Journal of Banking and Finance 73 (2016) 1–15

Contents lists available at ScienceDirect

Journal of Banking and Finance

journal homepage: www.elsevier.com/locate/jbf

Idiosyncratic risk, costly arbitrage, and the cross-section of stock returns☆

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ARTICLE INFO

Article history:
Received 26 December 2014
Accepted 10 August 2016
Available online 18 August 2016

JEL Classification:
G11
G12
G14

Keywords:
Costly arbitrage
Idiosyncratic risk
Mispricing

ABSTRACT

We test a new cross-sectional relation between expected stock return and idiosyncratic risk implied by the theory of costly arbitrage. If arbitrageurs find it more difficult to correct the mispricing of stocks with high idiosyncratic risk, there should be a positive (negative) relation between expected return and idiosyncratic risk for undervalued (overvalued) stocks. We combine several well-known anomalies to measure stock mispricing and proxy stock idiosyncratic risk using an exponential GARCH model for stock returns. We confirm that average stock returns monotonically increase (decrease) with idiosyncratic risk for undervalued (overvalued) stocks. Overall, our results support the importance of idiosyncratic risk as an arbitrage cost.

1. Introduction

It has been well established that stock returns are predictable in the cross-section (e.g., momentum and book-to-market effect). Researchers still debate whether such return predictabilities reflect stock mispricing caused by various psychological biases. In a frictionless market, sophisticated traders fully eliminate mispricing. However, in the presence of frictions, arbitrage is costly and mispricing is not fully eliminated (e.g., Shleifer and Vishny, 1997). Mispricing persists because a variety of costs affects the ability of rational agents to profit from mispricing. Under costly arbitrage, arbitrageurs and mispricing co-exist in equilibrium. The equilibrium amount of mispricing depends on the arbitrage costs, or frictions that prevent rational traders from exerting price pressure that totally eliminates mispricing. Stocks that are more costly to arbitrage should face greater mispricing.

Arbitrage costs include both transaction costs and holding costs. Transaction costs are incurred when a transaction occurs such as brokerage fees, commissions and market impact. Holding costs are costs that are incurred every period that a position remains open. Examples of holding costs include the opportunity cost of capital, the opportunity cost of not receiving full interest on short-sale proceeds, and idiosyncratic risk exposure. Holding costs force arbitrageurs to take limited positions in mispriced securities. They make even seemingly riskless arbitrage risky, since losses will be incurred if the mispricing does not dissipate quickly enough (Tuckman and Vila, 1992).

Shleifer and Vishny (1997) and Pontiff (2006) identify idiosyncratic risk as the primary arbitrage holding cost.[2] Conceptually, if

☆ We thank Carol Alexander (Editor), two anonymous referees, Tarun Chordia, Fangjian Fu, Lorenzo Garlappi, John Griffin, Danling Jiang, Alok Kumar, Si Li, Stephen Magee, Rudi Schadt, Clemens Sialm, Laura Starks, Sheridan Titman, and seminar participants at Chinese University of Hong Kong, City University of Hong Kong, Hong Kong Baptist University, Hong Kong University of Science and Technology, Korea University, National University of Singapore, Peking University, Renmin University, Shanghai University of Finance and Economics, Shanghai Jiaotong University, Singapore Management University, University of Hong Kong, University of Melbourne, and Wharton Research Data Service for helpful comments. We have also benefited from the comments of participants at Chulalongkorn Accounting and Finance Symposium at Bangkok, 17th Conference on the Theories and Practices of Securities and Financial Market, and Financial Management Association Annual Meeting. We acknowledge the financial support from Rega Technologies Limited and two grants from the Research Grant Council of the Hong Kong Special Administrative Region, China (Project No. CUHK 440410 and 14501115). We are grateful for research assistance from Xintong (Eunice) Zhan. All errors are our own.

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[2] Based on a survey of empirical findings in the literature, Pontiff (2006) conclude that “A common theme that unifies this literature is that the primary source of arbitrage costs occurs from holding costs, and in particular, idiosyncratic risk. All of the above papers that simultaneously estimate the impact of idiosyncratic risk and transaction costs on mispricing, find that the impact of idiosyncratic risk on mispricing dwarfs the impact of transaction costs on mispricing.”

http://dx.doi.org/10.1016/j.jbankfin.2016.08.004
0378-4266/

the arbitrageur cannot perfectly hedge the fundamental value of the arbitrage position, then arbitrage involves risk. Unhedgeable fundamental risk imposes a cost. Arbitrageurs are unable to hedge idiosyncratic risk, and thus they must trade off between the expected profit from a position and the idiosyncratic risk to which the position exposes them. Although a mispricing opportunity is a benefit to the arbitrageur, yet she does not take an infinite position in the mispriced security. The size of the arbitrage position will be chosen such that the marginal profit of the arbitrage position is zero. All else equal, a risk-averse arbitrageur will take a relatively small position in a high idiosyncratic risk stock. Thus, idiosyncratic risk hinders the correction of price errors by effectively imposing a holding cost on arbitrageurs (Pontiff, 2006).[3]

Motivated by the theory of costly arbitrage, this paper examines the impact of idiosyncratic risk (as part of arbitrage holding costs) on expected stock returns. Stocks that are more costly to arbitrage tend to have greater mispricing, since costs affect sophisticated traders' ability to exert corrective price pressure. In other words, the greater the cost, the greater the equilibrium mispricing. Because idiosyncratic risk is a cost to the arbitrageurs, arbitrageurs will push stock mispricing towards zero, but do so less for high idiosyncratic risk stocks, as arbitrageurs take smaller positions in these stocks. Hence the largest mispricing should be found in the highest idiosyncratic risk stocks, as these stocks receive the least arbitrage resources. If the abnormal returns associated with various anomalies reflect mispricing, then they should be greater in magnitudes among stocks with high idiosyncratic risk. This is our first test hypothesis.

Second, under the costly arbitrage theory, the equilibrium relation between idiosyncratic risk and expected stock return should vary with the direction and magnitude of mispricing: expected stock return should monotonically increase in idiosyncratic risk among undervalued stocks and monotonically decrease in idiosyncratic risk among overvalued stocks; among fairly-valued stocks, the costly arbitrage theory implies no significant relation between idiosyncratic risk and expected stock return. This non-monotonic relation between idiosyncratic risk and expected stock return is new and unique to the theory of costly arbitrage. All cross-sectional asset pricing models whether risk-, tax-, or transaction cost based, rely on monotonic relations between expected returns and the variables that drive expected returns.[4] As Pontiff and Schill (2004) show, costly arbitrage tests are useful because they test a specific relation between the mispricing and costly arbitrage proxies that is dictated by the behavior of economically rational traders.

Our second hypothesis is an equilibrium consequence of how idiosyncratic risk (as an arbitrage cost) affects the arbitraguers' demand for stocks when their prices deviate from the fair values. For stocks with high idiosyncratic risk, the corrective price pressure from the arbitrageurs is weaker and thus arbitrage is less effective at pushing the price back to the fundamental value, compared to the case of stocks with low idiosyncratic risk. Idiosyncratic risk deters rational traders from short selling overvalued stocks and buying undervalued stocks. When the high idiosyncratic risk stocks are overvalued, the weaker arbitrage activities imply that their market prices exceed the corresponding fair values by a larger amount compared to the low idiosyncratic risk stocks, leading to lower (more negative) expected future returns. Equivalently, shorting high idiosyncratic risk stocks when they are overvalued delivers higher average future returns, which is necessary in order to attract arbitraguers to these more costly and risky trades. Similarly, among undervalued stocks, those with high idiosyncratic risk would have higher expected returns in equilibrium to make arbitraguers indifferent between buying stocks with high idiosyncratic risk versus those with low idiosyncratic risk. Thus, among undervalued stocks, expected returns increase with stock idiosyncratic risk.

To measure the amount of idiosyncratic risk exposure that arbitrageurs are expected to endure after they establish a position in a given stock, we first fit an exponential GARCH model to historical stock returns and then estimate the expected idiosyncratic volatility over the investment horizon.[5] To measure relative stock mispricing, we define an arbitrage score for each stock in each week by combining four well known anomalies. At the beginning of each week, all stocks are independently sorted into deciles from low to high, based on book-to-market, the compounded gross return from t-52 weeks to t-4 weeks, negative size and negative return of the previous week.[6] Stocks are assigned the corresponding score of its decile rank. Arbitrage score is the sum of these four ranks and ranges from 4 to 40.

We find that the arbitrage score strongly forecasts the cross section of future returns. The portfolio of stocks with high arbitrage score on average outperform the portfolio of stocks with low score by 0.88% (0.59%) per week on equal-weighted (value-weighted) basis. The differences are statistically significant and cannot be explained by the Fama and French (1993) three-factors and the Carhart (1997) four-factors model. These findings support our arbitrage score as an accurate measure of relative stock mispricing. Stocks with high arbitrage score are undervalued while those with low arbitrage score are overvalued.

Further, we find that the return spread between high arbitrage score stocks and low arbitrage score stocks is significantly higher among stocks with high idiosyncratic risk. For example, when value weighted, high arbitrage score stocks on average outperform low arbitrage score stocks by 0.44% per week among stocks with low idiosyncratic risk. In contrast, high arbitrage score stocks on average outperform low arbitrage score stocks by 1.36% per week among stocks with high idiosyncratic risk. The difference is 0.92% and significant. Similarly, the equal-weighted return of our strategy sorting on arbitrage score is 2% (0.3%) for the high (low) high idiosyncratic risk stocks. These findings are consistent with our first hypothesis that the magnitude of mispricing and abnormal returns are larger for stocks with high idiosyncratic risk because they face more limits to arbitrage.

To test the second hypothesis, we examine the conditional relation between expected stock return and idiosyncratic risk. We find that within the highest arbitrage score quintile (relatively undervalued stocks), high idiosyncratic risk stocks *outperform* low idiosyncratic risk stocks by 0.96% (0.34%) per week for equal-weighted (value-weighted) returns. In contrast, within the lowest arbitrage score quintile (relatively overvalued stocks), high idiosyncratic risk stocks *underperform* low idiosyncratic risk stocks by 0.67% (0.36%) per week for equal-weighted (value-weighted) returns. Within the middle arbitrage score quintile (neither undervalued nor overvalued), there is no significant difference between the returns of high idiosyncratic risk stocks and low idiosyncratic risk stocks. These results strongly support our second hypothesis that there is a positive relation between expected return and expected

[3] The active portfolio management theory (e.g., Treynor and Black, 1973) also shows that the portfolio weights chosen by an informed arbitrageur are negatively related to a security's idiosyncratic risk.

[4] For example, Merton (1987) argues for a monotonic positive relation between idiosyncratic risk and expected return when investors can only invest in a subset of stocks due to informational frictions. Different from our hypotheses, Merton's prediction does not consider mispricing or limits to arbitrage.

[5] Exponential GARCH models are capable of capturing both the clustering and asymmetric properties of time-varying volatility. The idiosyncratic volatility is estimated relative to the Fama and French (1993) three-factor model.

[6] These four firm characteristics are known to forecast future returns and cannot be fully accounted for by risk-based explanations.

idiosyncratic volatility for relatively undervalued stocks, but a negative relation for relatively overvalued stocks.

