Housing Price and Fundamentals in a Transition Economy: The Case of the Beijing Market∗

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Abstract

This paper develops a dynamic rational expectations general equilibrium framework that links house value to fundamental economic variables such as income growth, demographics, migration and land supply. Our framework handles non-stationary dynamics as well as structural changes in fundamentals that are commonplace in transition economies. Applying the framework to Beijing, we find that the equilibrium house price and rent under reasonable parametrizations of the model are substantially lower than the data. We explore potential explanations for the discrepancies between the model and the data.

Running Head: Housing Price in Transition Economy

Keywords: Housing price, Economic fundamentals, Dynamic general equilibrium model, Transition economy, Non-stationary dynamics

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1 Introduction

Economists have long been interested in the relationship between house price and economic fundamentals. With a few exceptions, much of the literature has focused on developed economies. Recently, the unprecedentedly high and rising house price in major Chinese markets has attracted global attention (Fang et al., 2015). For example, between 2006 and 2014, real house price in Beijing appreciated by 19.8 percent per year (Wu et al., 2016). This is more than double the annual price growth in Las Vegas – one of the fastest appreciating markets in the U.S. in the recent boom. Yet Beijing has also experienced an extraordinary growth in income and population – with an average annual real disposable income growth rate of approximately 9 percent and a net annual inflow of migrants that is about 4 percent of the incumbent population during the last decade. While qualitatively these growing fundamentals may support the high level of Beijing house price today, it is challenging to evaluate such justification quantitatively, especially given the non-stationarity in the economic fundamentals often encountered in a transition economy. As noted by Robert Shiller in April 2014, “China is in such a rapid growth period. It is very hard to price assets when growth is at the high level. The future matters more. In a stable economy that is not going anywhere, you have a pretty good idea of what they are worth.” In a transition economy, the historical relationship between house price and fundamentals is unlikely to repeat itself. Conventional house pricing methods, such as price-income and price-rent ratios, often fail to capture the changing dynamics in the transition phase and hence are insufficient for assessing house price in transition economies.

In this paper, we develop a dynamic rational expectations general equilibrium model with the goal of understanding the relationship between house price and fundamentals in a transition economy. The model is general and flexible enough to deal with situations

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2 See, e.g., Poterba et al. (1991), Capozza et al. (2002), Case and Shiller (2003), Glaeser and Gyourko (2005), Himmelberg et al. (2005), Hott and Monnin (2009), and Ambrose et al. (2013).

3 The statistics are computed from the National Bureau of Statistics of China.

4 Throughout this paper, we refer an economy that is undergoing structural transformations as a transition economy.
where fundamentals can be non-stationary and rapidly changing, yet retains tractability for deriving the price-fundamental relation endogenously and dynamically along the transition path.

The model features overlapping-generations of heterogeneous households on the demand side and a representative housing production firm on the supply side. Housing in our model is not only consumption but also investment. Housing demand is determined both by the extensive margin (renting vs. owning) and the intensive margin (the amount of housing to consume/invest). Leverage is allowed through mortgage for households who have enough savings to meet the down payment. There is no aggregate risk in the model – agents have perfect foresight about the future dynamics of the fundamentals and take them into account when making current consumption, investment and production decisions. Following Aiyagari (1994), the model admits two types of idiosyncratic risks: household’s income shocks and medical expense shocks. Thus heterogeneity among households stems not only from age but also from income and expenses. The idiosyncratic risks are uninsurable and lead to additional precautionary demand for housing.

We apply this model to Beijing. In doing so, we are able to capture a wide range of important and well-documented features of the Beijing housing market: (i) income growth has been high but is expected to slow down (Pritchett and Summers, 2014); (ii) urbanization process is ongoing but is expected to stabilize eventually (Garriga et al., 2016); (iii) population is aging in the near term (Song et al., 2015); (iv) access to mortgage loans is allowed with a relatively high down payment requirement (Fang et al., 2015); (v) urban land supply is at the control of the government (Cai et al., 2013); and (vi) investment vehicles are limited (Chen and Wen, 2017). These features interact in a general equilibrium setting, yielding evolving dynamics of the fundamentals over the course of the transition period. Once the economic transition is complete, the economy enters a balanced growth path (BGP). Using the equilibrium quantities in BGP as the terminal conditions and backward inductions, we
solve for the paths of equilibrium house prices and rents during the transition phase.\footnote{In the application of our model to the Beijing housing market, the transition phase lasts 110 years. We develop an efficient numerical method to compute equilibrium house prices and rents during this period.}

Our analysis produces consistent and robust findings. First, using Hong Kong as a reference city for the future BGP, the baseline model predicts that the equilibrium Beijing house price in 2014 is about 30 percent below the observed market price. To probe the robustness of the finding, we experiment with alternative assumptions about land supply, income growth, initial wealth, population structure, mortgage rate, and down payment constraints. We further extend the model to allow residents in second-tier cities to endogenously choose between staying in home cities and moving to Beijing. In all these cases, we observe moderate changes in the equilibrium Beijing house price in 2014, indicating that the gap between the predicted and actual house price remains substantial.

Second, despite limited land supply and influx of migrant workers that keep up the housing demand, the Beijing market becomes more affordable over time, as evidenced by the increasing ratio of average annual income to price per square meter. Thus, high price-income ratio alone does not indicate that house is overvalued. Instead it may be consistent with the evolution of economic fundamentals and may converge to those found in developed markets as the economy matures. It is therefore misleading to compare price-income and price-rent ratios in a transition economy over time or across countries because these ratios are evidently not stationary during the economic transitions.

We stress that our analysis is meant to develop a dynamic equilibrium framework to examine house price in a non-stationary environment with changing fundamentals rather than to fully account for every single factor in the Beijing market. The discrepancy between the implied and observed prices could be driven by various factors that are left out of the current model including omitted market features and reporting errors. We abstract from market features such as government interventions (Wu et al., 2012), endogenous rural-to-urban migration (Garriga et al., 2016), illiquidity of housing (Haurin and Gill, 2002;
Han, 2008), demand shocks and search frictions. Modelling each of these features could bring additional dynamics into the housing market. We find that all else equal, the actual average income in Beijing would need to be about 60 percent over the officially reported income to justify the observed house price. This is not entirely unrealistic in light of the recent evidence suggesting that the National Bureau of Statistics may significantly understate income for Chinese government officials (Deng et al., 2016). However, such substantial income understatement cannot represent the overall population, nor can it be sustainable in the long run.

Another possibility that is worth future investigation is the existence of a housing bubble. Although our findings say little about a bubble, one could extend our framework by dropping the assumption of perfect rationality and exploring how the belief formation process interacts with institutional features of Chinese housing markets. In a transition economy where income growth is historically high, households who are extrapolating recent income growth would fail to recognize the mean reversion of income growth in the longer run. In addition, the short history of the Chinese private housing market may lead households to fail to internalize the longer term impact of housing supply. These deviations from rational expectations about the changing values of fundamentals could lead to excessive optimism among homebuyers, giving rise to the formation of a housing bubble. While accounting for other market features, reporting errors, and irrationality is beyond the scope of this paper, we believe it would be a promising avenue for future research. In this sense, our model serves as a useful starting point for studying the implications of evolving housing markets in transition economies.

2 Literature Review

This paper is most closely related to an important and growing literature that aims to understand house price growth in developing economies like China. On the empirical side,

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6As shown in the recent search literature (Genesove and Han, 2012; Head et al., 2014), search frictions can amplify the demand shocks and thereby generate large momentum in prices at least in the short run.
Wu et al. (2012) provide an assessment for eight major Chinese housing markets during 2003-2010. They find that high expected house price appreciation is needed to justify the low rental return and that much of the increase in the Beijing house price occurs in its land values; both findings are equilibrium outcomes in our structural model. Using an independent data source, Fang et al. (2015) document the patterns of house price growth and local per capita aggregate income growth for 120 Chinese cities during 2003–2013. Among other things, they find that Beijing house price levels increased 660 percent accompanied by rapid but declining income growth and changing demographic trends. Their findings provide a natural motivation for our model.

Theoretical work that models house price path in a transition economy has been relatively limited. Two notable exceptions are Chen and Wen (2017) and Garriga et al. (2016). Chen and Wen (2017) present a theory of rational bubble and show that high house price growth relative to income growth in China indicates a bubble that emerges from resource reallocation from the traditional low-productivity sector to the emerging high-productivity sector. Garriga et al. (2016) show that the quick rise of house price in China between 1998 and 2007 can be largely explained by the process of rural migrants entering cities driven by ongoing structural transformations.

Built upon this line of literature, our study focuses on the equilibrium relationship between house price and fundamentals in a non-stationary economy. Unlike Chen and Wen (2017) and Garriga et al. (2016), we focus on one city in China—Beijing, and take its population inflow as exogenous in the main analysis. While this prevents us from investigating the labor market dynamics, it helps us examine the role of a rich set of fundamentals. In addition, our setting is flexible enough to account for the unique features of housing itself. For example, both consumption and investment roles of housing are important in our model. This is in contrast with Chen and Wen (2017) who treat housing as an investment only and Garriga et al. (2016) who mainly focus on the consumption role of housing.

