

Fluctuating Attention and Financial Contagion*

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Abstract

Financial contagion occurs when return and volatility transmit between fundamentally unrelated sectors. We develop an equilibrium model showing that contagion arises because investors pay fluctuating attention to news. As a negative shock hits one sector, investors pay more attention to it. This raises the volatility of equilibrium discount rates resulting in simultaneous spikes in cross-sector correlations and volatilities. We test the economic mechanism of our model on large U.S. industries by decomposing their returns into discount-rate and cash-flow components. Consistent with the model's predictions, we find that fluctuating attention generates return and volatility spillovers between discount-rate components of industry returns.

Keywords: Learning, Attention to News, Contagion, Return and Volatility Spillovers

JEL Classification. D51, G12

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1 Introduction

Contagion in financial markets has been extensively documented in the empirical literature (e.g., Hamao, Masulis, and Ng, 1990; Lin, Engle, and Ito, 1994).¹ Indeed, there is ample evidence of return and volatility spillovers between two fundamentally unrelated securities. Such phenomena have become increasingly important in light of the subprime and sovereign debt crises. This is because simultaneous spikes in return volatilities and cross-return correlations significantly alter risk management strategies, optimal portfolio choices, and the trading of derivatives.

In this paper, we provide theoretical and empirical evidence that investors' fluctuating attention to news is an important channel through which contagion arises in financial markets. We show that when investors' attention to a particular sector increases, risk-adjusted discount rates become more volatile. As a result, return volatilities and cross-return correlations increase simultaneously in the entire market, despite the fact that cash-flows and news associated to each sector are independent from one another.

We consider a pure-exchange economy *à la* Lucas (1978) with two risky assets—*sectors*—that are claims to two exogenous and independent dividend streams. The economy is populated by a representative investor who needs to estimate both unobservable expected dividend growth rates (henceforth *fundamentals*). The investor has two different types of relevant information at hand: information provided by the observation of dividends, and information provided by the observation of news. The key innovation here is that the investor pays fluctuating attention to news, which is supported by recent empirical evidence in Sichernman, Loewenstein, Seppi, and Utkus (2015).² In other words, there are periods when she is well focused and capable of processing many news sources, and periods when she is not. Motivated by the empirical findings of Andrei and Hasler (2015), we let investor's attention to a given sector depend on the past performance of that sector's dividend growth. We emphasize that attention to one sector is independent from attention to the other sector because their dividend dynamics are independent.

¹See also King and Wadhvani (1990), King, Sentana, and Wadhvani (1994), Karolyi and Stulz (1996), Fleming, Kirby, and Ostdiek (1998), Kaminsky and Reinhart (2000), Bae, Karolyi, and Stulz (2003), Bekaert, Harvey, and Ng (2005), Kallberg and Pasquariello (2005, 2008), Barberis, Shleifer, and Wurgler (2005), Boyer, Kumagai, and Yuan (2006), and Diebold and Yilmaz (2009).

²See also Da, Engelberg, and Gao (2011) and Vlastakis and Markellos (2012).

The main prediction of our model is that fluctuating attention implies return and volatility spillover effects among fundamentally unrelated sectors. The intuition is as follows. As a negative shock hits one sector, more attention is paid to news on that sector. Since the content of news is used to estimate the economic fundamental, a rise in attention implies a faster transmission of news, thereby increasing the volatility of that sector’s estimated fundamental. In equilibrium, a more volatile estimated fundamental endogenously generates more volatile equilibrium discount rates. This in turn implies increases in the volatility of the sector hit by the shock, the volatility of the sector that is unrelated to it, and the cross-sector return correlation. The key mechanism of how shocks propagate from one sector to another is therefore through attention and discount rates.

When the fluctuating attention feature is shut down, the model reduces to a standard two-tree exchange economy with learning. In this case, dividend shares are the main drivers of return volatilities and cross-return correlation. Since an increase in the dividend share of one sector mechanically decreases that of the other, return volatilities move in opposite directions which makes a simultaneous increase in return volatilities and cross-return correlation—contagion—inconceivable. Therefore, fluctuating investor’s attention to news offers an explanation for the observed return and volatility spillover effects between unrelated sectors in financial markets.

We empirically test the economic mechanism of our model by examining contagion between sixteen large U.S. industries from 1985 through 2014. In the model, contagion between sectors with unrelated dividend streams (i.e., fundamentals) arises due to shocks to the equilibrium discount rates. We test this economic mechanism by decomposing quarterly industry expected returns into two components reflecting the change in future discount rates (discount-rate return) and the change in future dividends (cash-flow return). Following [Campbell \(1991\)](#) and [Campbell and Ammer \(1993\)](#), among others, we estimate a vector autoregression (VAR) from a large panel of industries and apply the return-decomposition. We then focus our empirical analysis on financial contagion between the discount-rate component of industry returns. This method helps remove the direct impact of shocks to dividend growth rates—fundamentals—that may cause returns and the volatilities of different industries to co-move. Importantly, our return-decomposition results show that the correlation between discount-rate and cash-flow returns of large U.S. industries is small and statistically

insignificant.³ Therefore, we expect shocks to dividend growth rates to have little to no impact on financial contagion observed through the discount-rate component of large industry sectors.

Our empirical analysis is based on quarterly time-series observations from 1985–2014. In order to construct a relatively long time-series of investor attention measure, we draw from various information sources. Specifically, we use the following four variables: trading volume, the number of institutional owners, the number of analyst followings, and the number of analysts’ forecast revisions.⁴ We choose these four variables because prior studies have associated them with changes in investor attention and their firm-level data are available over a long time period.⁵ Each variable is first calculated at the firm level over each quarter and then value-weighted averaged across firms in the industry to yield the industry-level variable. The time series of these four variables are positively correlated, and their values should collectively increase with investor attention. To make these four variables comparable in magnitude, we normalize them after correcting for their time-trend and seasonality effects. We then combine them into a single time series and remove its persistence using an autoregressive model. We refer to the resulting time series as the attention measure, where its level represents an *unexpected change* in investor attention to the industry over each quarter.

We test the impact of fluctuating attention on time-varying volatilities and correlations of discount-rate returns between sixteen large industries. We use a panel-regression framework. Each cross section corresponds to a unique industry pairing and consists of 120 quarterly observations from 1985–2014. We include fixed effects and various control variables to

³This finding is consistent with [Vuolteenaho \(2002\)](#) and [Campbell and Vuolteenaho \(2004\)](#) who find statistically insignificant correlation between discount-rate and cash-flow returns for large stocks and the market index.

⁴[Da, Engelberg, and Gao \(2011\)](#) propose using Google search volume as a more direct measure of investor attention. However, Google search data is available only after 2004, thus, relying on this information variable would shorten our time-series length by two-third (or down to 44 quarters). Nevertheless, we construct a quarterly attention measure using the Google search volume at the industry level from 2004–2014 and find that its correlation with our attention measure is 21%.

⁵[Fisher, Martineau, and Sheng \(2016\)](#) find that aggregate trading volume is highly correlated with media attention to macro-fundamentals. [Boone and White \(2015\)](#) show that firms owned by more institutional investors elicits more public information production. [Womack \(1996\)](#), among others, shows that analysts’ recommendations and earnings forecasts significantly influence investor attention to the firm. [Jacob, Lys, and Neale \(1999\)](#) argues that the frequency of forecast revisions proxies for an analyst’s effort in paying attention to the latest information on the firm that she covers.

account for time-varying aggregate market conditions. We calculate time-varying discount-rate volatilities and discount-rate-return correlations using two approaches. The first is an exponentially-weighted moving-average (EWMA) model, which can be easily implemented given the small number of time-series observations.⁶ The second approach that we use is simply their realized values. That is, we use the absolute discount-rate return as the measure of volatility, and the signed correlation between discount-rate returns (i.e., -1 , 0 , or $+1$) as the measure of correlation. In both approaches, we find empirical evidence supporting the model’s predictions.

The model predicts that fluctuating investor attention generates time-varying volatility co-movements (volatility spillover) and time-varying return correlations (return spillover) between two unrelated sectors through the discount-rate channel. We confirm these empirical predictions. Examining the co-movement in volatilities, we find that an unexpected increase in attention to one industry is related to an increase in its discount-rate volatility as well as those of other industries. As for cross-sector return correlations, we find that discount-rate returns from two industries are more positively correlated when investor attention to each of the two industries, or both, increases.

This paper contributes to two strands of literature. The first is the asset pricing literature examining how contagion arises in financial markets. It is generally difficult to explain both return and volatility spillover effects in a unified general equilibrium framework. For instance, in [Cochrane, Longstaff, and Santa-Clara \(2008\)](#) market volatilities and cross-market correlations are driven by dividend shares. An increase in the dividend share of one market mechanically decreases that of another causing volatilities to move in opposite directions. Therefore, although the model implies a positive cross-market correlation, it cannot generate simultaneous increases in market volatilities and cross-market correlations. We show theoretically that such simultaneous increases—contagion—are observed when investors pay fluctuating attention to news, and we provide empirical evidence supporting the model’s prediction that contagion is driven by fluctuating investors’ attention.

[Yuan \(2005\)](#) shows that asymmetric information and financial constraints lead to contagion. In [Pasquariello \(2007\)](#), contagion is implied by asymmetric information and systemic

⁶We cannot estimate time-varying volatilities using the GARCH model due to the relatively small number of time-series observations (120 quarters). We prefer the EWMA model because no further parameter estimation is required.

risk. In these studies, contagion is defined as the correlation in excess of a benchmark model. Such definition differs slightly from ours in that we require return volatilities and cross-return correlations of fundamentally unrelated sectors to increase simultaneously. Using the same definition of contagion as ours, [Kyle and Xiong \(2001\)](#) show that contagion is implied by wealth effects. Our paper is complementary because it shows both theoretically and empirically that investors' fluctuating attention to news leads to contagion in financial markets.

Our paper also contributes to the growing literature examining how investor attention affects asset prices. [Huberman and Regev \(2001\)](#) provide evidence that new information can only influence prices if investors pay attention to it. More recently, [Dellavigna and Pollet \(2009\)](#) find the post-earnings announcement drift on stock returns is stronger subsequent to earnings announced on Friday relative to other weekdays, suggesting that inattention associated with Friday earnings announcements delays information from being reflected in stock prices. [Da, Engelberg, and Gao \(2011\)](#) proxy investors' attention using Google search frequencies on companies' names and uncover that attention predicts short-term stock returns. [Garcia \(2013\)](#) finds that news recorded from the *New York Times* explain market returns better in recessions than in expansions, suggesting that investors' attention to news concentrates in down markets. [Andrei and Hasler \(2015\)](#) find that stock-return volatility and risk premia increase with both investors' attention and uncertainty. In a limited information processing capacity framework, [Peng and Xiong \(2006\)](#) show that investors optimally choose to gather market and industry-specific information and not firm-specific information.⁷ Our results add to this growing literature by showing the effect of fluctuating attention on the cross-section of returns and volatilities, generating a new and complementary contribution.

The remainder of the paper is organized as follows. Section 2 describes the model and derives the equilibrium variables. Section 3 exposes the model's predictions. Section 4 discusses our empirical tests and results. Section 5 concludes. Derivations and computational considerations are provided in Appendix A.

⁷In a similar framework, [van Nieuwerburgh and Veldkamp \(2010\)](#) show that limited information processing capacity leads to under-diversification, whereas [Mondria and Quintana-Domeque \(2013\)](#) show that high cashflow volatility in one market attracts investors' attention to that market, and therefore implies a low asset price in the other market.

2 The Economy

In this section, we describe the economic environment as well as the learning and optimization problems faced by a representative investor who pays fluctuating attention to news. We then solve the learning and optimization problems and characterize the corresponding equilibrium asset prices.

We consider an infinite horizon economy populated by a representative investor who consumes the sum of two output streams (henceforth the *dividends*) with unobservable expected growth rates (henceforth the *fundamentals*). All quantities are expressed in units of a single perishable good with price equal to unity. The set of securities available for investment consists in one riskless asset in zero net supply and two risky assets (stocks) in positive supply of one unit. The riskless asset is locally deterministic and pays a riskless rate r to be determined in equilibrium. The two stocks are claims to the exogenous dividends δ_1 and δ_2 and have prices P_1 and P_2 , respectively. Dividends dynamics are written as

$$\frac{d\delta_{it}}{\delta_{it}} = f_{it}dt + \sigma_{\delta}dW_{it}^{\delta}, \quad i \in \{1, 2\} \quad (1)$$

where $(W_1^{\delta}, W_2^{\delta})^{\top}$ is a standard Brownian motion.

Although the investor does not observe fundamentals f_1 and f_2 , she knows that these processes follow

$$df_{it} = \lambda(\bar{f} - f_{it})dt + \sigma_f dW_{it}^f, \quad i \in \{1, 2\} \quad (2)$$

where $(W_1^f, W_2^f)^{\top}$ is a standard Brownian motion. Hence fundamentals mean-revert to their long-term means \bar{f} at speed λ .

The investor has four pieces of information available to estimate the value of the fundamentals. The first two pieces consist in the dividend growth rates $\frac{d\delta_1}{\delta_1}$ and $\frac{d\delta_2}{\delta_2}$. Because fundamentals drive dividends, observing dividend growth rates provides valuable information about the level of fundamentals.

The remaining two pieces of information are signals denoted by s_1 and s_2 . Their dynamics

are

$$ds_{it} = \Phi_{it}dW_{it}^f + \sqrt{1 - \Phi_{it}^2}dW_{it}^s, \quad i \in \{1, 2\} \quad (3)$$

where $\Phi_1, \Phi_2 \in (0, 1)$ represent fluctuating accuracies of the signals. We assume that Brownian shocks in Equations (1)-(3) are independent. This assumption implies that markets are perfectly symmetric and fundamentally unrelated, which allows us to precisely determine the mechanism leading to contagion.

The dynamics of the information signals in Equation (3) are motivated as follows. Assume the investor collects m_{it} , $i \in \{1, 2\}$ signals s_{it}^j , $j = 1, \dots, m_{it}$ at time t . s_{it}^j is the j -th noisy signal providing information on fundamental i . These publicly available sources of information represent, for instance, *CNN Money*, *Financial Times*, *Bloomberg*, *Wall Street Journal*, etc. For simplicity, let assume that accuracies of these individual signals are given by a similar process as follows $ds_{it}^j = a dW_{it}^f + \sqrt{1 - a^2} dW_{it}^j$, where $0 < a < 1$ is the accuracy of the individual signals and all Brownian motions are uncorrelated. By aggregating, the investor can summarize these m_i sources of information into two signals s_i whose dynamics are

$$ds_{it} = \Phi_{it}dW_{it}^f + \sqrt{1 - \Phi_{it}^2}dW_{it}^s, \quad (4)$$

where $\Phi_{it} = \frac{a}{\sqrt{\frac{1}{m_{it}}(1+(m_{it}-1)a^2)}}$. Comparing (3) and (4) shows that both specifications are equivalent. That is, the investor can change the accuracy of information Φ_i by choosing the number of signals m_i she acquires. When the investor is very attentive to news, the number of individual signals collected is large and leads to high accuracy. When the investor is inattentive to news, the number of signals acquired is small and leads to low accuracy. For this reason, we call Φ_i the *attention* to news associated to stock i .

Our specification for the information signal in (3) follows that in [Scheinkman and Xiong \(2003\)](#), [Dumas, Kurshev, and Uppal \(2009\)](#), and [Xiong and Yan \(2010\)](#). It shows that signals, s_1 and s_2 , provide information on the unexpected shocks driving fundamentals and not on their levels, as in [Detemple and Kihlstrom \(1987\)](#) and [Veronesi \(2000\)](#) among others. Although we adopt the former specification, our results also hold under the alternative.

2.1 Definition of Fluctuating Attention

Attention to stock i is defined as follows:

$$\Phi_{it} = \frac{\Psi}{\Psi + (1 - \Psi)e^{\Lambda\pi_{it}}}, \quad i \in \{1, 2\} \quad (5)$$

$$\pi_{it} = \int_0^t e^{-\omega(t-u)} \left(\frac{d\delta_{iu}}{\delta_{iu}} - \widehat{f}_{iu} du \right), \quad (6)$$

where $\Psi > 0$, $\omega > 0$, $\Lambda > 0$, and $\Lambda \in \mathbb{R}$.