We conduct extensive analysis to document the robustness of the conditional relation between expected return and expected idiosyncratic volatility. For example, when we look at subsamples of stocks stratified by size, book-to-market ratio, turnover, price level or the exchanges they trade on, we continue to find a positive relation among undervalued stocks and negative relation among overvalued stocks that is significant both economically and statistically. Our results hold consistently over subperiods as well. We find similar patterns in each of the Fama-French 12 industries. The effect of idiosyncratic risk as an impediment to arbitrage remains strong and significant after we control for other arbitrage costs such as bid-ask spreads, illiquidity and short-sale constraints. Most of our results are based on portfolio sorts, but we also verify our results using Fama–MacBeth regressions. Our results are also robust to alternative estimates of expected idiosyncratic risk.

This paper contributes to a growing literature that examines the impact of limits to arbitrage on the cross-section of stock returns. As far as we are aware, we are the first to propose arbitrage score as an effective and simple measure of relative stock mispricing. Although there are studies on how arbitrage risk affects individual anomaly (e.g., Ali et al., 2003 for book-to-market effect, Mendenhall, 2004 for post-earnings-announcement drift, Ang et al., 2006 for momentum, McLean, 2010 for long-term reversal, Cao et al., 2016a for the partner-based trading effect), few papers examine the cross-sectional relation between idiosyncratic risk and stock returns conditioning on the relative mispricing.[7] A rare exception is Brav et al. (2010), who document that value-weighted annual returns do not increase in annually updated idiosyncratic volatility among "undervalued" stocks. In their study, undervalued stocks are either small firms, or value firms or recent winners. In contrast, we measure the relative mispricing of stocks by combining different anomalies into one measure. Stambaugh et al. (2015) follow our approach to aggregate different anomalies to define relative mispricing. They do not find a strong positive relation between idiosyncratic risk and monthly returns among undervalued stocks. Different from the annual or monthly horizon of these studies, we focus the weekly horizon for both expected idiosyncratic volatility and stock return.[8] Duan et al. (2010) also argues that idiosyncratic risk deters arbitrages. They focus on the most heavily shorted stocks, and find a negative relation between idiosyncratic risk and returns among high short interest stocks. In contrast, we study the full cross-section of stocks, and examine the impact of idiosyncratic risk on stock returns separately for undervalued and for overvalued stocks.

Our paper focuses on the cross-sectional relation between expected return and expected idiosyncratic volatility *conditional* on the relative mispricing of stocks. Our work is related to but differs from recent studies that examine the *unconditional* relation between stock return and idiosyncratic volatility (e.g., Ang et al., 2006; 2009; Bali and Cakici, 2008; Bali et al., 2011; Fu, 2009; Huang et al., 2010; Khovansky and Zhylyevskyy, 2013; Kumar and Han, 2013; Loh and Hou, 2016).[9] Prior empirical evidence on the idiosyncratic volatility premium is mixed, some reporting a positive relation between stock return and idiosyncratic volatility while others find a negative relation. Our paper helps reconcile the seemingly contradictory evidence in the previous studies on the idiosyncratic volatility premium. The limits to arbitrage theory predicts, and our empirical results verify a positive (negative) relation between stock return and idiosyncratic volatility for undervalued (overvalued) stocks. Therefore, the net effect of idiosyncratic volatility on stock return in a given sample would crucially depend on whether most stocks are undervalued or overvalued.

The rest of paper is as follows: Section 2 describes the data and the measure for expected idiosyncratic risk as well as for relative stock mispricing. Section 3 presents the results for our main test hypotheses. Section 4 presents the results from robustness tests and Section 5 concludes.

2. Data and variables

We focus on weekly stock returns because arbitrageurs care about short-term performances of their positions (see Shleifer (2000)). Even when the signals that forecast returns are updated every year (e.g. the book-to-market ratio), arbitrageurs can change the portfolio weights of mispriced stocks more frequently, because expected idiosyncratic risk varies over the short-horizon.

We obtain daily stock returns for NYSE, AMEX, and NASDAQ stocks from the Center for Research in Security Prices (CRSP), and compounded them to obtain weekly returns (Monday–Friday) from July 1963 to December 2006.[10] The weekly returns of common risk factors and risk-free rate are taken from Kenneth French's website. We also obtain stock price, trading volume and shares outstanding from CRSP. Annual accounting data and quarterly earnings-announcement data are obtained from Compustat. We obtain analyst coverage and earnings forecasts data from I/B/E/S. The quarterly institutional holding data are from CDA/Spectrum Institutional (13f) database.

2.1. Idiosyncratic risk

Following recent studies, we measure a stock's idiosyncratic risk as the standard deviation of residuals from fitting the Fama and French (1993) three-factor model to the stock's realized returns.[11] Risk-averse arbitrageurs care about the expected idiosyncratic risk exposure they face over the holding period. We apply exponential GARCH (EGARCH) models to capture the time-variation of idiosyncratic risk.[12] Assuming that arbitrageurs could utilize available historical information to forecast the expected idiosyncratic risk, we estimate the following EGARCH (1,1) model with weekly returns and then obtain an out-of-sample forecast of the conditional idiosyncratic volatility over the next week:

$$R_{it} - r_t = \alpha_i + \beta_i (R_{mt} - r_t) + s_i SMB_t$$
$$+ h_i HML_t + \varepsilon_{it}, \varepsilon_{it} \sim N(0, \sigma_{it}^2)$$

[7] Our paper differs significantly from, Doukas et al. (2010) who conduct time-series tests of the impact of limits to arbitrage. They find that the average return difference between undervalued and overvalued stocks is larger among the subsample of stocks with high arbitrage cost than among the subsample of stocks with low arbitrage cost. Their paper is not about the conditional or unconditional relation between idiosyncratic risk and stock returns.

[8] Khovansky and Zhylyevskyy (2013) show that the sign of idiosyncratic volatility premium importantly depends on the frequency of the return data used in the tests.

[9] Recent studies have also investigated the relation between idiosyncratic volatility and future returns of options. See e.g., Cao et al. (2016b); Cao and Han (2013).

[10] To be consistent with Fama-French weekly factors, we measure weekly stock returns from Monday to Friday. To mitigate nonsynchronous trading or bid-ask bounce effects in daily prices, we also use Thursday–Wednesday weekly stock return and reconstruct corresponding Fama-French weekly factors using the daily data. All results remain robust.

[11] Practitioners may use models with extra factors (such as industry factors) to help to model the risk of under-diversified portfolio. However, in the context of arbitrage, these models are most likely over-specified since it is difficult to hedge out the systematic risk associated with so many common factors.

[12] GARCH models have been widely used to model the conditional volatility of returns. Pagan and Schwert (1990) fit a number of different models to monthly U.S. stock returns and find that (Nelson, 1991)'s EGARCH model is the best in overall performance. EGARCH models are able to capture the asymmetric effects of volatility, and they do not require restricting parameter values to avoid negative variance as do other ARCH and GARCH models. Fu (2009) and Spiegel and Wang (2006) also use exponential GARCH models to estimate idiosyncratic volatility.

$$\ln \sigma_{it}^2 = a_i + b_i \cdot \ln \sigma_{i,t-1}^2 + c_i \cdot \theta_i \cdot \left(\frac{\varepsilon_{i,t-1}}{\sigma_{i,t-1}}\right) + c_i \cdot \left[\left|\frac{\varepsilon_{i,t-1}}{\sigma_{i,t-1}}\right| - (2/\pi)^{1/2}\right] \quad (1)$$

For each stock and in each week, we estimate the EGARCH (1, 1) model using all the available historical returns data.[13] To obtain accurate results, we require at least 260 weekly return observations for the estimations.[14] The full sample is from July 1963 to December 2006. Since five-year return history is required to estimate the EGARCH model and forecast volatility, the testing period starts from June 1968, covering 2010 weeks. A total of 7.4 million EGARCH estimations are conducted with a mean (median) sample size of 724 (592) weekly observations. Table A2 in the Appendix reports summary statistics of the EGARCH(1,1) model parameters along with summary statistics of contemporaneous idiosyncratic volatility and one-week ahead forecast of expected idiosyncratic volatility (Eidio). The latter is our main measure of idiosyncratic risk, which is part of the arbitrage holding costs.

2.2. Measures of arbitrage costs

Throughout this study, idiosyncratic risk is our main proxy for arbitrage costs. For robustness and comparisons, we also use alternative measures of arbitrage costs such as proxies for transaction costs, short-sale constraints and information uncertainty. One measure of direct transaction costs is bid-ask spreads, defined as 2(ask-bid)/(ask+bid). We also use *stock price* at the end of the previous week as an alternative measure. We measure indirect transaction costs by *illiquidity*, defined as the average daily Amihud measure over previous week. We follow Nagel (2005) and use the *percentage of institutional ownership* at the end of the most recent quarter as a proxy for short-sale constraints.

Information uncertainty is a risk that arbitrageurs are uncertain about the true fundamental value of their arbitrage positions due to information or valuation uncertainty. Following Zhang (2006), we evaluate this risk with three proxies. The first proxy is *firm age*, which is the number of years since the firm first appeared in the CRSP database. The second proxy is *analyst coverage*, measured as the number of analysts following the firm in the previous month. The third proxy is *dispersion in analyst earnings forecasts*, defined as the standard deviation of analyst forecasts in the previous month scaled by the prior year-end stock price.[15]

In Section 4.2, we check the robustness of our results controlling for these alternative measures of arbitrage costs. In addition, we compare the strength of asset pricing impact of idiosyncratic risk and other arbitrage costs. We find that idiosyncratic risk has a more significant and consistent impact on stock prices than these alternative measures of arbitrage costs.

[13] In robustness checks, we estimate alternative EGARH (p, q) models, for p and q up to 3. Using weekly returns to estimate idiosyncratic risk is a compromise between monthly and daily frequency. The monthly return history is short for GARCH based estimations and thus weekly returns could offer improved estimation accuracy (See Table A1 of the Appendix). Although daily or even intra-daily returns can improve precision further, they may also introduce confounding microstructure influences (such as bid-ask bounce and nonsynchronous trading). In addition, it is difficult to use GARCH based conditional volatility models on daily returns to provide precise volatility forecasts of longer horizons such as one week.

[14] This is a compromise between having precise volatility estimates and keeping enough young firms. Table A2 shows the relation between regression sample size and the accuracy of EGARCH estimations.

[15] Our results remain the same if we use the standard deviation of analyst forecasts scaled by the absolute value of the mean forecast as an alternative measure of dispersion in analyst earnings forecasts.

2.3. Measure of relative mispricing: arbitrage score

Our main empirical test concerns how expected stock return is related to idiosyncratic risk (Eidio). Under the theory of costly arbitrage, this relation crucially depends on whether the stocks are undervalued or overvalued (see our second hypothesis).

We construct an arbitrage score measure of relative stock mispricing based on a mixture of both quantitative and fundamental information, both long-term and short-term information. Our arbitrage score aggregates four well-known anomalies: short-term return reversals, size, book-to-market, and momentum effects.[16] Specifically, at the beginning of each week, all stocks are independently sorted into deciles from low to high, based on BE/ME, the compounded gross return from t-52 week to t-4 week, negative size and negative return of the previous week. Each stock is given the corresponding score of its decile rank. We define the arbitrage score as the sum of the four scores so that it ranges from 4 to 40. In Section 3.2, we verify that stocks with high arbitrage scores tend to be relatively undervalued, while stocks with low arbitrage scores tend to be relatively overvalued.

3. Empirical results

3.1. Summary statistics

Table 1 presents some properties of EGARCH (1,1) estimated conditional expected idiosyncratic volatility over the next week (Eidio). Panel A reports the cross-sectional distribution of the autocorrelation functions of Eidio, up to five lags. The median correlation is 0.51 for the first lag and decays slowly to 0.24 for the fifth lag. However, the persistence of Eidio varies significantly across individual stocks. Panel B shows that the likelihood of staying in the same Eidio-ranked decile is not very high even after just one week, and it decreases further as the horizon extends. For example, 13.47% of the stocks in decile 1 and 28.66% of the stocks in decile 10 move to other deciles after one week. Overall, the results in Table 1 suggest that there is significant cross-sectional and time-series variation in stock's idiosyncratic risk, and weekly updating the Eidio measure is necessary for cross-sectional analysis.