More broadly, this paper is related to a rich body of literature that studies the structural
link between house price and fundamentals in developed economies. Much of the research in this area focuses on the U.S. market.\textsuperscript{7} The innovation in our paper lies in its focus on the long-run trends of house price and rent in a non-stationary environment with changing fundamental variables such as income, population, and land supply.

Our emphasis on the changing demographics also links this paper to Mankiw and Weil (1989) and voluminous ensuing empirical studies on the impact of demographics on house price. An important difference is that our model endogenizes housing supply and provides useful counterfactual exercises.

Finally, our modelling strategy resembles Kiyotaki et al. (2011) in that we have a representative firm that issues equity to finance land purchase and new capital in order to produce houses. Rather than studying the perturbation of the economy around the balanced growth path, we focus on the equilibrium house price and rent dynamics during the economic transition phase.

3 Model

In the model economy, there exist overlapping generations of households and a long-lived representative firm that produces houses. There are two financial assets that a household can invest savings into – a risk-free asset and a housing equity.\textsuperscript{8} The risk-free rate of return is exogenous, but the return on housing equity is endogenously determined in the equilibrium. The dynamics of macroeconomic fundamentals, including aggregate income, population structure and land supply, are exogenously specified and common knowledge. There exists no aggregate uncertainty in this economy and no productivity shock. Consequently house price is non-stochastic.\textsuperscript{9}

\textsuperscript{7}See, e.g., Campbell and Shiller (1988); Campbell et al. (2006); Brunnermeier and Julliard (2008); Ambrose et al. (2013); Sommer et al. (2013); Chu (2014).

\textsuperscript{8}Our model excludes risky stocks as investment vehicles. Fang et al. (2015) find that stock investment in China is “still small by size relative to the pool of savings and has not offered attractive returns in the last two decades”.

\textsuperscript{9}The net effect of aggregate uncertainty on housing demand depends on the local regulations, institutional arrangements, the availability of alternative financial instruments, and nature and extent of uncertainty (Han,
3.1 Firm

Housing supply is endogenously determined by the optimal decision of a representative firm that combines land and capital to produce housing units. In each period, the firm issues equity to finance land purchases and capital investment in order to maximizes its shareholder’s value.

3.1.1 Production Function

Letting $K$ and $L$ denote capital and land input, the firm’s production function is

\[ H_t = Z L_t^\theta K_t^{1-\theta}, \]

where $Z$ is a scaling parameter, and $\theta \in (0,1)$ measures the relative importance of land in housing construction. As in Kiyotaki et al. (2011), the firm can continuously adjust the housing stock without any cost.

We abstract away from labor input in housing construction for simplicity and transparency. This is consistent with the empirical finding in the previous studies that land price is much more important than labor cost in determining house price in big cities.\(^{10}\) In our model, although land supply is exogenous, land price is endogenously determined to clear the land market.

3.1.2 Timing and Flow of Funds

At the beginning of period $t$, denote per unit house price as $p_{t-1}$, per unit rent as $r_t$, and housing units of the firm as $H_{t-1}$. Without loss of generality, we normalize the number of shares in firm equity to be the same as the number of housing units, thus $p_{t-1}$ is also price per share of housing equity and $r_{t-1}$ is also dividend per share.

\(^{10}\)See Davis and Heathcote (2007) for evidence on the US market and Wu et al. (2012) for evidence on the Chinese market. In an extended version of the model, we include labor input in the model and find its impact on equilibrium house price for Beijing to be minor (see Section 7 of the online Appendix for details).
The firm issues new shares to raise capital and purchase land. The issuance price $p_t$ (also housing price) is determined endogenously. After the issuance of new shares, the number of total housing units becomes $H_t$. Thus the firm’s flow of funds in period $t$ is

\begin{equation}
    p_t(H_t - H_{t-1}) = K_t - (1 - \delta)K_{t-1} + q_t(L_t - L_{t-1}),
\end{equation}

where $\delta$ is the depreciation rate of capital, and $q_t$ is land price in period $t$. The left side of the equation represents the source of funds, and the right side is the use of funds.

3.1.3 Optimization Problem

The firm collects rental income $r_tH_{t-1}$ and pays it out to shareholders as dividends. At the beginning of period $t$, the firm maximizes the value of the existing shareholders, $p_tH_{t-1}$. Using equation (2), we obtain

\begin{equation}
    p_tH_{t-1} = p_tH_t - [K_t - (1 - \delta)K_{t-1}] - q_t(L_t - L_{t-1}) \nonumber
\end{equation}

\begin{equation}
    = \frac{r_{t+1} + p_{t+1}}{(r_{t+1} + p_{t+1})/p_t}H_t - [K_t - (1 - \delta)K_{t-1}] - q_t(L_t - L_{t-1}) \nonumber
\end{equation}

\begin{equation}
    = \tilde{r}_tH_t - [K_t - (1 - \delta)K_{t-1}] - q_t(L_t - L_{t-1}) + \frac{1}{R_{t+1}}p_{t+1}H_t
\end{equation}

where $R_{t+1} = \frac{r_{t+1} + p_{t+1}}{p_t}$ is the total return on housing equity between $t$ and $t + 1$. We define $\tilde{r}_t = \frac{r_{t+1}}{R_{t+1}}$ so $\tilde{r}_tH_t$ is the present value of rental income to be collected at the beginning of next period.

The firm solves a dynamic programming problem with the following value function in terms of the state vector $(K_{t-1}, L_{t-1})$:

\begin{equation}
    V(K_{t-1}, L_{t-1}) = \max_{K_t, L_t} \tilde{r}_tH_t - [K_t - (1 - \delta)K_{t-1}] - q_t(L_t - L_{t-1}) + \frac{1}{R_{t+1}}V(K_t, L_t)
\end{equation}

s.t. $H_t = ZK_t^{1-\theta}L_t^\theta$. 

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First order conditions with respect to $K_t$ and $L_t$ are

\begin{align}
Z(1 - \theta)\check{r}_t \left(\frac{K_t}{L_t}\right)^{-\theta} &= 1 - \frac{1}{R_{t+1}} \frac{\partial V(K_t, L_t)}{\partial K_t} = 1 - \frac{1 - \delta}{R_{t+1}} \\
Z\theta\check{r}_t \left(\frac{K_t}{L_t}\right)^{1-\theta} &= q_t - \frac{1}{R_{t+1}} \frac{\partial V(K_t, L_t)}{\partial L_t} = q_t - \frac{q_{t+1}}{R_{t+1}}
\end{align}

3.1.4 Optimal Housing Supply

The optimal level of capital can be obtained from the first order condition (5) as:

\begin{equation}
K^*_t = \left[ \frac{Z(1 - \theta)\check{r}_t}{1 - (1 - \delta)/R_{t+1}} \right]^{1/\theta} L^*_t,
\end{equation}

where $L^*_t$ is the exogenous land supply in period $t$. Plugging this expression into the housing production equation (1), we derive the following housing supply as a function of the exogenous land supply:

\begin{equation}
H_t = Z^{1/\theta} \left[ \frac{(1 - \theta)\check{r}_t}{1 - (1 - \delta)/R_{t+1}} \right]^{(1-\theta)/\theta} L^*_t,
\end{equation}

thus housing supply depends critically on land supply. When more land is supplied by the government, the firm optimally chooses more capital investment (equation 7), which further increases housing supply.

3.1.5 Market Clearing Land Price

We obtain the following dynamic relation from equations (6) and (8):

\begin{equation}
q_t - \frac{q_{t+1}}{R_{t+1}} = \theta Z^{1/\theta} \check{r}_t^{1/\theta} \left[ \frac{1 - \theta}{1 - (1 - \delta)/R_{t+1}} \right]^{(1-\theta)/\theta}.
\end{equation}

We will show that a BGP exists in which land price grows at a constant factor ($G_q$) and
housing investment return is time invariant, denoted by $R_{BGP}$, thus in the BGP,

$$q_t = M \frac{\tilde{r}_t^{1/\theta}}{1 - G_q/R_{BGP}},$$

where $M = \theta Z^{1/\theta} \left[ \frac{1-\theta}{1-(1-\delta)/R_{BGP}} \right]^{(1-\theta)/\theta}$ is an increasing function of $R_{BGP}$.

Replacing $q_t$ and $\tilde{r}_t$ with $q_{BGP}$ and $r_{BGP}$, the above equation becomes an expression for land price when the economy enters the BGP. In the quantitative analysis, we first derive $q_{BGP}$ from $r_{BGP}$ and $R_{BGP}$ which in turn are obtained from a set of regularity conditions. Then we compute equilibrium paths of land price via backward induction using equation (9).

### 3.2 Households

The economy is populated by a growing mass of households that start to work at age $J_0$ and retire at $J_1$, then live up to a maximum age of $J$. (In the numerical analysis, $J_0 = 21$, $J_1 = 60$ and $J = 96$.) Households choose housing and non-housing consumptions as well as housing investments to maximize life-time utility. Home ownership is an endogenous outcome in the model.