The parameter Ψ is the long-run level of attention paid to each stock. Attention paid to stock i depends on the process π_i , which measures the performance of dividend i 's past growth rate relative to the investor's estimate of the fundamental, \widehat{f}_i .^{8,9} For this reason, we refer to π_i as dividend i 's *performance index*. In order to map the level of the performance index, $\pi_i \in \mathbb{R}$, to the level of attention, $\Phi_i \in (0, 1)$, we use the logistic transformation as described in equation (5).

The coefficient Λ indicates how the level of attention Φ_i changes in relation to the dividend performance index π_i . If Λ is positive, a positive shock to π_i , i.e., a positive dividend growth surprise, decreases the level of attention. That is, attention decreases when the investor underestimates the dividend growth rate (when $\widehat{f}_i < d\delta_i/\delta_i$). If Λ is negative, a positive shock to π_i increases Φ_i , implying that attention increases when the investor underestimates the growth rate. The magnitude of Λ determines the range of attention. If Λ is large, attention effectively belongs to the entire interval $(0, 1)$. If instead Λ is relatively small, the range of attention is narrow. If Λ is zero, attention is constant and equal to the long-run level Ψ .

The parameter ω in (6) controls the importance of past dividend growth surprises relative to the current dividend growth surprise. If ω is small, past dividend surprises matter in the determination of the current performance index. If instead ω is large, past realizations of dividend surprises do not significantly alter the value of the performance index. Applying

⁸The dividend performance index is inspired by [Kojien, Rodriguez, and Sbuelz \(2009\)](#) who assume that expected stock returns depend on the past performance of realized returns.

⁹The investor's estimate of the fundamental, \widehat{f}_i , is described in details in [Section 2.2](#).

Itô's lemma to (6) yields the following dynamics for the performance indices

$$d\pi_{it} = -\omega\pi_{it}dt + \sigma_{\delta}dW_{it}, \quad i \in \{1, 2\}, \quad (7)$$

where $dW_{it} = \frac{1}{\sigma_{\delta}} \left(\frac{d\delta_{it}}{\delta_{it}} - \widehat{f}_i dt \right)$ is dividend i 's scaled surprise at time t . The dynamics of the performance indices in (7) show that π_i reverts to 0 at speed ω .

Evidence of how attention fluctuates in relation to the dividend performance index is provided in [Andrei and Hasler \(2015\)](#). In their study, the parameters driving the attention dynamics are estimated using U.S. GDP data from 1969 to 2012. [Table 1](#) reports their parameters estimated using the Generalized Method of Moments. The parameter Λ is positive, suggesting that the investor is more attentive to news when the dividend performance index is low than when it is high. That is, attention is high when the investor tends to overestimate (when $\widehat{f}_i > d\delta_i/\delta_i$) the dividend growth rate, and low when the investor tends to underestimate it (when $\widehat{f}_i < d\delta_i/\delta_i$). Furthermore, the parameter Λ is large, which implies that the range of attention is wide.

Equations (5) and (6) show a one-to-one mapping between attention and dividend performance indices. Thus, attention is observable and the vector of state variables is conditionally Gaussian. This implies that standard Bayesian filtering techniques can be applied to our model, a task that we undertake in the next section.

2.2 Filtered State Variables

The investor learns about the fundamental f_i , $i \in \{1, 2\}$ by observing two different sources of information: the dividend δ_i and the signal s_i .¹⁰ [Proposition 1](#) describes the dynamics of the state variables inferred using these two sources of information.

Proposition 1. *Following [Liptser and Shiryaev \(2001\)](#), the dynamics of the state variables*

¹⁰Note that the dividend performance index π_i , or equivalently the attention process Φ_i , does not provide more information than the dividend δ_i and is therefore not part of the filtering problem.

inferred by the investor satisfy

$$\frac{d\delta_{it}}{\delta_{it}} = \widehat{f}_{it}dt + \sigma_{\delta}dW_{it}, \quad (8)$$

$$d\widehat{f}_{it} = \lambda(\bar{f} - \widehat{f}_{it})dt + \frac{\gamma_{it}}{\sigma_{\delta}}dW_{it} + \sigma_f\Phi_{it}dW_{i+2,t}, \quad (9)$$

$$d\pi_{it} = -\omega\pi_{it}dt + \sigma_{\delta}dW_{it}, \quad (10)$$

$$d\gamma_{it} = \left(-\frac{\gamma_{it}^2}{\sigma_{\delta}^2} - 2\lambda\gamma_{it} + \sigma_f^2(1 - \Phi_{it}^2) \right) dt, \quad i \in \{1, 2\}. \quad (11)$$

The 4-dimensional innovation process W is a standard Brownian motion defined by

$$dW_t \equiv \begin{pmatrix} dW_{1t} & dW_{2t} & dW_{3t} & dW_{4t} \end{pmatrix}^{\top} = \begin{pmatrix} \frac{1}{\sigma_{\delta}} \left(\frac{d\delta_{1t}}{\delta_{1t}} - \widehat{f}_{1t}dt \right) & \frac{1}{\sigma_{\delta}} \left(\frac{d\delta_{2t}}{\delta_{2t}} - \widehat{f}_{2t}dt \right) & ds_{1t} & ds_{2t} \end{pmatrix}^{\top}. \quad (12)$$

Proof. See Theorem 12.7 in [Liptser and Shiryaev \(2001\)](#).

The dynamics of the dividend process in (8) follow closely those in (1), but with the filtered fundamental \widehat{f}_i replacing the true fundamental f_i . The dynamics of the filtered fundamentals, however, differ from those in (2). When the investor is able to learn about the fundamental by observing the dividend and the signal, the dynamics in (9) show that the volatility of the filtered fundamental is stochastic and driven by two components. The first, $\gamma_{it} \equiv \mathbb{E} \left((f_{it} - \widehat{f}_{it})^2 | \mathcal{O}_t \right)$, is the *uncertainty* about the current value of the fundamental given the investor's current observation filtration. The second is *attention* Φ_i .

From (9) and (12), we see that uncertainty loads on the dividend innovation while attention loads on the news signal innovation. As attention increases, the investor perceives the news source as more important relative to the dividend source. Conversely, reduced attention pushes the investor to weight the information content of the dividend performance more than that of the news signal. This naturally implies that an increase (decrease) in attention weakens (strengthens) the correlation between dividends and fundamentals.

The dynamics in (11) show that attention also impacts the level of uncertainty. As attention increases, the investor gathers more accurate information. Therefore, the learning procedure becomes more efficient and uncertainty decreases. Conversely, as attention drops investors acquire less accurate information and uncertainty rises. For sufficiently high (low)

attention level, the third component in (11) decreases (increase) enough to generate lower (higher) uncertainty. Interestingly, there is a lag between a change in attention and a change in uncertainty because uncertainty is locally deterministic. That is, high attention implies low future uncertainty, whereas low attention is followed by high uncertainty.

Table 1: Calibration.

This table reports the parameters determining the dynamics of dividends and fundamentals. The parameters are borrowed from [Andrei and Hasler \(2015\)](#) who calibrate a fluctuating attention model with a single dividend and fundamental to match several moments of the U.S. GDP growth rate and its corresponding forecast from 1969 to 2012. The last two parameters are the investor’s relative risk aversion α and subjective discount rate Δ . Those are chosen and not estimated.

σ_δ	\bar{f}	λ	σ_f	ω	Λ	Ψ	α	Δ
1.4%	2.8%	0.42	2.9%	4.74	286	0.368	5	1%

Table 2: Initial state values.

This table reports the initial values of the state variables that we use to generate the model implications. Initial dividends are set to unity, i.e., $\zeta_i \equiv \log \delta_i = 0$. Therefore, $Q \equiv \log(\delta_1/(\delta_1 + \delta_2)) = \log 0.5$. The other variables are set to their long-term levels. Long-term levels of the fundamentals and the performance indices are derived in [Appendix A.1](#). Long-term uncertainty, γ_{ss} , is defined and derived in [Appendix A.2](#).

ζ_1	Q	$\hat{f}_1 = \hat{f}_2$	$\pi_1 = \pi_2$	$\gamma_1 = \gamma_2$
0	$\log(0.5)$	\bar{f}	0	γ_{ss}

2.3 Equilibrium

The representative investor has CRRA utility over consumption. Since the investment horizon is assumed to be infinite, the investor maximizes her expected lifetime utility of con-

sumption subject to a budget constraint

$$\begin{aligned} & \sup_{C,h} \mathbb{E}_t \left(\int_t^\infty e^{-\Delta(s-t)} \frac{C_s^{1-\alpha}}{1-\alpha} ds \right) \\ & \text{s.t. } dV_t = (r_t V_t + h_t \text{diag}(P_t) (\mu_t - r_t \mathbb{1}_{2 \times 1}) - C_t) dt + h_t \text{diag}(P_t) D_t dW_t, \end{aligned}$$

where C is consumption, V is wealth, μ is the 2×1 vector of expected stock returns, h is the 1×2 vector of risky asset holdings, D is the 2×4 matrix of stock return diffusion, Δ is the subjective discount rate, and α is the coefficient of relative risk aversion. The risk-free rate r and the 2×1 vector of stock prices P are determined in equilibrium.

Solving the optimization problem and clearing markets yields the following state-price density

$$\xi_t = e^{-\Delta t} \left(\frac{C_t}{C_0} \right)^{-\alpha} = e^{-\Delta t} \left(\frac{\delta_{1t} + \delta_{2t}}{\delta_{1,0} + \delta_{2,0}} \right)^{-\alpha}. \quad (13)$$

Equation (13) shows that the state-price density depends on dividend 1 and dividend 2. As either fundamental 1 or fundamental 2 increases, the expected value of discount factors decreases or, in other words, the expected discount rates increase. As will be explained further, the interactions between fundamentals and discount rates implied by fluctuating attention to news are key determinants of the contagion phenomenon.

Since the state-price density ξ prices future cash-flows, the price P_i^T of a security paying a single-dividend δ_{iT} at time T is defined by

$$P_{it}^T = e^{-\Delta(T-t)} \mathbb{E}_t \left(\left(\frac{\delta_{1T} + \delta_{2T}}{\delta_{1t} + \delta_{2t}} \right)^{-\alpha} \delta_{iT} \right).$$

Proposition 2 characterizes the price of the single-dividend paying securities.

Proposition 2. *At time t , the prices P_{1t}^T and P_{2t}^T of the securities paying the single-dividends δ_{1T} and δ_{2T} at time T satisfy*

$$P_{1t}^T = e^{-\Delta(T-t)} e^{\alpha(\zeta_{1t} - Q_t)} \mathbb{E}_t \left(e^{(1-\alpha)\zeta_{1T} + \alpha Q_T} \right) \quad (14)$$

$$P_{2t}^T = e^{-\Delta(T-t)} e^{\alpha(\zeta_{1t} - Q_t)} \mathbb{E}_t \left(e^{(1-\alpha)\zeta_{1T} + (\alpha-1)Q_T} \right) - P_{1t}^T, \quad (15)$$

where $\zeta_i \equiv \log \delta_i$ is the log-dividend, and $Q = \log \frac{\delta_1}{\delta_1 + \delta_2}$ the log-dividend share.

Proof. See Appendix A.3.

The stock price P_{it} , $i \in \{1, 2\}$ at current time t is defined as the sum of the single-dividend paying securities P_{it}^T over maturities T

$$P_{it} = \int_t^\infty P_{it}^T dT. \quad (16)$$

Equations (14) and (15) show that the single-dividend paying securities are determined by moment-generating functions (henceforth *transforms*) of the vector $(\zeta_1, Q)^\top$. Computing these transforms is challenging, as the vector of state variables is not affine-quadratic. Appendix A.4 exposes a simple methodology that allows us to accurately approximate them.

3 Model Implications

In this section, we investigate the implications of fluctuating attention on the dynamics of return volatilities and cross-return correlation of two fundamentally unrelated stocks. We show that increased attention to one stock raises return volatilities on both stocks, as well as their cross-return correlation. That is, fluctuating attention implies return and volatility spillover effects among fundamentally unrelated stocks.

3.1 Volatility Spillovers

In order to understand the mechanism leading to contagion, let us first characterize the components of the stock-return diffusion. Since the innovation process (12) driving the filtered state variables consists of four components, applying Itô's lemma to stock prices in (16) yields the following stock-return diffusion matrix D

$$D \equiv \begin{pmatrix} D_{11} & D_{12} & D_{13} & D_{14} \\ D_{21} & D_{22} & D_{23} & D_{24} \end{pmatrix},$$

where D_{ij} is the j -th component of the return diffusion of stock i . The components of the diffusion matrix D are provided in Proposition 3 below.

Proposition 3. *The diffusion components of stock i satisfy*

$$D_{i1} = \frac{P_{i\hat{f}_1}}{P_i} \frac{\gamma_1}{\sigma_\delta} + \sigma_\delta \left(1 + \frac{P_{i\pi_1}}{P_i} + \frac{P_{iQ}}{P_i} (1 - e^Q) \right), \quad D_{i2} = \frac{P_{i\hat{f}_2}}{P_i} \frac{\gamma_2}{\sigma_\delta} + \sigma_\delta \left(\frac{P_{i\pi_2}}{P_i} + \frac{P_{iQ}}{P_i} (e^Q - 1) \right),$$

$$D_{i3} = \frac{P_{i\hat{f}_1}}{P_i} \sigma_f \Phi_1, \quad D_{i4} = \frac{P_{i\hat{f}_2}}{P_i} \sigma_f \Phi_2,$$

where P_{iy} stands for the derivative of stock i with respect to the state variable y .

For brevity, we drop the time t notation when writing the stock price P_i and its partial derivative P_{iy} with respect to state y . Nevertheless, we note that all notations regarding the stock price should be referenced against the current time t .

Definition 1. *The return variance of stock i , σ_i^2 , satisfies*

$$\sigma_i^2 = \sum_{j=1}^4 D_{ij}^2,$$

where the component of the stock-return diffusion D_{ij} is provided in Proposition 3.

Proposition 3 and Definition 1 show that a change in attention to stock 1 impacts the return volatility of both stock 1 and stock 2 through the stock-return diffusion component D_{i3} . The significance of the impact is determined by the sensitivity of stock prices P_1 and P_2 to a shock in the filtered fundamental \hat{f}_1 . The properties of the sensitivity, $P_{i\hat{f}_1}$, are discussed below.

A positive shock in the filtered fundamental \hat{f}_1 has two opposite effects on stock 1. First, dividend δ_1 is expected to increase. This is the *direct channel*. Second, discount rates rise (see Equation (13)), which represent the *indirect channel*. The direct channel pushes stock price P_1 up, while the indirect channel pushes stock price P_1 down. As explained in Veronesi (2000), the discounting effect is stronger than the dividend effect as long as risk aversion is sufficiently large. An increase in expected future consumption increases current consumption because the investor smoothes consumption over time. This implies that savings (and investments) decrease as do the demands for risky assets and the riskless asset. As a result, the prices of stock 1 and stock 2 decline while the risk-free rate rises. The

decline in stock price P_2 , however, can be more pronounced than the decline in stock price P_1 because stock 2 is only impacted by the discounting effect. Note that if the representative agent had either a risk aversion smaller than one or recursive utility (Epstein and Zin, 1989) with an elasticity of intertemporal substitution larger than one, then stock 1 would be more sensitive to a change in fundamental 1 than stock 2.

In order to see the effect of attention on volatilities visually, Figure 1 plots the relationship between attention to stock 1 and return volatilities of both stocks. The model parameters and the initial values of the state variables that we use are reported in Tables 1 and 2, respectively.

Panels A and B of Figure 1 plot the impact of increasing attention on volatilities of stock 1 and stock 2, holding all other variables constant. We see that an increase in attention to stock 1 increases both return volatilities. That is, fluctuating attention implies volatility spillover effects. The volatility of stock 2 increases more than that of stock 1 because stock 2 is influenced by the indirect discounting channel only. Regarding stock 1, the direct channel of dividend δ_1 dampens the indirect discounting channel and therefore implies a weaker increase in its volatility. Symmetrically, the volatility of stock 1 increases more than that of stock 2 as attention to stock 2 increases.