Table 2 Panel A presents the pooled descriptive statistics of Eidio and other variables used in our empirical tests.[17] In Panel B, we sort stocks by Eidio into five quintiles, and report the time-series averages of several firm characteristics (including firm size, book-to-market ratio, past returns, institutional ownership and liquidity) averaged across all stocks in a given Eidio quintile. The average weekly idiosyncratic volatility in the top Eidio quintile is 0.11, almost five times higher than that of the bottom Eidio quintile. Low Eidio stocks are much larger in market capitalization. On average, Quintile 1 has a market share of 51.76% while quintile 5 contains only 1.87% of the CRSP market capitalization. High Eidio stocks tend to have high market beta, low price, poor liquidity, less coverage by analysts, and high analyst dispersion.

The correlations between Eidio and other variables are further documented in Table 3. We estimate both Pearson and nonparametric Spearman correlations each week and report their time-series means. Consistent with Table 2, Eidio is not significantly correlated with book-to-market ratio or past stock returns, but it is correlated with variables such as stock price level, institutional

[16] For these anomalies, see e.g., Fama and French (2008); Jegadeesh (1990); Jegadeesh and Titman (1993). These four variables are available for most of the stocks and for a long sample period. Including other anomalies such as accruals and post-earning-announcements drift would reduce the sample size significantly.

[17] To avoid the impact of extreme outliers, the observations on Eidio and several other variables are winsorized each week at 0.5% level. See Table 2 legend.

Table 1
Time-Series Properties of Expected Idiosyncratic Risk. We estimate the conditional expected idiosyncratic volatility (Eidio) of each stock in each week using a EGARCH(1,1) specification on Fama-French 3-factor model and all the historical weekly returns data. Estimates are only conducted if at least 260 observations exist. Panel A reports the cross-sectional distribution of the autocorrelation of Eidio. Panel B reports the likelihood that a stock in a given Eidio decile will stay in the same Eidio decile over various future horizons. The sample period is from July 1963 to December 2006. The expected idiosyncratic volatility measure starts from June 1968.

Panel A: Cross-Sectional Distribution of Autocorrelation Function of Eidio									
LAG	P10	P20	P30	P40	P50	P60	P70	P80	P90
1	−0.12	0.05	0.20	0.36	0.51	0.65	0.76	0.85	0.92
2	0.04	0.15	0.26	0.37	0.48	0.59	0.70	0.80	0.89
3	−0.08	0.01	0.09	0.19	0.31	0.45	0.59	0.72	0.84
4	−0.01	0.07	0.14	0.23	0.34	0.45	0.57	0.69	0.82
5	−0.06	0.00	0.07	0.14	0.24	0.36	0.50	0.64	0.78

Panel B: Likelihood (%) of Staying in the Same Eidio Decile							
Eidio Decile Week t	Week t + 1	Week t + 2	Week t + 3	Week t + 4	Week t + 9	Week t + 13	Week t + 52
1-Low	86.53	84.45	81.50	80.49	75.55	73.54	65.69
2	71.70	66.52	61.64	59.47	51.76	48.84	40.01
3	64.06	58.08	52.69	50.39	42.52	39.91	32.05
4	59.26	53.40	47.89	45.81	38.44	36.12	28.86
5	55.74	50.36	44.75	43.06	36.07	33.92	27.23
6	52.94	48.20	42.80	41.46	35.09	33.12	26.79
7	51.03	46.95	41.80	40.77	34.94	33.10	27.16
8	51.04	47.73	42.61	41.93	36.38	34.74	29.30
9	55.04	52.58	47.39	46.95	41.38	39.86	34.05
10-High	71.34	71.11	65.87	66.30	60.81	59.19	51.69

Table 2
Summary Statistics. This table reports the summary statistics of variables used in our empirical study for stocks traded in the NYSE, AMEX, or Nasdaq. Panel A presents the pooled descriptive statistics. Panel B reports the average firm characteristics for quintile portfolios sorted by the expected idiosyncratic risk (Eidio). Time-series averages of the cross-sectional mean (median for BM) are reported in Panel B. The sample period is from July 1963 to December 2006. Stocks are included if they have at least 260 weeks of return data. Eidio is the estimated weekly expected idiosyncratic volatility from EGARCH(1,1) on Fama-French 3-factor model. ME is the firm's market capitalization at the end of last week. BE/ME (BM) is the fiscal-yearend book value of common equity divided by the calendar-yearend market value of equity. $Ret_{(-52,-4)}$ is the compound gross return from t-52 weeks to t-4 weeks. $Ret_{(-1,0)}$ is the raw return of previous week. Price is the closing price at the end of last week. Illiquidity is the Amihud (2002)'s illiquidity measure of last week. IO is institutional ownership defined as the percentage of common stocks owned by institutions in the previous quarter. Analyst Cov. is analyst coverage defined as the number of analysts following the firm in the previous month. Analyst Disp. is the analyst dispersion defined as the standard deviation of analyst forecasts in the previous month scaled by the prior year-end stock price. Market Shares (%) is the time-series average of portfolio market value relative to total market value. The observations on Eidio, Ln(ME), BM, $Ret_{(-52,-4)}$, $Ret_{(-1,0)}$, Illiquidity, Institutional Ownership, and Analyst Dispersion are winsorized each week at 0.5% level.

Variables	Panel A pooled descriptive statistics					Panel B across idiosyncratic risk quintiles				
	Mean	Std Dev	Q1	Median	Q3	Low Eidio	2	3	4	High Eidio
Eidio	0.06	0.04	0.03	0.05	0.07	0.02	0.04	0.05	0.07	0.11
Ln(ME)	4.66	2.13	3.12	4.55	6.12	5.78	5.45	4.69	3.88	2.86
BM	1.79	7.52	0.43	0.75	1.25	0.89	0.83	0.86	0.86	0.83
Ret (−52,−4) (%)	15.16	53.14	−14.99	7.88	33.33	13.62	15.09	15.59	15.34	10.85
Ret (−1, 0) (%)	0.36	8.00	−2.64	0.00	2.72	0.22	0.25	0.24	0.25	0.69
Stock Price	18.94	18.18	5.88	14.01	26.25	30.40	27.43	19.48	12.66	6.59
Age	13.93	8.15	7.63	11.38	17.96	14.78	14.96	13.36	11.76	10.45
Ln(Volume+1)	4.29	2.35	2.66	4.23	5.92	3.93	4.12	4.10	3.94	3.81
Illiquidity	4.95	23.83	0.01	0.10	0.93	0.53	1.06	2.31	5.37	16.43
Institutional Ownership	0.31	0.27	0.07	0.24	0.50	0.29	0.39	0.36	0.27	0.15
Analyst Coverage	7.41	7.33	2.00	5.00	11.00	10.30	8.40	6.67	5.08	3.39
Analyst Dispersion (%)	1.19	4.88	0.08	0.21	0.63	0.34	0.54	1.06	2.06	5.45
Market Shares (%)						51.76	27.48	13.17	5.72	1.87

ownership, the Amihud illiquidity, analyst coverage and analyst forecast dispersion. These variables proxy for transaction costs, short-sale constraints and information uncertainty.

3.2. Arbitrage score strategy

Our arbitrage score aggregates four well-known anomalies: short-term return reversals, size, book-to-market, and momentum effects. As shown in Table 3, the correlations among these four anomalies are low, with the highest spearman correlation of −0.32 between size and BE/ME, and the lowest correlation of 0.01 between last week return and BE/ME. Therefore, using individual anomaly to define mispricing could be problematic because the same stock could be simultaneously subject to several anomalies, which are not perfectly correlated.[18]

Table 4 Panel A reports various firm characteristics across arbitrage score quintiles. Arbitrage score is weakly related to idiosyncratic risk. The average Eidio is 0.05 in arbitrage score quintile 1 and increases to 0.07 in quintile 5. The difference (0.02) is small relative to the standard deviation of Eidio (0.04). As expected by construction of arbitrage score, it is significantly correlated with size, book-to-market ratio and past stock returns. The correlation of arbitrage score with its four components ranges from 0.36 (BE/ME) to 0.56 (Size). As shown by the average decile ranks, arbitrage score generates appropriate dispersion for all component

[18] For instance, small stocks could contain both recent losers and growth firms. As a result, simply attributing all small stock as undervalued is not precise.

Table 3
Correlations. This table reports the time-series average of cross-sectional correlations among listed variables. The Pearson correlations are shown above the diagonal with Spearman correlations below the diagonal. Eidio is the estimated weekly expected idiosyncratic volatility from EGARCH(1,1) on Fama-French 3-factor model. CAPM Beta is estimated weekly from the market model using previous 104 weeks returns data. ME is the firm's market capitalization at the end of last week. BE/ME (BM) is the fiscal-yearend book value of common equity divided by the calendar-yearend market value of equity. $\text{Ret}_{(-52,-4)}$ is the compound gross return from t-52 weeks to t-4 weeks. Ret (−1,0) is the raw return of previous week. Price is the closing price at the end of last week. VOL is weekly total trading volume of last week. Illiquidity is the Amihud (2002)'s illiquidity measure of last week. IO is institutional ownership defined as the percentage of common stocks owned by institutions in the previous quarter. Analyst Cov. is analyst coverage defined as the number of analysts following the firm in the previous month. Analyst Disp. is the analyst dispersion defined as the standard deviation of analyst forecasts in the previous month scaled by the prior year-end stock price.

	Eidio	Beta	LnME	LnBM	Ret (−52,−4)	Ret (−1, 0)	Price	LnVOL	Illiquidity	IO	Analyst Cov.	Analyst Disp.
Eidio	1	0.17	−0.51	−0.08	−0.07	0.03	−0.45	−0.08	0.27	−0.28	−0.29	0.27
Beta	0.22	1	0.11	−0.08	0.00	0.00	−0.01	0.25	−0.05	0.15	0.05	0.10
LnME	−0.53	0.14	1	−0.29	0.15	0.02	0.70	0.70	−0.33	0.61	0.78	−0.20
LnBM	−0.05	−0.10	−0.32	1	0.02	0.01	−0.16	−0.29	0.13	−0.01	−0.19	0.09
Ret (−52,−4)	−0.17	−0.02	0.21	0.03	1	0.01	0.21	0.07	−0.12	0.06	−0.03	−0.17
Ret (−1, 0)	−0.05	−0.01	0.06	0.01	0.03	1	0.03	0.05	−0.02	0.00	−0.01	−0.01
Price	−0.62	0.02	0.80	−0.23	0.34	0.09	1	0.32	−0.20	0.50	0.47	−0.21
LnVOL	−0.10	0.28	0.70	−0.32	0.06	0.06	0.36	1	−0.28	0.51	0.66	0.00
Illiquidity	0.43	−0.12	−0.80	0.28	−0.19	−0.06	−0.65	−0.72	1	−0.17	−0.13	0.11
IO	−0.25	0.21	0.65	−0.11	0.11	0.03	0.58	0.53	−0.56	1	0.46	−0.14
Analyst Cov.	−0.32	0.10	0.79	−0.23	0.02	0.01	0.51	0.69	−0.74	0.55	1	−0.10
Analyst Disp.	0.28	0.14	−0.28	0.30	−0.29	−0.03	−0.47	−0.02	0.23	−0.17	−0.15	1

Table 4
The Arbitrage Score Strategy. At the beginning of each week, all stocks are independently sorted into deciles from low to high, based on BE/ME, the compound gross return from t-52 weeks to t-4 weeks, negative size and negative return of previous week. A stock's arbitrage score is the sum of its decile rank in each of the four rankings. Each week, stocks are sorted on their arbitrage scores into quintiles. Panel A reports the mean value and average decile rank of several firm characteristics for stocks in each arbitrage score quintile. Panel B reports the average equal-weighted and value-weighted portfolio weekly returns for each arbitrage score quintile. CAPM alphas, FF-3 alphas and Carhart-4 alphas are calculated using the CAPM, Fama-French 3-factor model and Carhart (1997) 4-factor model, respectively. The Sharpe ratio is defined as average portfolio excess return over the standard deviation of portfolio raw returns. The sample period is from July 1963 to December 2006 and the testing period starts from June 1968. To adjust for serial correlation, robust Newey and West (1987) t-statistics are reported in brackets. The symbols *, **, *** denote significance at the 10%, 5% and 1% levels, respectively.