The households have homogenous preferences and beliefs, but they are heterogeneous in their age and initial wealth. Households are subject to idiosyncratic shocks to income and medical expense. These shocks generate within-cohort heterogeneity in income, consumption, savings, home ownership and housing size.

#### 3.2.1 Utility function and bequest value

We assume the Cobb-Douglas utility for households in each period:

$$u(c, h) = \frac{c^{1-\omega} h^\omega}{1 - \gamma},$$
where $h$ is the housing consumption, $c$ is the non-housing consumption, $\omega$ is the housing share in utility, and $\gamma$ is the inverse of intertemporal elasticity of substitution (EIS).

At the end of period $t$, if a household of age $a$ dies, the value of bequeathing $W_a$ amount of wealth is

$$V_b(W_a) = \max_{c,h} B u(c, h),$$

$$s.t. \quad c + r_t h = W_a.$$ 

where parameter $B$ determines the strength of the bequest motive. With the Cobb-Douglas preference, the bequest value has the following analytical form:

$$(12) \quad V_b(W_a) = B \left[ (1 - \omega)^{1-\omega} \omega \right]^{1-\gamma} \left( \frac{1}{r_t} \right)^{\omega(1-\gamma)} \frac{W_a^{1-\gamma}}{1 - \gamma}.$$ 

### 3.2.2 Household Income and Medical Expense

Income of the $i^{th}$ household at age $a \leq J_1$ and year $t$ is

$$(13) \quad y(i, a, t) = \tilde{y}(i, a, t) \times \overline{y}(a, t), \forall a \leq J_1,$$

where $\tilde{y}(i, a, t)$ and $\overline{y}(a, t)$ are the stochastic and the deterministic components respectively. The deterministic income includes an age effect ($a$) capturing the hump-shaped life-cycle profile of income and a time effect ($t$) for the growth of the aggregate income. We assume an AR(1) process for the logarithm of the stochastic component of income:

$$(14) \quad \ln \tilde{y}(i, a, t) = \rho_y \ln \tilde{y}(i, a - 1, t - 1) + \epsilon(i, a, t), \forall a \leq J_1,$$

where $\epsilon_{i,a,t}$ is the idiosyncratic income shock drawn from a normal distribution with a mean of zero and a standard deviation of $\sigma_y$.\textsuperscript{11}

\textsuperscript{11}For a household just entering the labor market, we assume $\tilde{y}(i, J_0, t) = \epsilon(i, J_0, t)$. After retirement, households are no longer subject to income shocks, and their income grows at the same rate as the aggregate income.
Motivated by the strong link between medical expenses and wealth accumulation of retirees (De Nardi et al., 2010), we assume that after retirement, households face out-of-pocket medical expenses $m(i, a, t)$ that is also comprised of a deterministic component and a stochastic component whose logarithm follows an AR(1) process.

### 3.2.3 Housing Investment and Leverage

A household enters the economy as a renter and saves via the risk-free asset before becoming a homeowner. To buy a house of size $s$ at price $p$, a household needs to pay down at least $d \times p \times s$ where $d$ is the minimum down payment, and $s \geq s_0$, the minimum housing size. Homeowners can hold leveraged investments in housing equity by using mortgages to borrow up to $1 - d$ of the house values. The mortgage interest rate $r_m$ is exogenous. We assume mortgage adjustment, such as refinancing, is costless.

In our model, leveraged return on housing investment is always higher than the risk-free rate since house price and rent adjust until the leveraged return on the housing investment is sufficiently high for housing investment demand to equal housing supply. Therefore, homeowners invest in housing asset only. Renters save in the form of the risk-free asset. After they have saved enough for the minimum down payment, they can become homeowners and invest in the housing equity.

### 3.2.4 Household’s Optimization Problem

For ease of presentation, we omit the household index $i$ and the time index $t$ from the state and control variables of households, but keep the age index $a$. For aggregate variables such as price and rent, the time index is still used.

A household of age $a$ holds $s_{a-1, t-1}$ shares of housing equity at the end of $t - 1$. At the beginning of the period $t$, after the income and medical expense are revealed, the renters decide whether to become a homeowner, while the existing homeowners can adjust the income.
holdings of housing equity. Under some circumstances it may be optimal for homeowners to exit the housing market and become renters. In addition to investment choices, households need to decide on the quantity of housing consumption $h_{a,t}$ and nonhousing consumption $c_{a,t}$.

**The homeowner’s problem** We use $V^{own}(s_{a-1}, y_a, m_a)$ to denote a homeowner’s value if she stays in the housing market, and $V^{rent}(s_{a-1}, y_a, m_a)$ to denote a homeowner’s value if she exits the housing market and becomes a renter. The value function of the homeowner is

$$V(s_{a-1}, y_a, m_a) = \max \{V^{rent}(s_{a-1}, y_a, m_a), V^{own}(s_{a-1}, y_a, m_a)\}.$$ 

Specifically,

$$V^{own}(s_{a-1}, y_a, m_a) = \max_{s_a, h_a} u(c_a, h_a) + \beta \mathcal{E} [(1 - \nu_a)V(s_a, y_{a+1}, m_{a+1}) + \nu_a V_b(s_a)],$$

s.t. $r_t h_a + c_a = y_a - m_a + [p_t + r_t - LTV_{t-1}p_{t-1}(1 + r_m)]s_{a-1} - (1 - LTV_t)p_t s_a$, and $s_a \geq \underline{s}$.

where $\mathcal{E}$ is the expectation operator, $\nu_a$ is the probability of death by the end of the period for households of age $a$, $V_b(s_a)$ is the bequest value given in (12), and $LTV_t$ is the loan-to-value ratio in period $t$:

$$LTV_t = \begin{cases} 
1 - d, & \text{if } r_m < R_{t+1}; \\
0, & \text{if } r_m \geq R_{t+1},
\end{cases}$$

i.e., homeowners use leverage via the mortgage and borrow up to the limit (i.e., only pays the minimum down payment) if the mortgage rate is lower than the housing equity return, and borrow nothing otherwise.
In a similar vein, if the homeowner chooses to become a renter, the value is given by:

\[ V_{\text{rent}}(s_{a-1}, y_a, m_a) = \max_{b_a, h_a} \ u(c_a, h_a) + \beta \mathbb{E} \left[ (1 - \nu_a)W(b_a, y_{a+1}, m_{a+1}) + \nu_a V_b(b_a) \right], \]

s.t. \[ r_t h_a + c_a = y_a - m_a + [p_t + r_t - LTV_{t-1}p_{t-1}(1 + r_m)]s_{a-1} - b_a, \]

and \( b_a > 0, \)

where \( W(b_a, y_{a+1}, m_{a+1}) \), to be defined below, is the next period’s value function of a renter given the state vector \( (b_a, y_{a+1}, m_{a+1}) \).

**The renter’s problem**  Let \( W_{\text{rent}}(b_{a-1}, y_a, m_a) \) denote the renter’s value if she continues to rent in the current period, and \( W_{\text{own}}(b_{a-1}, y_a, m_a) \) denote her value if she enters the housing market. Overall, the value function of a renter given the state vector \( (b_{a-1}, y_a, m_a) \) is

\[ W(b_{a-1}, y_a, m_a) = \max \{ W_{\text{rent}}(b_{a-1}, y_a, m_a), W_{\text{own}}(b_{a-1}, y_a, m_a) \}. \]

Specifically,

\[ W_{\text{rent}}(b_{a-1}, y_a, m_a) = \max_{b_a, h_a} \ u(c_a, h_a) + \beta \mathbb{E} \left[ (1 - \nu_a)W(b_a, y_{a+1}, m_{a+1}) + \nu_a V_b(b_a) \right], \]

s.t. \[ r_t h_a + c_a = y_a - m_a + (1 + r_b)b_{a-1} - b_a, \]

and \( b_a > 0 \)

where \( V_b(b_a) \) is the bequest value given in (12), and \( r_b \) is the exogenous risk-free rate.

\[ W_{\text{own}}(b_{a-1}, y_a, m_a) = \max_{s_a, h_a} \ u(c_a, h_a) + \beta \mathbb{E} \left[ (1 - \nu_a)V(s_a, y_{a+1}, m_{a+1}) + \nu_a V_b(s_a) \right], \]

s.t. \[ r_t h_a + c_a = y_a - m_a + (1 + r_b)b_{a-1} - (1 - LTV_t)p_t s_a, \]

and \( s_a \geq s, \)

where \( V(s_a, y_{a+1}, m_{a+1}) \) is the next period’s value function of a homeowner defined above.
3.3 General Equilibrium

The equilibrium comprises the paths of house price $p_t$, rent $r_t$, and land price $q_t$, as well as the choices made by the firm and the households that satisfy the following conditions: (i) the firm’s choices of $L_t$, $K_t$, $H_t$ are consistent with the firm’s optimization problem; (ii) the household’s choices of consumptions and investments are consistent with the household’s optimization problem; and (iii) the price and rent satisfy market clearing conditions.