The joint effect of increasing attention paid to each stock 1 and stock 2, are illustrated in Panels C and D of Figure 1. The results show that an increase in attention to any stock, i.e., either Φ_1 and Φ_2 , will increase both stock-return volatilities. Importantly, increasing attention to both stock 1 and stock 2 appear to work together by pushing stock-return volatilities higher. In other words, when total attention, i.e., $\Phi_1 + \Phi_2$, paid to the stocks increases, the volatility spillover effects become most evident. Overall, Figure 1 shows that an increase in attention to any stock leads to contagion in the form of increased stock-return volatilities in the entire market. However, the effect is largest when the total attention paid to the whole market increases.

While Figure 1 describes the static relationship between attention and volatilities, we next examine how stock-return volatilities change relative to attention in a dynamic setting. Such analysis is useful for understanding the economic relevance of fluctuating attention on volatilities under the presence of noises generated by other state variables. We simulate the model at the monthly frequency for 20 years, using the parameters and initial variables reported in Tables 1 and 2. Figure 2 depicts the dependence of stock-return volatilities on

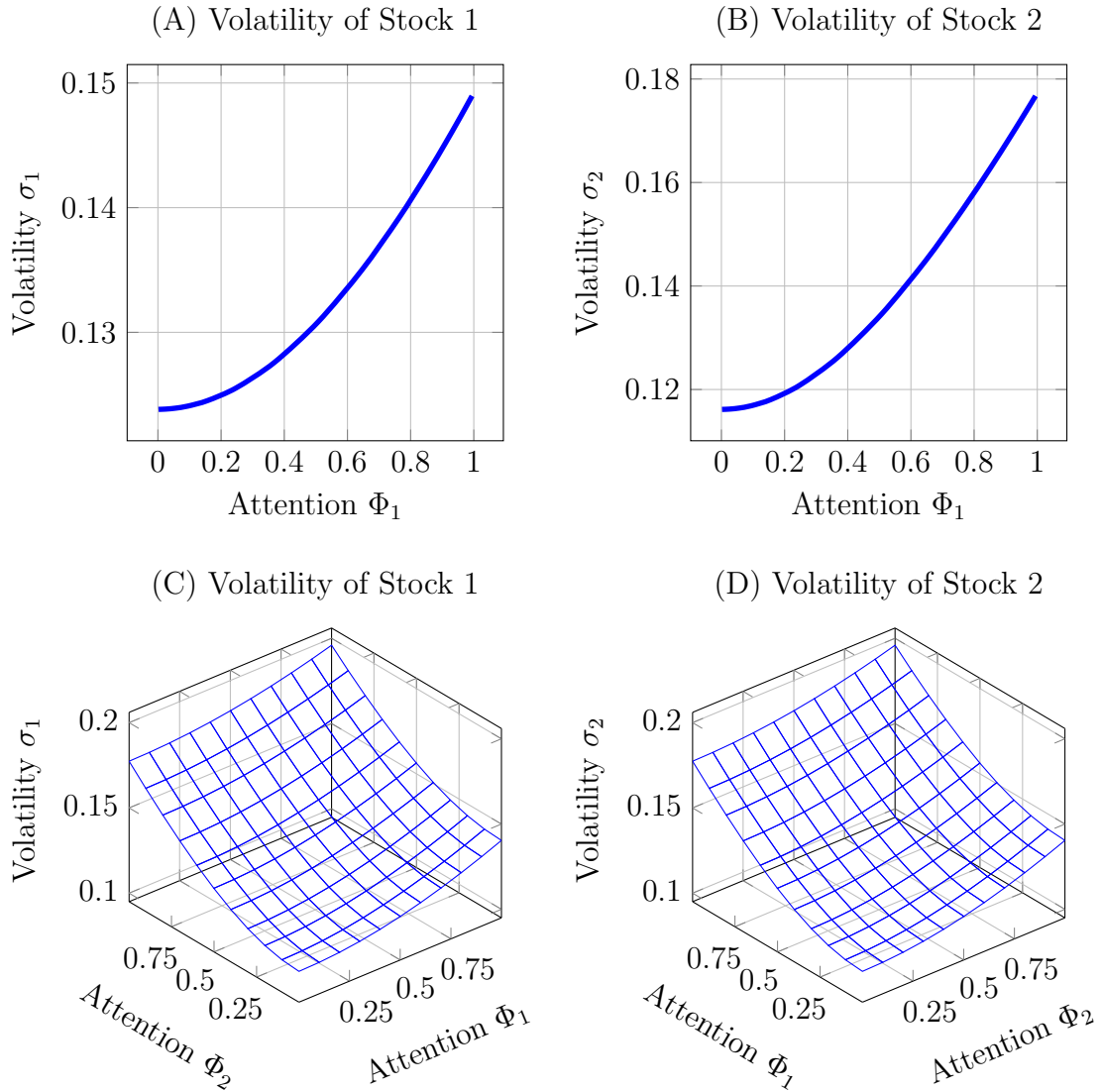


Figure 1: Stock-return Volatility vs. Attention.

This figure provides a static analysis of our model's implications on the volatility spillover effects. Panels A and B plot the relationship between attention paid to stock 1 and the return volatilities of stock 1 and 2, respectively. In Panels C and D, we plot the joint relationship of attention paid to stock 1 and stock 2 on the return volatilities of stock 1 and stock 2, respectively. The model parameters and the initial values of the state variables used to generate the results are reported in Tables 1 and 2, respectively.

aggregate attention paid to stocks 1 and 2. Consistent with our previous results showing that stock-return volatilities increase with attention to stocks 1 and 2 separately, there is a strong

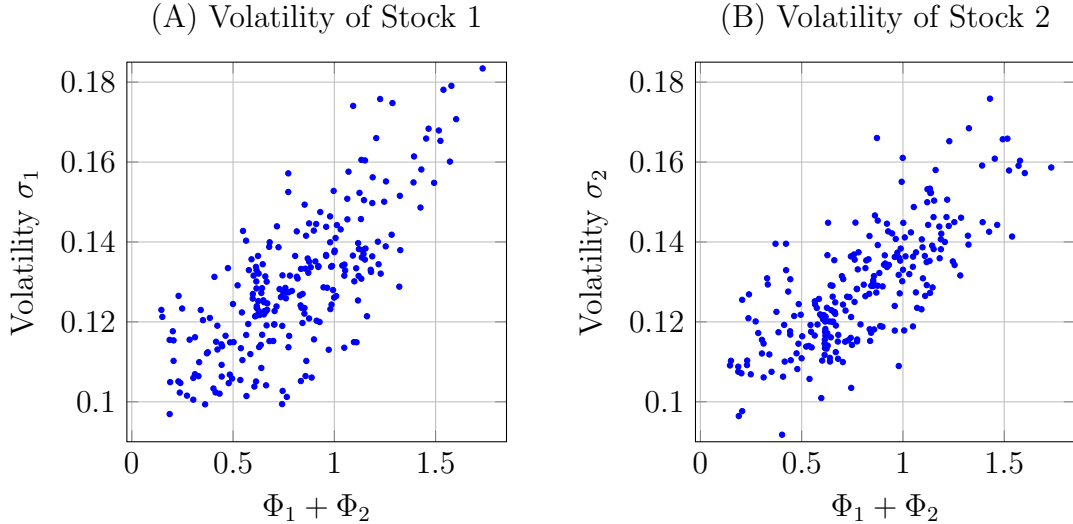


Figure 2: Stock-return Volatility vs. Aggregate Attention.

This figure plots simulation results showing the dynamic relationship between volatilities and aggregate attention. We simulate 20-year of data at the monthly frequency using the model introduced in Section 2. In both panels, the x-axis indicates the aggregate attention paid to stocks 1 and 2. The left panel plots the stock-return volatility for stock 1, while the right panel plots the stock-return volatility for stock 2. Each dot represents one simulated monthly observation. The model parameters and the initial values of the state variables used to generate the results are reported in Tables 1 and 2, respectively.

positive relationship between the volatilities of both stocks and the aggregate attention. This shows that the dynamics of the other state variables have very little influence on the dynamics of stock-return volatilities, and consequently that aggregate attention is the main driver of return volatilities.

To compare the dynamics of volatilities in a fluctuating attention model to those in an otherwise equivalent constant attention model, we depict in Figure 3 a 20-year simulation of volatilities in both models. To obtain the constant attention model, we turn off the fluctuating attention feature by setting $\Lambda = 0$ in Equation (5). Figure 3 shows that the volatility dynamics significantly differ between the fluctuating and constant attention models. In the fluctuating attention model (Panel A), volatilities swing rapidly from month-to-month and simultaneous spikes in volatilities of stock 1 and stock 2 are often observed. On the other hand, results from the constant attention model (Panel B) show that volatilities of the two stocks move smoothly and, on average, in opposite directions. In order to quantify the

(A) Volatility with Fluctuating Attention (B) Volatility with Constant Attention

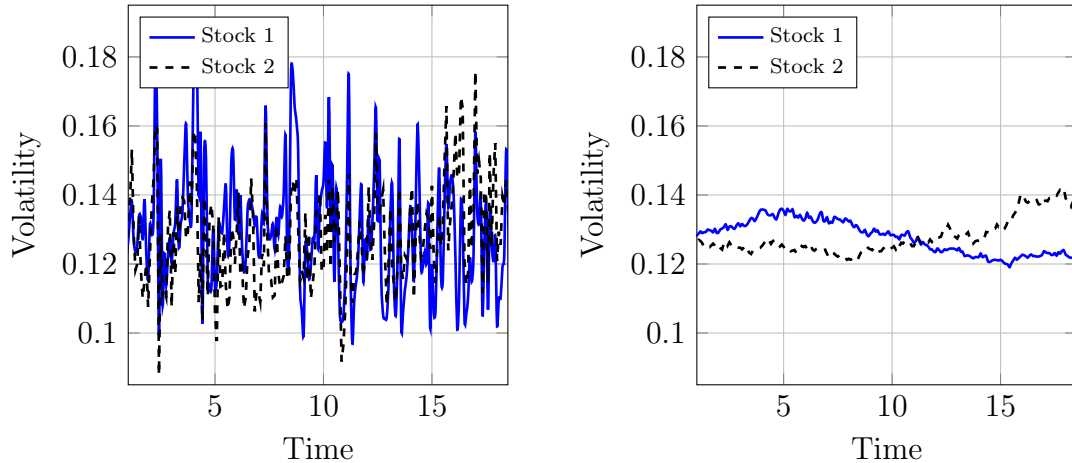


Figure 3: Stock-return Volatility and Attention: A Simulation Study

We plot time-series results of monthly volatilities from a 20-year simulation of the economy introduced in Section 2. Monthly volatilities are calculated following Definition 1, and reported in annualized terms. Panel A plots monthly volatilities of stock 1 and stock 2 for an economy with fluctuating investor attention. In Panel B, we turn off the fluctuating attention feature of the model by setting $\Lambda = 0$ in Equation (5), and plot the monthly volatilities for an economy with constant investor attention ($\Phi_1 = \Phi_2 = 0.368$). In each panel, the blue line indicates volatilities of stock 1, while the dashed black line indicates volatilities of stock 2. The model parameters and the initial values of the state variables used to generate the results are reported in Tables 1 and 2, respectively.

average correlation between volatilities of stocks 1 and 2, we repeat this simulation study 10,000 times. We find that the volatility of volatility in the fluctuating attention model is about 1.8% for both stocks, while the correlation between the volatilities of stock 1 and stock 2 is around 0.65. In the constant attention model, however, volatility of volatility falls to 0.4%, while the correlation between volatilities of stock 1 and stock 2 is -0.35 .

The economic intuition for the significantly different dynamics of volatilities obtained in the fluctuating and constant attention models is as follows. When attention is constant, volatilities are principally driven by dividend shares (Cochrane, Longstaff, and Santa-Clara, 2008). As the dividend share of stock 1 increases, the dividend share of stock 2 mechanically decreases. As a result, volatilities move in opposite directions. On the other hand, in our fluctuating attention model, variations in dividend shares are outweighed by variations in aggregate attention and therefore have little influence on volatilities (see Figure 2). This

implies an amplification of the fluctuations in stock-return volatilities and a positive co-movement between them. To summarize, a standard model with learning (e.g. the constant attention model) does not help understand the dynamics of stock-return volatilities in the cross-section. However, a model in which the investor learns by paying fluctuating attention to news does. In this case, volatilities among fundamentally unrelated sectors fluctuate strongly and co-vary positively, consistent with empirical findings in Fleming, Kirby, and Ostdiek (1998).

3.2 Return Spillovers

We now turn to the relationship between attention and the co-movement of stock returns. The covariance and correlation between returns of stock 1 and stock 2 are described Definition 2 below.

Definition 2. *The cross-return covariance, σ_{12} , and the cross-return correlation, ρ_{12} , satisfy*

$$\sigma_{12} = \sum_{j=1}^4 D_{1j} D_{2j}, \quad \rho_{12} = \frac{\sigma_{12}}{\sigma_1 \sigma_2} = \frac{\sum_{j=1}^4 D_{1j} D_{2j}}{\sqrt{\sum_{j=1}^4 D_{1j}^2} \sqrt{\sum_{j=1}^4 D_{2j}^2}},$$

where the component of the stock-return diffusion D_{ij} is provided in Proposition 3.

We plot the static relationship between the level of attention paid to stock 1 and the cross-return correlation in Panel A of Figure 4. We note that by symmetry, the relationship between Φ_2 and ρ_{12} would look identical. Panel A shows that the cross-return correlation between stock 1 and stock 2 is positive and increases with attention even though dividends, fundamentals, and signals are uncorrelated. Panel B shows the static joint impact of attention Φ_1 and Φ_2 on the cross-return correlation ρ_{12} . Similar to our results for volatilities, we find that an increase in attention to stock 1 and stock 2, together, further strengthens the return spillover effects. Thus, contagion arises most prominently when attention paid to the whole market increases.

We now discuss the economic mechanism leading to a positive cross-return correlation in our model. We first explain how a non-zero cross-return correlation can arise in a standard equilibrium model with learning and unrelated fundamentals, and then discuss how the fluctuating attention feature produces the positive cross-return correlation observed in our

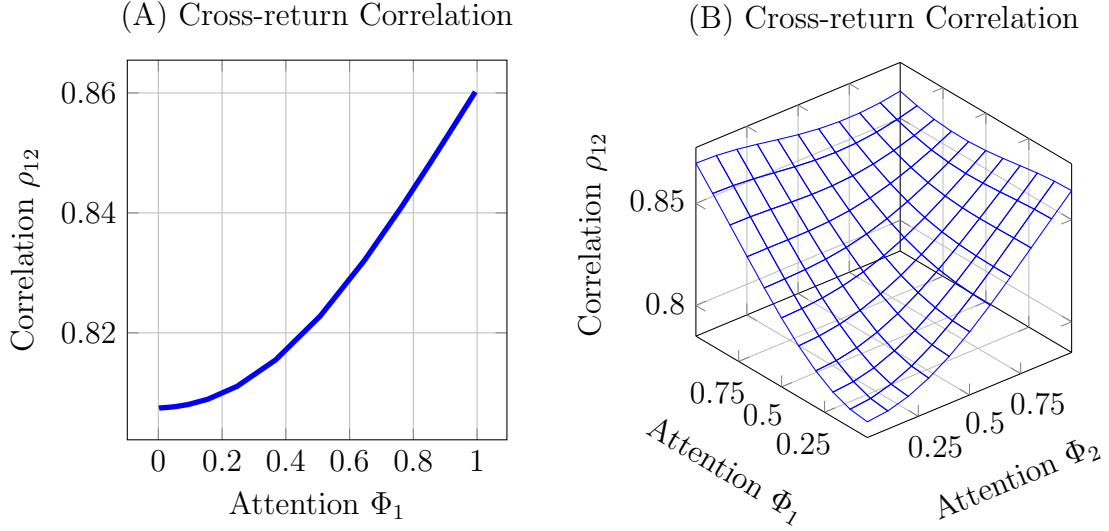


Figure 4: Cross-return Correlation vs. Attention.