Panel A: Firm characteristics across arbitrage score quintiles

Arbitrage Score Quintile	1-Low	2	3	4	5-High	1-Low	2	3	4	5-High	Spearman Correlations
	Mean value (Median for BM)					Average decile ranks					Arbitrage Score
Eidio	0.05	0.05	0.06	0.06	0.07	5.00	5.00	5.34	5.68	6.49	0.18
Ln(ME)	6.13	5.36	4.63	3.95	3.01	7.62	6.61	5.56	4.55	3.10	−0.56
BM	0.44	0.68	0.86	1.08	1.51	2.90	4.48	5.52	6.65	8.12	0.64
$\text{Ret}_{(-52,-4)}$ (%)	−5.32	7.19	12.62	20.93	38.82	3.99	5.05	5.46	6.00	7.03	0.36
$\text{Ret}_{(-1,0)}$ (%)	4.00	1.54	0.37	−0.88	−3.41	7.52	6.17	5.47	4.81	3.50	−0.47

Panel B: Weekly Portfolio Returns (%) across Arbitrage Score Quintiles

Arbitrage Score Quintile	1-Low	2	3	4	5-High	H-L	t-stat	1-Low	2	3	4	5-High	H-L	t-stat
	Equal-weighted returns (%)							Value-weighted returns (%)						
Raw returns	−0.03	0.15	0.28	0.44	0.86	0.88***	(21.04)	0.12	0.30	0.38	0.47	0.71	0.59***	(14.65)
CAPM α	−0.24	−0.05	0.08	0.24	0.67	0.91***	(21.92)	−0.10	0.09	0.17	0.26	0.50	0.59***	(14.58)
FF-3 α	−0.26	−0.10	0.02	0.17	0.60	0.85***	(23.22)	−0.08	0.06	0.12	0.18	0.42	0.50***	(15.46)
Carhart-4 α	−0.19	−0.07	0.04	0.18	0.60	0.79***	(23.92)	−0.05	0.05	0.09	0.14	0.38	0.43***	(15.55)
Sharpe Ratio	−0.06	0.02	0.08	0.15	0.33	0.59		0.00	0.09	0.12	0.15	0.23	0.35	

anomalies, while no single anomaly dominates others. This pattern is further confirmed by the Spearman correlations.

Panel B of Table 4 examines whether arbitrage score predicts future returns. We find that stocks with higher arbitrage scores exhibit significantly higher returns next week. The (5–1) difference is 0.88% per week for equal-weighted returns (t-statistic 21.04) and 0.59% per week (t-statistic 14.65) for value-weighted returns. Both are economically large and statistically significant. The results change little for CAPM alphas, Fama-French three-factor alphas and Carhart (1997) four-factor alphas. These results support using the arbitrage score as proxy for relative stock mispricing.

If idiosyncratic risk deters arbitrage, then the magnitude of mispricing and thus abnormal returns should be larger for stocks with high idiosyncratic risk (our first hypothesis). To test this hypothesis, we examine how the return spread between high arbitrage score stocks and low arbitrage score stocks depend on idiosyncratic risk. We conduct standard 5 × 5 independent double sorts on arbitrage score and idiosyncratic risk, and then form either equal-weighted or value-weighted portfolios. Table 5 report the average returns of these portfolios.[19]

Consistent with our first hypothesis, Table 5 shows that the return spread between high arbitrage score stocks and low arbitrage score stocks is significantly higher among stocks with high idiosyncratic risk. For example, when value weighted, high arbitrage score stocks on average outperform low arbitrage score stocks by 0.44% per week among stocks with low idiosyncratic risk. In contrast, high arbitrage score stocks on average outperform low arbitrage score stocks by 1.36% per week among stocks with high idiosyncratic risk. The difference is 0.92% and significant. Similarly,

[19] Each of the double-sorted portfolios contains enough stocks to be reasonably diversified. Among the 20% of stocks with highest arbitrage score (580 stocks on average each week), the number stocks in the high and low idiosyncratic risk portfolio is 91 and 135, respectively. Among the 20% of stocks with lowest arbitrage score (582 stocks on average each week), the number stocks in the high and low idiosyncratic risk portfolio is 167 and 65, respectively.

Table 5
Independent Sort of Arbitrage Score and Idiosyncratic Risk. This table reports the equal-weighted (Panel A) and value-weighted (Panel B) average returns of portfolios independently double sorted by arbitrage score and idiosyncratic risk (Eidio). It also reports the average return differences between the top and the bottom arbitrage score quintiles within each Eidio quintile. Eidio is the estimated weekly expected idiosyncratic volatility from EGARCH(1,1) on Fama-French 3-factor model. A stock's arbitrage score is the sum of its decile rank in each of the four anomalies as defined in Table 4. The sample period is from July 1963 to December 2006 and the testing period starts from June 1968. To adjust for serial correlation, robust Newey and West (1987) t-statistics are reported in brackets. The symbols *, **, *** denote significance at the 10%, 5% and 1% levels, respectively.

Panel A: equal-weighted portfolio returns (%) of independent sorting						
Arbitrage Scores	G1-Low Eidio	G2	G3	G4	G5-High Eidio	G5-G1 raw returns
P1 Low	0.16	0.11	0.02	−0.14	−0.67	
(Most Overpriced)	(3.67)	(1.96)	(0.30)	(−1.57)	(−6.27)	
P2	0.27	0.27	0.21	0.07	−0.22	
	(6.50)	(4.79)	(3.23)	(0.90)	(−2.11)	
P3	0.30	0.33	0.30	0.25	0.23	
	(7.40)	(6.31)	(4.55)	(3.17)	(2.28)	
P4	0.34	0.42	0.42	0.41	0.61	
	(8.73)	(8.15)	(6.66)	(5.38)	(6.21)	
P5 High	0.46	0.53	0.67	0.87	1.33	
(Most Underpriced)	(11.53)	(10.97)	(11.43)	(11.63)	(13.62)	
P5-P1 raw returns	0.30***	0.42***	0.65***	1.01***	2.00***	1.70***
t-stat	(9.27)	(12.70)	(17.43)	(21.14)	(28.22)	(25.25)
Panel B: value-weighted portfolio returns (%) of independent sorting						
P1 Low	0.15	0.10	0.06	0.01	−0.41	
(Most Overpriced)	(3.31)	(1.77)	(0.90)	(0.08)	(−3.94)	
P2	0.30	0.34	0.32	0.30	0.07	
	(7.28)	(6.13)	(4.77)	(3.57)	(0.65)	
P3	0.35	0.44	0.46	0.45	0.35	
	(8.40)	(7.95)	(6.93)	(5.06)	(3.35)	
P4	0.42	0.51	0.55	0.51	0.56	
	(9.14)	(9.08)	(7.96)	(6.22)	(5.77)	
P5 High	0.59	0.65	0.70	0.84	0.95	
(Most Underpriced)	(11.54)	(11.23)	(10.57)	(10.51)	(9.73)	
P5-P1 raw returns	0.44***	0.55***	0.64***	0.83***	1.36***	0.92***
t-stat	(9.96)	(12.55)	(14.57)	(14.36)	(18.48)	(11.74)

the equal-weighted return of our strategy sorting on arbitrage score is 2% (0.3%) for the high (low) high idiosyncratic risk stocks. The difference is even larger at 1.7%. These findings strongly support the idea that idiosyncratic risk is an important measure of arbitrage cost and arbitrage score measures relative stock mispricing.

In an internet appendix, we replicate the tests in Table 5 for the largest 1000 or 500 stocks of the CRSP universe. We find that the predictive power of arbitrage score remains intact after controlling for Eidio in bivariate independent sorts. The magnitudes of our results are substantially lower for these relatively big stocks because they face low arbitrage costs.[20]

3.3. Conditional relation between expected return and idiosyncratic risk

Next we examine the relation between expected return and idiosyncratic risk conditional on relative stock mispricing proxied by arbitrage score. As shown in Tables 4 and 5, stocks within the high arbitrage score quintile are undervalued and have high future returns. Because idiosyncratic risk is an arbitrage cost, risk-averse arbitrageurs are reluctant to buy stocks with high idiosyncratic risk. Thus, high idiosyncratic risk stocks with high arbitrage score are more undervalued and have even higher expected returns than low idiosyncratic risk stocks with high arbitrage score. In other words, stock returns should increase with idiosyncratic risk among high arbitrage score stocks. On the other hand, Table 4 shows that stocks with low arbitrage score are overvalued and have low future returns. However, risk-averse arbitrageurs are reluctant to short sell low arbitrage score stocks with high idiosyncratic risk. Therefore, these stocks should have even lower returns among low arbitrage score stocks. We expect a negative relation between stock returns and idiosyncratic risk among low arbitrage score stocks.

To test the hypothesis above (our second hypothesis), we conduct dependent double sort on idiosyncratic risk and arbitrage score. Each week, stocks are first sorted by arbitrage score into quintiles and then sorted within each quintile into five portfolios based on proxy of idiosyncratic risk (Eidio). Table 6 Panel A reports the equal-weighted returns and Panel B reports the value-weighted returns for the dependent sorting. The table also reports the average return differences between the top and the bottom Eidio quintiles within each arbitrage score quintile.

Table 6 shows that when moving from the low arbitrage score quintile to the high arbitrage score quintile, the return differences between high and low idiosyncratic risk stocks change from being significantly negative to significantly positive. For instance, within arbitrage score quintile 1, stock returns monotonically decrease in idiosyncratic risk. The average return spread between high and low idiosyncratic risk stocks is −0.67% per week (t-statistic −8.36) for equal-weighted returns and −0.36% (t-statistic −4.75) for value-weighted returns. In contrast, within arbitrage score quintile 5, stock returns monotonically increase in idiosyncratic risk. The average return difference between high idiosyncratic risk stocks and low idiosyncratic risk stocks is 0.96% per week (t-statistic 11.79) for equal-weighted return and 0.34% (t-statistic 4.07) for value-weighted returns. Within arbitrage score quintile 3, there is no significant relation between idiosyncratic risk and returns for either equal-weighted and value-weighted returns.

[20] For example, amongst the largest 1000 stocks, when value weighted, high arbitrage score stocks on average outperform low arbitrage score stocks by 0.76% (resp. 0.30%) per week among stocks with high (resp. low) idiosyncratic risk. The difference is 0.46%. The same difference when we use only the largest 500 stocks is 0.38%. Both are statistically significant but smaller than the corresponding number (0.92%) for the full sample of stocks.