Households are distributed in the state space of $B \times S \times A \times Y \times M$, with $b \in B$, $s \in S$, $a \in A$, $y \in Y$, $m \in M$, and the probability of distribution denoted by $\lambda_t(b, s, a, y, m)$.

The housing consumption market clears if:

(15) $H_t = \int_{(b, s, a, y, m) \in B \times S \times A \times Y \times M} h'(b, s, a, y, m) d\lambda_t(b, s, a, y, m)$

where $h'(b, s, a, y, m)$ is the optimal housing consumption given the state $(b, s, a, y, m)$. The equity market clears if:

(16) $H_t = \int_{(b, s, a, y, m) \in B \times S \times A \times Y \times M} s'(b, s, a, y, m) d\lambda_t(b, s, a, y, m)$

where $s'(b, s, a, y, m)$ is the optimal housing investment. Finally, the land market clearing condition is:

$L_t = L_t^*$

where $L_t$ is the firm’s land demand and $L_t^*$ is the exogenous land supply in period $t$.

3.4 Balanced Growth Path

Assume that from year $T_{BGP}$ forward, aggregate income, land supply and population grow at the fixed factors of $G_Y$, $G_L$ and $G_N$ respectively. In addition, age distribution of population no longer changes over time.

**Proposition** A BGP exists and is characterized by the following:
1. Aggregate capital growing at a factor of $G_K = G_Y$;

2. Aggregate housing supply growing at a factor of $G_H = G_1^{-\theta}G_L^\theta$;

3. Housing investment demand and consumption per capita growing at $G_s = (G_Y/G_N)^{1-\theta}(G_L/G_N)^\theta$

   and $G_h = G_s$;

4. Demand for risk-free asset per capita growing at a factor of $G_Y$;

5. Non-housing consumption per capita growing at $G_c = G_Y/G_N$;

6. House price growing at $G_p = (G_Y/G_L)^\theta$;

7. House rent growing at $G_r = (G_Y/G_L)^\theta$;

8. Land price growing at $G_q = G_Y/G_L$;

9. Floor-area ratio, defined as $H/L$, growing at $G_{FAR} = (G_Y/G_L)^{1-\theta}$;

10. Constant price-income ratio ($pH/Y$) and price-rent ratio ($p/r$).

Equilibrium in the BGP has a set of properties that are consistent with stylized facts. For example, house price is driven by income growth and land supply. The importance of income is shown in Case and Shiller (2003), and the importance of land supply is emphasized in Glaeser et al. (2005) and Saiz (2010). It has been observed that real house price can exhibit extremely low growth rates (Shiller, 2007), which can be generated in our model when land supply and aggregate income grow at similar rates. On the other hand, when land supply grows at a lower rate than income, the model predicts a growing trend of house price.

The sixth and the eighth points in the Proposition indicate that under our model, the growth rate of land price is always higher than that of house price since $\theta < 1$. This is consistent with the observations of major cities in both the U.S. (Davis and Heathcote, 2007) and China (Deng et al., 2012).

Since key variables grow at constant rates in the BGP, they can be re-scaled so that the economy operates as if it is in a steady state. It should be noted that the economy does
not operate in the BGP immediately after the stabilization of the exogenous variables. It needs to wait until the age distribution and the asset distribution of households become time-invariant. In the quantitative analysis below, we assume that after year 2044, all exogenous variables grow at constant rates. After another 70 years, i.e., after 2114, the age distribution and asset distribution will be time-invariant. Therefore, the Beijing housing market enters into BGP in 2114 under our model.

4 Quantitative Analysis: Projection and Calibration

In the quantitative analysis, we start with finding the house price and rent in the BGP; using these terminal conditions and backward inductions, we then solve for house price and rent trajectories that clear the housing equity market, rental market and land market during the transition periods. For this purpose, we develop an efficient method to solve the dynamic equilibrium numerically. We also need to: (i) project the future paths of population structure, income, and land supply; (ii) specify the initial distribution of household wealth; and (iii) calibrate the model parameters. More details about our computation strategy, projection of future paths of the fundamentals and calibration of model parameters can be found in the online Appendix.

4.1 Projections of Population, Income, and Land Supply

Figure 1 plots projections of the two dimensions of the population structure: the population size and the age distribution. The former directly affects housing demand, and the latter matters because housing demand is age-specific. The population data are obtained from the 2010 Census, and from Sample Survey on Population Change for other years between 2005 and 2013. Based on the 2013 data on population structure and using the projected fertility rate, mortality rate and migration rate, we extrapolate the population structure after 2013. Upon the completion of urbanization which is represented by a zero migration
This figure shows the evolution of the size and age distribution of population.

Figure 1: Size and Age Distribution of Population

rate after 2044, the peak age of the population moves to 65 in 2060. This causes a period of population decline as shown in the right panel. In year 2100, the population structure stabilizes to a profile that decreases with age, due to the increasing age-profile of the mortality rate.

The left panel of Figure 2 shows the projected land supply. Evolution of land per capita is given by the solid line which falls gradually due to the inflow of migrants during the urbanization process. As the growth of population plateaus, land supply per capita becomes time-invariant. The two broken lines show the evolution of land per capita when the growth of aggregate land supply is either 1 percent or 0 percent, both of which will be used in the sensitivity analyses.

The right panel of Figure 2 plots the projected evolution of the per capita income under the baseline where aggregate income growth rate falls linearly from 7 percent in 2015 to 3 percent over the next 30 years, and two alternative scenarios where income growth rate falls to 3 percent after either 20 or 40 years.

4.2 Other Exogenous Inputs

Using the 2012 China Household Finance Survey (CHFS), we estimate the ratio of financial wealth to income and the age profiles of financial asset and housing equity for urban households in China. The estimated ratio of financial wealth to income is 3.68.
This figure shows the projected land supply and income. “growth” refers to the growth rate of land per capita in the left panel, and number of years it takes before the growth rate of aggregate income plateaus in the right panel.

Figure 2: Projected land supply and income per capita

Data from the Beijing Bureau of Statistics show that the disposable income per capita in 2005 is 20.02 thousand (in terms of 2014 RMB). Therefore in 2005, the estimated average financial wealth for Beijing residents is 73.65 thousand RMB. In addition, the average housing size for Beijing residents in 2005 is 19.5 square meters. We distribute these assets across different age cohorts, assuming the same age profiles of financial wealth and housing equity as in the 2012 wave of CHFS. These are the initial assets of households who enter the economy at 2005. For the post-2005 entrants, the initial assets are bequests from those who die in the previous period. The total amount of bequest wealth is endogenously determined in the model and distributed evenly among new entrants in the economy.

The age profile of income, i.e., the term \( \bar{y}(a, t) \) in equation (13) is estimated using panel data from China Health and Nutrition Survey between 1989-2011. The estimated parameter values for the AR(1) model of \( \ln \bar{y}(i, a, t) \) in equation (14) are \( \rho_y = 0.864 \) and \( \sigma_y = 0.253 \).

The medical expense process is estimated from the 2011 wave of China Health and Retirement Longitudinal Study. The estimated parameter values for the process of idiosyncratic medical expense are \( \rho_m = 0.922 \), and \( \sigma_m = 0.498 \).

4.3 Calibration of Model Parameters

Our model has three parameters related to the housing production \((Z, \theta, \delta)\) and four preference parameters \((\gamma, B, \beta, \omega)\). To pin down these parameter values, we assume that Beijing
housing market in the BGP will resemble the current state of Hong Kong in terms of: a price-income ratio of 11.88, a price-rent ratio of 35.6, a growth rate of real house price of 2.14 percent and a growth rate of land price of 2.95 percent per annum.\textsuperscript{12} The resulting parameter values are reported in Table 1.

<table>
<thead>
<tr>
<th>Table 1: Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Production parameters</strong></td>
</tr>
<tr>
<td>land share in production</td>
</tr>
<tr>
<td>capital depreciation rate</td>
</tr>
<tr>
<td>scaling parameter in production</td>
</tr>
<tr>
<td><strong>Preference parameters</strong></td>
</tr>
<tr>
<td>inverse of EIS</td>
</tr>
<tr>
<td>discount factor</td>
</tr>
<tr>
<td>housing share in utility</td>
</tr>
<tr>
<td>strength of bequest motive</td>
</tr>
<tr>
<td><strong>Asset market parameters</strong></td>
</tr>
<tr>
<td>down payment requirement</td>
</tr>
<tr>
<td>minimum housing size</td>
</tr>
<tr>
<td>risk-free rate</td>
</tr>
<tr>
<td>mortgage rate</td>
</tr>
</tbody>
</table>

In addition, we need to choose proper values for asset market related parameters, including minimum down payment $d$, minimum housing size $s$, risk-free return $r_b$, and mortgage rate $r_m$. Between 1990 and 2014, the one-year bank deposit (or the 90-day treasury-bill) in China has an average real annual return of 1.8 percent (or 1.75 percent).\textsuperscript{13} Chinese banks also offer various “wealth management products” with higher returns than bank deposits. Therefore, we set $r_b = 2\%$.