This figure provides a static analysis of our model’s implications on the return spillover effect. Panel A plots the relationship between attention paid to stock 1 and the level of cross-return correlation between stock 1 and stock 2, holding other variables fixed. In Panel B, we plot the joint relationship between attention paid to stock 1 and stock 2 of the level of their cross-return correlation, while holding other variables fixed. The model parameters and the initial values of the state variables used to generate the results are reported in Tables 1 and 2, respectively.

model. Following the exposition in Cochrane, Longstaff, and Santa-Clara (2008), the cross-return correlation between stock 1 and stock 2 can arise due to co-movements in one of the following four relationship pairs. The first is the co-movement between price-dividend ratio $\frac{P_1}{\delta_1}$ and dividend δ_2 . The second is the co-movement between price-dividend ratio $\frac{P_2}{\delta_2}$ and dividend δ_1 . The third is the co-movement between price-dividend ratios $\frac{P_1}{\delta_1}$ and $\frac{P_2}{\delta_2}$. Finally, the fourth, is the co-movement between dividends δ_1 and δ_2 . Because our model assumes that dividend δ_1 and dividend δ_2 are uncorrelated, the fourth relationship source can be eliminated.

In our model, the positive correlation between returns of stock 1 and stock 2 arises through discount rates. To understand the economic mechanism, let us consider a negative shock in the dividend of stock 1, i.e., the shock dW_{1t} in (8) is negative. This shock decreases the performance of dividend 1, thereby raising the attention paid to stock 1 (see Equations (10) and (5)). In the meantime, the negative shock also pushes the investor to decrease

her estimation of the fundamental \widehat{f}_1 (see Equation (9)). This decrease in the expectation of future dividend growth causes discount rates to fall and triggers an increase in price-dividend ratio $\frac{P_2}{\delta_2}$. As for stock 1, because risk aversion is sufficiently large, the discounting channel outweighs the dividend channel and implies an increase in price-dividend ratio $\frac{P_1}{\delta_1}$. The end result is a positive co-movement between price-dividend ratios $\frac{P_1}{\delta_1}$ and $\frac{P_2}{\delta_2}$, while the co-movement between δ_1 and $\frac{P_2}{\delta_2}$ is negative. The former effect dominates the latter and implies a positive cross-return correlation because the discounting channel is the strongest.

In the previous paragraph we explain how, given a sufficiently large risk aversion, returns of two fundamentally unrelated stocks co-move positively due to the the discount rate channel. However, an important result from our model is that the magnitude of cross-return correlation depends on the level of investor attention (see Figure 4). This result can also be directly seen from Proposition 3, which shows that when attention paid to stock 1 increases, the cross-return correlation rises because the diffusion components D_{13} and D_{23} increase (in absolute value). The intuition to why attention impacts the magnitude of return spillover effects is discussed below.

Recall that the diffusion of the filtered fundamentals reflects two pieces of information: dividend innovations (dW_{1t}, dW_{2t}) and signal innovations (dW_{3t}, dW_{4t}). It follows from (9) that the uncertainty γ_i loads on dividend innovations, while attention Φ_i loads on signal innovations. As a negative shock hits dividend 1, attention to stock 1 increases but uncertainty γ_1 remains currently unchanged because it is locally deterministic. In other words, the weight assigned to signal innovations rises while the weight assigned to dividend innovations remains unchanged. Consequently, the variance of the filtered fundamental increases while the covariance between the dividend growth and the filtered fundamental, i.e., $\text{cov}_t \left[\frac{d\delta_{1t}}{\delta_{1t}}, d\widehat{f}_{1t} \right]$, remains constant. This means that an increase in attention disconnects the dividend from the filtered fundamental. Since the filtered fundamental drives discount rates, the correlation between discount rates and dividend 1 is reduced in absolute terms. This implies that the negative co-movement between price-dividend ratio 2 and dividend 1 is less pronounced when attention to stock 1 is high, and therefore that the cross-return correlation is larger.

This mechanism explains why the cross-return correlation increases with attention to stock 1, and by symmetry, with attention to stock 2. As a result, the cross-return correlation rises, exactly like stock-return volatilities, with aggregate attention. Panel A of Figure 5 plots monthly cross-return correlations against aggregate attention obtained from a 20-year

simulation. We observe a strong positive relationship between correlation ρ_{12} and aggregate attention $\Phi_1 + \Phi_2$, similar to the finding for volatilities. This result shows that aggregate attention is the main driver of the variation in cross-return correlation, and that the other state variables have a minor impact on it.

Panel B of Figure 5 plots the monthly time series of cross-return correlations for the fluctuating attention model, as well as for the constant attention model. The correlation in the constant attention model is positive, consistent with the prediction that correlations between asset returns naturally arise due to the discount rate channel. However, monthly cross-return correlations for the constant attention model do not fluctuate significantly from month-to-month, suggesting that fluctuations in dividend shares alone cannot generate sudden cross-return spikes that are distinctive of financial contagion. In the fluctuating attention model, monthly cross-return correlations swing more intensely. This finding suggests that fluctuating attention is an important feature for generating the return spillover effects observed in financial markets.

Performing 10,000 simulations of our economy over 20 years, we find that fluctuating attention significantly amplifies the variations in cross-return correlation compared to an economy in which the investor pays constant attention. Simulation results show that correlation ranges from 0.38 to 0.47 in the constant attention model, and from 0.30 to 0.65 in the fluctuating attention model. In addition, volatilities and correlation co-move positively in the fluctuating attention model because those quantities are mainly driven by aggregate attention. This result, however, does not hold when attention is constant because, in that case, the dividend share is the main driver of volatility and correlation.

4 Empirical Evidence

We empirically test the model's predictions using large U.S. industry sectors. In the model, the mechanism of how time-varying attention generates contagion between sectors with independent dividend streams is through the equilibrium discount rates. We therefore examine the volatility- and return-spillover effects on the discount-rate-news component of industry returns.

Our empirical strategy builds on insights drawn from the return-decomposition framework of Campbell and Shiller (1988) and Campbell (1991). Following their work, we decompose

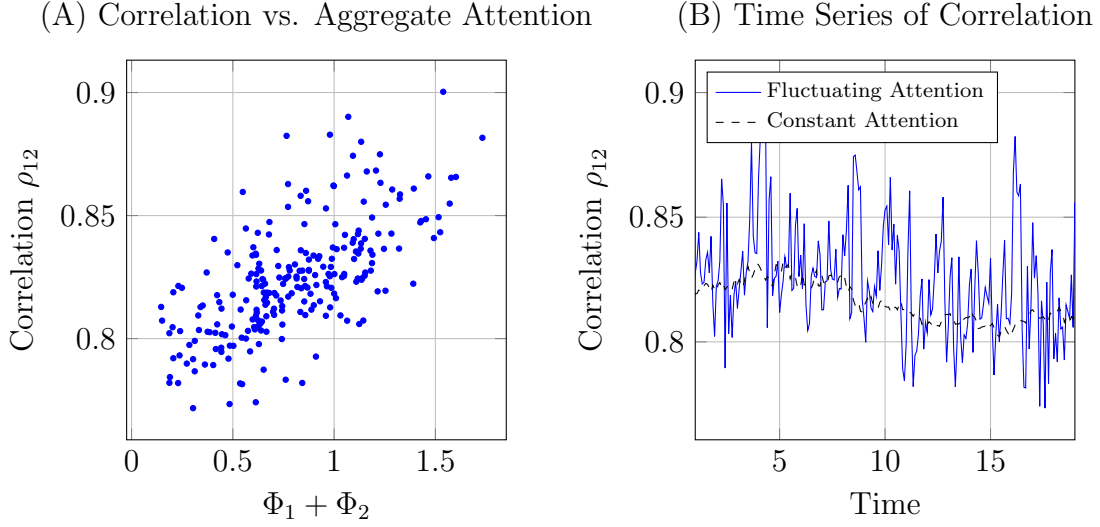


Figure 5: Cross-return Correlation and Attention: A Simulation Study

We plot cross-return correlations results generated from a 20-year simulation. Panel A plots the dynamic relationship between correlation and aggregate attention $\Phi_1 + \Phi_2$. In Panel B, we plot the time-series of monthly return correlations between stock 1 and stock 2 for an economy with fluctuating attention (blue line) and constant attention (dashed black line). Monthly return correlations are calculated following Definition 2. The model parameters and the initial values of the state variables used to generate the results are reported in Tables 1 and 2, respectively.

industry returns into two components corresponding to changes in future cash flows (i.e., dividends), and changes in future discount rates (i.e., expected returns). We then focus on the discount-rate return. Using a panel regression, we show that fluctuating attention to one industry sector affects its discount-rate return volatility, discount-rate return volatilities of other industries, as well as correlations between its discount-rate return and those of other industries.

We first describe the data and the method used to decompose quarterly industry returns into two components reflecting the cash-flow change and the discount-rate change. We then discuss the estimation of time-varying investor attention, industry return volatilities, and cross-industry return correlations. After, we outline testable hypotheses based on the model's predictions and discuss their empirical findings.

4.1 Data and Sample

Our sample consists of U.S. incorporated firms that are traded on the NYSE/AMEX and NASDAQ. We require that firms have information available on both CRSP and COMPUSTAT databases. We group firms into different industries following the Fama-French's 38 industry portfolio definitions. We choose the 38-industry classification because each industry's size is sufficiently large to capture shocks to its sector, and because it is more refined than Fama-French's 12 and 17 portfolio definitions. We exclude financial industry from the sample. We further drop five industries from our analysis because they have missing observations.¹¹ We measure changes in quarterly investor attention from various sources namely CRSP, I/B/E/S, and Thomson Reuters 13F institutional ownership. The sample period for our main empirical tests is from 1985 to 2014. The main empirical analysis begins in 1985 due to data availability in I/B/E/S.

The general equilibrium model that we introduced in Section 2 provides predictions of contagion between two sectors with sizable market share. We therefore require that industries in our sample are sufficiently large. Using firm-level information from COMPUSTAT, we measure industry size based on three dimensions: market equity value, book equity value, and annual dividend payout. We use a simple filter requiring that each industry in the final sample contributes 1% or more to the aggregate U.S. market in all the three size dimensions. Time-series means of each industry are used in this filter. There are 16 industries that meet this size requirement. Together, they make up about 90% of the total market capitalization. Table 3 presents the 16 industry portfolios in our final sample. For each industry, we report its time-series median values for the number of firms, market equity value, book equity value, and annual dividend payout.

¹¹Following [Hong, Torous, and Valkanov \(2007\)](#) who study Fama-French 38 industry portfolios, we exclude GARBAGE (sanitary services), STEAM (steam supply), WATER (irrigation systems), GOVT (public administration), and OTHER (everything else).

4.2 Decomposing the Industry Return

By definition, a firm's stock return is driven by shocks to expected cash flows and/or shocks to discount rates.¹² Campbell and Shiller (1988) and Campbell (1991) show that a stock return r_t can be decomposed using the following log-linear approximation:

$$r_t - \mathbb{E}_{t-1}[r_t] = \Delta \mathbb{E}_t \sum_{j=0}^{\infty} \rho^j \Delta d_{t+j} - \Delta \mathbb{E}_t \sum_{j=1}^{\infty} \rho^j r_{t+j} \quad (17)$$

$$= \text{Ncf}_t - \text{Ndr}_t, \quad (18)$$

where d_t is the log dividend paid by the stock. We denote by $\Delta \mathbb{E}_t = \mathbb{E}_t(\cdot) - \mathbb{E}_{t-1}(\cdot)$ the change in expectations from $t-1$ to t , and ρ is the discount factor. Equation (17) shows that an unexpected change in return is associated with changes in expectation of future cash flows (i.e., dividends) or with changes in expectation of future discount rates (i.e., expected returns). We can write the return-decomposition more compactly as shown in Equation (18) by defining N_{cf} and N_{dr} as the cash-flow news and discount-rate news components of the return, respectively.

The variance of unexpected return can be decomposed into three components as follows:

$$\text{var}(r_t - E_{t-1}) = \text{var}(\text{Ncf}_t) + \text{var}(\text{Ndr}_t) - 2 \text{cov}(\text{Ndr}_t, \text{Ncf}_t). \quad (19)$$

For our empirical analysis, we focus on financial contagion between the discount-rate component (Ndr) of industry returns. This helps mitigate the likelihood of fundamental-based contagion (i.e. dividend shocks) from driving the results. Although we cannot rule out the effect of cash-flow news due to the potential correlation between Ndr and Ncf, our estimate based on large U.S. industries shows that it is small and is statistically insignificant. This finding is consistent with Vuolteenaho (2002) and Campbell and Vuolteenaho (2004) who find statistically insignificant correlation between Ndr and Ncf for both large stocks and the market index.

Following Campbell (1991) among others, we use a vector autoregressive (VAR) model to estimate Ndr time series. The estimation frequency is quarterly because some industry-level

¹²There is a substantial body of research that studies the decomposition of aggregate market returns (e.g., see Campbell and Ammer (1993), and Campbell and Vuolteenaho (2004)) and individual stock returns (see Vuolteenaho (2002)).

variables that we use are calculated using COMPUSTAT data. We obtain monthly industry portfolio returns data from Kenneth French’s website and compound them over each quarter to obtain quarterly returns for each industry.¹³

The VAR methodology assumes that expected returns of industry i can be estimated and that the data are generated by a first-order VAR model:

$$\mathbf{z}_{i,t} = \mathbf{A} + \Gamma \mathbf{z}_{i,t-1} + u_{i,t}, \quad \Sigma = E [u_{i,t} u'_{i,t}],$$

where $\mathbf{z}_{i,t}$ is an m -by-1 state vector with a log industry excess return over the 3-month Treasury rate, $r_{i,t}$, as its first element. The remaining elements in $\mathbf{z}_{i,t}$ contain industry-level and macro variables that can predict quarterly industry returns. \mathbf{A} and Γ are m -by-1 vectors, and m -by- m matrix $u_{i,t}$ is an m -by-1 vector of i.i.d. shocks with a covariance matrix Σ .

We use five forecasting variables for the industry return, which include industry-specific variables and macro variables. Our variable choice is motivated by previous studies, as well as parsimony. For industry-specific variables, we use the industry excess return (one and two lags), the log dividend-price ratio, and the log return on equity. The macro variables that we use are the term yield spread defined as the difference between 10-year and 3-month Treasury rates, and the log excess return of CRSP value-weighted index, i.e., log market excess return. We include the dividend-price ratio following Campbell (1991), which is the firm’s past 12-month dividend payout over its last quarter equity price. We include return on equity following Vuolteenaho (2002); it is calculated as the firm’s past 12-month earnings over its last quarter book equity. Industry-level dividend-price ratio and return on equity are calculated as their value-weighted averages across firms in the industry. Portfolio weights are updated once per year using the firm’s market capitalization from the most recent June. We explain the implementation in greater details in Appendix A.5.

Estimation results and variance decomposition of industry returns are reported in Appendix Table A1. We find that all forecasting variables that we use significantly predict the next-quarter industry return with an adjusted R^2 of 2.2%. Variance decomposition following Equation (19) shows that the industry return is mostly explained by shocks to its future dividends (i.e., cash-flow news). The ratios of discount-rate-news variance and cash-flow-news variance to total-unexpected-return variance are about 25% and 79%, respectively. This

¹³The findings are virtually unaffected when using industry returns calculated by ourselves.

finding is highly consistent with Vuolteenaho (2002) who applies the return-decomposition framework to individual stock returns and finds that stock returns are mainly driven by cash-flow news.

In line with the literature, we find that cash-flow news (Ncf) and discount-rate news (Ndr) of large U.S. industries are positively correlated. Its estimate is about 3.9% and is statistically insignificant.¹⁴ This weak correlation implies that shocks to cash-flow news have an economically small impact on discount-rate news, and vice versa. Our main empirical tests for the financial contagion focus on the discount-rate-news return and volatility. This approach helps mitigate the impact of shocks to cash-flow news that may cause the return and the volatility of different sectors to move in the same direction.