Table 6
Dependent Sort of Arbitrage Score and Idiosyncratic Risk. This table examines the relation between expected returns and idiosyncratic risk (Eidio) conditional on the relative stock mispricing proxied by arbitrage score. Eidio is the estimated weekly expected idiosyncratic volatility from EGARCH(1,1) on Fama-French 3-factor model. A stock's arbitrage score is the sum of its decile rank in each of the four anomalies as defined in Table 4. Each week, stocks are first sorted by arbitrage score into quintiles and then sorted within each quintile into five portfolios based on expected idiosyncratic volatility (Eidio). Panel A reports the equal-weighted returns and Panel B reports the value-weighted returns for the dependent sorting. The table also reports the average return differences between the top and the bottom Eidio quintiles within each arbitrage score quintile. FF-3 alphas are calculated using Fama-French 3-factor model. Characteristics-adjusted returns are calculated using DGTW (1997) benchmarks. Panel C reports the average arbitrage scores of the double sorted portfolios. The sample period is from July 1963 to December 2006 and the testing period starts from June 1968. To adjust for serial correlation, robust Newey and West (1987) t-statistics are reported in brackets. The symbols *, **, *** denote significance at the 10%, 5% and 1% levels, respectively.

Panel A: equal-weighted portfolio returns (%) of dependent sorting								
Arbitrage Scores	G1-Low Eidio	G2	G3	G4	G5-High Eidio	G5-G1 raw returns	G5-G1 FF-3 α	G5-G1 DGTW α
P1 Low	0.17	0.15	0.07	−0.02	−0.50	−0.67***	−0.67***	−0.66***
(Most Overpriced)	(3.76)	(2.75)	(1.09)	(−0.22)	(−4.94)	(−8.36)	(−13.66)	(−14.88)
P2	0.27	0.27	0.24	0.16	−0.17	−0.43***	−0.45***	−0.43***
	(6.55)	(5.3)	(3.87)	(2.03)	(−1.71)	(−5.49)	(−9.60)	(−9.78)
P3	0.29	0.33	0.29	0.26	0.23	−0.06	−0.08	−0.08
	(7.30)	(6.43)	(4.6)	(3.39)	(2.28)	(−0.72)	(−1.44)	(−1.55)
P4	0.34	0.41	0.41	0.40	0.61	0.27***	0.25***	0.26***
	(8.36)	(7.78)	(6.34)	(5.27)	(6.14)	(3.50)	(4.88)	(4.89)
P5 High	0.47	0.63	0.76	1.00	1.43	0.96***	0.91***	0.90***
(Most Underpriced)	(11.43)	(11.28)	(10.97)	(12.15)	(13.54)	(11.79)	(14.68)	(13.93)
Panel B: value-weighted portfolio returns (%) of dependent sorting								
P1 Low	0.15	0.13	0.10	0.05	−0.22	−0.36***	−0.36***	−0.36***
(Most Overpriced)	(3.31)	(2.44)	(1.45)	(0.66)	(−2.20)	(−4.75)	(−6.55)	(−7.13)
P2	0.30	0.33	0.35	0.31	0.17	−0.12	−0.11**	−0.12**
	(7.20)	(6.36)	(5.52)	(3.97)	(1.80)	(−1.54)	(−2.19)	(−2.30)
P3	0.34	0.44	0.46	0.47	0.35	0.01	0.04	0.00
	(8.27)	(8.05)	(7.12)	(5.51)	(3.34)	(0.10)	(0.60)	(0.08)
P4	0.44	0.52	0.55	0.50	0.53	0.10	0.12**	0.05
	(9.25)	(8.92)	(7.81)	(6.09)	(5.35)	(1.20)	(2.17)	(0.98)
P5 High	0.60	0.69	0.78	0.90	0.95	0.34***	0.34***	0.33***
(Most underpriced)	(12.03)	(10.79)	(10.26)	(10.26)	(9.08)	(4.07)	(5.48)	(4.95)
Panel C: average arbitrage score across 5x5 dependently sorted portfolios								
P1 Low	14.10	13.81	13.64	13.61	13.74			
P2	18.63	18.62	18.65	18.67	18.71			
P3	21.91	21.94	21.97	21.98	21.97			
P4	25.19	25.26	25.31	25.34	25.36			
P5 High	29.66	29.99	30.35	30.72	31.05			

These results are illustrated in Fig. 1. The results are robust to controlling for Fama-French three-factors.[21] Adjusting for DGTW (1997) firm characteristics benchmark returns does not change the results either.[22] Therefore, results in Table 6 strongly support our second hypothesis on the conditional relation between expected return and idiosyncratic risk.

4. Robustness

We have shown that within low arbitrage score (i.e., overvalued) stocks, expected stock return significantly increases with idiosyncratic risk. On the other hand, within high arbitrage score (i.e., overvalued) stocks, expected stock return significantly decreases with idiosyncratic risk. This section provides various robustness checks, including subsample analysis, controlling for other proxies of arbitrage costs, using alternative econometric methodology as well as alternative measures of mispricing or idiosyncratic volatility.

4.1. Subsample analysis

We first repeat the tests for stocks traded at NYSE only and NASDQ only, for stocks with price over $5 only, and for stocks with different levels of size, book-to-market ratio and trading volume. There are two reasons to check the robustness among these subsamples. On one hand, our results could be driven by low-priced, tiny, or illiquid stocks which are more prone to microstructure effects and too costly to trade. On the other hand, arbitrageurs are not identical in practice. Due to competition, some arbitrageurs may specialize in stocks with specific characteristics such as small cap stocks and growth stocks. Because of regulations, some arbitrageurs may be prevented from holding illiquid stocks.

Panel A of Table 7 presents the (5–1) spreads between high and low Eidio stocks in value-weighted three-factor alphas within each arbitrage score quintile. For all subsamples, the average (5–1) spread is significantly negative in arbitrage score quintile 1, but significantly positive in arbitrage score quintile 5. The conditional relation between expected stock return and idiosyncratic risk is robust in all subsamples. Our results are not driven by low-priced, tiny, or illiquid stocks.

Although for all subsamples, the relation between expected stock return and idiosyncratic risk depends on relative stock mispricing (proxied by arbitrage score) qualitatively in the same manner, the magnitudes of the impact of idiosyncratic risk on stock returns differ across subsamples. Table 7 Panel A shows that the magnitudes of return spreads between high and low Eidio stocks among both undervalued and overvalued stocks are larger for NASDAQ stocks, small firms and firms with low trading volume than for NYSE stocks, big firms and firms with high trading volume. For example, among NYSE stocks with high arbitrage score, the return spreads between high and low Eidio stocks have a mean of 0.23% (t-statistic 3.97), compared with 0.57% (t-statistic 5.67) in the case of NASDAQ stocks with high arbitrage score. This is consistent with

[21] The results are very similar for CAPM alphas and Carhart (1997) four-factor alphas.

[22] We follow Daniel et al. (1997) to construct characteristics (Size-BM-Momentum) adjusted abnormal returns. Specifically, we subtract from each stock return the return on a portfolio of firms matched on market equity (NYSE breakpoints), market-book (industry adjusted), and prior one-year return quintiles. The details are available at the appendix of Daniel et al. (1997).

Table 7

Conditional Relation between Returns and Idiosyncratic Risk: Subsample Evidence. This table examines the relation between expected returns and idiosyncratic risk (Eidio) conditional on proxies of relative stock mispricing (arbitrage score) over various subsamples. Each number in this table is the alpha of a value-weighted portfolio with respect to the Fama-French three factor models. To obtain the numbers reported in Panel A, each week we sort stocks belonging to a specific subset of the stock universe by arbitrage score into quintiles and then further sort stocks within each arbitrage score quintile into five portfolios based on Eidio. For example, the first number in Panel A is the Fama-French three factor alpha of a value-weighted portfolio that is long the high idiosyncratic risk stocks and short the high idiosyncratic risk stocks within low arbitrage score quintile for all NYSE stocks. Size is the firm's market capitalization at the end of last week. BE/ME (BM) is the fiscal-yearend book value of common equity divided by the calendar-yearend market value of equity. VOL is weekly total trading volume of last week. Panel B reports the (5–1) spreads sorted on Edio in value-weighted FF-3 alphas within each arbitrage score quintile over different subperiods. The low and high idiosyncratic risk periods refer respectively to the weeks when the cross-sectional average of Eidio is lower (higher) than the 33 (66) percentile of its time-series distribution. The sample period is from July 1963 to December 2006 and the testing period starts from June 1968. To adjust for serial correlation, robust Newey and West (1987) t-statistics are reported in brackets. The symbols *, **, *** denote significance at the 10%, 5% and 1% levels, respectively.

Panel A: FF-3 α of (5–1) spread sorted by eidio within arbitrage score quintile: subsamples					
Value-Weighted (5–1) Spread (Sorted by Edio) in FF-3 α (%)					
	Arbitrage scores low	2	3	4	Arbitrage scores high
NYSE Stocks Only	−0.22***	−0.10**	−0.01	0.11**	0.23***
	(−4.90)	(−2.29)	(−0.27)	(2.05)	(3.97)
NASDAQ Stocks Only	−0.91***	−0.25**	0.10	0.14	0.57***
	(−7.75)	(−2.04)	(0.88)	(1.13)	(5.67)
Price > $5	−0.21***	−0.08*	0.02	0.09*	0.19***
	(−4.52)	(−1.84)	(0.36)	(1.76)	(3.62)
Size-Small	−0.89***	0.04	0.16**	0.36***	0.74***
	(−11.15)	(0.58)	(2.08)	(5.20)	(10.50)
Size-Medium	−0.68***	−0.15***	−0.06	0.09	0.29***
	(−11.69)	(−2.73)	(−1.13)	(1.59)	(5.29)
Size-Big	−0.21***	0.05	0.01	0.15***	0.20***
	(−4.29)	(0.98)	(0.29)	(3.09)	(3.97)
Volume-Low	−0.99***	−0.59***	−0.30***	−0.10*	0.57***
	(−15.20)	(−9.44)	(−5.52)	(−1.70)	(9.27)
Volume-Medium	−0.65***	−0.30***	−0.06	0.08	0.39***
	(−11.11)	(−5.30)	(−0.93)	(1.34)	(4.26)
Volume-High	−0.39***	−0.05	0.04	0.19***	
	(−6.33)	(−0.91)	(0.67)	(2.82)	(4.80)
BM-Low	−0.53***	−0.23***	−0.23***	−0.14*	0.16*
	(−7.42)	(−3.10)	(−3.28)	(−1.65)	(1.71)
BM-Medium	−0.36***	−0.09	−0.06	−0.11*	0.28***
	(−5.65)	(−1.44)	(−0.97)	(−1.76)	(3.61)
BM-High	−0.58***	−0.18**	−0.15**	0.05	0.47***
	(−7.97)	(−2.26)	(−2.10)	(0.56)	(6.41)
Panel B: FF-3 α of (5–1) spread sorted by eidio within arbitrage score quintile: subperiods					
1968–1980	−0.47***	−0.16**	0.02	0.01	0.31***
	(−6.80)	(−2.03)	(0.20)	(0.08)	(3.15)
1981–1993	−0.49***	−0.12*	−0.09	0.13	0.28***
	(−6.25)	(−1.67)	(−1.08)	(1.48)	(3.00)
1994–2006	−0.18*	−0.08	0.15	0.18*	0.41***
	(−1.71)	(−0.81)	(1.20)	(1.96)	(3.30)
Low Eidio Periods	−0.42***	−0.05	0.01	0.05	0.18**
	(−5.59)	(−0.66)	(0.16)	(0.58)	(2.07)
High Eidio Periods	−0.35***	−0.14	0.19	0.23**	0.55***
	(−3.13)	(−1.34)	(1.42)	(2.30)	(4.23)

the notion that there is more mispricing among small or illiquid stocks.

