theta}\sigma}\right) + e^{-\alpha\left(v_0-\alpha\frac{(1-\theta)\sigma^2}{2}\right)}\left(1-\Phi\left(\frac{r-v_0}{\sqrt{1-\theta}\sigma}+\alpha\sqrt{1-\theta}\sigma\right)\right)\right). \end{aligned} \tag{28}$$

Then, the first and second partial derivatives of $p(r, v_0)$ can be compactly written as

$$\partial_r p(r-v_0) = \frac{\dfrac{\phi\left(\frac{r-v_0}{\sqrt{1-\theta\sigma}}+\alpha\sqrt{1-\theta}\sigma\right)}{1-\Phi\left(\frac{r-v_0}{\sqrt{1-\theta\sigma}}+\alpha\sqrt{1-\theta}\sigma\right)}}{\dfrac{\phi\left(\frac{r-v_0}{\sqrt{1-\theta\sigma}}+\alpha\sqrt{1-\theta}\sigma\right)}{1-\Phi\left(\frac{r-v_0}{\sqrt{1-\theta\sigma}}+\alpha\sqrt{1-\theta}\sigma\right)} + \dfrac{\phi\left(\frac{r-v_0}{\sqrt{1-\theta\sigma}}\right)}{\Phi\left(\frac{r-v_0}{\sqrt{1-\theta\sigma}}\right)}}. \tag{29}$$

$$\partial_r^2 p(r-v_0) = \partial_r p(r-v_0)\left(\frac{\phi\left(\frac{r-v_0}{\sqrt{1-\theta\sigma}}\right)}{\Phi\left(\frac{r-v_0}{\sqrt{1-\theta\sigma}}\right)} - \alpha\sqrt{1-\theta}\sigma(1-\partial_r p(r-v_0))\right). \tag{30}$$

$$\partial_{v_0} p(r-v_0) = \frac{\dfrac{\phi\left(\frac{r-v_0}{\sqrt{1-\theta}\sigma}\right)}{\Phi\left(\frac{r-v_0}{\sqrt{1-\theta}\sigma}\right)}}{\dfrac{\phi\left(\frac{r-v_0}{\sqrt{1-\theta}\sigma}+\alpha\sqrt{1-\theta}\sigma\right)}{1-\Phi\left(\frac{r-v_0}{\sqrt{1-\theta}\sigma}+\alpha\sqrt{1-\theta}\sigma\right)}+\dfrac{\phi\left(\frac{r-v_0}{\sqrt{1-\theta}\sigma}\right)}{\Phi\left(\frac{r-v_0}{\sqrt{1-\theta}\sigma}\right)}} = 1-\partial_r p(r-v_0). \tag{31}$$

Note that, as opposed to $p(r,v_0)$, the expressions of $\partial_r p(r,v_0)$, $\partial_r^2 p(r,v_0)$, $\partial_{v_0} p(r,v_0)$ do not depend on the absolute value of the refund r, but only on its difference with respect to v_0, namely on $(r-v_0)$. Therefore, we slightly abuse the notation by making their dependence on $(r-v_0)$ explicit in their argument.

Now we formalize some useful properties of these functions.

Lemma 6. *For any refund r, and estimated valuation v_0, we have that $\partial_r p(v_0-r) = 1-\partial_r p(r-v_0-\alpha(1-\theta)\sigma^2)$, and $\partial_r^2 p(v_0-r) = \partial_r^2 p(r-v_0-\alpha(1-\theta)\sigma^2)$.*

Proof. The proof of the first statement follows directly from the symmetry properties of $\phi(x)$ and $\Phi(x)$. Namely, from $\phi(-x)=\phi(x)$ and $\Phi(-x)=1-\Phi(x)$. Specifically,

$$\partial_r p(v_0-r) = \frac{\frac{\phi\left(\frac{v_0-r}{\sqrt{1-\theta}\sigma}+\alpha\sqrt{1-\theta}\sigma\right)}{1-\Phi\left(\frac{v_0-r}{\sqrt{1-\theta}\sigma}+\alpha\sqrt{1-\theta}\sigma\right)}}{\frac{\phi\left(\frac{v_0-r}{\sqrt{1-\theta}\sigma}+\alpha\sqrt{1-\theta}\sigma\right)}{1-\Phi\left(\frac{v_0-r}{\sqrt{1-\theta}\sigma}+\alpha\sqrt{1-\theta}\sigma\right)}+\frac{\phi\left(\frac{v_0-r}{\sqrt{1-\theta}\sigma}\right)}{\Phi\left(\frac{v_0-r}{\sqrt{1-\theta}\sigma}\right)}} = \frac{\frac{\phi\left(\frac{r-v_0}{\sqrt{1-\theta}\sigma}-\alpha\sqrt{1-\theta}\sigma\right)}{\Phi\left(\frac{r-v_0}{\sqrt{1-\theta}\sigma}-\alpha\sqrt{1-\theta}\sigma\right)}}{\frac{\phi\left(\frac{r-v_0}{\sqrt{1-\theta}\sigma}-\alpha\sqrt{1-\theta}\sigma\right)}{\Phi\left(\frac{r-v_0}{\sqrt{1-\theta}\sigma}-\alpha\sqrt{1-\theta}\sigma\right)}+\frac{\phi\left(\frac{r-v_0}{\sqrt{1-\theta}\sigma}\right)}{1-\Phi\left(\frac{r-v_0}{\sqrt{1-\theta}\sigma}\right)}} = 1-\partial_r p(r-v_0-\alpha(1-\theta)\sigma^2).$$