The mortgage rate in China has ranged from 4.5 percent to 7 percent during the past decade. Controlling for inflation, it is about 3-4 percent. In the baseline model, we set $r_m$ to 4 percent. In the sensitivity analysis, we use $r_m = 3\%$. Minimum housing size is assumed to be 30 square meters per household during the transition period. We also use $s = 20$ and $s = 40$ in the sensitivity analysis.\textsuperscript{12}

\textsuperscript{12}Beijing and Hong Kong have similar cultural background and a similar land lease policy. Anglin et al. (2014) show the land lease policy affects house price dynamics and city structures.

\textsuperscript{13}The average annual return on bank deposits is available from the website of People’s Bank of China.
The minimum down payment requirement \( d \) is 50 percent in the baseline analysis.\(^{14}\) In the sensitivity analysis we also consider \( d = 30\% \). For home buyers in Beijing, this requirement is typically 30 percent for the first home, between 50-60 percent for the second home, and even higher for the third home.

5 Quantitative Analysis: Results

In this section, we first compare the model-implied price and rent in 2014 with the data counterparts. We also compare the model-implied outcomes for the pre-2014 period with the historical housing market data. Next, we conduct a wide range of sensitivity analysis to examine how the equilibrium house price changes under alternative model specifications and assumptions. Finally, we explore ways to narrow the gap between the model-implied house price and the data.

To ensure proper mapping between the model and the data, we obtain the price between 2005 and 2014 as the weighted average prices of newly-built and existing homes. For each year the weights are the shares of new and existing home transactions in the total value of transactions. The 2014 average house price and rent in Beijing are 28,194 RMB per square meter and 744 RMB per square meter per year respectively, both in terms of 2014 RMB. Details on the data source and the construction of price and rent are reported in Appendix.

5.1 Equilibrium Price, Rent and Other Market Outcomes

Figure 3 plots the equilibrium path of house price, rent and land price (all in terms of 2014 RMB) in the baseline model.

The baseline model implies an equilibrium house price in 2014 of 19.41 thousand per square meter, which is significantly lower than the 28.194 thousand per square meter observed in the data, despite the fact that price-rent and price-income ratios of Beijing in the BGP

\(^{14}\)For home buyers in the first-tier cities, the average down payment ratio is between 45-50 percent in 2012, as shown in Figure 11 of Fang et al. (2015).
This figure plots the paths of house price, rent and land price (in 2014 RMB) under the baseline model. The thick line in the upper-right panel is the growth rate of house price, plotted along with the projected growth rates of population, income and land supply.

Figure 3: Equilibrium house price, rent and land price under the model match with the high values of the Hong Kong market. Similarly, the equilibrium annual rent under our baseline model is about 560 RMB per square meter in 2014 – lower than the 744 RMB per square meter in the data.

As can be seen from the upper-right panel of Figure 3, the growth of real house price is relatively sluggish between 2040 and 2080, due to an aging population and a period of slow growth of population size after the completion of urbanization. It eventually converges to 2.14 percent – the growth rate of real house price in the BGP. The growth rates of population, aggregate income, and aggregate land supply are also plotted. The correlations between these fundamentals and house price are clearly discernible.\(^{15}\)

The upper panels of Figure 4 plot two affordability measures: price-income ratio and the ratio of annual income over house price (i.e., how many square meters of housing can be purchased by one year of income). It is clear that the high price-income ratio experienced in the early years of the economic transition period does not necessarily indicate a price bubble.

\(^{15}\)The relations between house price and economic fundamentals are unstable under our model. The regression coefficients of house price growth or rent growth on income growth and land supply growth vary considerably during the economic transition (see Section 5 of the online Appendix for details). Therefore, while regression analysis is useful for studying price-fundamentals relations in the developed markets, it can produce misleading results for transition economies where the relations between house price and economic fundamentals are not stable.
Intuitively, high price-income ratio in the early period is supported by high expected growth rate of income. When income is expected to grow quickly, households’ actual ability to pay is much higher than what is captured by the current income. The ratio of annual income over house price displays an increasing time trend. Thus, housing affordability improves over time, because the income growth rate is higher than the house price growth rate. In the proposition, we have shown that aggregate income grows more quickly than house price in the BGP. This is also true during the transition period.

The middle panels of Figure 4 plot the price-rent ratio and return on housing investment. Similar to the price-income ratio, the model-implied equilibrium price-rent ratio declines quickly between 2005 and 2045, then rises gradually until it reaches the level in the BGP. The early period of the declining price-rent ratio coincides with a rapid increase in house price and a slower growth in the rent (see Figure 3). It declines after 2014, which is consistent with the evolution of the exogenous fundamental variables: income growth and the migration.
rate both decline over time in our model. The housing return converges to 4.95 percent in
the BGP.

The bottom left panel of Figure 4 shows that the ratio of land price to house price rises
steadily. This indicates the increasing importance of land relative to structure in house price,
which is consistent with the pattern found in the major U.S. cities (Davis and Heathcote,
2007). Our model also implies that Beijing will witness increasingly higher density, reflected
by rising the floor-area ratio as shown in the bottom right panel of Figure 4.

5.2 Equilibrium Outcomes Compared with the Historical Data

We now compare the model-implied house price, price-income ratio and price-rent ratio with
what we observe in the pre-2014 data. Two series of house price are constructed, referred
to as the NBS series and the THU series respectively. The former is quality-unadjusted
while the latter is quality-adjusted. Both series are calculated as the weighted averages of
newly-built and existing homes.

![Figure 5: Model-Data Comparison](image)

This figure compares equilibrium house price, price-rent ratio and price-income ratio from the
model (solid lines) with those from the data. The “data(NBS)” series is quality-unadjusted
and the “data(THU)” series is quality-adjusted.

The left panel of Figure 5 plots the equilibrium house price under the baseline model.
The model-implied price is in line with the NBS data in 2005. However, the house price
growth rate between 2005 and 2014 is lower than that in the data. Consequently the model-
implied price in 2014 is significantly below the price in the data. One way to measure the
fit of the model is the normalized mean squared error (NMSE) proposed by Garriga, Tang
and Wang (2016). NMSE is defined as \( \frac{\sum_t(x^M_t - x^D_t)^2}{\sum_t(x^D_t)^2} \), where \( x^M_t \) and \( x^D_t \) are the
prices from the model and from the data respectively. The NMSE is 0.13 for the NBS series
and 0.08 for the THU series. Thus, the overall fit of house prices is reasonably good.

The right panels of Figure 5 show that the model does not capture the rising price-income
ratio and price-rent ratio in the data prior to 2010. One possible reason for this wedge is
that households in our model perfectly foresee that high income growth will slow down in
the future, which generates a declining price-income ratio. In reality, evidence suggests that
during 2005-2010, Chinese households likely adopted unrealistically optimistic expectations
of future income growth. High income growth expectations, combined with the access to
the mortgage market, make housing more affordable, causing a rising price-income ratio over
time as observed in the data. The unrealistic expectations about future income growth also
lead to a rising price-rent ratio, since the high growth rate of income is associated with high
capital gain in the housing market, which lowers the required rental return.

During 2010-2014, our model captures the declining patterns in the price-income and
price-rent ratios in the data, and thus the model performs better than in the first subpe-
riod. It is possible that the 2009 global financial crisis made households more aware of
market conditions or the potential for mean reversion in the income growth rate. Therefore,
expectations may have conformed more closely with what our model assumes.

Finally, we compare the age distribution of home ownership rate and the life-cycle profile
of housing assets in the data to those implied by the model. The left panel of Figure 6 plots
the model-generated age distribution of the home ownership rate in 2010 and that calculated
from the 2010 census data. In the data, the home ownership rate starts at about 31 percent

\[16\text{As noted in Fang et al. (2015), “such high income growth expectations might have resulted from extrap}
\text{olative behavior as emphasized by Barberis et al. (1998) and Shiller (2000), or from contagious social}
\text{dynamics between households as modeled by Burnside et al. (2016).”}\]

\[17\text{Our model ignores cyclical movements and hence does not capture the temporary rise in the price-
\text{income and price-rent ratios between 2012 and 2013 which could be caused by changes in down payment
\text{and mortgage rate due to government interventions.}\]
at 21, rises gradually to about 86 percent at age 75 and then falls slightly afterwards. The average home ownership rate is 66.5 percent. In the model, home ownership rate rises monotonically from about zero for those in their early twenties to almost 100 percent by age 65. The average home ownership rate under the model is 81.8 percent, higher than the data. This gap occurs because housing investment has a high return and no risk under our model; as a result, households become homeowners as soon as they can afford the minimum down payment.

The right panel of Figure 6 plots the life-cycle profile of housing assets from the model in 2012 along with the national level age distribution of housing size (in square meters) provided by the 2012 CHFS. Although we do not target to fit this age distribution in 2012, the model-generated age distribution of housing asset is highly correlated with the data. The magnitude of housing asset under our calibrated model is somewhat larger than that in the data, except for both very young and old households.

![Figure 6: Home Ownership Rate and Housing Asset: Model versus Data](image)

Note: This figure plots the age distribution of the home ownership rate and housing asset (in square meters per capita) in the data and implied from the model.