4.3 Measuring Investor Attention

There is no direct measure of investor attention. However, previous studies in finance and economic have used various information sources to proxy for it. For instance, Barber and Odean (2008), Gervais, Kaniel, and Mingelgrin (2001) and Hou, Xiong, and Peng (2009) use trading volume as a proxy for investor attention. On the other hand, Grullon, Kanatas, and Weston (2004) and Chemmanur and Yan (2009) use advertising expense to proxy for investor attention. More recently, Da, Engelberg, and Gao (2011) propose a novel measure of investor attention using the search frequency in Google. Because our sample period is from 1985–2014 with quarterly observations, we face a challenge of finding an attention measure that can be reliably constructed dating back to more than 30 years. In this case, using Google search volume proves challenging as the data is only available from 2004 onwards. Thus, relying on Google search data would cut down our time-series length by two-third, or equivalently down to 44 quarters. In order to construct a measure of investor attention over a relatively long time period, we draw from various information sources. We discuss the methodology below.

We construct the quarterly measure of investor attention from the following four variables: the trading volume, the number of institutional owners, the number of analyst followings,

¹⁴Vuolteenaho (2002) finds that the correlation between Ncf and Ndr depends on firm size; it is weak and statistically insignificant among big firms. Our industry-level results can be compared against his results for very large firms.

and the number of analysts' forecast revisions.¹⁵ We choose these four variables because they have been associated with changes in investor attention and their data are available over a sufficiently long time period. Each variable is first calculated at the firm level over each quarter and then value-weighted averaged across firms in the industry to yield the industry-level variable.¹⁶

We use trading volume because it is easily measurable, and as argued by Hou, Xiong, and Peng (2009), it should be highly correlated with attention because investors cannot actively trade a stock if they do not pay attention to it. Similarly, Fisher, Martineau, and Sheng (2016) study time-varying media attention to macroeconomic fundamentals and find that it is linked to variations in aggregate trading volume. We use the number of institutional owners because it directly measures the number of sophisticated investors trading on the firm. As shown by Boone and White (2015), firms owned by more institutional investors elicits more public information production produced by the firms' management, media, and analysts. We use the number of analysts following the firm as it has been shown in an extensive literature that their recommendations and earnings forecasts significantly influence investor to trade and pay attention to the firm.¹⁷ Finally, we use the average number of analyst forecasts' revisions. This variable measures how frequently an analyst revises her estimate on the firm's quarterly earnings. Jacob, Lys, and Neale (1999) argues that the frequency of forecast revisions proxies for an analyst's effort in paying attention to the latest information on the firm that she covers. In all four variables described above, their higher value should proxy for increased investor attention to the firm.

We find that the industry-level time series of trading volume, number of institutional owners, number of analyst followings, and number of forecast revisions are persistent and exhibit a time trend. To circumvent this issue, we log transform each time series and adjust for its time-trend and seasonality effects using a linear regression model. We retain the re-

¹⁵Grullon, Kanatas, and Weston (2004) suggest using firms' advertising expense as a proxy for their investor attention. We exclude advertising expense because advertising expense data in COMPUSTAT is available annually, while our observation frequency is quarterly.

¹⁶We construct a quarterly attention measure using the Google search volume at the industry level from 2004–2014. The method that we use is similar to the one described in this section. We find the correlation between the attention measure constructed based on the four variables and the attention measure constructed using only Google search index is about 21%.

¹⁷For instance, Womack (1996) and Irvine (2004) documents significant abnormal trading volume following analysts' recommendations and forecasts.

gression residuals and normalize them using their time-series mean and standard deviation. These procedures produce four time series of *normalized* industry variables that are stationary and comparable in magnitude per each industry. Table 4 presents sample descriptives of the four normalized industry variables. Panel A reports their summary statistics from January 1985 to December 2014. There are a total of 1920 industry-quarter observations making up of sixteen industries over 120 quarters. As expected, the mean and the standard deviation of each time-series means are approximately 0 and 1, respectively. Panel B reports correlations of the four normalized industry variables. We first calculate correlations of the four time series for each industry, and then report their cross-industry averages. Clearly, we find that the four industry variables are positively correlated with their correlation estimates falling between 15 and 65%.

For each industry, we construct the measure of quarterly time-varying investor attention as follows. First, we combine information from the four normalized industry variables by calculating their cross-sectional equally-averaged values. The last row in Panel A reports time-series properties of the quarterly averaged industry variable. We find that by averaging across the four industry variables, the resulting time series is significantly less dispersed with the minimum value of -2.02 and the maximum value of 1.90 . Finally, we apply a first-order autoregressive model to the averaged time series and use the residuals as our quarterly attention measure Φ_t . This step ensures that the attention measure is stationary, and its level represents an *unexpected change* in investor attention over each quarter.¹⁸ Panel C of Table 4 presents time-series statistics of Φ_t across all industries. The mean and the standard deviation of the time series are 0.02 and 0.33 , respectively. Industry-by-industry summary statistics of the attention measure are reported in the Appendix Table A2. We find that the distributions of Φ_t are fairly comparable across industries.

4.4 Time-varying Volatilities and Correlations

We calculate time-varying volatilities of discount-rate returns using two approaches. The first is the exponentially weighted moving average (EWMA) model, where the current-quarter variance of the i th industry, $\sigma_{i,t}^2$, is a moving-average value of past squared discount-rate

¹⁸Although the four normalized industry variables that we use in the cross-sectional averaging are stationary, their combined time series can be persistent due to potential co-integration. We apply unit-root tests to verify that all time series of Φ_t are stationary.

returns with exponentially-decaying weights. We prefer the EWMA model for volatilities because it is simple and requires no further estimation. An alternative approach is a GARCH model for volatility modeling, but this is not feasible for our short time-series data with 120 quarterly observations. The EWMA variance is updated using a recursive relation:

$$\sigma_{i,t}^2 = \lambda \text{Ndr}_{i,t}^2 + (1 - \lambda) \sigma_{i,t-1}^2,$$

where $\text{Ndr}_{i,t}^2$ is the squared of discount-rate return from the current quarter, and λ is the weight. We set λ to 0.20, which we find provides the best one-period ahead volatility forecast.¹⁹ The second approach for calculating time-varying volatilities is the absolute value of discount-rate return, i.e., $|\text{Ndr}_{i,t}|$. This approach is very straightforward and provides an estimate of the realized volatility over each quarter.

Appendix Table A2 presents time-series statistics of discount-rate-news returns and their annualized volatility calculated using the EWMA model for each industry. We multiply quarterly volatilities by $\sqrt{4}$ to convert them to annualized terms. The standard deviations of Ndr are roughly around 0.5 across industries; they range between 0.38 and 0.67. Phone (Telephone and Communication) is the most volatile industry, while TV (Radio and Television Broadcasting) is the least volatile. The means of annualized volatilities across industries are roughly 10%, which is consistent with the standard deviations of quarterly discount-rate returns that we report.

Similar to volatilities, we use two approaches to calculate time-varying correlations of discount-rate returns between industry pairs. There are 120 unique industry pairs between the sixteen industries. The first approach we use is based on the EWMA model. We calculate time-varying covariances $\sigma_{ij,t}$ between industries i and j using a recursive relation:

$$\sigma_{ij,t} = \lambda (\text{Ndr}_{i,t} \text{Ndr}_{j,t}) + (1 - \lambda) \sigma_{ij,t-1},$$

where $\text{Ndr}_{i,t} \text{Ndr}_{j,t}$ is the product of discount-rate returns of the two industries over the current quarter.²⁰ Discount-rate return correlations are then calculated as $\rho_{ij,t} = \sigma_{ij,t} / (\sigma_{i,t} \sigma_{j,t})$, where $\sigma_{i,t}$ and $\sigma_{j,t}$ are EWMA volatilities that we calculated previously. The correlations are trimmed to be between -1 and 1 to remove erroneous outliers. Appendix Table A3

¹⁹Our conclusions are robust to a reasonably large range of λ .

²⁰The EWMA weight for covariance is set to 0.3, which provides the best one-period covariance forecast.

presents summary statistics of cross-industry correlations for the 120 industry pairs. For each correlation time series, we report the mean, the standard deviation, the minimum, and the maximum. Correlations of discount-rate returns are all positive. Importantly, we find that there are substantial fluctuations in the correlation time series.

The second approach that we use for time-varying correlations is the realized sign correlation, which is calculated as $\text{Ndr}_{i,t}\text{Ndr}_{j,t}/|\text{Ndr}_{i,t}\text{Ndr}_{j,t}|$. The benefit of this approach is that it is very simple. However, it measures correlations between discount-rate returns over each quarter as a discrete variable. That is, a negative correlation is represented by -1 , while a positive correlation is represented by $+1$. When the discount-rate return of either industry i or j , or both, is zero, the realized sign correlation is equal to 0.

4.5 Hypotheses and Empirical Findings

We outline four empirical hypotheses based on the model’s predictions that we test and report their findings.

The *first hypothesis* that we test is how unexpected changes in attention to one industry increase its discount-rate-news volatilities and those of other industries. As illustrated in Figure 1, the model predicts that an increase in the volatility of one industry is related to an increase in investor attention to that industry, as well as an increase in attention to other industries. To test this, we estimate the following panel regression on unique industry pairs:

$$\sigma_{i,t} = \alpha_{ij} + \beta_1\Phi_{i,t} + \beta_2\Phi_{j,t} + \sum_{j=0}^p\gamma'_jX_{t-j} + \varepsilon_{i,t}, \quad (20)$$

where $j < i$. The dependent variable $\sigma_{i,t}$ is the annualized discount-rate-news volatility of industry i in quarter t , and $\varepsilon_{i,t}$ is the regression residual. Our sample consists of sixteen large industries from 1985–2014. This results in 120 unique industry pairings each consisting of 120 quarterly observations. Our independent variables of interest are $\Phi_{i,t}$ and $\Phi_{j,t}$, representing unexpected changes in attention to industries i and j in quarter t , respectively. The results are reported in Columns (1) and (3) of Table 5. Industry-pair fixed effects, α_{ij} , as well as year fixed effects are included.

We include various contemporaneous and lagged control variables in the regression model shown in Equation (20), which are represented by $\sum_{j=0}^p\gamma'_jX_{t-j}$. These control variables are applied to all regression specifications in Tables 5 and 6. We include two lags of the attention

variables, Φ_i and Φ_j , and two lags of the dependent variable. We control for the volatility of the aggregate stock market by including the contemporaneous and two lags of *Market volatility*. The NYSE/NASDAQ value-weighted index is used as our proxy for the market portfolio. Consistent with how we calculate the discount-rate volatility for each industry, the market volatility is calculated using either the EWMA model or as the absolute value of the index return, i.e., realized volatility. We control for the performance of the aggregate stock market and Treasury bond markets by including the contemporaneous and two lags of the NYSE/NASDAQ value-weighted index return (*Market return*), and the bond portfolio return (*Bond return*). The bond portfolio that we use corresponds to the Fama bond portfolio with 7–12 month to maturity obtained from CRSP.²¹ Finally, we include two lags of the log of market capitalization for each industry to control for the difference in sizes between each industry pair. We adjust for heteroskedasticity in the coefficient estimates using White’s (1980) standard errors.

Columns (1) and (3) report regression results obtained using EWMA volatilities and realized volatilities, respectively. For brevity, only coefficient estimates on the contemporaneous variables are reported. In both columns, we find the coefficients on attention measures $\Phi_{i,t}$ and $\Phi_{j,t}$ are positive and statistically significant. This finding supports the model’s prediction that we show in Figure 1. Table 5 shows that the adjusted- R^2 is much higher (about 82%) when we use EWMA volatilities, while it is only about 18% when we use realized volatilities. This finding is expected because EWMA volatilities are moving averages of past discount-rate volatilities, and these lagged volatilities are included in our list of time-varying control variables. We find the relationship between attention measures on discount-rate volatilities are positive and statistically significant up to one-period lag (not reported), but with no significant relationship found at further lags. That is, the coefficients on $\Phi_{i,t-1}$ and $\Phi_{j,t-1}$ are positive and statistically significant, but with smaller magnitudes.

We find the coefficient estimates on control variables shown in Table 5 are all statistically significant. A positive coefficient on *Market volatility* indicates that the discount-rate volatility of each industry is increasing with the stock market volatility. We find that the estimate on *Market return* is negative. This indicates that volatility spillover, i.e., financial contagion,

²¹We verify that our conclusions are unaffected by either using the equally weighted NYSE/NASDAQ index, or the S&P 500 index as our proxy for the stock market portfolio. We also verify that our conclusions are robust to using Fama bond portfolios with different bond maturities, e.g., 1–6 month, 13–18 month, 19–24 month, and 25–30 month.

is more likely to occur during the market downturn. Finally, we find that the discount-rate volatility of each industry is increasing with *Bond return*. This finding is intuitive as the bond and stock markets often move in opposite directions. An increase in a bond portfolio return indicates a fall in bond yields, and hence a decrease in equilibrium discount rates. This effect generally generates co-movement in asset returns and asset volatilities.

The *second hypothesis* that we test is how unexpected changes in total attention to both industries increase their discount-rate-news volatilities. As illustrated in Figure 2, we expect a positive shock to aggregate attention of both industries to increase their discount-rate volatilities. We test this hypothesis by running the following panel-regression model:

$$\sigma_{i,t} = \alpha_{ij} + \beta (\Phi_{i,t} + \Phi_{j,t}) + \sum_{j=0}^p \gamma'_j X_{t-j} + \varepsilon_{i,t+1}, \quad (21)$$

where $j < i$. The above regression model is very similar to that shown in Equation (20). However, our independent variable of interest here is the total unexpected change in attention $\Phi_{i,t} + \Phi_{j,t}$ to industries i and j . The lagged time-varying control variables are adjusted accordingly to include two lags of the total attention measure. Control variables in Equation (21) are identical to those described earlier in Equation (20). Columns (2) and (4) of Table 5 report the results. We find that the coefficients on $\Phi_{i,t} + \Phi_{j,t}$ are positive and statistically significant in both columns, which is consistent with our model's prediction. The magnitude of the coefficients on $\Phi_{i,t} + \Phi_{j,t}$ reflects the average effect of the two attention measures separately on discount-rate volatilities. For instance, we can compare coefficients estimates in Column (4) to those in Column (3). Here, the estimate on $\Phi_{i,t} + \Phi_{j,t}$ is 0.008, which is roughly the average value of coefficient estimates 0.010 and 0.006 found on $\Phi_{i,t}$ and $\Phi_{j,t}$, respectively.²²

The *third hypothesis* that we test is how an unexpected change in investor attention to each of the two industries affects the correlation of their discount-rate returns. As illustrated in Figure 4, the model predicts that the correlation between two unrelated industries is increasing with changes in investor attention to each of the two industries. We test this prediction using the following panel-regression model on unique industry pairs:

²²We observe the same finding when using EWMA volatilities in Columns (1) and (2) of Table 5. However, the rounding of significant digits makes it difficult to see.

$$\rho_{ij} = \alpha_{ij} + \beta_1 \Phi_{i,t} + \beta_2 \Phi_{j,t} + \sum_{j=0}^p \gamma'_j X_{t-j} + \varepsilon_{i,t+1}, \quad (22)$$

where $i < j$. The dependent variable is the correlation between discount-rate returns of industry i and j in quarter t . Our independent variables of interest are unexpected changes in attention to the two industries in quarter t . Table 6 reports the results. Column (1) reports results obtained using EWMA correlations, while Column (3) reports results obtained using realized sign correlations. Fixed effects and control variables that we include in Equation (22) are identical to those described earlier in Equation (20).

Consistent with the model's prediction, Column (1) shows that the coefficients on $\Phi_{i,t}$ and $\Phi_{j,t}$ are positive and statistically significant. The adjusted R^2 is about 45.9%, which is relatively large reflecting the fact that EWMA correlations are filtered and smoothed using past information. Turning to the results in Column (3), we find that our estimates on the attention measures are positive, but the estimate on $\Phi_{j,t}$ is not statistically significant at the conventional level; its t-statistic is 1.64. We believe that the weaker result found in Column (3) is due to the noisiness of the realized sign correlation measure, which takes discrete values of +1, 0, or -1. In fact, the adjusted R^2 in Column (3) is only 6.4%, indicating a relatively low explanatory power on the realized sign correlation.

Finally, the *fourth hypothesis* that we test is how changes in aggregate investor attention to two industries affect the correlation of their discount-rate returns. As shown in Figure 5, the model predicts that an increase in total attention to the two industries should increase their return correlations. We estimate the following pair-wise panel-regression model:

$$\rho_{i,t+1} = \alpha_{ij} + \beta (\Phi_{i,t} + \Phi_{j,t}) + \sum_{j=1}^p \gamma'_j X_{t-j} + \varepsilon_{i,t+1},$$

where $i < j$. We report the results in Columns (2) and (4) of Table 6. Consistent with the model's prediction, we find a positive and significant coefficient on $\Phi_{i,t} + \Phi_{j,t}$ in both Columns (2) and (3). We conclude that these findings provide the evidence in support of our fourth hypothesis.