The proof of the second statement follows from the first statement, and the definition of $\partial_r p(r-v_0)$ and $\partial_r^2 p(r-v_0)$ in equations (29) and (30), respectively. Namely,

$$\begin{aligned}
\partial_r^2 p(v_0-r) &= \partial_r p(v_0-r)\left(\frac{\phi\left(\frac{v_0-r}{\sqrt{1-\theta}\sigma}\right)}{\Phi\left(\frac{v_0-r}{\sqrt{1-\theta}\sigma}\right)} - \alpha\sqrt{1-\theta}\sigma(1-\partial_r p(v_0-r))\right)\\
&= (1-\partial_r p(r-v_0-\alpha(1-\theta)\sigma^2))\left(\frac{\phi\left(\frac{r-v_0}{\sqrt{1-\theta}\sigma}\right)}{1-\Phi\left(\frac{r-v_0}{\sqrt{1-\theta}\sigma}\right)} - \alpha\sqrt{1-\theta}\sigma\partial_r p(r-v_0-\alpha(1-\theta)\sigma^2)\right)\\
&= \partial_r p(r-v_0-\alpha(1-\theta)\sigma^2)\left(\frac{\phi\left(\frac{r-v_0}{\sqrt{1-\theta}\sigma}-\alpha\sqrt{1-\theta}\sigma\right)}{\Phi\left(\frac{r-v_0}{\sqrt{1-\theta}\sigma}-\alpha\sqrt{1-\theta}\sigma\right)} - \alpha\sqrt{1-\theta}\sigma(1-\partial_r p(r-v_0-\alpha(1-\theta)\sigma^2))\right)\\
&= \partial_r^2 p(r-v_0-\alpha(1-\theta)\sigma^2).
\end{aligned}$$

□

Lemma 7. *For any refund r, estimated valuation v_0, and constant absolute risk aversion parameter $\alpha>0$, we have that $\Phi\left(\frac{r-v_0}{\sqrt{1-\theta}\sigma}\right) < \partial_r p(r-v_0) < \Phi\left(\frac{r-v_0}{\sqrt{1-\theta}\sigma}+\alpha\sqrt{1-\theta}\sigma\right)$.*

Proof. First note that from the definition of $p'(r-z)$ in equation (29) we have that

$$\partial_r p(r-v_0) - \Phi\left(\frac{r-v_0}{\sqrt{1-\theta}\sigma}\right) = \left(1-\Phi\left(\frac{r-v_0}{\sqrt{1-\theta}\sigma}\right)\right)\frac{\frac{\phi\left(\frac{r-v_0}{\sqrt{1-\theta}\sigma}+\alpha\sqrt{1-\theta}\sigma\right)}{1-\Phi\left(\frac{r-v_0}{\sqrt{1-\theta}\sigma}+\alpha\sqrt{1-\theta}\sigma\right)} - \frac{\phi\left(\frac{r-v_0}{\sqrt{1-\theta}\sigma}\right)}{1-\Phi\left(\frac{r-v_0}{\sqrt{1-\theta}\sigma}\right)}}{\frac{\phi\left(\frac{r-v_0}{\sqrt{1-\theta}\sigma}+\alpha\sqrt{1-\theta}\sigma\right)}{1-\Phi\left(\frac{r-v_0}{\sqrt{1-\theta}\sigma}+\alpha\sqrt{1-\theta}\sigma\right)}+\frac{\phi\left(\frac{r-v_0}{\sqrt{1-\theta}\sigma}\right)}{\Phi\left(\frac{r-v_0}{\sqrt{1-\theta}\sigma}\right)}} > 0,$$

where the inequality follows from the failure rate of a standard normal being increasing.

To conclude note that the upper bound follows from the lower bound and Lemma 6. Specifically, we have that $\Phi\left(\frac{v_0-r}{\sqrt{1-\theta}\sigma}\right) < \partial_r p(v_0-r)$ is equivalent to $1-\Phi\left(\frac{r-v_0}{\sqrt{1-\theta}\sigma}\right) < 1-\partial_r p(r-v_0-\alpha(1-\theta)\sigma^2)$, which is equivalent to the upper bound in the statement of the lemma. □

6.2.3. Some properties of the probability of return $\mathbb{P}(V_0 \geq v_0, V \leq r)$. First, note that

$$\begin{aligned}\mathbb{P}(V_0 \geq v_0, V \leq r|\theta) &= \int_{v_0}^{\infty}\int_{-\infty}^{r-v} \phi\left(\frac{x}{\sqrt{1-\theta}\sigma}\right)\frac{1}{\sqrt{1-\theta}\sigma}dx\phi\left(\frac{v-\mu}{\sqrt{\theta}\sigma}\right)\frac{1}{\sqrt{\theta}\sigma}dv \\ &= \int_{v_0}^{\infty}\Phi\left(\frac{r-v}{\sqrt{1-\theta}\sigma}\right)\phi\left(\frac{v-\mu}{\sqrt{\theta}\sigma}\right)\frac{1}{\sqrt{\theta}\sigma}dv.\end{aligned} \tag{32}$$

Where we have used that $V_0 \sim N(\mu, \theta\sigma^2)$, $\epsilon = V - V_0 \sim N(0,(1-\theta)\sigma^2)$.