Next, we examine the robustness of the conditional relation over different subperiods. Panel B of Table 7 presents the Fama-French three-factor alphas of the value-weighted return spreads sorted by Eidio within each arbitrage score quintile. The pattern is strikingly similar during 1968–1980, 1981–1993, and 1994–2006 subperiods. In each subperiod, the return spreads between high and low Eidio stocks are significantly negative for overvalued stocks and significantly positive for undervalued stocks. The influence of idiosyncratic risk on undervalued stocks increases over time while its impact on overvalued stocks turns weaker in the most recent subperiod. This seems consistent with more short-selling activities in the most recent subperiod.

Further, to address the concern that our results only derive from the periods when the aggregate idiosyncratic risk is high, we repeat the tests separately over the period of high average idiosyncratic risk and over the period of low average idiosyncratic risk. A week belongs to the period of high (low) average idiosyncratic risk if that week's cross-sectional average of idiosyncratic risk is higher (lower) than the 66 (33) percentile of its time-series distribution. Again, we find that for both the period of high average idiosyncratic risk and the period of low average idiosyncratic risk, the relation between stock returns and idiosyncratic risk is significantly positive for undervalued stocks and significantly negative for overvalued stocks.

Finally, we examine the conditional relation between stock returns and idiosyncratic risk within each industry. We employ Fama-French three-factor model to estimate idiosyncratic risk. If the true asset pricing model contains industry factors, then our Eidio measure may be contaminated by industry effects. At the end of June of each year from 1963 to 2006, we assign all firms in our sample to one of 12 industries based on their four-digit SIC code, following the industry definitions obtained from Ken French's website.[23] Each week, stocks within each industry are first sorted based on their arbitrage scores into quintiles

[23] Assigning firms into 12 industries represents a compromise between having a reasonable number of distinct industries and having enough firms within each industry so that sorting within industries will not produce portfolios that are too thin.

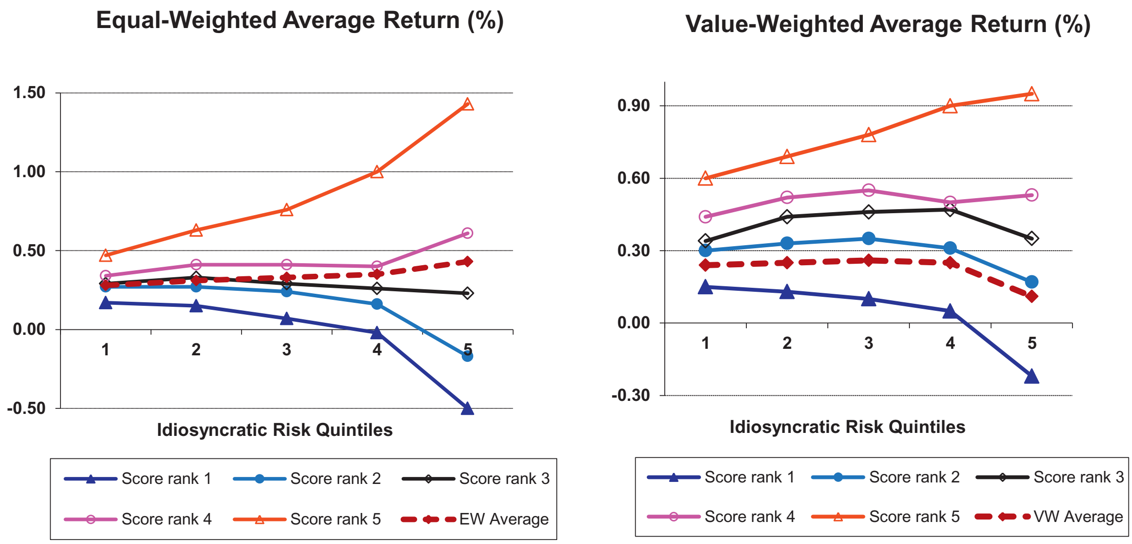

Fig. 1. Cross-Section of Stocks Returns and Idiosyncratic Risk across Arbitrage Score Quintiles. At the beginning of each week, all stocks are independently sorted into deciles from low to high, based on BE/ME, the compound return from t-52 weeks to t-4 weeks, negative size and negative previous week return. A stock's arbitrage score is the sum of its decile rank in each of the four rankings. Each week, stock are first sorted on their arbitrage scores into quintiles and then within each arbitrage score stocks are further sorted into five portfolios based on expected idiosyncratic volatility (Eidio). Eidio is the estimated weekly expected idiosyncratic volatility from EGARCH(1,1) on Fama-French 3-factor model. The left and right figures correspond to Table 6 Panel A (equal-weighted return) and Panel B (value-weighted return), respectively.

Table 8

Conditional Relation between Returns and Idiosyncratic Risk: Controlling for Industry effect This table examines the relation between expected returns and idiosyncratic risk (Eidio) conditional on proxies of relative stock mispricing (arbitrage score) for each of the Fama-French 12 industries. Eidio is the estimated weekly expected idiosyncratic volatility from EGARCH(1,1) on Fama-French 3-factor model. A stock's arbitrage score is the sum of its decile rank in each of the four anomalies as defined in Table 4. Each week, stocks within each industry are first sorted on their arbitrage scores into quintiles and then sorted within each quintile into five portfolios based on Eidio. We then form portfolios that are long the top Eidio stocks and short the bottom Eidio stocks for each arbitrage score quintile within each of the Fama-French 12 industries. This table reports the alpha of value-weighted returns of these portfolios with respect to the Fama-French three factor models. The sample period is from July 1963 to December 2006 and the testing period starts from June 1968. To adjust for serial correlation, robust Newey and West (1987) t-statistics are reported in brackets. The symbols *, **, *** denote significance at the 10%, 5% and 1% levels, respectively.

FF-3 α of (5–1) spread sorted by eidio within arbitrage score quintile for industries Value-Weighted (5–1) Spread (Sorted by Edio) in FF-3 α (%)					
	Arbitrage scores low	2	3	4	Arbitrage scores high
Consumer NonDurables	−0.38***	−0.38***	−0.50***	−0.17*	0.35***
	(−5.01)	(−4.25)	(−5.00)	(−1.71)	(3.17)
Consumer Durables	−0.71***	−0.49***	−0.24	−0.29**	0.64***
	(−5.08)	(−3.85)	(−1.58)	(−2.09)	(4.02)
Manufacturing	−0.33***	−0.24***	−0.16*	−0.04	0.55***
	(−4.30)	(−3.14)	(−1.86)	(−0.44)	(5.39)
Energy	−0.50***	−0.40***	−0.35***	−0.35***	0.58***
	(−4.09)	(−3.08)	(−2.96)	(−2.66)	(4.08)
Chemicals	−0.30***	−0.34***	0.13	−0.11	0.61***
	(−2.75)	(−2.87)	(0.97)	(−0.75)	(3.61)
Business Equipment	−0.55***	−0.53***	−0.23**	0.03	0.56***
	(−5.78)	(−4.93)	(−1.98)	(0.29)	(4.49)
Telecom	−0.37**	−0.07	0.15	0.53***	0.51***
	(−2.11)	(−0.44)	(0.85)	(2.66)	(2.71)
Utilities	−0.04	0.07	0.07	0.04	0.11
	(−0.57)	(1.07)	(1.02)	(0.52)	(1.32)
Wholesale & Retail	−0.44***	−0.23***	−0.13	−0.12	0.39***
	(−5.09)	(−2.65)	(−1.28)	(−1.20)	(3.82)
Healthcare	−0.36***	−0.43***	−0.14	0.02	0.39**
	(−2.80)	(−3.73)	(−1.05)	(0.17)	(2.48)
Finance	−0.26***	−0.10	−0.21**	−0.21**	0.18
	(−3.20)	(−1.23)	(−2.2)	(−2.34)	(1.49)
Others	−0.68***	−0.42***	−0.24**	−0.08	0.51***
	(−7.38)	(−4.22)	(−2.21)	(−0.84)	(4.77)

and then sorted within each quintile into five portfolios based on proxy of idiosyncratic risk (Eidio). Table 8 reports the return spread between high and low Eidio stocks in value-weighted three-factor alphas within each arbitrage score quintile for Fama-French 12 industries. The results of equal-weighted returns are similar.

As shown in Table 8, within the arbitrage score quintile 1, the return spread between high and low Eidio stocks is significantly negative for all industries except for the utility sector. Within the arbitrage score quintile 5, the return spread between high and low Eidio stocks is significantly positive for all industries except for finance and utility sectors. Since utility firms are subject to regulations and finance firms have extremely high leverage ratios, stocks within these two industries might behaved differently. Overall, Table 8 shows that the conditional relation between idiosyncratic risk and return are significant and consistent across industries.

Table 9
Idiosyncratic Risk and Alternative Measures of Arbitrage Costs. This table compares the asset pricing impact of idiosyncratic risk and other measures of arbitrage costs. It reports the average value-weighted returns of portfolios independently double sorted on idiosyncratic risk (Eidio) and an alternative measure of arbitrage cost, separately for the undervalued stocks (top arbitrage score quintile) and the overvalued stocks (bottom arbitrage score quintile). Eidio is the estimated weekly expected idiosyncratic volatility from EGARCH(1,1) on Fama-French 3-factor model. A stock's arbitrage score is the sum of its decile rank in each of the four anomalies as defined in Table 4. Price level proxies for direct transaction cost, and is measured as the closing price at the end of week t-1. The proxy for indirect transaction cost is illiquidity, calculated as the average of daily Amihud (2002) measure over week t-1. IO is the percentage of common stocks owned by institutions in the previous quarter, and proxies for short-sale constraints. The sample period is from July 1963 to December 2006 and the testing period starts from June 1968. To adjust for serial correlation, robust Newey and West (1987) t-statistics are reported in brackets. The symbols *, **, *** denote significance at the 10%, 5% and 1% levels, respectively.