5 Conclusion

Recent empirical studies shed light on how investors focus on public information. These studies document that investor attention to news is fluctuates. This paper shows both theoretically and empirically that fluctuating attention implies return and volatility spillover effects among fundamentally unrelated sectors. Indeed, a negative shock affecting one sector propagates to other sectors through an increase in investor attention, which simultaneously raises each sector's volatility and cross-sector correlation. The model predicts that the shock propagates from one sector to another through discount rates. We empirically test the key mechanism of our model on sixteen large U.S. industries. We apply Campbell's (1991) framework to decompose quarterly industry returns from 1985–2015 into two components reflecting changes to the discount-rate news and changes to the cash-flow news. We then focus on the discount-rate-return component. Using a panel regression, we find that shocks propagate from one industry to another through the effect of time-varying attention on the discount-rate component of industry returns.

Our paper contributes to the existing literature by offering an explanation for financial contagion through the well-documented observation that investors pay fluctuating attention to news. Possible extensions of our work include incorporating more flexible investor preferences, as well as allowing for jumps in the dividend dynamics.

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Table 3: Industry Descriptions

This table summarizes the 16 industry portfolios that we study. For each industry, we report its time-series median values for the number of firms, the total market equity value (in millions \$), the total book equity value (in millions \$), and the total annual dividend payout (in millions\$). The sample consists of U.S. firms in the CRSP database from 1985 through 2014 with non-missing information in COMPUSTAT. See text in Section 4.2 for additional filters applied to the data. We classify firms into different industry portfolios following the Fama-French 38-industry classifications. We drop the following industries from our analysis: *Government, Money, Water, Steam, Garbage, and Other*. Additionally, we only keep large industries for our empirical analysis. We exclude industries that contribute less than 1% to the total market share in at least one of the following dimensions: market equity value, book equity value, and annual dividend payout. The remaining 16 industries that we keep in the sample make up for about 90% of the total market capitalization.

Industry code	Descriptions	Sample Medians: 1985–2014			
		Number of firms	Market equity value (mils)	Book equity value (mils)	Annual dividend payout (mils)
Cars	Transportation Equipment	58	250,651	124,179	51.62
Chems	Chemicals and Allied Products	195	1,210,968	196,991	92.66
Elctr	Electrical and Electronic Equipment	167	434,758	143,352	32.58
Food	Food and Kindred Products	52	341,791	79,890	30.74
Instr	Instruments and Related Products	138	210,579	57,756	27.77
Machn	Machinery, Except Electrical	148	260,256	102,109	38.30
Oil	Oil and Gas Extraction	77	105,466	47,110	18.02
Phone	Telephone and Communication	38	346,063	152,728	35.68
Print	Printing and Publishing	44	591,21	29,531	28.39
Ptrlm	Petroleum and Coal Products	21	395,290	169,605	28.69
Rtail	Retail stores	103	180,236	509,38	21.65
Srvc	Services	369	983,598	228,055	68.42
TV	Radio and Television Broadcasting	38	156,044	84,250	9.10
Trans	Transportation	75	127,192	84,321	26.08
Utils	Electric, Gas, and Water Supply	141	331,499	314,136	565.43
Whlsl	Wholesale	84	101,106	45,965	26.67

Table 4: Measuring Investor Attention: Sample Descriptives

This table presents summary statistics of quarterly industry-level variables associated with investor attention on the 16 industries (see Table 3) from 1985 to 2014. Panel A reports the four normalized industry variables that we use to calculate the attention measure. Panel B reports the average cross-correlation matrix of the four normalized industry variables. Panel C reports the aggregate investor attention measure Φ calculated from the four industry variables. The industry variables that we use are the total trading volume, the number of institutional owners, the number of analyst followings, and the number of analyst forecast revisions over each quarter; they are calculated as value-weighted averages across firms in the industry. We de-trend each industry variable using a log-linear model adjusting for the time-trend and seasonality effects. After, we normalize each de-trended industry variable using its time-series mean and standard deviation. For each industry, we equally average the normalized industry variables to calculate the aggregate measure of investor attention. We apply an AR(1) model to reduce the time-series persistence in the aggregate attention measure and use the model’s residual as our proxy for time-varying investor attention Φ . In Panels A and C, the summary statistics that we report are mean, standard deviation, minimum, 25th percentile, median, 75th percentile, and maximum values.

Panel A. Normalized Industry Variables: Summary Statistics

	Nobs	Mean	Stdev	Minimum	P25	P50	P75	Maximum
Trading volume	1920	0.00	0.99	-2.89	-0.70	-0.01	0.70	3.34
Num institutional owners	1920	0.00	0.99	-3.34	-0.67	0.03	0.70	3.99
Num analyst followings	1920	0.00	0.98	-5.71	-0.62	0.06	0.69	2.46
Num forecast revisions	1920	0.00	0.98	-5.04	-0.64	0.02	0.64	3.17
Average value	1920	0.02	0.65	-2.02	-0.43	0.02	0.52	1.90

Panel B. Normalize Industry Variables: Pearson Correlations

	Trading Volume	Num institutional owners	Num analyst followings	Num forecast revisions
Trading volume	1			
Num institutional owners	0.401	1		
Num analyst followings	0.155	0.371	1	
Num forecast revisions	0.152	0.258	0.654	1

Panel C. Attention Measures: Summary Statistics

	Nobs	Mean	Stdev	Minimum	P25	P50	P75	Maximum
Attention measure Φ	1920	0.02	0.33	-2.20	-0.17	0.02	0.21	1.42

Table 5: Time-varying Attention and Cross-industry Volatility Spillover

This table reports quarterly panel-regression results for the effect of time-varying attention on discount-rate news volatility. Each panel represents a unique industry-pair combinations from the 16 industry portfolios. The sample period is from 1985 through 2014. The dependent variable in each column is the quarterly discount-rate news volatility $\sigma_{i,t}$ at time t for industry i . The general regression model is:

$$\sigma_{i,t} = \alpha_{ij} + \beta_1 \Phi_{i,t} + \beta_2 \Phi_{j,t} + \sum_{j=0}^p \gamma'_j X_{t-j} + \varepsilon_{i,t},$$

for $i = 1, \dots, 16$, and $j < i$. We define $\Phi_{i,t}$ and $\Phi_{j,t}$ as unexpected changes in attention to industry i and industry j , respectively, at time t . Panel A reports results where the discount-rate news volatility is calculated using the exponentially weighted moving average (EWMA) model. In Panel B, the volatility is calculated as the absolute value of the discount-rate news component, i.e., realized volatility. Columns (1) and (3) report results examining the impact of shocks to industry each industry i and j separately. Columns (2) and (4) examine the impact of unexpected change in total attention to the two industries, which is accomplished by restricting $\beta_1 = \beta_2$ in the regression model. We include a number of contemporaneous and lagged time-series variables as controls; they are represented by $\sum_{j=0}^p \gamma'_j X_{t-j}$. See text for complete details. Two lags of the attention variables and two lags of the dependent variable are included. The NYSE/NASDAQ value-weighted index is used as the proxy for market portfolio when calculating the market return and the market volatility. Bond return corresponds to the Fama bond portfolio return with 7-12 month maturity obtained from CRSP. For brevity, only estimates on contemporaneous independent variables are reported. Industry pair-, year-fixed effects are included. *No. of cross sections* reports the number of industry pairs in the panel. *Time-series length* reports the number of observations in each cross section. T-statistic adjusted for Heteroscedasticity (White, 1980) is shown in parentheses below each estimate. ***, ** and * indicate statistical significance at the 1, 5 and 10 percent levels, respectively.

	Discount-rate Volatility $\sigma_{i,t}$			
	Panel A. EWMA Volatility		Panel B. Realized Volatility	
	(1)	(2)	(3)	(4)
$\Phi_{i,t}$	0.003*** (3.77)		0.010*** (4.20)	
$\Phi_{j,t}$	0.002*** (2.80)		0.006*** (3.40)	
$\Phi_{i,t} + \Phi_{j,t}$		0.003*** (4.63)		0.008*** (5.29)
Market volatility	0.240*** (15.72)	0.239*** (16.21)	0.220*** (15.93)	0.219*** (15.90)
Market return	-0.039*** (-14.02)	-0.039*** (-14.40)	-0.243*** (-16.27)	-0.242*** (-16.17)
Bond return	0.841*** (16.30)	0.839*** (16.35)	2.474*** (12.01)	2.469*** (11.98)
Industry pair-fixed effects	✓	✓	✓	✓
Year-fixed effects	✓	✓	✓	✓
Control vars	✓	✓	✓	✓
Adjusted R^2	82.1%	82.2%	18.1%	18.0%
No. of cross sections	120	120	120	120
Time-series length	120	120	120	120

Table 6: Time-varying Attention and Cross-industry Return Correlations

This table reports quarterly panel-regression results for the effect of time-varying attention on the correlation of discount-rate returns of two industries. Each panel represents a unique industry-pair combinations from the 16 industry portfolios. The sample period is from 1985 through 2014. The dependent variable in each column is the correlation between discount-rate-news returns of industry i and j , i.e., $\rho_{ij,t}$, at time t . The general regression model is:

$$\rho_{ij,t} = \alpha_{ij} + \beta_1 \Phi_{i,t} + \beta_2 \Phi_{j,t} + \sum_{j=0}^p \gamma'_j X_{t-j} + \varepsilon_{i,t},$$

for $i = 1, \dots, 16$, and $j < i$. In the above regression, $\Phi_{i,t}$ and $\Phi_{j,t}$ are unexpected changes in attention to industry i and industry j , respectively, at time t . Panel A reports results where the cross-industry discount-rate return correlation is calculated using the exponentially weighted moving average (EWMA) model. In Panel B, the discount-rate return correlation is the realized signed correlation calculated as $Ndr_{i,t+1}Ndr_{j,t+1}/|Ndr_{i,t+1}Ndr_{j,t+1}|$; it takes the value of +1, 0, or -1. Columns (1) and (3) report results examining the impact of unexpected change in attention to each industry separately. Columns (2) and (4) examines the impact of unexpected change in total attention to the two industries, which is accomplished by restricting $\beta_1 = \beta_2$ in the above regression model. We include a number of contemporaneous and lagged time-series variables as controls; they are represented by $\sum_{j=0}^p \gamma'_j X_{t-j}$. See text for complete details. Two lags of the attention variables and two lags of the dependent variable are included. The NYSE/NASDAQ value-weighted index is used as the proxy for market portfolio when calculating the market return and the market volatility. Bond return corresponds to the Fama bond portfolio return with 7-12 month maturity obtained from CRSP. For brevity, only estimates on contemporaneous independent variables are reported. Industry pair-, year-fixed effects are included. *No. of cross sections* reports the number of industry pairs in the panel. *Time-series length* reports the number of observations in each cross section. T-statistic adjusted for Heteroscedasticity (White, 1980) is shown in parentheses below each estimate. ***, ** and * indicate statistical significance at the 1, 5 and 10 percent levels, respectively.

	Discount-rate Return Correlations $\rho_{ij,t}$			
	Panel A. EWMA correlation		Panel B. Realized Correlation	
	(1)	(2)	(3)	(4)
$\Phi_{i,t}$	0.028*** (2.59)		0.056** (2.13)	
$\Phi_{j,t}$	0.024*** (2.48)		0.040 (1.64)	
$\Phi_{i,t} + \Phi_{j,t}$		0.026*** (3.63)		0.048*** (3.16)
Market volatility	4.717*** (17.73)	3.751*** (16.21)	0.167 (1.38)	0.177 (1.45)
Market return	-0.174*** (-4.11)	-0.170*** (-4.02)	-0.049 (-0.30)	-0.060 (-0.37)
Bond return	22.229*** (14.78)	22.183*** (14.91)	32.737*** (8.16)	32.185*** (7.85)
Industry pair-fixed effect	✓	✓	✓	✓
Year-fixed effect	✓	✓	✓	✓
Lagged control vars	✓	✓	✓	✓
Adjusted R^2	45.9%	45.9%	6.4%	6.4%
No. of cross sections	120	120	120	120
Time-series length	120	120	120	120

A Appendix

A.1 Long-term Means and Variances

Let us consider the 4-dimensional vector $Y = \left(\hat{f}_1 \ \hat{f}_2 \ \pi_1 \ \pi_2 \right)^\top$. The dynamics of Y in vector notation is

$$dY_t = (A - BY_t) dt + C dW_t$$

where

$$A = \begin{pmatrix} \lambda \bar{f} & \lambda \bar{f} & 0 & 0 \end{pmatrix}^\top$$

$$B = \begin{pmatrix} \lambda & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 \\ 0 & 0 & \omega & 0 \\ 0 & 0 & 0 & \omega \end{pmatrix}$$

$$C = \begin{pmatrix} \frac{\gamma_{1t}}{\sigma_\delta} & 0 & \sigma_f \Phi_{1t} & 0 \\ 0 & \frac{\gamma_{2t}}{\sigma_\delta} & 0 & \sigma_f \Phi_{2t} \\ \sigma_\delta & 0 & 0 & 0 \\ 0 & \sigma_\delta & 0 & 0 \end{pmatrix}.$$

Applying Itô's lemma on $F \equiv e^{Bt}Y$ yields

$$dF = \begin{pmatrix} \frac{e^{\lambda t}(\lambda \bar{f} \sigma_\delta dt + \gamma_{1t} dW_{1t} + \sigma_f \sigma_\delta \Phi_{1t} dW_{3t})}{\sigma_\delta} \\ \frac{e^{t\lambda}(\lambda \bar{f} \sigma_\delta dt + \gamma_{2t} dW_{2t} + \sigma_f \sigma_\delta \Phi_{2t} dW_{4t})}{\sigma_\delta} \\ e^{\omega t} \sigma_\delta dW_{1t} \\ e^{\omega t} \sigma_\delta dW_{2t} \end{pmatrix}.$$

Integrating from 0 to t and taking expectation yields

$$\begin{pmatrix} e^{\lambda t} \mathbb{E} \left(\widehat{f}_{1t} \right) - \widehat{f}_{10} \\ e^{\lambda t} \mathbb{E} \left(\widehat{f}_{2t} \right) - \widehat{f}_{20} \\ e^{\omega t} \mathbb{E} \left(\pi_{1t} \right) - \pi_{10} \\ e^{\omega t} \mathbb{E} \left(\pi_{2t} \right) - \pi_{20} \end{pmatrix} = \begin{pmatrix} (e^{\lambda t} - 1) \bar{f} \\ (e^{\lambda t} - 1) \bar{f} \\ 0 \\ 0 \end{pmatrix}.$$

Therefore, the long-term means satisfy

$$\lim_{t \rightarrow +\infty} \begin{pmatrix} \mathbb{E} \left(\widehat{f}_{1t} \right) \\ \mathbb{E} \left(\widehat{f}_{2t} \right) \\ \mathbb{E} \left(\pi_{1t} \right) \\ \mathbb{E} \left(\pi_{2t} \right) \end{pmatrix} = \begin{pmatrix} \bar{f} \\ \bar{f} \\ 0 \\ 0 \end{pmatrix}.$$

Similar computations yield the long-term variances

$$\lim_{t \rightarrow +\infty} \begin{pmatrix} \text{Var} \left(\widehat{f}_{1t} \right) \\ \text{Var} \left(\widehat{f}_{2t} \right) \\ \text{Var} \left(\pi_{1t} \right) \\ \text{Var} \left(\pi_{2t} \right) \end{pmatrix} = \begin{pmatrix} \frac{\sigma_f^2}{2\lambda} \\ \frac{\sigma_f^2}{2\lambda} \\ \frac{\sigma_\delta^2}{2\omega} \\ \frac{\sigma_\delta^2}{2\omega} \end{pmatrix}.$$

A.2 Long-term Uncertainty

The dynamics of the uncertainty γ_i conditional on $\pi_i = 0$ is

$$d\gamma_{it} = \left(-\frac{\gamma_{it}^2}{\sigma_\delta^2} - 2\lambda\gamma_{it} + \sigma_f^2 (1 - \Psi^2) \right) dt.$$