Then, the first and second partial derivatives of $\mathbb{P}(V_0 \geq v_0, V \leq r)$ can be compactly written as follows. First, from the dominated convergence theorem, it follows that we can interchange the integral and derivative and conclude

$$\begin{aligned}\partial_r\mathbb{P}(V_0 \geq v_0, V \leq r) &= \int_{v_0}^{\infty}\phi\left(\frac{r-v}{\sqrt{1-\theta}\sigma}\right)\frac{1}{\sqrt{1-\theta}\sigma}\phi\left(\frac{v-\mu}{\sqrt{\theta}\sigma}\right)\frac{1}{\sqrt{\theta}\sigma}dv \\ &= \int_{v_0}^{\infty}\phi\left(\frac{v-\theta r-(1-\theta)\mu}{\sqrt{(1-\theta)\theta}\sigma}\right)\frac{1}{\sqrt{(1-\theta)\theta}\sigma}dv\phi\left(\frac{r-\mu}{\sigma}\right)\frac{1}{\sigma} \\ &= \left(1-\Phi\left(\frac{v_0-\theta r-(1-\theta)\mu}{\sqrt{(1-\theta)\theta}\sigma}\right)\right)\phi\left(\frac{r-\mu}{\sigma}\right)\frac{1}{\sigma}.\end{aligned} \tag{33}$$

Additionally, we have that

$$\begin{aligned}\partial_r^2\mathbb{P}(V_0 \geq v_0, V \leq r) &= \phi\left(\frac{v_0-\theta r-(1-\theta)\mu}{\sqrt{(1-\theta)\theta}\sigma}\right)\phi\left(\frac{r-\mu}{\sigma}\right)\sqrt{\frac{\theta}{1-\theta}}\frac{1}{\sigma^2} \\ &\quad - \left(1-\Phi\left(\frac{v_0-\theta r-(1-\theta)\mu}{\sqrt{(1-\theta)\theta}\sigma}\right)\right)\phi\left(\frac{r-\mu}{\sigma}\right)\frac{r-\mu}{\sigma^2} \\ &= \phi\left(\frac{v_0-\theta r-(1-\theta)\mu}{\sqrt{(1-\theta)\theta}\sigma}\right)\phi\left(\frac{r-\mu}{\sigma}\right)\sqrt{\frac{\theta}{1-\theta}}\frac{1}{\sigma^2} - \partial_r\mathbb{P}(V_0 \geq v_0, V \leq r)\frac{r-\mu}{\sigma}.\end{aligned} \tag{34}$$

Now we prove a couple of useful bounds.

Lemma 8. *For any refund r, and estimated valuation v_0, we have that*

$$\mathbb{P}(V_0 \geq v_0, V \leq r) < \partial_r p(r-v_0)\mathbb{P}(V_0 \geq v_0).$$

Proof. We have that $\mathbb{P}(V_0 \geq v_0, V \leq r) = \int_{v_0}^{\infty}\Phi\left(\frac{r-v}{\sqrt{1-\theta}\sigma}\right)\phi\left(\frac{v-\mu}{\sqrt{\theta}\sigma}\right)\frac{1}{\sqrt{\theta}\sigma}dv < \Phi\left(\frac{r-v_0}{\sqrt{1-\theta}\sigma}\right)\left(1-\Phi\left(\frac{v_0-\mu}{\sqrt{\theta}\sigma}\right)\right) < \partial_r p(r-v_0)\left(1-\Phi\left(\frac{v_0-\mu}{\sqrt{\theta}\sigma}\right)\right) = \partial_r p(r-v_0)\mathbb{P}(V_0 \geq v_0)$. The first inequality follows from taking the integration variable v equal to its lower bound v_0 in the first term in the integrand. The second inequality follows from Lemma 7. □

Lemma 9. *For any refund r, and estimated valuation v_0, we have that*

$$\partial_r \mathbb{P}(V_0 \geq v_0, V \leq r) > \frac{\phi\left(\frac{r-v_0}{\sqrt{1-\theta}\sigma}\right)}{\Phi\left(\frac{r-v_0}{\sqrt{1-\theta}\sigma}\right)} \mathbb{P}(V_0 \geq v_0, V \leq r).$$

Proof. We have that

$$\begin{aligned}
\partial_r \mathbb{P}(V_0 \geq v_0, V \leq r) &= \int_{v_0}^{\infty} \phi\left(\frac{r-v}{\sqrt{1-\theta}\sigma}\right) \frac{1}{\sqrt{1-\theta}\sigma} \phi\left(\frac{v-\mu}{\sqrt{\theta}\sigma}\right) \frac{1}{\sqrt{\theta}\sigma} dv \\
&= \int_{v_0}^{\infty} \frac{\phi\left(\frac{r-v}{\sqrt{1-\theta}\sigma}\right)}{\Phi\left(\frac{r-v}{\sqrt{1-\theta}\sigma}\right)} \frac{1}{\sqrt{1-\theta}\sigma} \Phi\left(\frac{r-v}{\sqrt{1-\theta}\sigma}\right) \phi\left(\frac{v-\mu}{\sqrt{\theta}\sigma}\right) \frac{1}{\sqrt{\theta}\sigma} dv \\
&> \frac{\phi\left(\frac{r-v}{\sqrt{1-\theta}\sigma}\right)}{\Phi\left(\frac{r-v}{\sqrt{1-\theta}\sigma}\right)} \frac{1}{\sqrt{1-\theta}\sigma} \mathbb{P}(V_0 \geq v_0, V \leq r)
\end{aligned}$$

The inequality follows from noticing that $\frac{\phi(x)}{\Phi(x)}$ is decreasing, and taking the integration variable v equal to its lower bound v_0 in the first term in the integrand. □

Lemma 10. *For any refund r, and estimated valuation v_0, we have that*

$$\frac{\alpha\sqrt{1-\theta}\sigma \partial_r p(r-v_0)(1-\partial_r p(r-v_0))\mathbb{P}(V_0 \geq v_0)}{\partial_r p(r-v_0)\mathbb{P}(V_0 \geq v_0) - \mathbb{P}(V_0 \geq v_0, V \leq r)} - \frac{\phi\left(\frac{r-v_0}{\sqrt{1-\theta}\sigma}\right)}{\Phi\left(\frac{r-v_0}{\sqrt{1-\theta}\sigma}\right)} < \frac{\partial_r \mathbb{P}(V_0 \geq v_0, V \leq r) - \partial_r^2 p(r-v_0)\mathbb{P}(V_0 \geq v_0)}{\partial_r p(r-v_0)\mathbb{P}(V_0 \geq v_0) - \mathbb{P}(V_0 \geq v_0, V \leq r)}.$$

