Review of Quantitative Finance and Accounting, 16: 191–203, 2001

Linear Accounting Valuation When Abnormal Earnings Are AR(2)

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Abstract. The Ohlson (1995) model assumes that abnormal earnings follow an AR(1) process primarily for reasons of mathematical tractability. However, the empirical literature on the Garman and Ohlson (1980) model finds that the data support an AR(2) lag structure for earnings, book values and dividends. Moreover, the AR(2) process encompasses a far richer variety of time series patterns than does the AR(1) process and includes the AR(1) process as a special case. This paper solves the Ohlson model directly for an AR(2) abnormal earnings dynamic. The model is estimated on a time series firm-level basis following the approach used by Myers (1999). It is found that, like the Ohlson AR(1) model, the Ohlson AR(2) model severely underestimates market prices even relative to book values. These results further bring into question the empirical validity of the Ohlson model.

Key words: linear accounting valuation models, Ohlson model

JEL Classification: M41, G12

1. Introduction

Ohlson's (1995) valuation model (hereinafter the Ohlson model) is important to accounting research not only because it relates security prices to accounting numbers in a rigorous fashion but also because it has become widely adopted by accounting empiricists.[1] There appear to be three principal reasons as to why accounting empiricists have embraced the model as wholeheartedly as they have. First, the few alternative models suggested by the accounting literature are either ad hoc, not having been developed from underlying primitives, or they are difficult to implement empirically.[2] Second, on a more positive note, Ohlson's model, by focusing on the Clean Surplus relationship, shifts the overemphasis on the income statement by the earlier accounting literature, and brings the balance sheet back

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into focus. Third, firm value is derived from accounting variables such as earnings and book value, constructs over which accounting researchers have a comparative advantage.

Although the Ohlson model is the fundamental accounting paradigm relating accounting information to security prices and is widely used by accounting researchers, the model has not fared particularly well in empirical testing. Both the cross-sectional tests of Dechow, Hutton and Sloan (1999) and the time series tests of Morel (1998, 1999a) and Myers (1999) appear to reject the model as an adequate description of security pricing. This raises the issue: can the assumptions of the Ohlson model be relaxed to yield an empirically robust framework for understanding security prices. Of the three major assumptions underlying the model, two do not lend themselves to change. Relaxing the assumption that price is the present value of future expected dividends (conditional on current information) is tantamount to the existence of arbitrage opportunities and, therefore, untenable.[3] Also, it makes little sense to relax the Clean Surplus relationship since not only is it an accounting identity that financial statements must satisfy but it also drives the accounting content in the Ohlson model.

However, the third assumption underlying the Ohlson model, namely, that abnormal earnings follows a simple AR(1) process, is ad hoc and begs investigation. Although this assumption simplifies the mathematics of the model considerably, thereby providing the theorist with a simple framework that yields insights into the structure of accounting-based linear valuation models, it is a great leap of faith to then suppose that the AR(1) assumption has empirical content. Contrary evidence is provided by Fama and French (2000) who find that while earnings changes mean revert, "there is a rich nonlinear pattern in the autocorrelation of successive changes in earnings." Such patterns cannot be explained by a simple AR(1) process. Moreover, those papers that have addressed the lag issue directly, tend to reject the assumption that the linear valuation dynamic is single lagged and that the concomitant linear pricing rule has no lags. Specifically, Bar-Yosef, Callen and Livnat (1996) find that the one lag autoregressive information dynamic is rejected by the data when the dynamic is evaluated within the context of a Garman-Ohlson (1980) framework. Extending Bar-Yosef et al. (1996) by using a lag structure specification criterion that mitigates against overfitting in small samples, Morel (1999a, 1999b) finds that the one lag autoregressive dynamic is rejected *both* for the information dynamic and the pricing equation. Interestingly, both Bar-Yosef et al. (1996) and Morel (1999a, 1999b) find that a two period autoregressive lag structure best characterizes the underlying dynamics of earnings, book-values and dividends.

Although the results of these latter studies are suggestive, they are not definitive regarding the Ohlson model per se. Neither study addresses the abnormal earnings dynamic directly preferring instead to determine the value relevance of (ordinary) earnings, book values and dividends through the prism of the Garman-Ohlson framework. The assumptions underlying the Garman-Ohlson framework, however, are sufficiently more general than those of the Ohlson model so that the results obtained using the former need not generalize to the latter.

The purpose of this study is to evaluate the ability of the Ohlson model to predict security prices on a time series firm-level basis under the assumption that abnormal earnings follow an AR(2) process. As is well-known from the time series literature, the AR(2) process encompasses a far richer variety of time series patterns than does the AR(1) process and, yet, it includes the AR(1) process as a special case.[4] Thus, a priori, the AR(2) model should yield better predictions of security prices than the AR(1) model.

This study is structured as follows. First, the equity valuation equation is derived assuming that the abnormal earnings dynamic can be described by an AR(2) process. Subsequently, following Myers (1999), time varying cost of capital estimates are obtained from a Fama and French (1997) three factor industry asset pricing model. These costs of capital are used to calculate abnormal earnings. The parameters of the abnormal earnings AR(2) dynamic are estimated on a time series firm-level basis. The estimates of the costs of capital and the estimated abnormal earnings parameters are then substituted into the valuation equation to obtain a time series of estimated equity values for each firm. Finally, the resulting equity value estimates are compared to actual market values in order to evaluate the predictive ability of the Ohlson model.

In what follows, Section 2 briefly describes the Ohlson AR(1) model and solves for the linear equity valuation equation under the assumption that the abnormal earnings dynamic follows an AR(2) process. Section 3 describes the data. Section 4 presents the empirical results. Section 5 concludes.