The dynamics of the uncertainty at the “steady-state” is

$$\frac{d\gamma_{ss}}{dt} = 0.$$

Solving yields

$$\gamma_{ss} = \sigma_\delta \sqrt{\sigma_f^2 (1 - \Psi^2) + \lambda^2 \sigma_\delta^2} - \lambda \sigma_\delta^2.$$

A.3 Proof of Proposition 2

The price of the single-dividend paying securities S_1^T is defined by

$$P_{1t}^T = \mathbb{E}_t \left(\frac{\xi_T}{\xi_t} \delta_{1T} \right). \quad (23)$$

Substituting Equation (13) in Equation (23) yields

$$\begin{aligned} P_{1t}^T &= e^{-\Delta(T-t)} (\delta_{1t} + \delta_{2t})^\alpha \mathbb{E}_t \left(\delta_{1T} \left(\frac{1}{\delta_{1T} + \delta_{2T}} \right)^\alpha \right) \\ &= e^{-\Delta(T-t)} (\delta_{1t} + \delta_{2t})^\alpha \mathbb{E}_t \left(\delta_{1T}^{1-\alpha} \left(\frac{\delta_{1T}}{\delta_{1T} + \delta_{2T}} \right)^\alpha \right) \\ &= e^{-\Delta(T-t)} e^{\alpha(\zeta_{1t} - Q_t)} \mathbb{E}_t \left(e^{(1-\alpha)\zeta_{1T} + \alpha Q_T} \right) \end{aligned}$$

where $\zeta_i \equiv \log \delta_i$ is the log-dividend and $Q = \log \frac{\delta_1}{\delta_1 + \delta_2}$ the log-dividend share. Similarly, the price of the single-dividend paying security P_2^T satisfies

$$\begin{aligned} P_{2t}^T &= \mathbb{E}_t \left(\frac{\xi_T}{\xi_t} \delta_{2T} \right) \\ &= e^{-\Delta(T-t)} (\delta_{1t} + \delta_{2t})^\alpha \mathbb{E}_t \left(\delta_{2T} \left(\frac{1}{\delta_{1T} + \delta_{2T}} \right)^\alpha \right) \\ &= e^{-\Delta(T-t)} (\delta_{1t} + \delta_{2t})^\alpha \mathbb{E}_t \left(\delta_{1T}^{-\alpha} \delta_{2T} \left(\frac{\delta_{1T}}{\delta_{1T} + \delta_{2T}} \right)^\alpha \right) \\ &= e^{-\Delta(T-t)} e^{\alpha(\zeta_{1t} - Q_t)} \mathbb{E}_t \left(e^{-\alpha\zeta_{1T}} (e^{-Q_T} - 1) e^{\zeta_{1T}} e^{\alpha Q_T} \right) \\ &= e^{-\Delta(T-t)} e^{\alpha(\zeta_{1t} - Q_t)} \mathbb{E}_t \left(e^{(1-\alpha)\zeta_{1T} + (\alpha-1)Q_T} - e^{(1-\alpha)\zeta_{1T} + \alpha Q_T} \right) \\ &= e^{-\Delta(T-t)} e^{\alpha(\zeta_{1t} - Q_t)} \mathbb{E}_t \left(e^{(1-\alpha)\zeta_{1T} + (\alpha-1)Q_T} \right) - P_{1t}^T \end{aligned}$$

□

A.4 Approximation of the Transforms

The idea consists in approximating the dynamics of the state-vector, and then computing the transforms appearing in Equations (14) and (15) by applying the theory on affine processes (e.g., Duffie, Pan, and Singleton, 2000). An accurate approximation of the dynamics includes second-order terms. Consequently, before approximating we augment the state-vector by these second order terms (Cheng and Scaillet, 2007). Then, we compute the drift and variance-covariance matrix of the augmented state-vector. Finally, we approximate the augmented drift and variance-covariance matrix by performing a Taylor expansion.

Because the dividend share belongs to the interval $(0, 1)$, the log-dividend share Q belongs to $(-\infty, 0)$. Therefore, the dynamics of Q cannot be accurately approximated by performing Taylor expansions. To overcome this problem we perform the following change of variable²³

$$\tilde{Q} \equiv \log \left(1 + \frac{\delta_1}{\delta_1 + \delta_2} \right) = \log (1 + e^Q)$$

where $\tilde{Q} \in]0, \log(2)[$. The dynamics of \tilde{Q} are

$$\begin{aligned} d\tilde{Q}_t = & e^{-2\tilde{Q}_t} \left(e^{\tilde{Q}_t} - 2 \right) \left(e^{\tilde{Q}_t} - 1 \right) \left(\left(e^{2\tilde{Q}_t} - 2 \right) \sigma_\delta^2 + e^{\tilde{Q}_t} \left(\hat{f}_{2t} - \hat{f}_{1t} \right) \right) dt \\ & + \left(\sigma_\delta \left(3 - 2e^{-\tilde{Q}_t} - e^{\tilde{Q}_t} \right) \quad \sigma_\delta \left(-3 + 2e^{-\tilde{Q}_t} + e^{\tilde{Q}_t} \right) \quad 0 \quad 0 \right) dW_t. \end{aligned}$$

Proposition 4. *Under the change of variable stated above and the assumption that the coefficient of relative risk aversion α is an integer, the single-dividend paying securities appearing in Equations (14) and (15) satisfy*

$$P_{1t}^T = e^{-\Delta(T-t)} \left(\frac{e^{\zeta_{1t}}}{e^{\tilde{Q}_t} - 1} \right)^\alpha \sum_{j=0}^{\alpha} \binom{\alpha}{j} (-1)^{\alpha-j} \mathbb{E}_t \left(e^{(1-\alpha)\zeta_{1T} + j\tilde{Q}_T} \right) \quad (24)$$

$$P_{2t}^T = e^{-\Delta(T-t)} \left(\frac{e^{\zeta_{1t}}}{e^{\tilde{Q}_t} - 1} \right)^\alpha \sum_{j=0}^{\alpha-1} \binom{\alpha-1}{j} (-1)^{\alpha-1-j} \mathbb{E}_t \left(e^{(1-\alpha)\zeta_{1T} + j\tilde{Q}_T} \right) - S_{1t}^T. \quad (25)$$

²³Note that this change of variable could be omitted. If it was, then the approximation of the transforms would be slightly less accurate.

Proof. The single-dividend paying securities price P_1^T satisfies

$$\begin{aligned}
P_{1t}^T &= e^{-\Delta(T-t)} e^{\alpha(\zeta_{1t}-Q_t)} \mathbb{E}_t \left(e^{(1-\alpha)\zeta_{1T} + \alpha Q_T} \right) \\
&= e^{-\Delta(T-t)} \left(\frac{e^{\zeta_{1t}}}{e^{\tilde{Q}_t} - 1} \right)^\alpha \mathbb{E}_t \left(e^{(1-\alpha)\zeta_{1T}} \left(e^{\tilde{Q}_T} - 1 \right)^\alpha \right) \\
&= e^{-\Delta(T-t)} \left(\frac{e^{\zeta_{1t}}}{e^{\tilde{Q}_t} - 1} \right)^\alpha \mathbb{E}_t \left(e^{(1-\alpha)\zeta_{1T}} \sum_{j=0}^{\alpha} \binom{\alpha}{j} (-1)^{\alpha-j} e^{j\tilde{Q}_T} \right) \\
&= e^{-\Delta(T-t)} \left(\frac{e^{\zeta_{1t}}}{e^{\tilde{Q}_t} - 1} \right)^\alpha \sum_{j=0}^{\alpha} \binom{\alpha}{j} (-1)^{\alpha-j} \mathbb{E}_t \left(e^{(1-\alpha)\zeta_{1T} + j\tilde{Q}_T} \right). \tag{26}
\end{aligned}$$

Similarly, P_2^T satisfies

$$\begin{aligned}
P_{2t}^T &= e^{-\Delta(T-t)} e^{\alpha(\zeta_{1t}-Q_t)} \mathbb{E}_t \left(e^{(1-\alpha)\zeta_{1T} + (\alpha-1)Q_T} \right) - P_{1t}^T \\
&= e^{-\Delta(T-t)} \left(\frac{e^{\zeta_{1t}}}{e^{\tilde{Q}_t} - 1} \right)^\alpha \mathbb{E}_t \left(e^{(1-\alpha)\zeta_{1T}} \left(e^{\tilde{Q}_T} - 1 \right)^{\alpha-1} \right) - P_{1t}^T \\
&= e^{-\Delta(T-t)} \left(\frac{e^{\zeta_{1t}}}{e^{\tilde{Q}_t} - 1} \right)^\alpha \sum_{j=0}^{\alpha-1} \binom{\alpha-1}{j} (-1)^{\alpha-1-j} \mathbb{E}_t \left(e^{(1-\alpha)\zeta_{1T} + j\tilde{Q}_T} \right) - P_{1t}^T. \tag{27}
\end{aligned}$$

□

We now proceed with the approximation method that allows us to compute the transforms appearing in Equations (26) and (27). Let the state-vector x be defined by

$$\begin{aligned}
x &\equiv (x_i)_{i=1}^8 \\
&= \left(\zeta_1 \quad \tilde{Q} \quad \hat{f}_1 \quad \hat{f}_2 \quad \pi_1 \quad \pi_2 \quad \gamma_1 \quad \gamma_2 \right)^\top,
\end{aligned}$$

with the dynamic

$$dx_t \equiv \mu(x_t) + \sigma(x_t) dW_t. \tag{28}$$

In Equation (28), the state-vector x has a non-affine dynamic with a non-affine drift $\mu(x)$ and a non-affine variance-covariance matrix $\sigma(x)\sigma(x)^\top$. Given the structure of $\mu(x)$ and

$\sigma(x)\sigma(x)^\top$, the augmented state-vector X is chosen to be

$$\begin{aligned} X &\equiv (X_i)_{i=1}^{17} \\ &= \left(\zeta_1 \quad \tilde{Q} \quad \hat{f}_1 \quad \hat{f}_2 \quad \pi_1 \quad \pi_2 \quad \gamma_1 \quad \gamma_2 \quad \dots \right. \\ &\quad \left. \dots \quad \tilde{Q}^2 \quad \tilde{Q}\hat{f}_1 \quad \tilde{Q}\hat{f}_2 \quad \tilde{Q}\gamma_1 \quad \tilde{Q}\gamma_2 \quad \pi_1^2 \quad \pi_2^2 \quad \gamma_1^2 \quad \gamma_2^2 \right)^\top \\ dX_t &\equiv \mu(X_t) + \sigma(X_t)dW_t. \end{aligned}$$

Approximated expressions for the augmented drift $\mu(X)$ and the variance-covariance matrix $\Sigma(X) \equiv \sigma(X)\sigma(X)^\top$ are derived using a Taylor expansion around the reference vector x_0 . We discuss the procedure below in Definition 3.

Definition 3. *The reference vector x_0 satisfies*

$$\begin{aligned} x_{02} &= \log(1.5) & x_{03} &= x_{04} = \bar{f} \\ x_{05} &= x_{06} = 0 & x_{07} &= x_{08} \equiv \gamma_{ss}. \end{aligned}$$

Note that x_{01} is not defined because ζ_1 neither shows up in the drift $\mu(X)$ nor in the variance-covariance matrix $\Sigma(X)$. \bar{f} is the long-term mean of \hat{f}_1 and \hat{f}_2 , 0 is the long-term mean of π_1 and π_2 , and $\gamma_{ss} = \sigma_\delta \sqrt{\sigma_f^2(1 - \Psi^2) + \lambda^2\sigma_\delta^2 - \lambda\sigma_\delta^2}$ is the uncertainty conditional on $\pi_i = 0$. The derivations of the long-term means are provided in Appendix A.1. The long-term uncertainty γ_{ss} is computed in Appendix A.2.

The drift $\mu(X)$ and the variance-covariance matrix $\Sigma(X)$ are expanded around the reference vector x_0 defined in 3. More precisely, $\mu(X)$ and $\Sigma(X)$ are written

$$\begin{aligned} \mu(X) &\approx K_0 + K_1 X \\ \Sigma(X) &\approx H_0 + \sum_{i=1}^{17} H_i X_i, \end{aligned}$$

where K_1 and H_i , $i = 0, \dots, 17$ are 17-dimensional squared matrices and K_0 a 17-dimensional vector. K_0 , K_1 , and H_i , $i = 0, \dots, 17$ are available upon request.

Using the approximation, the theory on affine processes applies. Following Duffie, Pan, and Singleton (2000), the transforms defined in Equations (24) and (25) are approximated

by

$$\mathbb{E}_t \left(e^{\epsilon\zeta_{1T} + \chi\tilde{Q}_T} \right) \approx e^{\bar{\alpha}(T-t) + \sum_{i=1}^{17} \bar{\beta}_i(T-t)X_i}, \quad (29)$$

where the functions $\bar{\alpha}(\cdot)$ and $\bar{\beta}_i(\cdot)$, $i = 1, \dots, 17$, solve a set of 18 Riccati equations subject to $\bar{\alpha}(0) = 0$, $\bar{\beta}_1(0) = \epsilon$, $\bar{\beta}_2(0) = \chi$, and $\bar{\beta}_i(0) = 0$, $i = 3, \dots, 17$.

The system of Riccati equations is

$$\begin{aligned} \bar{\beta}'(\tau) &= K_1^\top \bar{\beta}(\tau) + \frac{1}{2} \bar{\beta}(\tau)^\top H_+ \bar{\beta}(\tau) \\ \bar{\alpha}'(\tau) &= K_0^\top \bar{\beta}(\tau) + \frac{1}{2} \bar{\beta}(\tau)^\top H_0 \bar{\beta}(\tau), \end{aligned}$$

where $\tau = T - t$.²⁴ The set of Riccati equations is solved numerically. Then, substituting Equation (29) in Equations (24) and (25) determines the single-dividend paying securities prices P_1^T and P_2^T . As described in Equation (16), stock prices are obtained by numerically integrating over the single-dividend paying securities.

A.5 Return-decomposition using VAR

We decompose quarterly returns of sixteen industries using a VAR model from January 1970 to December 2014.²⁵ We assume that the vector of state variables $\mathbf{z}_{i,t}$ describing an industry i at time t are generated by the following first-order VAR model:

$$\mathbf{z}_{i,t} = \mathbf{A} + \Gamma \mathbf{z}_{i,t-1} + u_{i,t} \quad \Sigma = E \left[u_{i,t} u_{i,t}' \right], \quad (30)$$

²⁴Note that the matrix H_+ is 3-dimensional. It consists in the concatenation of the matrices H_i , $i = 1, \dots, 17$. This notation is used to avoid writing an equation for each $\bar{\beta}$.

²⁵Our main empirical tests begin in 1985 due to the data availability for calculating the attention measure. However, data for the return decomposition is available earlier. We use data starting in 1970 to improve the precision of VAR estimates.

where

$$\mathbf{z}_{i,t} = \begin{bmatrix} r_{i,t} \\ r_{i,t-1} \\ dp_{i,t} \\ roe_{i,t} \\ ty_t \\ rm_t \end{bmatrix} \quad \mathbf{A} = \begin{bmatrix} a_1 \\ 0 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \end{bmatrix} \quad \mathbf{\Gamma} = \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} & b_{15} & b_{16} \\ 1 & 0 & 0 & 0 & 0 & 0 \\ b_{31} & b_{32} & b_{33} & b_{34} & b_{35} & b_{36} \\ b_{41} & b_{42} & b_{43} & b_{44} & b_{45} & b_{46} \\ 0 & 0 & 0 & 0 & b_{55} & b_{56} \\ 0 & 0 & 0 & 0 & b_{56} & b_{66} \end{bmatrix}, \quad \text{and} \quad \mathbf{u}_{i,t} = \begin{bmatrix} u_{i1,t} \\ 0 \\ u_{i3,t} \\ u_{i4,t} \\ u_{i5,t} \\ u_{i6,t} \end{bmatrix}.$$

The error term $u_{i,t}$ is assumed to have a covariance matrix Σ . The first four elements in $\mathbf{z}_{i,t}$ are industry-specific variables, where $r_{i,t}$, $dp_{i,t}$, and $roe_{i,t}$ denote the log industry excess return, the log dividend-price ratio, and the log return on equity, respectively. The lagged industry excess return $r_{i,t-1}$ is stacked as a second element in $\mathbf{z}_{i,t}$ so that the VAR system in a first-order form (see [Campbell \(1991\)](#)). The remaining two elements in $\mathbf{z}_{i,t}$ are macro variables, where ty_t is the term yield spread, and rm_t is the log market excess return, respectively.