Proof. We have that

$$\begin{aligned}
&\frac{\partial_r \mathbb{P}(V_0 \geq v_0, V \leq r) - \partial_r^2 p(r-v_0)\mathbb{P}(V_0 \geq v_0)}{\partial_r p(r-v_0)\mathbb{P}(V_0 \geq v_0) - \mathbb{P}(V_0 \geq v_0, V \leq r)} \\
&= \frac{\alpha\sqrt{1-\theta}\sigma \partial_r p(r-v_0)(1-\partial_r p(r-v_0))\mathbb{P}(V_0 \geq v_0)}{\partial_r p(r-v_0)\mathbb{P}(V_0 \geq v_0) - \mathbb{P}(V_0 \geq v_0, V \leq r)} - \frac{\frac{\phi\left(\frac{r-v_0}{\sqrt{1-\theta}\sigma}\right)}{\Phi\left(\frac{r-v_0}{\sqrt{1-\theta}\sigma}\right)} \partial_r p(r-v_0) - \partial_r \mathbb{P}(V_0 \geq v_0, V \leq r)}{\partial_r p(r-v_0)\mathbb{P}(V_0 \geq v_0) - \mathbb{P}(V_0 \geq v_0, V \leq r)} \\
&> \frac{\alpha\sqrt{1-\theta}\sigma \partial_r p(r-v_0)(1-\partial_r p(r-v_0))\mathbb{P}(V_0 \geq v_0)}{\partial_r p(r-v_0)\mathbb{P}(V_0 \geq v_0) - \mathbb{P}(V_0 \geq v_0, V \leq r)} - \frac{\phi\left(\frac{r-v_0}{\sqrt{1-\theta}\sigma}\right)}{\Phi\left(\frac{r-v_0}{\sqrt{1-\theta}\sigma}\right)}.
\end{aligned}$$

The equality follows from equation (30). The inequality follows from Lemma 9. □

Lemma 11. *For any refund r, and estimated valuation v_0, we have that*

$$\partial_r p(r-v_0) - \mathbb{P}(V \leq r | V_0 \geq v_0) \leq \left(1 - \Phi\left(\frac{r-v_0}{\sqrt{1-\theta}\sigma}\right)\right) \frac{\frac{\phi\left(\frac{r-v_0}{\sqrt{1-\theta}\sigma}+\alpha\sqrt{1-\theta}\sigma\right)}{1-\Phi\left(\frac{r-v_0}{\sqrt{1-\theta}\sigma}+\alpha\sqrt{1-\theta}\sigma\right)} - \frac{\phi\left(\frac{r-v_0}{\sqrt{1-\theta}\sigma}\right)}{1-\Phi\left(\frac{r-v_0}{\sqrt{1-\theta}\sigma}\right)}}{\frac{\phi\left(\frac{r-v_0}{\sqrt{1-\theta}\sigma}+\alpha\sqrt{1-\theta}\sigma\right)}{1-\Phi\left(\frac{r-v_0}{\sqrt{1-\theta}\sigma}+\alpha\sqrt{1-\theta}\sigma\right)} + \frac{\phi\left(\frac{r-v_0}{\sqrt{1-\theta}\sigma}\right)}{\Phi\left(\frac{r-v_0}{\sqrt{1-\theta}\sigma}\right)}}.$$

Proof. We have that

$$\begin{aligned}
&\partial_r p(r-v_0) - \mathbb{P}(V \le r | V_0 \ge v_0) \\
&= \frac{\frac{\phi\left(\frac{r-v_0}{\sqrt{1-\theta}\sigma}+\alpha\sqrt{1-\theta}\sigma\right)}{1-\Phi\left(\frac{r-v_0}{\sqrt{1-\theta}\sigma}+\alpha\sqrt{1-\theta}\sigma\right)}(1-\mathbb{P}(V\le r|V_0\ge v_0)) - \mathbb{P}(V\le r|V_0\ge v_0)\frac{\phi\left(\frac{r-v_0}{\sqrt{1-\theta}\sigma}\right)}{\Phi\left(\frac{r-v_0}{\sqrt{1-\theta}\sigma}\right)}}{\frac{\phi\left(\frac{r-v_0}{\sqrt{1-\theta}\sigma}+\alpha\sqrt{1-\theta}\sigma\right)}{1-\Phi\left(\frac{r-v_0}{\sqrt{1-\theta}\sigma}+\alpha\sqrt{1-\theta}\sigma\right)} + \frac{\phi\left(\frac{r-v_0}{\sqrt{1-\theta}\sigma}\right)}{\Phi\left(\frac{r-v_0}{\sqrt{1-\theta}\sigma}\right)}} \\
&\le \left(1-\Phi\left(\frac{r-v_0}{\sqrt{1-\theta}\sigma}\right)\right)\frac{\frac{\phi\left(\frac{r-v_0}{\sqrt{1-\theta}\sigma}+\alpha\sqrt{1-\theta}\sigma\right)}{1-\Phi\left(\frac{r-v_0}{\sqrt{1-\theta}\sigma}+\alpha\sqrt{1-\theta}\sigma\right)} - \frac{\phi\left(\frac{r-v_0}{\sqrt{1-\theta}\sigma}\right)}{1-\Phi\left(\frac{r-v_0}{\sqrt{1-\theta}\sigma}\right)}}{\frac{\phi\left(\frac{r-v_0}{\sqrt{1-\theta}\sigma}+\alpha\sqrt{1-\theta}\sigma\right)}{1-\Phi\left(\frac{r-v_0}{\sqrt{1-\theta}\sigma}+\alpha\sqrt{1-\theta}\sigma\right)} + \frac{\phi\left(\frac{r-v_0}{\sqrt{1-\theta}\sigma}\right)}{\Phi\left(\frac{r-v_0}{\sqrt{1-\theta}\sigma}\right)}}.
\end{aligned}$$