5.3 Sensitivity Analyses

We conduct a wide range of sensitivity analyses and confirm a robust finding: the equilibrium price and rent under reasonable parameterizations of our model are substantially lower than the data. Model-implied house price would be higher under alternative assumptions such as more stringent land supply or limit on floor-area ratio, higher income growth, larger
initial wealth, longer period of population inflow, lower mortgage rate or down payment requirement, but there still remains a significant gap between the model-implied house price and that in the data. Additional robustness checks are provided in the online Appendix.

Table 2: Price and rent under alternative assumptions (in thousands of 2014 RMB)

<table>
<thead>
<tr>
<th></th>
<th>Price 2014</th>
<th>Price 2114</th>
<th>Rent 2014</th>
<th>Rent 2114</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>28.19</td>
<td>n.a.</td>
<td>0.74</td>
<td>n.a.</td>
</tr>
<tr>
<td>Baseline</td>
<td>19.41</td>
<td>153.54</td>
<td>0.56</td>
<td>4.31</td>
</tr>
<tr>
<td>(1) Growth of land supply = 1%</td>
<td>18.10</td>
<td>86.40</td>
<td>0.57</td>
<td>2.72</td>
</tr>
<tr>
<td>(2) Growth of land supply = 0%</td>
<td>22.33</td>
<td>207.21</td>
<td>0.62</td>
<td>5.90</td>
</tr>
<tr>
<td>(3) Income stabilizes in 20 years</td>
<td>18.55</td>
<td>128.87</td>
<td>0.54</td>
<td>3.64</td>
</tr>
<tr>
<td>(4) Income stabilizes in 40 years</td>
<td>20.06</td>
<td>183.81</td>
<td>0.60</td>
<td>5.32</td>
</tr>
<tr>
<td>(5) Migration stops in 40 years</td>
<td>20.39</td>
<td>203.24</td>
<td>0.59</td>
<td>5.56</td>
</tr>
<tr>
<td>(6) Initial financial wealth = 2 M</td>
<td>20.01</td>
<td>161.07</td>
<td>0.59</td>
<td>4.42</td>
</tr>
<tr>
<td>(7) FAR (\leq 3)</td>
<td>21.43</td>
<td>289.87</td>
<td>0.61</td>
<td>7.94</td>
</tr>
<tr>
<td>(8) Minimum down payment = 30%</td>
<td>24.62</td>
<td>179.96</td>
<td>0.63</td>
<td>4.66</td>
</tr>
<tr>
<td>(9) Mortgage rate = 3%</td>
<td>21.96</td>
<td>176.41</td>
<td>0.62</td>
<td>4.65</td>
</tr>
<tr>
<td>(10) Minimum housing size = 40 (m^2)</td>
<td>18.89</td>
<td>153.36</td>
<td>0.57</td>
<td>4.36</td>
</tr>
</tbody>
</table>

This table reports the equilibrium price and rent for the Beijing housing market in 2014 and 2114 under alternative model assumptions.

5.3.1 Alternative Projection of Land Supply

In the baseline model, we have assumed that aggregate land supply grows at a rate of 0.5 percent after 2014. With more stringent land supply, house price and rent as well as their growth rates are uniformly higher compared with the baseline case. However, even under zero land supply growth, house price in 2014 is 22.33 thousand RMB per square meter, which is 15 percent higher than the baseline case but still significantly lower than the data.

5.3.2 Alternative Projection of Income Growth

The baseline model scenario assumes that the growth rate of aggregate income declines linearly from 7 percent in 2015 to 3 percent in 2045. In the sensitivity analyses, we consider two alternative scenarios where income growth plateaus to 3 percent after either 20 or 40
years. When the high income growth lasts for a longer (shorter) period, the equilibrium house price and rent are higher (lower) throughout the transition period, but the magnitude of the effect is small.\textsuperscript{18} For example, Table 2 rows (3)-(4) show that the model-implied house price in 2014 is 18.55 (resp. 20.06) thousand RMB per square meter corresponding to a shorter (20 years) or a longer (40 years) period of high income growth. These price levels are within 5 percent of the Beijing house price under our baseline model.

5.3.3 Alternative Projection of Migration

In the baseline model we assume the rate of migration to Beijing decreases linearly from 2.88 percent in 2014 to zero after 30 years. Extending the period of migration generates more demand for both housing consumption and housing investment, hence it yields higher house price and rent compared with the baseline results, especially in the long-run. For example, row (5) of Table 2 shows that with 10 additional years of migration, the house price in 2014 (resp. 2114) is 5 percent (resp. 32 percent) higher than that under the baseline model.

5.3.4 Alternative Initial Wealth

Row (6) of Table 2 reports the results where we raise average household initial financial wealth (in the year 2005) from 73.65 thousand RMB in the baseline scenario to 200 thousand RMB. The implied house price is now 20 thousand RMB per square meter in 2014, a rise of 3 percent relative to the baseline. Correspondingly, the rent increases by 5.35 percent. Thus, the effect of a large increase (almost threefold) in the initial household wealth on equilibrium house price and rent is very small. Intuitively, households use the extra wealth for more consumptions over time.

\textsuperscript{18}Consider the case of lower future income path relative to the baseline scenario. This has two counter-vailing effects with opposing implications for housing investment demand and thus the house price. On one hand, due to the substitution effect, households now have a stronger incentive to save for the future leading to a higher housing investment demand but weaker housing consumption demand. On the other hand, due to the income effect, there would be less housing demand – for both consumption and investment purposes. The net impact on house price depends on the relative strength of the two effects.
5.3.5 Upper Bound on Floor-area Ratio

What happens if the government imposes an upper-limit on FAR, denoted $FAR$, so that urban density is restricted? In row (7) of Table 2, we assume $FAR = 3.0$ and re-compute the equilibrium house prices and rents.\textsuperscript{19} By potentially limiting housing supply, the policy leads to an increase in house price and rent by 10.4 percent and 8.9 percent respectively in 2014, and for house price by 89 percent and for rent by 84 percent in 2114 relative to the baseline scenario.

5.3.6 Alternative Housing Market Assumptions

The baseline model uses a mortgage borrowing rate $r_m$ of 4 percent and a minimum down payment $d$ of 50 percent. The minimum housing size $s$ during the transition periods is 30 square meters. To check the sensitivity of our results, we change these parameters one at a time, setting $d = 30\%$, or $r_m = 3\%$, or $s = 40$ respectively. Results are reported in rows (8)-(10) of Table 2.

Lowering the mortgage rate or down payment requirement makes housing more affordable, leading to an increase in housing demand and house price. The effect is quite significant. For example, Row (8) of Table 2 shows that by lowering the down payment from 50 percent to 30 percent, house price and rent in 2014 increase by about 26.8 percent and 12.5 percent respectively. Similarly, row (9) shows that house price increases by about 13 percent when the mortgage rate is cut from 4 percent to 3 percent.\textsuperscript{20} Nevertheless, even with easier access to the housing market, the equilibrium house price in 2014 is still substantially lower than the data.

\textsuperscript{19}In March 2008, the Beijing City Planning Committee and Beijing City Land Resources Bureau jointly issued a policy called "Beijing City Construction Land Saving Standards", stipulating that the $FAR$ of residential area should not exceed 2.8. In the absence of building restriction, the floor-area ratio in Beijing should be between 5 and 6 when the economy enters the BGP. Afterwards, it will grow at the constant annual rate of $(G_Y/G_L)^{1-\theta} - 1$ which is about 0.82 percent according to our calibration.

\textsuperscript{20}In an unreported robustness exercise, we examine the impact of an increase in the risk-free interest rate. When the risk-free rate rises from 2 percent to 2.5 percent, the equilibrium house price decreases by 1.54 percent, and the rent increases by 0.65 percent. The change of risk-free rate has both an income effect and a substitution effect. These two effects appear to largely cancel each other out.
Increasing the minimum housing size serves to depress the equilibrium house price but increase the rent, because more households have to rent which reduces the housing demand. But quantitatively, changing the minimum housing size has a rather small effect on the equilibrium house price. Row (10) of Table 2 shows that when the minimum housing size is increased from 30 to 40 square meters, price drops by 2.6 percent from 19.41 thousand RMB to 18.89 thousand RMB per square meter in 2014.

5.4 Reconciling the Model-data Discrepancy

We have shown that the model-implied equilibrium house price and rent are substantially lower than the actual data. The discrepancy could come from a number of channels, including: (i) measurement errors in the government data on housing fundamentals (e.g. household income) we use; (ii) strong model assumptions (e.g. exogenous population inflow); (iii) certain factors or market features that our model abstracts from (e.g. aggregate uncertainty, demand shocks, search frictions in the housing market); (iv) housing bubbles. While channels (iii) and (iv) are beyond the scope of our paper, in this subsection we experiment with channels (i) and (ii). Specifically, we study the impact of the potential understatement of household income. We also examine an extended model that allows sorting of households into Beijing as an outcome of migration choices.