	Within arbitrage score quintile 1 (Most overvalued stocks)							Within arbitrage score quintile 5 (Most undervalued stocks)						
	1-Low Eidio	2	3	4	5-High Eidio	H-L	t-stat	1-Low Eidio	2	3	4	5-High Eidio	H-L	t-stat
Panel A: idiosyncratic risk and direct transaction cost (price level) value-weighted raw returns (%)														
1-Low Price	0.02	0.21	−0.05	−0.28	−0.78	−0.84***	(−6.12)	0.46	0.66	0.80	0.93	1.09	0.61***	(6.84)
2	0.20	0.18	0.08	−0.00	−0.38	−0.57***	(−6.03)	0.54	0.65	0.67	0.78	0.78	0.24***	(2.77)
3-High Price	0.14	0.08	0.06	0.01	−0.22	−0.38***	(−3.33)	0.57	0.61	0.67	0.77	0.60	0.08	(0.53)
3-1	0.02	−0.08	0.11	0.30***	0.54***				0.10	−0.05	−0.13*	−0.15*	−0.38***	
t-stat	(0.23)	(−0.80)	(1.14)	(3.21)	(4.39)			(1.42)	(−0.64)	(−1.87)	(−1.65)	(−2.74)		
Panel B: idiosyncratic risk and indirect transaction cost (Illiquidity) value-weighted raw returns (%)														
1-Low Illiquidity	0.15	0.10	0.06	0.05	−0.19	−0.33***	(−3.48)	0.60	0.61	0.73	0.79	0.73	0.10	(0.74)
2	0.05	−0.00	−0.02	−0.20	−0.64	−0.69***	(−7.45)	0.54	0.65	0.70	0.88	0.90	0.36***	(4.07)
3-High Illiquidity	0.22	−0.23	−0.17	−0.59	−1.31	−1.41***	(−11.87)	0.54	0.55	0.69	0.81	1.02	0.49***	(6.96)
3-1	0.01	−0.32***	−0.24***	−0.64***	−1.12***			−0.05	−0.07	−0.05	0.02	0.30**		
t-stat	(0.14)	(−4.61)	(−2.95)	(−7.71)	(−10.57)			(−0.75)	(−0.90)	(−0.65)	(0.24)	(2.44)		
Panel C: idiosyncratic risk and short-sale constraints (institutional ownership): 1980–2006 value-weighted raw returns (%)														
1-Low IO	0.22	0.16	0.07	−0.00	−0.75	−0.96***	(−6.80)	0.61	0.55	0.67	0.83	0.94	0.33***	(3.19)
2	0.25	0.19	0.12	0.06	−0.33	−0.58***	(−4.25)	0.50	0.59	0.61	0.82	1.02	0.52***	(4.41)
3-High IO	0.20	0.14	0.10	0.07	−0.04	−0.24**	(−2.17)	0.59	0.67	0.74	0.88	0.83	0.24	(1.53)
3-1	−0.02	−0.02	0.03	0.07	0.71***			−0.02	0.12	0.07	0.05	−0.11		
t-stat	(−0.38)	(−0.23)	(0.32)	(0.68)	(5.98)			(−0.23)	(1.61)	(1.03)	(0.49)	(−0.65)		

4.2. Controlling for other measures of arbitrage costs

In this subsection, we check the robustness of our results controlling for alternative arbitrage cost measures including stock price level, Amihud illiquidity and institutional ownership. We also compare the strength of idiosyncratic risk and other arbitrage costs in deterring arbitrage. To investigate the impacts of these arbitrage costs on undervalued and overvalued stocks, we focus on stocks within arbitrage score quintile 1 and 5. Table 9 presents the portfolio sorting results using value-weighted raw returns.

In Panel A of Table 9, we compare price level with idiosyncratic risk. We find that the influence of idiosyncratic risk seems to be stronger than price level. For example, within the arbitrage score quintile 1 (overvalued stocks), returns strongly decrease in Eidio across all three price level groups. In contrast, price level predicts returns only when Eidio is high. Previous studies usually emphasize the impact of transaction costs on short-term mispricing and not much attention is given to idiosyncratic risk. However, our results suggest that idiosyncratic risk is equally or more important than transaction costs, even at the short horizon.

Table 9 Panel B reports the results comparing illiquidity with idiosyncratic risk. Among overvalued stocks, both measures strongly predict returns in the anticipated directions. Among undervalued stocks, the impact of Eidio exceeds illiquidity. In both cases, illiquidity cannot explain the role of Eidio. In Panel C, we compare institutional ownership (IO) with idiosyncratic risk. Different from other measures, short-sale constraints only prevent arbitrageurs from short-selling overvalued stocks. Consistent with this notion, IO does not predict returns in undervalued stocks. Among overvalued stocks, IO only matters when idiosyncratic risk is high. In either case, IO cannot account for the role of Eido.

In summary, the horse races between idiosyncratic risk and other arbitrage costs indicate that the asset pricing impact of idiosyncratic risk is more significant and consistent than transaction costs and short-sale constraints.

4.3. Fama-MacBeth regression results

Previous tables use the portfolio sorts approach and documents robust results concerning how the relation between idiosyncratic risk and returns depends on among relative stock mispricing. The shortcoming of portfolio sorting approach is that it is difficult to control for multiple alternative variables. In this subsection, we reexamine the the relation between idiosyncratic risk and returns using Fama-MacBeth cross-sectional regressions. Each week and for each arbitrage score quintile, we regress next week's stock returns on Eidio as well as other variables that may be correlated with both Eidio and stock returns. Table 10 reports the time-series average of regression coefficients and their Newey and West (1987) t-statistics.

Panel A presents the coefficients of Eidio without other control variables. Consistent with the portfolio sorting results, the coefficient for Eidio is significantly negative among low arbitrage score quintile and significantly positive among high arbitrage score quintile. In the middle arbitrage score quintile, Eidio cannot predict returns. In more details, in arbitrage score quintile 1, Eidio has a coefficient of −10.979 with a t-stat of −10.55. In arbitrage score quintile 5, Eidio has a coefficient of 9.702 with a t-stat of 12.83. In arbitrage score quintile 3, Eidio has a coefficient of −1.381 with an insignificant t-stat of −1.46.

Since idiosyncratic risk is positively correlated with CAPM beta as shown in Table 2, we then control for the CAPM beta and the results are reported in Panel B. Including beta almost causes no change to the coefficients of Eidio and the associated t-statistics. In addition, beta itself has a significantly positive coefficient from quintile 2 to quintile 5. These results are consistent with the idea that idiosyncratic risk is an arbitrage cost while systematic risk can be hedged out by arbitrageurs.

In Panel C, we further control for firm characteristics that are known to forecast returns including Ln(ME), Ln(BE/ME), $Ret_{(-52,-4)}$, and $Ret_{(-1,0)}$. Controlling for these four variables reduce the magnitude and t-statistics of the coefficients for Eidio but cannot fully explain the effect of Eidio on returns. For

Table 10
Weekly Fama-MacBeth Regressions. Each week, stocks are sorted by their arbitrage scores into quintiles and the Fama-MacBeth regressions are conducted within each quintile. A stock's arbitrage score is the sum of its decile rank in each of the four anomalies as defined in Table 4. The table reports the time-series averages of the cross-sectional regression coefficients for expected idiosyncratic risk (Eidio) and control variables. The dependent variable is stock return in week t. Panel A corresponds to univariate regression on Eidio (constant term is included but not reported). In Panel B, we include CAPM beta as a regressor. In Panel C, we also control for several firm characteristics. In Panel D, we further control for several alternative measures of arbitrage costs. Eidio is the weekly expected idiosyncratic volatility estimated from a EGARCH(1,1) specification on Fama-French 3-factor model, using all historical weekly returns data. Beta is the CAPM beta estimated weekly from the market model using weekly returns over previous 104 weeks. ME is the firm's market capitalization at the end of last week. BE/ME (BM) is the fiscal-yearend book value of common equity divided by the calendar-yearend market value of equity. Ret (−52,−4) is the compound return from t-52 weeks to t-4 weeks. Ret (-1,0) is the raw return of previous week. Price is the closing stock price at the end of last week. Firm age is defined as the number of years since a stock first appeared in the CRSP. Illiquidity is Amihud's illiquidity measure measured at last week. Institutional Ownership is the percentage of common stocks owned by institutions in the previous quarter. Analyst Coverage is the number of analysts following the firm in the previous month. Analyst Dispersion is the standard deviation of analyst forecasts in previous month scaled by the prior year-end stock price. All independent variables are winsorized at 0.5% level. The sample period is from July 1963 to December 2006 and the testing period starts from June 1968. Robust Newey and West (1987) t-statistics are reported in brackets. The symbols *, **, *** denote significance at the 10%, 5% and 1% levels, respectively.

Panel A: Stocks within Arbitrage score quintiles					
	Arbitrage scores quintile 1	Arbitrage scores quintile 2	Arbitrage scores quintile 3	Arbitrage scores quintile 4	Arbitrage scores quintile 5
Eidio	−10.979***	−7.019***	−1.381	3.289***	9.702***
	(−10.55)	(−7.19)	(−1.46)	(3.55)	(12.83)
Average Adj. R^2	2.20%	1.80%	1.50%	1.20%	1.10%
Panel B: control for systematic risk					
Eidio	−10.861***	−7.178***	−1.524	3.159***	10.585***
	(−10.59)	(−7.48)	(−1.64)	(3.53)	(12.75)
Beta	0.003	0.073***	0.058**	0.068***	0.052**
	(0.11)	(2.93)	(2.49)	(3.14)	(2.47)
Average Adj. R^2	3.00%	2.60%	2.20%	1.80%	1.50%
Panel C: control for firm characteristics					
Eidio	−5.964***	−3.627***	0.217	1.853**	2.900***
	(−6.87)	(−4.50)	(0.26)	(2.21)	(3.92)
Beta	−0.015	0.023	−0.007	0.010	0.014
	(−0.59)	(0.94)	(−0.31)	(0.44)	(0.68)
Ln(ME)	−0.048***	−0.046***	−0.071***	−0.082***	−0.146***
	(−4.29)	(−4.49)	(−6.50)	(−7.20)	(−10.59)
Ln(BM)	0.045**	0.072***	0.107***	0.124***	0.086***
	(2.42)	(3.63)	(5.18)	(6.47)	(5.64)
Ret (−52, −4)	0.229***	0.379***	0.370***	0.324***	0.071*
	(3.31)	(7.08)	(7.13)	(7.17)	(1.94)
Ret (-1, 0)	−6.024***	−7.451***	−10.049***	−10.943***	−16.244***
	(−20.28)	(−20.39)	(−27.9)	(−26.36)	(−32.54)
Average Adj. R^2	5.70%	5.20%	5.10%	4.40%	4.60%
Panel D: Control for other arbitrage cost measures basic control variables include Beta, Ln(ME), Ln(BM), $Ret_{(-52,-4)}$ and $Ret_{(-1,0)}$					
Control for Price, Firm Age and Illiquidity: 1968–2006					
Eidio	−4.783***	−2.426***	0.367	1.259*	2.105***
	(−5.46)	(−2.86)	(0.43)	(1.91)	(3.02)
Control for Institution Ownership, Analyst Coverage and Analyst Dispersion: 1980–2006					
Eidio	−3.381***	−1.937*	−0.162	2.583**	2.916***
	(−3.09)	(−1.84)	(−0.15)	(2.32)	(2.92)
Control for All Above: 1980–2006					
Eidio	−3.096***	−2.148**	−0.361	1.879*	2.975***
	(−2.83)	(−2.04)	(−0.33)	(1.70)	(2.87)

instance, in arbitrage score quintile 1, the coefficient of Eidio reduces to −5.964 with a t-stat of −6.87. In arbitrage score quintile 5, Eidio has a smaller coefficient of 2.900 with a t-stat of 3.92. In arbitrage score quintile 3, Eidio does not predict returns. Consistent with Fama and French (1992), beta loses explanatory power in the presence of firm characteristics. In contrast, Ln(ME), Ln(BE/ME), $Ret_{(-52,-4)}$, and $Ret_{(-1,0)}$ all have significant impacts on returns across all five groups. Specifically, returns increase in Ln(BE/ME) and $Ret_{(-52,-4)}$, while decreases in Ln(ME) and $Ret_{(-1,0)}$.

Moreover, idiosyncratic risk is highly correlated with many other arbitrage cost measures, the conditional relation between idiosyncratic risk and stocks returns may reflect the impacts of other arbitrage costs such as transaction costs, short-sale constraints and information uncertainty on returns. To address this concern, we repeat the regressions in Panel C and control for several other arbitrage costs.