The equality follows from the definition of $\partial_r p(r-v_0)$ in (29). The inequality follows from $\mathbb{P}(V \le r|V_0 \ge v_0) \ge \Phi\left(\frac{r-v_0}{\sqrt{1-\theta}\sigma}\right) = \mathbb{P}(V \le r|V_0 = v_0)$. □

Proposition 2. *The retailer's profit $\Pi_R(r, v_0)$ is quasi-concave in the refund r, for any given estimated valuation v_0. Moreover, for any v_0, let r^* be the unique maximizer of $\Pi_R(r, v_0)$. Then, $0 \le u < r^* < p(r^*, v_0)$. Namely, partial refunds are optimal for the retailer.*

Proof. We show that $\Pi_R(r, v_0)$ is quasi-concave in r, for any v_0, $\alpha > 0$, and $u \le \mu$.

Specifically, $\partial_r \Pi_R(r, v_0) = \partial_r \left((p(r, v_0) - c)\mathbb{P}(V_0 \ge v_0) - (r-u)\mathbb{P}(V_0 \ge v_0, V \le r)\right) = \partial_r p(r - v_0)\mathbb{P}(V_0 \ge v_0) - (r-u)\partial_r \mathbb{P}(V_0 \ge v_0, V \le r) - \mathbb{P}(V_0 \ge v_0, V \le r)$. From Lemma 8 it follows that

$$\partial_r \Pi_R(r, v_0) > 0 \text{ if and only if } 1 > \frac{(r-u)\partial_r \mathbb{P}(V_0 \ge v_0, V \le r)}{\partial_r p(r-v_0)\mathbb{P}(V_0 \ge v_0) - \mathbb{P}(V_0 \ge v_0, V \le r)}. \tag{35}$$

The quasi-concavity of $\Pi_R(r, v_0)$ with respect to r follows from showing that there exists a unique r^*, such that (35) is satisfied if and only if $r < r^*$. Specifically, the right hand side of inequality (35) is continuous, from Lemma 8 it is negative for any $r < u$, and we show next that it is non-negative and increasing in r for any $r \ge u$. Moreover, it grows without limit as r increases, i.e. it intersects with one exactly once. To see the latter notice that we have

$$\frac{(r-u)\partial_r \mathbb{P}(V_0 \ge v_0, V \le r)}{\partial_r p(r-v_0)\mathbb{P}(V_0 \ge v_0) - \mathbb{P}(V_0 \ge v_0, V \le r)} \ge \frac{\left(\frac{r-\mu}{\sigma}\right)\phi\left(\frac{r-\mu}{\sigma}\right)}{\partial_r p(r-v_0)\mathbb{P}(V_0 \ge v_0) - \mathbb{P}(V_0 \ge v_0, V \le r)} \ge \frac{\left(\frac{r-\mu}{\sigma}\right)\phi\left(\frac{r-\mu}{\sigma}\right)}{1-\Phi\left(\frac{r-\mu}{\sigma}\right)}.$$

The first inequality follows from the assumption $u < v_0$, and the expression for $\partial_r \mathbb{P}(V_0 \ge v_0, V \le r)$ in equation (33). The second inequality is equivalent to $1 \ge \partial_r p(r-v_0)\mathbb{P}(V_0 \ge v_0) + \mathbb{P}(V_0 \le v_0, V \le r)$, which clearly holds since $\partial_r p(r - v_0) < 1$ from Lemma 7. From the observation that the generalized failure rate of a standard Normal random variable is increasing and goes to infinity, it follows that $\frac{(r-u)\partial_r \mathbb{P}(V_0 \ge v_0, V \le r)}{\partial_r p(r-v_0)\mathbb{P}(V_0 \ge v_0) - \mathbb{P}(V_0 \ge v_0, V \le r)}$ goes to infinity as well, as r increases without limit.

To complete the quasi-concavity proof for $\Pi_R(r, v_0)$ with respect to r, we now show that $\frac{(r-u)\partial_r \mathbb{P}(V_0 \ge v_0, V \le r)}{\partial_r p(r-v_0)\mathbb{P}(V_0 \ge v_0) - \mathbb{P}(V_0 \ge v_0, V \le r)}$ is increasing for any $r > u$.

Note that

$$\partial_r\left(\frac{(r-u)\partial_r\mathbb{P}(V_0\geq v_0, V\leq r)}{\partial_r p(r-v_0)\mathbb{P}(V_0\geq v_0)-\mathbb{P}(V_0\geq v_0,V\leq r)}\right)$$
$$=\frac{(r-u)\partial_r\mathbb{P}(V_0\geq v_0, V\leq r)}{\partial_r p(r-v_0)\mathbb{P}(V_0\geq v_0)-\mathbb{P}(V_0\geq v_0,V\leq r)}\left(\frac{\partial_r\mathbb{P}(V_0\geq v_0,V\leq r)-\partial_r^2 p(r-v_0)\mathbb{P}(V_0\geq v_0)}{\partial_r p(r-v_0)\mathbb{P}(V_0\geq v_0)-\mathbb{P}(V_0\geq v_0,V\leq r)}-\frac{r-\mu}{\sigma}\right)$$
$$+\frac{\partial_r\mathbb{P}(V_0\geq v_0,V\leq r)+(r-u)\phi\left(\frac{v_0-\theta r-(1-\theta)\mu}{\sqrt{(1-\theta)\theta\sigma}}\right)\phi\left(\frac{r-\mu}{\sigma}\right)\sqrt{\frac{\theta}{1-\theta}}\frac{1}{\sigma^2}}{\partial_r p(r-v_0)\mathbb{P}(V_0\geq v_0)-\mathbb{P}(V_0\geq v_0,V\leq r)} \tag{36}$$

Where the equality follows from recognizing terms and the expression (34).