We assume that the VAR coefficients \mathbf{A} and $\mathbf{\Gamma}$ are constant, both over time and across industries. The elements in \mathbf{A} represent time-series means of the state variables. The elements in the lower-left-hand corner of $\mathbf{\Gamma}$ are set equal to 0. This ensures the uniqueness in the dynamic of macro variables as they cannot be generated by a data generating process that differs from one industry to the next; see also Table 5 in [Vuolteenaho \(2002\)](#) for a similar setup. We estimate the VAR model in Equation (30) using the weighted least squares (WLS) approach. The number of industries in each cross section are used as weights. Panel A in Appendix Table A1 reports estimation results for the VAR model. For brevity, only coefficient estimates in $\mathbf{\Gamma}$ are reported.

Estimates from the VAR model can be conveniently applied to implement the return decomposition. Define $e1 \equiv [1 \ 0 \ 0 \ 0 \ 0 \ 0]'$ and

$$\lambda' = e1' \rho \mathbf{\Gamma} (\mathbf{I} - \rho \mathbf{\Gamma})^{-1},$$

where we recall that ρ is the discount factor in Equation (17). We follow [Vuolteenaho \(2002\)](#) and set $\rho = 0.967$ per annum, or $0.967^{1/4}$ for our quarterly frequency. [Campbell \(1991\)](#) shows that the discount-rate-news return can be expressed as $\lambda' u_{i,t}$ and the cash-flow-news

return as $(e1' + \lambda') u_{i,t}$. Components in the variance decomposition of unexpected industry return as shown in Equation (17) can be written as follows:

$$\begin{aligned}\text{var}(\text{Ndr}) &= \lambda' \Sigma \lambda \\ \text{var}(\text{Ncf}) &= (e1' + \lambda') \Sigma (e1 + \lambda) \\ \text{cov}(\text{Ndr}, \text{Ncf}) &= \lambda' \Sigma (e1 + \lambda).\end{aligned}$$

Table A1: VAR Estimates and Variance Decomposition of Industry Returns

This table reports the parameter estimates for the first-order VAR (Panel A) and presents the variance decomposition of unexpected industry returns in a spirit of Campbell (1991) (Panel B). We estimate the following VAR system for the sixteen industries:

$$\mathbf{z}_{i,t} = A + \Gamma \mathbf{z}_{i,t} + u_{i,t}, \quad \Sigma = E [u_{i,t} u'_{i,t}],$$

where $\mathbf{z}_{i,t} = [r_{i,t} \ r_{i,t-1} \ dp_{i,t} \ roe_{i,t} \ ty_t \ rm_t]'$. The model includes industry-specific variables and aggregate variables. The industry-specific variables included are excess log return, r ; log dividend-to-price ratio, dp ; and log return on equity, roe . We include two lags of industry excess log return in the VAR system. The aggregate variables are term yield spread, ty , defined as the difference between 10-year and 3-month treasury rates; and excess log market return, rm . We estimate the VAR system using quarterly data from 1970 to 2014. Panel A reports the estimates and robust Jackknife t-statistics of the parameters in Γ . ***, ** and * indicate statistical significance at the 1, 5 and 10 percent levels, respectively. Adjusted R^2 for each dependent variable is reported in the far-right column. Panel B reports a variance decomposition of quarterly industry excess returns using the VAR estimates. For each industry, we decompose the unexpected industry return, r_e , into a discount-rate news component, Ndr, and a cash-flow news component, Ncf. The variance of the total unexpected return can be split up into three terms $\text{var}(\text{Ndr})$, $\text{var}(\text{Ncf})$, $\text{cov}(\text{Ndr}, \text{Ncf})$; see Equation (19) for a reference. The far-right column in Panel B reports the correlation between Ndr and Ncf. For each variable, we report the cross-sectional mean and its standard error, as well as the minimum and maximum values across the sixteen industries.

Panel A. Coefficient Estimates of the First-order VAR model

$\mathbf{z}_{i,t}$	$\mathbf{z}_{i,t-1}$						Adjusted
	$r_{i,t-1}$	$r_{i,t-2}$	$dp_{i,t-1}$	$roe_{i,t-1}$	ty_{t-1}	rm_{t-1}	R^2
$r_{i,t}$	-0.0359* (-1.80)	-0.0581*** (-4.44)	0.6410*** (6.35)	0.0048* (1.85)	0.8263*** (6.48)	0.0892*** (3.23)	2.2%
$r_{i,t-1}$	1	0	0	0	0	0	
$dp_{i,t}$	0.0023** (2.52)	0.0008 (1.43)	0.9409*** (17.47)	-0.0003** (-2.37)	-0.0255*** (-4.47)	-0.0068*** (-5.49)	88.7%
$roe_{i,t}$	0.1321** (2.54)	-0.0041 (-0.12)	-0.6023** (-2.30)	0.8606*** 28.1100	0.0602 (0.18)	-0.2608*** (-3.63)	74.5%
ty_t	0	0	0	0	0.7980*** (18.38)	-0.0080*** (-6.88)	63.3%
rm_t	0	0	0	0	0.8802*** (9.57)	0.0401*** (3.04)	1.9%

Panel B. Descriptives of the Variance Decomposition

	$\text{var}(r_e)$	$\text{var}(\text{Ndr})$	$\text{var}(\text{Ncf})$	$-2\text{cov}(\text{Ndr}, \text{Ncf})$	$\text{corr}(\text{Ndr}, \text{Ncf})$
Mean	0.0122	0.0030	0.0097	-0.0004	0.0309
Standard error	0.0004	0.0003	0.0010	0.0006	0.0295
Minimum	0.0060	0.0017	0.0031	-0.0048	-0.0987
Maximum	0.0184	0.0052	0.0182	0.0013	0.2890

Table A2: Industry-level Summary: Discount-rate Return, Discount-rate Volatility, and Investor Attention

This table reports summary statistics of quarterly variables used in the panel-regression model for each industry. See Table 3 for industry descriptions. Industry-level variables reported are the discount-rate return, Ndr; the discount-rate news volatility (in annualized term), σ ; and the attention measure, Φ . For each variable, we report its time-series mean, standard deviation, minimum, and maximum values. Ndr each industry is calculated using a VAR model in the spirit of Campbell and Shiller (1988) and Campbell (1991); see Table A1 for the VAR estimates. Quarterly volatility of the discount-rate return σ that we report is calculated using the exponentially weighted moving-average (EWMA) model. Quarterly investor attention measure Φ is derived based on four industry-level characteristics: trading volume, number of institutional investors, number for analyst followings, and number of analysts' forecast revisions. See text for details.

Industry	Discount-rate news: Ndr				Discount-rate vol (annualized): σ				Attention measure: Φ			
	Mean	Stdev	Min	Max	Mean	Stdev	Min	Max	Mean	Stdev	Min	Max
Cars	0.002	0.056	-0.117	0.220	0.110	0.036	0.044	0.225	0.037	0.375	-0.773	1.355
Chemts	-0.004	0.043	-0.130	0.134	0.085	0.022	0.037	0.151	0.044	0.264	-0.559	0.860
Electr	-0.003	0.056	-0.103	0.299	0.100	0.032	0.057	0.275	0.018	0.324	-1.089	1.125
Food	-0.004	0.054	-0.134	0.170	0.101	0.040	0.046	0.213	0.038	0.288	-0.965	0.682
Instr	-0.006	0.043	-0.173	0.165	0.082	0.031	0.025	0.183	0.037	0.264	-0.592	0.749
Machn	-0.006	0.055	-0.182	0.171	0.107	0.031	0.050	0.196	0.015	0.319	-2.056	0.860
Oil	0.000	0.058	-0.216	0.204	0.110	0.042	0.054	0.245	-0.012	0.312	-1.517	0.867
Phone	0.007	0.067	-0.332	0.156	0.130	0.053	0.068	0.357	0.017	0.331	-0.823	0.928
Print	0.002	0.057	-0.103	0.224	0.110	0.034	0.044	0.217	0.019	0.345	-1.478	0.844
Ptrim	0.004	0.047	-0.096	0.157	0.095	0.025	0.058	0.184	0.015	0.338	-1.148	0.975
Rtail	-0.008	0.041	-0.103	0.165	0.082	0.025	0.033	0.170	0.039	0.311	-0.598	0.835
Srvc	-0.009	0.048	-0.199	0.147	0.092	0.037	0.033	0.215	0.015	0.358	-2.696	1.339
TV	-0.004	0.038	-0.083	0.147	0.077	0.023	0.040	0.152	0.001	0.346	-0.947	0.803
Trans	-0.008	0.043	-0.134	0.096	0.088	0.026	0.047	0.160	-0.012	0.275	-0.931	1.006
Utils	0.015	0.057	-0.189	0.162	0.115	0.032	0.061	0.195	0.027	0.434	-0.977	1.421
Whlsl	-0.004	0.040	-0.187	0.131	0.078	0.030	0.029	0.172	0.030	0.305	-0.702	0.700

Table A3: Cross-industry Correlations of Discount-rate-news Returns

This table reports the correlation matrix of the discount-rate return component (Ndr) in quarterly returns of the sixteen industry portfolios from 1970 to 2014. There are 120 unique industry pairs. We estimate Ndr for each industry using a VAR model in the spirit of Campbell (1991). Quarterly time-varying correlations are calculated using the Exponentially-Weighted Moving Average (EWMA) model. Each cell consists of three rows. The first and second rows report the time-series mean and standard deviation (in parentheses) of the Ndr correlations. The third row reports the range of each correlation time series (in brackets), i.e., the minimum value and the maximum value.

Industry	Cars	Chem	Electr	Food	Instr	Machn	Oil	Phone	Print	Ptblm	Rtail	Srvc	TV	Trans	Utils	Whlsl	
Cars	1	0.68 (0.17) [0.25,1.00]	0.72 (0.17) [0.26,1.00]	0.55 (0.15) [0.19,0.87]	0.69 (0.18) [0.24,1.00]	0.69 (0.18) [0.26,1.00]	0.48 (0.14) [0.12,0.75]	0.49 (0.14) [0.12,0.73]	0.65 (0.17) [0.25,1.00]	0.52 (0.13) [0.14,0.76]	0.71 (0.19) [0.29,1.00]	0.51 (0.14) [0.13,0.77]	0.62 (0.16) [0.22,0.95]	0.62 (0.16) [0.22,0.95]	0.75 (0.20) [0.28,1.00]	0.55 (0.15) [0.18,0.86]	0.71 (0.19) [0.33,1.00]
Chem		1	0.73 (0.17) [0.33,1.00]	0.86 (0.21) [0.37,1.00]	0.89 (0.23) [0.36,1.00]	0.79 (0.20) [0.33,1.00]	0.64 (0.16) [0.24,0.96]	0.57 (0.14) [0.23,0.86]	0.68 (0.16) [0.34,0.99]	0.65 (0.15) [0.20,0.94]	0.78 (0.20) [0.33,1.00]	0.65 (0.17) [0.17,0.97]	0.75 (0.18) [0.33,1.00]	0.75 (0.18) [0.33,1.00]	0.88 (0.21) [0.33,1.00]	0.69 (0.17) [0.29,1.00]	0.87 (0.22) [0.39,1.00]
Electr			1	0.63 (0.15) [0.26,0.98]	0.77 (0.19) [0.31,1.00]	0.78 (0.19) [0.31,1.00]	0.52 (0.13) [0.15,0.74]	0.52 (0.13) [0.14,0.84]	0.67 (0.16) [0.26,0.98]	0.54 (0.13) [0.13,0.79]	0.80 (0.19) [0.34,1.00]	0.57 (0.15) [0.15,0.82]	0.72 (0.17) [0.28,1.00]	0.72 (0.17) [0.28,1.00]	0.76 (0.18) [0.29,1.00]	0.56 (0.14) [0.23,0.90]	0.78 (0.19) [0.36,1.00]
Food				1	0.78 (0.20) [0.30,1.00]	0.62 (0.17) [0.26,0.95]	0.53 (0.14) [0.19,0.79]	0.54 (0.14) [0.19,0.79]	0.63 (0.16) [0.28,0.92]	0.54 (0.13) [0.19,0.79]	0.68 (0.18) [0.28,1.00]	0.55 (0.14) [0.15,0.80]	0.62 (0.15) [0.27,0.96]	0.62 (0.15) [0.27,0.96]	0.74 (0.18) [0.30,1.00]	0.66 (0.17) [0.29,1.00]	0.71 (0.18) [0.31,1.00]
Instr					1	0.86 (0.22) [0.34,1.00]	0.64 (0.16) [0.25,0.99]	0.49 (0.13) [0.15,0.71]	0.63 (0.15) [0.14,0.92]	0.64 (0.16) [0.19,0.92]	0.82 (0.22) [0.30,1.00]	0.65 (0.18) [0.15,0.97]	0.75 (0.19) [0.29,1.00]	0.75 (0.19) [0.29,1.00]	0.82 (0.21) [0.26,1.00]	0.65 (0.16) [0.28,1.00]	0.89 (0.24) [0.32,1.00]
Machn						1	0.70 (0.18) [0.23,1.00]	0.53 (0.14) [0.22,0.83]	0.68 (0.18) [0.24,1.00]	0.68 (0.17) [0.21,0.99]	0.76 (0.19) [0.33,1.00]	0.60 (0.16) [0.16,0.88]	0.73 (0.18) [0.34,1.00]	0.80 (0.20) [0.30,1.00]	0.62 (0.16) [0.27,0.99]	0.84 (0.21) [0.32,1.00]	
Oil							1	0.40 (0.11) [0.12,0.59]	0.48 (0.12) [0.15,0.72]	0.78 (0.20) [0.22,1.00]	0.61 (0.17) [0.24,0.92]	0.44 (0.11) [0.07,0.62]	0.63 (0.16) [0.26,0.92]	0.70 (0.18) [0.24,1.00]	0.54 (0.15) [0.25,0.82]	0.66 (0.18) [0.22,1.00]	
Phone								1	0.48 (0.12) [0.19,0.74]	0.48 (0.12) [0.11,0.69]	0.56 (0.15) [0.25,0.83]	0.40 (0.11) [0.07,0.58]	0.56 (0.15) [0.23,0.87]	0.57 (0.15) [0.19,0.86]	0.71 (0.19) [0.29,1.00]	0.52 (0.14) [0.19,0.77]	
Print									1	0.50 (0.12) [0.12,0.75]	0.70 (0.17) [0.32,1.00]	0.49 (0.13) [0.11,0.74]	0.67 (0.16) [0.30,1.00]	0.70 (0.17) [0.28,1.00]	0.54 (0.13) [0.25,0.82]	0.68 (0.17) [0.30,0.98]	
Ptblm										1	0.53 (0.14) [0.07,0.80]	0.46 (0.12) [0.09,0.63]	0.58 (0.14) [0.15,0.84]	0.72 (0.18) [0.16,1.00]	0.67 (0.16) [0.25,1.00]	0.62 (0.16) [0.16,0.87]	
Rtail											1	0.56 (0.16) [0.06,0.83]	0.77 (0.20) [0.34,1.00]	0.64 (0.16) [0.26,1.00]	0.83 (0.22) [0.35,1.00]	0.66 (0.17) [0.31,1.00]	
Srvc												1	0.46 (0.13) [0.04,0.68]	0.54 (0.15) [0.09,0.77]	0.41 (0.11) [0.10,0.61]	0.65 (0.15) [0.15,0.97]	
TV													1	0.78 (0.20) [0.28,1.00]	0.57 (0.15) [0.24,0.92]	0.75 (0.19) [0.33,1.00]	
Trans														1	0.69 (0.17) [0.31,1.00]	0.87 (0.24) [0.31,1.00]	
Utils															1	0.66 (0.17) [0.29,1.00]	
Whlsl																1	