In Panel D, we further add several alternative arbitrage cost measures as regressors. First, we control for price level, firm age and illiquidity since they are available for the full sample. The coefficient of Eidio monotonically increases from −4.783 (t-statistic −5.46) in arbitrages score quintile 1, to 2.105 (t-statistic 3.02) in arbitrage score quintile 5. Second, we control for institution ownership, analyst coverage and analyst dispersion for the subsample of 1980–2006. This sample is much smaller due to the availability of analyst forecasts data. The coefficients of Eidio monotonically increase from -3.381 (t-statistic -3.09) in the bottom arbitrage score quintile to 2.916 (t-statistic 2.92) in the bottom arbitrage score quintile. The results are similar after including all these variables for the 1980–2006 subsample.

In summary, the multivariate regression tests are consistent with the results of portfolio sorts. At the individual stock level, expected returns is positive related to idiosyncratic risk among relatively undervalued stock but negatively related to idiosyncratic risk among relatively overvalued stocks. This pattern cannot be explained by systematic risk exposure, firm characteristics, or other arbitrage costs such as transaction costs, short-sale constraints and information uncertainty.

Table 11
Alternative Aggregate Mispricing, Alternative Idiosyncratic Risk Measures, and Longer-Horizon Return. This table examines the relation between expected returns and expected idiosyncratic risk (Eidio) conditional on proxies of relative stock mispricing (arbitrage score). Each week, stocks are first sorted on their arbitrage scores into quintiles and then sorted within each quintile into five portfolios based on expected idiosyncratic risk. The baseline specification (see Table 6) uses four anomalies to construct arbitrage score, uses Eidio to proxy for expected idiosyncratic risk, and look at next one week return. Panel A adds Total Accrual and PEAD into the construction of arbitrage score. Panel B uses two alternative idiosyncratic risk measures. IVOL_AHXZ is the standard deviation of the residuals of the Fama-French 3-factor model estimated using the daily stock returns over the previous month. IVOL_Week is the standard deviation of the residuals of the Fama-French 3-factor model estimated using the weekly stock returns over previous 104 months. Panel C reports the results of longer-horizon holding period returns. The sample period is from July 1963 to December 2006 and the testing period starts from June 1968. To adjust for serial correlation, robust Newey and West (1987) t-statistics are reported in brackets. The symbols *, **, *** denote significance at the 10%, 5% and 1% levels, respectively.

	(5–1) Raw return spread (%) sorted by idiosyncratic risk measure within arbitrage score quintile				
	Arbitrage scores low	2	3	4	Arbitrage scores high
Panel A: additional set of anomalies for aggregate mispricing					
Total Accrual &	−0.37***	−0.41***	−0.06	−0.44***	1.04***
PEAD	(−4.36)	(−4.94)	(−0.77)	(5.20)	(12.89)
Panel B: Alternative Expected Idiosyncratic Risk Measures					
IVOL_AHXZ	−0.55***	−0.58***	−0.20***	0.41***	1.08***
	(−6.98)	(−7.59)	(−2.73)	(5.20)	(13.65)
IVOL_Week	−0.42***	−0.39***	−0.14*	0.32***	0.70***
	(−4.85)	(−4.28)	(−1.71)	(3.83)	(8.86)
Panel C: longer-horizon cumulative holding period returns					
$Ret_{(t,t+2)}$	−0.72***	−0.53***	−0.05	0.50***	1.24***
	(−4.80)	(−3.38)	(−0.31)	(3.05)	(8.27)
$Ret_{(t,t+3)}$	−0.92***	−0.56**	−0.05	0.55**	1.25***
	(−4.24)	(−2.48)	(−0.20)	(2.21)	(5.74)
$Ret_{(t,t+4)}$	−1.09***	−0.59**	−0.00	0.62*	1.22***
	(−3.95)	(−1.96)	(−0.00)	(1.86)	(4.40)
$Ret_{(t,t+5)}$	−1.22***	−0.75**	−0.08	0.63	1.28***
	(−3.61)	(−2.12)	(−0.21)	(1.56)	(3.77)

4.4. Alternative aggregate mispricing, alternative idiosyncratic risk measures, and longer-horizon returns

Alternative way of combining anomalies into aggregate mispricing: The arbitrage score measure we have used assigns an equal weight to short-term return reversals, size, book-to-market and momentum effects. For robustness, we measure whether stocks are overvalued or undervalued using the multivariate regression framework, allowing each anomaly to have its own marginal effect. Specifically, each week we use previous 52-week data and run a Fama-MacBeth regression of stock returns on ex-ante variables: firm size, book-to-market, compounded return from 52 weeks ago to 4 weeks ago and the previous week's return. Then we predict next week's return of each stock based on the current week's firm characteristics, and the time-series average of historical regression coefficients. Finally, we sort stocks based on their predicted returns. In unreported tests, our main results do not change materially under this alternative aggregate measure of stock mispricing.

Additional sets of anomalies for aggregate mispricing: In another robustness check, we add total accrual and post-earning-announcements drift so that we compute the arbitrage score using six anomalies.[24] As shown in Panel A of Table 11, the arbitrage score based on six anomalies generates the same conditional relation between expected stock return and idiosyncratic risk, i.e., positive (negative) among undervalued (overvalued) stocks. In the internet appendix, we replicate Table 6 using a dependent bivariate sort of idiosyncratic risk and MAX(5), a proxy for lottery stock demand which has been shown to explain various existing anomalies such as the abnormal negative returns of high idiosyncratic volatility stocks and high beta stocks (Bali et al., 2011) and (Bali et al., 2014).

Alternative EGARCH specifications for the estimation of idiosyncratic volatility: We further conduct several robustness checks to ensure that our results are not driven by specific model of the EGARCH volatility estimation. First, our results are robust when we measure idiosyncratic volatility relative to a six-factor model: three Fama-French factors and their first lag. The lagged Fama-French factors are included to control for potential lead-lag effect. Second, our results are robust when we estimate alternative $EGARH(p, q)$ models, for p and q up to 3. Third, we tweak the estimation of $EGARCH(1, 1)$ models and our results do not change materially.[25]

Alternative measures of idiosyncratic risk: Panel B of Table 11 shows that our results are robust to two alternative idiosyncratic risk measures: as defined in Ang et al., *IVOL_AHXZ* is the standard deviation of the residuals of the Fama-French 3-factor model estimated using the daily stock returns over the previous month; *IVOL_Week* is the standard deviation of the residuals of the Fama-French 3-factor model estimated using the weekly stock returns over previous 104 months.

Longer holding horizons: Panel C of Table 11 reports the results of next two to five weeks' cumulative returns after weekly portfolio formation. The conditional relation between expected stock return and idiosyncratic risk changes little as holding horizon increases.

5. Conclusion

The evidence presented in this paper supports the limits to arbitrage theory, an important building block of behavioral finance. The most commonly used proxy for arbitrage cost is idiosyncratic risk. If idiosyncratic risk indeed prevents arbitrageurs from buying undervalued stocks and short selling overvalued stocks, then the cross-sectional relation between expected stock return and idiosyncratic risk will depend on the direction of mispricing.

Empirically, we forecast expected idiosyncratic volatility by estimating an exponential GARCH for each stock each week. We measure the relative mispricing of stocks by an arbitrage score of each stock which combines the size effect, value premium, return momentum and short-term reversal. Consistent with the interpretation that high (low) arbitrage score stocks are relatively undervalued (overvalued), we find that stocks with high arbitrage scores subsequently significantly outperform stocks with low arbitrage scores, even after controlling for the usual risk factors.

Consistent with the prediction of the limits to arbitrage theory, we find that average stock returns monotonically increase with idiosyncratic risk for relatively undervalued stocks and monotonically decrease with idiosyncratic risk for relatively overvalued stocks. This pattern holds for both equal-weighted and value-weighted returns and is robust across various subsamples and industries. Furthermore, systematic risk exposures, firm characteristics and other arbitrage cost measures cannot account for the role of idiosyncratic

[24] Following Richardson et al. (2005), we measure the total accrual as changes in non-cash working capital minus depreciation expense scaled by average total assets of the previous two years. The quarterly earnings-announcement shocks are measured using the market model cumulative abnormal returns (CAR) for a [-1, 1] event window around the quarterly announcement dates. Using SUE, CAR(-2,2), CAR(-3,3), or abnormal returns based on Fama-French three-factor model does not change the results.

[25] For example, we only require 120 weeks (rather than 260 weeks originally) of historical return data before estimating the *EGARCH* model. Another change is that we use at most 520 weeks historical returns (rather than all previous returns data) in the estimation. Our results are not sensitive to these changes.

Table A1
Regression Sample Size and the Probability of Convergence for EGARCH Estimations. This table shows the relation between the accuracy of EGARCH(1,1) estimations and the number of observations used in the regression. The probability of convergence is defined as cross-sectional percentage of successful convergence in the MLE procedures. The time-series mean and other statistics are reported. The regression sample size intervals are listed in the square bracket, with a maximum of 2270 weeks. The sample period is from July 1963 to December 2006.

Probability of convergence	Sample size intervals		
	[52,120]	[120, 260]	[260, 360]
Min	60.64%	81.03%	91.47%
Max	84.93%	93.16%	98.90%
Median	73.49%	88.44%	95.21%
Mean	73.52%	88.29%	95.04%
Std	4.23%	2.30%	1.60%
	[52, 2270]	[120, 2270]	[260, 2270]
Min	60.64%	81.03%	91.47%
Max	99.62%	99.62%	99.62%
Median	90.87%	93.49%	97.75%
Mean	89.65%	93.20%	97.38%
Std	4.77%	2.41%	0.96%
Total observations	12.6 million	9.6 million	7.4 million

Table A2
Summary of EGARCH Estimation. This table shows the pooled summary of 7.4 million EGARCH(1,1) estimations. Stocks are included if they have at least 260 weeks of return data. Eidio_in is the contemporaneous idiosyncratic volatility from EGARCH(1,1) on Fama-French 3-factor model. Eidio_out is the estimated one-week ahead idiosyncratic volatility from EGARCH(1,1) on Fama-French 3-factor model. Other parameters are specified in Eq. (1). The sample period is from July 1963 to December 2006.

Variable	Mean	Std Dev	10th Pctl	25th Pctl	Median	75th Pctl	90th Pctl
Eidio_in	0.05990	0.03694	0.02528	0.03491	0.05055	0.07433	0.10486
Eidio_out	0.05989	0.03701	0.02525	0.03488	0.05051	0.07428	0.10481
β_i (MKT-Rf)	0.875	0.445	0.288	0.555	0.875	1.172	1.446
S_i (SMB)	0.761	0.622	0.025	0.300	0.695	1.142	1.595
h_i (HML)	0.167	0.546	−0.505	−0.090	0.215	0.472	0.742
a_i	−2.327	3.083	−7.436	−3.914	−0.613	−0.126	−0.036
c_i	0.229	0.201	0.059	0.106	0.181	0.316	0.485
b_i	0.590	0.533	−0.313	0.284	0.892	0.978	0.992
Θ	−0.069	3.258	−1.037	−0.604	−0.281	0.006	0.380

risk. We also find that for stocks which are neither undervalued nor overvalued, returns are unrelated to idiosyncratic risk.

We have used arbitrage score as a proxy for mispricing and idiosyncratic risk as a measure of arbitrage cost, and our empirical results are consistent with such interpretations. Our paper would stimulate further research related to the effect of stock idiosyncratic volatility on expected stock returns. In particular, a risk-based explanation for the positive (negative) relation between idiosyncratic volatility and expected stock returns among stocks with high (low) arbitrage scores would require that the premium of some risk factor to switch signs across subsamples of stocks. The search for such an asset pricing model would be a fruitful topic for future research.

Appendix A

Table A1 and A2

Supplementary material

Supplementary material associated with this article can be found, in the online version, at 10.1016/j.jbankfin.2016.08.004

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