To conclude, it is enough to show that

$$\frac{\partial_r\mathbb{P}(V_0\geq v_0,V\leq r)-\partial_r^2 p(r-v_0)\mathbb{P}(V_0\geq v_0)}{\partial_r p(r-v_0)\mathbb{P}(V_0\geq v_0)-\mathbb{P}(V_0\geq v_0,V\leq r)}\geq\frac{r-\mu}{\sigma} \tag{37}$$

Assume, in order to get a contradiction, that there exists an $\bar{r}$ such that $\frac{\partial_r\mathbb{P}(V_0\geq v_0,V\leq\bar{r})-\partial_r^2 p(\bar{r}-v_0)\mathbb{P}(V_0\geq v_0)}{\partial_r p(\bar{r}-v_0)\mathbb{P}(V_0\geq v_0)-\mathbb{P}(V_0\geq v_0,V\leq\bar{r})}<\frac{\bar{r}-\mu}{\sigma}\leq\frac{\bar{r}-v_0}{\sqrt{1-\theta}\sigma}$. Then, from Lemma 10 it follows that $\frac{\alpha\sqrt{1-\theta}\sigma\partial_r p(\bar{r}-v_0)(1-\partial_r p(\bar{r}-v_0))\mathbb{P}(V_0\geq v_0)}{\partial_r p(\bar{r}-v_0)\mathbb{P}(V_0\geq v_0)-\mathbb{P}(V_0\geq v_0,V\leq\bar{r})}-\frac{\phi\left(\frac{\bar{r}-v_0}{\sqrt{1-\theta}\sigma}\right)}{\Phi\left(\frac{\bar{r}-v_0}{\sqrt{1-\theta}\sigma}\right)}<\frac{\bar{r}-v_0}{\sqrt{1-\theta}\sigma}$, which is the starting point for the following chain of inequalities

$$1<\left(\frac{\bar{r}-v_0}{\sqrt{1-\theta}\sigma}+\frac{\phi\left(\frac{\bar{r}-v_0}{\sqrt{1-\theta}\sigma}\right)}{\Phi\left(\frac{\bar{r}-v_0}{\sqrt{1-\theta}\sigma}\right)}\right)\frac{\partial_r p(\bar{r}-v_0)\mathbb{P}(V_0\geq v_0)-\mathbb{P}(V_0\geq v_0,V\leq\bar{r})}{\alpha\sqrt{1-\theta}\sigma\partial_r p(\bar{r}-v_0)(1-\partial_r p(\bar{r}-v_0))\mathbb{P}(V_0\geq v_0)}$$
$$\leq\left(\frac{\bar{r}-v_0}{\sqrt{1-\theta}\sigma}+\frac{\phi\left(\frac{\bar{r}-v_0}{\sqrt{1-\theta}\sigma}\right)}{\Phi\left(\frac{\bar{r}-v_0}{\sqrt{1-\theta}\sigma}\right)}\right)\frac{1-\Phi\left(\frac{\bar{r}-v_0}{\sqrt{1-\theta}\sigma}\right)}{\phi\left(\frac{\bar{r}-v_0}{\sqrt{1-\theta}\sigma}\right)}\frac{\Phi\left(\frac{\bar{r}-v_0}{\sqrt{1-\theta}\sigma}\right)}{\partial_r p(\bar{r}-v_0)}\frac{\frac{\phi\left(\frac{\bar{r}-v_0}{\sqrt{1-\theta}\sigma}+\alpha\sqrt{1-\theta}\sigma\right)}{1-\Phi\left(\frac{\bar{r}-v_0}{\sqrt{1-\theta}\sigma}+\alpha\sqrt{1-\theta}\sigma\right)}-\frac{\phi\left(\frac{\bar{r}-v_0}{\sqrt{1-\theta}\sigma}\right)}{1-\Phi\left(\frac{\bar{r}-v_0}{\sqrt{1-\theta}\sigma}\right)}}{\alpha\sqrt{1-\theta}\sigma}$$
$$<\left(\frac{\bar{r}-v_0}{\sqrt{1-\theta}\sigma}+\frac{\phi\left(\frac{\bar{r}-v_0}{\sqrt{1-\theta}\sigma}\right)}{\Phi\left(\frac{\bar{r}-v_0}{\sqrt{1-\theta}\sigma}\right)}\right)\frac{1-\Phi\left(\frac{\bar{r}-v_0}{\sqrt{1-\theta}\sigma}\right)}{\phi\left(\frac{\bar{r}-v_0}{\sqrt{1-\theta}\sigma}\right)}\frac{\Phi\left(\frac{\bar{r}-v_0}{\sqrt{1-\theta}\sigma}\right)}{\partial_r p(\bar{r}-v_0)}\left(\frac{\phi\left(\frac{\bar{r}-v_0}{\sqrt{1-\theta}\sigma}+\alpha\sqrt{1-\theta}\sigma\right)}{1-\Phi\left(\frac{\bar{r}-v_0}{\sqrt{1-\theta}\sigma}+\alpha\sqrt{1-\theta}\sigma\right)}\right)'$$
$$=\left(\frac{\bar{r}-v_0}{\sqrt{1-\theta}\sigma}+\frac{\phi\left(\frac{\bar{r}-v_0}{\sqrt{1-\theta}\sigma}\right)}{\Phi\left(\frac{\bar{r}-v_0}{\sqrt{1-\theta}\sigma}\right)}\right)\frac{1-\Phi\left(\frac{\bar{r}-v_0}{\sqrt{1-\theta}\sigma}\right)}{\phi\left(\frac{\bar{r}-v_0}{\sqrt{1-\theta}\sigma}\right)}\Phi\left(\frac{\bar{r}-v_0}{\sqrt{1-\theta}\sigma}\right)\left(\frac{\phi\left(\frac{\bar{r}-v_0}{\sqrt{1-\theta}\sigma}+\alpha\sqrt{1-\theta}\sigma\right)}{1-\Phi\left(\frac{\bar{r}-v_0}{\sqrt{1-\theta}\sigma}+\alpha\sqrt{1-\theta}\sigma\right)}\right)'$$
$$+\left(\frac{\bar{r}-v_0}{\sqrt{1-\theta}\sigma}+\frac{\phi\left(\frac{\bar{r}-v_0}{\sqrt{1-\theta}\sigma}\right)}{\Phi\left(\frac{\bar{r}-v_0}{\sqrt{1-\theta}\sigma}\right)}\right)\left(1-\Phi\left(\frac{\bar{r}-v_0}{\sqrt{1-\theta}\sigma}\right)\right)\left(\frac{\phi\left(\frac{\bar{r}-v_0}{\sqrt{1-\theta}\sigma}+\alpha\sqrt{1-\theta}\sigma\right)}{1-\Phi\left(\frac{\bar{r}-v_0}{\sqrt{1-\theta}\sigma}+\alpha\sqrt{1-\theta}\sigma\right)}-\left(\frac{\bar{r}-v_0}{\sqrt{1-\theta}\sigma}+\alpha\sqrt{1-\theta}\sigma\right)\right)$$