5.4.1 Understatement of Income

The average income of Beijing residents as reported by the National Bureau of Statistics (NBS) can be understated for several reasons. First of all, a large number of rich households outside of Beijing purchase houses in Beijing for a variety of reasons including as an investment, and for better access to facilities, services and conveniences. Some of these home buyers live in Beijing for only short periods of time hence they are not included in the NBS survey. Second, government officials sometimes report extremely mediocre income although their actual income is considerably higher than the average. Deng et al. (2016) estimate that
the amount of unofficial income for an average government official is 60 percent of his or her official permanent income. Indeed, they argue that this may be an underestimation and the actual unofficial income could be even bigger.

Row (2) of Table 4 reports the results of our model when the level of income in Beijing is raised by 60 percent while the income growth rate remains the same as the baseline model. Given the uniformly higher income, both house price and rent increase unequivocally. The model-implied house price in 2014 is 27,040 RMB per square meter, which is close to the 28,194 RMB market price in the data. The equilibrium annual rent is 800 RMB per square meter, compared to 740 RMB observed in the data. In other words, the level of price and rent in 2014 are roughly consistent with the fundamentals if the average income of Beijing residents is about 60 percent higher than reported in the government statistics. Even if such a high level of income understatement is plausible for government officials, it seems unrealistic for the overall population and is certainly not sustainable in the long run. Thus one cannot rely on the income understatement alone to completely close the gap between the model-implied price and the market price.

5.4.2 Endogenous Migration

In the baseline model, we have taken the inflow of migrants to Beijing as exogenously given. In reality, the in-migrants are those who find that the benefits of migration outweigh the costs. In this subsection, we extend the baseline model to study the endogenous migration of households. Note that our goal here is not to formally investigate the driving force behind the labor mobility, rather to examine to what extent the main finding would change when the non-randomness of the population inflow is taken into account. Formally modelling the labor reallocation dynamics, particularly the urbanization process, could be revealing but is beyond the scope of this paper. In this regard, recent work by Garriga et al. (2016) has provided an elegant dynamic general equilibrium model that links urbanization to the ongoing structural transformation in China.
In each period, a potential migrant decides whether to move by comparing the life-time utility she receives if she stays in her hometown with the life-time utility if she moves to Beijing. As in the baseline model, the utility in each scenario is computed based on her optimal decisions. Our characterization of migration benefits and costs is consistent with the literature. A migrant moving to Beijing faces higher housing costs than in the home market. In return, she receives utility gains that capture improved amenities (Henderson and Becker, 2000)), better income prospects that capture the productivity difference (Garriga et al. (2016)), and higher return on housing equity in Beijing.

Specifically, the utility function becomes $u(\phi_c, \phi_h)$ where $\phi$ measures, in the form of consumption equivalence, the advantage of migrating to Beijing in the following way:

$$
\phi = \begin{cases} 
\phi_{\text{owner}}, & \text{if migrate to Beijing and own;} \\
\phi_{\text{renter}}, & \text{if migrate to Beijing and rent;} \\
1, & \text{otherwise.}
\end{cases}
$$

The utility gain parameter $\phi$ differs across renters and homeowners. Turning to income, we assume that if a household migrates to Beijing, her subsequent income is given by $y(i, a, t) = \tilde{y}(i, a, t) \times \bar{y}(a, t)$, where the deterministic component $\bar{y}(a, t)$ is the average income of Beijing residents of the same age, but the individual-specific stochastic component of income $\tilde{y}(i, a, t)$ is not affected by the migration decision.

When taking the model to the data, we restrict our attention to potential migrants who are from the so-called second-tier cities in China. The total population in these cities is about 10 times the Beijing population. Realistically, only a fraction of people outside Beijing consider moving, we therefore randomly draw one-third of the population of the second-tier

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21Thus we omit the rural-to-urban migration. If we replace some of the migrants from second-tier cities with those from the rural areas, it would lead not only to reduced home ownership among migrants, but also reduced home purchases conditional on owning. This would bring down the equilibrium price and magnify the gap between the predicted and actual prices even further. Thus, the conclusion of the main analysis remains robust.
cities as the pool of potential migrants to Beijing.\textsuperscript{22}

To obtain information about migrants’ home markets, we consider four representative second-tier cities: Wuhan, Chengdu, Dalian and Xi’an. As shown in the upper panel of Table 3, their average house price and annual rent in 2014 are around 9,000 RMB and 300 RMB per square meter respectively, less than half of the corresponding values for Beijing. The average disposable income per capita in these cities is about 3/4 of the Beijing income level.

<table>
<thead>
<tr>
<th>Data</th>
<th>Second-tier City</th>
<th>Beijing</th>
</tr>
</thead>
<tbody>
<tr>
<td>House Price (per $m^2$)</td>
<td>9,000</td>
<td>30,000</td>
</tr>
<tr>
<td>Rent (per $m^2$ per year)</td>
<td>300</td>
<td>700</td>
</tr>
<tr>
<td>Income</td>
<td>32,933</td>
<td>43,910</td>
</tr>
<tr>
<td>Initial Housing Asset</td>
<td>30 $m^2$</td>
<td>30 $m^2$</td>
</tr>
<tr>
<td>Initial Financial Asset</td>
<td>3.68 $\times$ income</td>
<td>3.68 $\times$ income</td>
</tr>
<tr>
<td>Assumptions</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Growth of Price (BGP)</td>
<td>1.62%</td>
<td>2.14%</td>
</tr>
<tr>
<td>Growth of Rent (BGP)</td>
<td>1.62%</td>
<td>2.14%</td>
</tr>
<tr>
<td>Price/Rent (BGP)</td>
<td>30</td>
<td>35.6</td>
</tr>
<tr>
<td>Growth of Rent (transition)</td>
<td>exogenous</td>
<td>endogenous</td>
</tr>
<tr>
<td>Growth of Price (transition)</td>
<td>exogenous</td>
<td>endogenous</td>
</tr>
<tr>
<td>Growth of Land supply</td>
<td>0.077%</td>
<td>0.05%</td>
</tr>
</tbody>
</table>

Further, we make the following assumptions about the second-tier cities in the BGP. First, the price-rent ratio is 30 which is slightly lower than that of Beijing. Second, return on housing investment is 4.95 percent, about the same as in Beijing. Together, these two assumptions imply that the growth rate of house price in the BGP is 1.62 percent for second-tier cities – lower than the 2.14 percent rate for Beijing. Finally, the income growth rate of the second-tier cities in the BGP is assumed to be the same as that of Beijing.

It follows from point 6 of the \textbf{Proposition} that the growth factors of land $(G^0_L)$ and

\textsuperscript{22}Our results are not sensitive to this choice as long as we allow enough households outside of Beijing to consider moving so that the migration rate target can be matched.

33
Table 4: Prices and rents (in thousands of 2014 RMB)

<table>
<thead>
<tr>
<th></th>
<th>Price</th>
<th>Rent</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2014</td>
<td>2114</td>
</tr>
<tr>
<td>Data</td>
<td>28.19</td>
<td>n.a.</td>
</tr>
<tr>
<td>(1) Baseline</td>
<td>19.41</td>
<td>153.54</td>
</tr>
<tr>
<td>(2) Income Raised by 60%</td>
<td>27.04</td>
<td>211.34</td>
</tr>
<tr>
<td>(3) Endogenous Migration</td>
<td>21.84</td>
<td>152.17</td>
</tr>
</tbody>
</table>

This table compares the equilibrium price and rent for Beijing housing market under alternative model specifications to the data.

The extended model has two additional parameters $\phi^{owner}$ and $\phi^{renter}$ that measure the utility gain of migrating to Beijing as a homeowner or a renter, respectively. We calibrate these parameters by targeting the following two moments based on the data: (i) the migration rate (number of migrants divided by Beijing population) equals 2.88 percent in 2014; (ii) the ratio of average income of migrants to those with Beijing Hukou is 0.86 (based on the 2005 mini-census data). The calibrated model parameters are $\phi^{owner} = 1.28$ and $\phi^{renter} = 1.23$. The difference captures benefits that are only available to homeowners in Beijing, such as child education benefits.

In the equilibrium, there are both rich and poor households that optimally choose to
move to Beijing, with the latter group starting out as renters, just like their local resident counterparts. The last row of Table 4 reports price and rent from the endogenous migration model. As of 2014, house price in Beijing is 21,840 RMB per square meter, which is 12.5 percent higher than the baseline model but 23 percent lower than the market price. The annual rent in 2014 is 510 RMB per square meter, lower than that in the baseline model (560 RMB) as well as in the data (700 RMB). Overall the equilibrium price and rent are still significantly lower than what we observed in the data. Thus our main finding about the gap between the model-implied and actual home price remains robust.

The lower equilibrium rent relative to the baseline model is expected. In the simulated model with endogenous migration, the average income of migrants who rent in Beijing is 83 percent of that of local renters. This leads to a lower housing consumption demand and hence a lower rent in the model with endogenous migration, compared to the baseline model where migrants and locals face the same income profile.