$$<\left(\frac{\bar{r}-v_0}{\sqrt{1-\theta}\sigma}+\frac{\phi\left(\frac{\bar{r}-v_0}{\sqrt{1-\theta}\sigma}\right)}{\Phi\left(\frac{\bar{r}-v_0}{\sqrt{1-\theta}\sigma}\right)}\right)\frac{1-\Phi\left(\frac{\bar{r}-v_0}{\sqrt{1-\theta}\sigma}\right)}{\phi\left(\frac{\bar{r}-v_0}{\sqrt{1-\theta}\sigma}\right)}\Phi\left(\frac{\bar{r}-v_0}{\sqrt{1-\theta}\sigma}\right)$$
$$+\left(\frac{\bar{r}-v_0}{\sqrt{1-\theta}\sigma}+\frac{\phi\left(\frac{\bar{r}-v_0}{\sqrt{1-\theta}\sigma}\right)}{\Phi\left(\frac{\bar{r}-v_0}{\sqrt{1-\theta}\sigma}\right)}\right)\left(1-\Phi\left(\frac{\bar{r}-v_0}{\sqrt{1-\theta}\sigma}\right)\right)\left(\frac{\phi\left(\frac{\bar{r}-v_0}{\sqrt{1-\theta}\sigma}\right)}{1-\Phi\left(\frac{\bar{r}-v_0}{\sqrt{1-\theta}\sigma}\right)}-\left(\frac{\bar{r}-v_0}{\sqrt{1-\theta}\sigma}\right)\right)$$
$$\leq 1,$$

a contradiction. The second inequality follows from Lemma 11 and the definition of $(1-\partial_r p(r-v_0))$ in equation (31). The third inequality follows from the convexity of the failure rate of a standard normal distribution, see Lemma 4. The equality follows from the definition of $\partial_r p(r-v_0)$ in equation (29). The fourth inequality follows from (i) $\left(\frac{\phi(r-z+\alpha)}{1-\Phi(r-z+\alpha)}\right)' < 1$, see Lemma 3, and (ii) $\left(\frac{\phi(x+\alpha)}{1-\Phi(x+\alpha)} - (x+\alpha)\right)$ being decreasing in α, and taking the lower bound $\alpha = 0$. Finally, the last inequality follows from Lemma 5.

We have shown that $\Pi_R(r, v_0)$ is quasi-concave in r, for any v_0. Therefore, there exists a unique r^* that maximizes it. Moreover, from equation (35) it follows that $r^* > u \geq 0$. Finally, from the definition of $p(r, v_0)$ in equation (16) we conclude that $p(r, v_0) > r$, for any r. In particular, $p(r^*, v_0) > r^*$. This completes the proof. □

Corollary 6. *Consider a market where consumers have no information about their type ($\theta = 0$), i.e. $V_0 = \mu$ with probability one. Let $z = \frac{\mu - c}{\sigma}$ denote the normalized margin of the product. Then, for any customers' absolute risk aversion parameter $\alpha > 0$, and refund to the retailer u, there exists a unique value $\bar{z}\left(\alpha, u, \frac{\mu}{\sigma}, \pi_r\right)$ such that $\max_r \Pi(r, \mu) < \pi_R$ if and only if $z < \bar{z}\left(\alpha, u, \frac{\mu}{\sigma}, \pi_r\right)$.*

Namely, if the distributor provides no additional information ($\theta = 0$), the product has a small enough normalized margin and the consumers are risk averse enough, ($z < \bar{z}\left(\alpha, u, \frac{\mu}{\sigma}, \pi_r\right)$), it is unprofitable for the retailer to carry the product.