On the other hand, in the extended model, the higher equilibrium price relative to the baseline model reflects migrants’ stronger incentives to save than Beijing residents, because moving to Beijing gives migrants higher utility (as captured by $\phi > 1$). Compared to an average local resident, an average migrant has accumulated more wealth by the time she purchases a house in Beijing.\footnote{In the simulated model, migrants who purchase houses in Beijing have an average housing size that is 15 percent larger than local homeowners in Beijing, although the average income of the former is 3 percent lower than the latter.} The positive effect of migrating homeowners’ larger asset holdings on housing investment demand dominates the negative effect due to their lower average income. As a result, the equilibrium house price in the endogenous migration model is higher than that in the baseline model.

To further assess the quantitative performance of the extended model, we compare the model predicted income ratio between migrants and locals as well as homeownership difference between them with the data counterparts. The model implies that the average income of migrants in 2014 is 92 percent that of incumbent Beijing residents, which is slightly higher
than the 86 percent income ratio as observed in the data. In the simulated model with endogenous migration, the homeownership rate is 74.8 percent for migrants and 85.6 percent for Beijing local residents in 2014. The corresponding values in the data are 42 percent and 82 percent. Thus overall the model does a reasonable job, particularly for local residents. The over-prediction of home ownership and income for migrants is expected given that the endogenous migration model excludes rural-to-urban migration.

6 Conclusion

This paper presents a dynamic rational-expectations general equilibrium model that links house price to economic fundamentals including income growth, land supply, population structure and migration. Our model is general enough to deal with non-stationary fundamentals in transition economies. It generates rich predictions about the dynamics of house price and housing affordability from variations in both the supply and demand sides of the housing market. We apply the model to the Beijing market, and examine to what extent current house prices are consistent with rapidly changing economic fundamentals. We also develop an efficient numerical method to compute the paths of equilibrium house price and rent during the economic transition.

Under reasonable parameterizations, the model-implied equilibrium house price for Beijing is significantly lower than the market price. This gap between model and market prices is robust to alternative assumptions about land supply, income growth, population structure, mortgage rate, and down payment requirement. The discrepancy between the predicted and actual prices could come from a number of channels, such as income underreporting, sorting of households as an outcome of migration choices, search frictions in the housing market, and housing bubbles. In one of the sensitivity analyses, we extend the model by allowing

24Home ownership rate for Beijing is proxied by the average rate in the first-tier cities from the 2012 wave of the CHFS. The home ownership rate for migrants is from http://www.360doc.com/content/10/0711/07/72265_38196048.shtml. In addition, Table 2 in Wu et al. (2012) shows that about one third of the housing units in China were purchased by migrants in 2009.
households to choose endogenously whether to migrate to Beijing, which generates a current price that is higher than in the baseline model but still substantially below the actual price in the data.

Since the model provides a structural link between price and fundamentals, it can be used to analyze the impacts of a number of policies, including the implementation of property tax, tightening of migration restrictions, provision of universal medical insurance. We leave these analyses for future studies.

Appendices

Data on House Price and Rent

To ensure proper mapping between the data and the model, we obtain the market house price for Beijing in 2014 as the average house price of both newly-built and existing homes without quality adjustment. Specifically, following Garriga et al. (2016) and others, we obtain the average sales price for new homes from the National Bureau of Statistics (NBS). Since the NBS does not provide the price level for existing homes, we obtain the 2014 existing home price from the WIND database – a well-known database that collects information from financial and real estate markets.\(^{25}\) The average house sales price in both NBS and WINDs are quality unadjusted, and calculated as total dollar sales of residential buildings divided by the floor space of residential buildings. We then impute the weighted average of the new home price and the existing home price, using the shares of new and existing home transactions in the total value of transactions in Beijing in 2014 as the weights.\(^{26}\) As shown in Table 5, for the Beijing market in 2014, the new home price reported by NBS is

\(^{25}\)See [http://www.wind.com.cn/](http://www.wind.com.cn/). Although NBS does not not provide the price level for existing homes, it provides a price index for existing homes, as well as a price index for new homes. Later, we use both price indices when measuring market prices in years prior to 2014.

\(^{26}\)We thank Jing Wu for generously providing the transaction shares of the new and existing homes over time. The original data are recorded in the Real Estate Market Information System (REMIS) administered by the municipal housing authority in Beijing.
18,499 RMB per square meter, and the existing home price is 34,106 RMB per square meter according to the WIND database. The weight on new homes is 37.9 percent in 2014. We take the weighted average number, 28,194 RMB, as the Beijing house price in 2014 (in terms of 2014 RMB).

Due to the data limitation, we do not always observe average house price in the years prior to 2014, so we use the new (resp. resale) house price level in 2014 obtained above, combined with the NBS new home price index (resp. the NBS resale home price index), to infer the nominal price for new (resp. resale) homes in earlier years, and then use CPI to convert nominal prices into real prices in terms of 2014 RMB. Finally, for each year, we impute the weighted average of new home price and resale home price, using the shares of new and resale home transactions in that year as weights. We label the Beijing house price series obtained this way as the “NBS” series in Table 5.

Although appealing, the NBS price series have some limitations due to their inconsistent sampling method over time and lack of control for house quality, as pointed out by Fang et al. (2015) and Wu et al. (2016) among others. For these reasons, we provide an additional price measure as a robustness check. We name the new measure as the “THU” series because it is based on the constant quality house price indices provided by the Hang Lung Center for Real Estate at Tsinghua University. The center provides price indices for new homes (since 2004) and price indices for resale homes (since 2009). The THU series we imputed uses the repeated sales index which does not adjust for land supply, floor-to-area ratio and suburbanization (through distance to city centre). In an ideal world where housing stock is truly homogenous, the repeated sales index would be equivalent to the average-price-based NBS index. By controlling for quality variations, the THU price measure reflects the average price of constant-quality homes in Beijing over time, consistent with the “homogenous housing stock” concept of our theoretic model.

27The data are publicly available at http://www.tsinghua.edu.cn/publish/creen/9569/index.html. There are three different measures of price indices: average sales price, hedonic index and repeated-sales index (see http://www.cre.tsinghua.edu.cn/publish/cre/9254/20150505/15251430808272951.pdf).
Table 5 also presents the annual house rent series based on the rental housing price index for Beijing as reported by the NBS. The NBS estimates the rent series using the rents of both new and existing homes. The average annual rent is 744 RMB per square meter in 2014.\textsuperscript{28}

To obtain conceptually consistent measures price-income (price-rent) ratio as in the model, we use the weighted average price (rent) of existing and new homes reported by the NBS. We define price-income ratio as \( (\text{price per square meter}) \times (\text{average number of square meters per capita}) / (\text{average income per capita}) \). Data on income and housing size per capita are also from the NBS. The last column of Table 5 shows housing size per capita in Beijing which has been increasing steadily. Price-rent ratio is calculated as \( (\text{average house price}) / (\text{average rent}) \).

### Table 5: House price, rent and housing size per capita

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>existing</td>
<td>new</td>
<td>existing</td>
<td>new</td>
<td>overall</td>
</tr>
<tr>
<td>2005</td>
<td>0.26</td>
<td>0.74</td>
<td>28478</td>
<td>7997</td>
<td>13411</td>
</tr>
<tr>
<td>2006</td>
<td>0.34</td>
<td>0.66</td>
<td>33889</td>
<td>9430</td>
<td>17797</td>
</tr>
<tr>
<td>2007</td>
<td>0.44</td>
<td>0.56</td>
<td>34621</td>
<td>13044</td>
<td>22567</td>
</tr>
<tr>
<td>2008</td>
<td>0.53</td>
<td>0.47</td>
<td>32561</td>
<td>13493</td>
<td>23692</td>
</tr>
<tr>
<td>2009</td>
<td>0.60</td>
<td>0.40</td>
<td>33567</td>
<td>15461</td>
<td>26266</td>
</tr>
<tr>
<td>2010</td>
<td>0.71</td>
<td>0.29</td>
<td>33276</td>
<td>19430</td>
<td>29263</td>
</tr>
<tr>
<td>2011</td>
<td>0.69</td>
<td>0.31</td>
<td>31143</td>
<td>16695</td>
<td>26684</td>
</tr>
<tr>
<td>2012</td>
<td>0.62</td>
<td>0.38</td>
<td>31576</td>
<td>17341</td>
<td>26101</td>
</tr>
<tr>
<td>2013</td>
<td>0.69</td>
<td>0.31</td>
<td>35249</td>
<td>18229</td>
<td>30044</td>
</tr>
<tr>
<td>2014</td>
<td>0.62</td>
<td>0.38</td>
<td>3106</td>
<td>18499</td>
<td>28194</td>
</tr>
</tbody>
</table>

Note: This table shows the weights of new and existing houses, house price, annual rent and housing size per capita in Beijing between 2005-2014. The weights are the fraction of new and existing home transactions in the total value of home transactions in a given a year.

\textsuperscript{28}Rental housing price index is available at \url{http://data.stats.gov.cn/easyquery.htm?cn=E0103&zb=A0902&reg=110000&sj=2013}. The 2014 rent level is from the Guoxinda database \url{http://www.gxdgroup.com.cn/}. 

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References