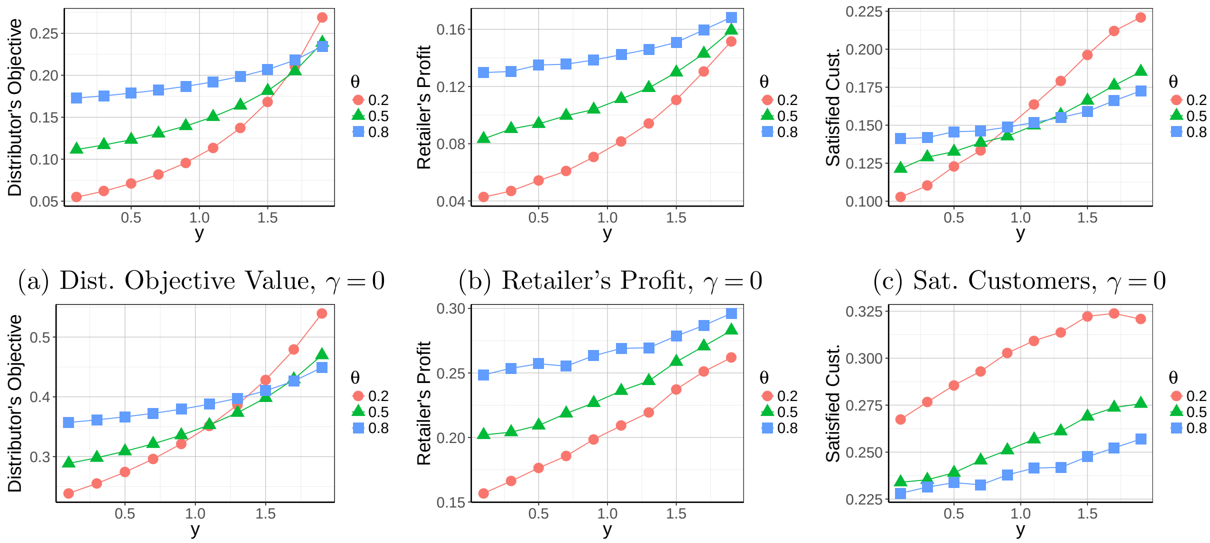
Proof. When $\theta = 0$, all the consumers get the same signal μ. Hence, for any refund r, all the consumers buy the product if $p \leq p(r, \mu)$, and no one buys the product otherwise. Therefore, the optimal price set by the retailer is $p(r, \mu)$, everyone buys the product, and the probability of returns is $\mathbb{P}(V \leq r) = \Phi\left(\frac{r-\mu}{\sigma}\right)$. The retailer's profit function in this case is

$$\Pi(r, \mu) = p(r, \mu) - c - (r-u)\Phi\left(\frac{r-\mu}{\sigma}\right) = \sigma\left(p\left(\frac{r-c}{\sigma}, z\right) - \frac{r-u}{\sigma}\Phi\left(\frac{r-c}{\sigma} - z\right)\right) = \Pi(r, z),$$

where we have used the following scaling properties of the retailer's pricing function: $p(r+h, v_0+h) = p(r, v_0) + h$, and $p(hr, hv_0, h\sigma, \frac{\alpha}{h}) = hp(r, v_0, \sigma, \alpha)$, for any r, v_0, σ, α, and h. Note that in the second property we have abused notation by including the consumers' constant risk aversion parameter α, and the standard deviation σ, in the argument of the retailer's pricing function. We are interested in finding necessary and sufficient conditions such that $\max_r \Pi(r, z) < 0$.

From Proposition 2 we know that $\Pi(r, z)$ is quasi-concave. Hence, we can define, and easily compute, $\Pi^*\left(z, \alpha, u, \frac{\mu}{\sigma}\right) = \max_r \Pi(r, z)$. To conclude, we now show that $\Pi^*\left(z, \alpha, u, \frac{\mu}{\sigma}\right)$ is increasing in z. Specifically, $\partial_z \Pi(r, z) = \partial_z\left(p(r, z) + (u-r)\Phi\left(\frac{r-c}{\sigma} - z\right)\right) = 1 - \partial_z p(r-z) + (r-u)\Phi(r-z) > 0$, where the second equality follows from equation (31), while from Lemma 7 it follows that the inequality holds for any $r > u$, and in particular for the maximizer r^*.

It follows that for any $\alpha > 0$ there exists a unique $\bar{z}\left(\alpha, u, \frac{\mu}{\sigma}, \pi_r\right)$ such that $\Pi^*\left(z, \alpha, u, \frac{\mu}{\sigma}\right) = \pi_r$, and $\Pi^*\left(z, \alpha, u, \frac{\mu}{\sigma}\right) < \pi_r$ if and only if $z < \bar{z}\left(\alpha, u, \frac{\mu}{\sigma}, \pi_r\right)$. □



(a) Dist. Objective Value, $\gamma = 0$ (b) Retailer's Profit, $\gamma = 0$ (c) Sat. Customers, $\gamma = 0$

(d) Dist. Objective Value, $\gamma = 1$ (e) Retailer's Profit, $\gamma = 1$ (f) Sat. Customers, $\gamma = 1$

Figure 9 **Simulation results for a moderate margin product, where $w = 2$, $\mu = 3$, $\sigma = 1.5$, and $\alpha = 3$. Plots of the distributor's objective value, retailer's profit, and fraction of satisfied customers as a function of the salvage value y, and the accuracy of the information to the consumers θ. Includes the case when the distributor does not value satisfied customers (top row, $\gamma = 0$), and when it does (bottom row, $\gamma = 1$).**

6.3. Simulations for a moderate margin product

We calibrated the simulations presented in this section using data from a popular solar flashlight sold by Essmart. This solar flashlight's wholesale price ₹150 (around US$ 4) and is high margin product for both the distributor and the retailer. It is also a product with a high adoption potential, since the majority of households targeted by Essmart have limited access to electricity. Given the price point of the solar flashlight, Essmart estimates that about 80% of their consumers would be willing to purchase this product. Essmart also perceives consumers to be moderately averse towards purchasing this product, since there are cheaper alternatives (such as kerosene lamps). Both returns and warranties are available for this product.

Figure 9 shows that all the insights discussed in Section 4.4 for a low margin product are even stronger for a moderate margin product. We avoid repeating the discussion here for the sake of brevity.