2. The Ohlson model: Empirical formulation

The Ohlson valuation model is based on three assumptions.[5] First, following the non-arbitrage model of Rubinstein (1976), Ohlson utilizes the well-known relationship that equity values are the present value of expected future dividends conditional on current information (PVED). Second, Ohlson assumes an AR(1) linear information dynamic, where the dynamic is comprised of abnormal earnings and a variable representing "other information" whose source is uncorrelated with accounting information. Since this latter variable is difficult to specify empirically for a large sample of data, we will assume that it enters the empirical estimation process as an intercept term.[6] Third, Ohlson utilizes the accounting Clean Surplus relationship to substitute accounting numbers for dividends, so that equity values can be written as the present value of future abnormal earnings. In this study, we will replace the second assumption by assuming that the abnormal earnings dynamic is AR(2).

The three standard assumptions of the Ohlson model can be written formally as:[7]

$$V_t = \sum R_f^{-\tau} E_t[d_{t+\tau}] \qquad \text{(PVED)} \tag{1}$$

$$x^a_{t+1} = \omega_0 + \omega_1 x^a_t + \epsilon_{t+1} \qquad \text{(AR(1) Linear Dynamic)} \tag{2}$$

$$d_t = x^a_t - y_t + R_f y_{t-1} \qquad \text{(Clean Surplus)} \tag{3}$$

where V_t = firm equity value at the beginning of period t.

d_t = dividends during period t.

x_t = earnings during period t.

y_t = book value of common equity at the end of period t.

ω_0, ω_1 = parameters of the linear dynamic.

$$\begin{aligned}
R_f &= \text{one plus the firm's cost of capital (or risk free rate).}^8 \\
x_t^a &= \text{abnormal earnings during period } t \\
&= x_t - (R_f - 1)y_{t-1} \\
\epsilon_{t+1} &= \text{white noise error term.} \\
E_t &= \text{expectations operator at time } t.
\end{aligned}$$

Substituting the clean surplus relationship [equation (3)] into valuation equation (1), thereby eliminating dividends, yields a valuation equation that is a function of accounting variables only, namely:[9]

$$V_t = y_t + \sum R_f^{-\tau} E_t\left[x_{t+\tau}^a\right] \tag{4}$$

This equation states that firm's equity value can be expressed as the sum of the book value of the firm's assets in place plus goodwill (the present value of expected future abnormal earnings).

Since equation (4) cannot be estimated empirically, Ohlson further assumes that abnormal earnings are described by an AR(1) process. Substituting the AR(1) process [equation (2)] into equation (4), yields the Ohlson valuation equation:

$$V_t^1 = y_t + \frac{R_f \omega_0}{(R_f - \omega_1)(R_f - 1)} + \frac{\omega_1}{(R_f - \omega_1)} x_t^a \tag{5}$$

Here, the value of the firm's equity is expressed as the sum of the book value of the assets in place, an intercept term and a multiple of current abnormal earnings. Equation (5) takes the regression form:

$$V_t^1 = B_0 + B_1 y_t + B_2 x_t^a \tag{6}$$

where the B_i are the coefficients to be estimated.

Replacing the AR(1) dynamic of equation (2) by the AR(2) dynamic:

$$x_{t+1}^a = \omega_0 + \omega_1 x_t^a + \omega_2 x_{t-1}^a + \epsilon_{t+1} \tag{7}$$

it is shown in the appendix that the valuation equation takes the form:

$$\begin{aligned}
V_t^2 = y_t &+ \frac{R_f^2 \omega_0}{\left(R_f^2 - \omega_1 R_f - \omega_2\right)(R_f - 1)} \\
&+ \frac{\omega_2 R_f}{\left(R_f^2 - \omega_1 R_f - \omega_2\right)} x_{t-1}^a + \frac{R_f \omega_1 + \omega_2}{\left(R_f^2 - \omega_1 R_f - \omega_2\right)} x_t^a
\end{aligned} \tag{8}$$

Thus, when the abnormal earnings dynamic is AR(2), firm value is expressed as the sum of the book value of assets in place, an intercept term, a multiple of prior year's abnormal earnings and a multiple of current year's abnormal earnings. Note that when $\omega_2 = 0$, so that the earnings dynamic is AR(1), equation (8) collapses to equation (5). In regression form, valuation equation (8) becomes

$$V_t^2 = C_0 + C_1 y_t + C_2 x_{t-1}^a + C_3 x_t^a \tag{9}$$

where the C_i are the coefficients to be estimated.

3. Sample selection

The sample is selected from the period 1962 to 1996, a maximum of thirty-four years of annual data. The sample selection criteria are as follows:

(1) Data must be available for each firm on at least 27 years of Compustat Primary, Secondary, and Tertiary, Full Coverage, and Research Annual Industrial Files.[10] The data include annual earnings, book values of equity, total liabilities, common equity shares outstanding, the adjustment factor for stock splits and dividends, and the closing price at the end of the third month after the firm's fiscal year-end;[11]
(2) Book values of equity must be greater than zero;
(3) Firms in the financial sector are excluded.

This selection process yields 676 firms with 27 years or more of data per firm, a total of 19,789 firm-years. The distribution of sample firm SIC codes is shown in Table 1. The largest proportion of sample firms is in the manufacturing sector. Table 2 shows selected descriptive statistics for the pooled sample data. The median market value for the pooled sample is 355 million dollars, the median return on equity is 13.3%, the median book to market value is 70% and the median abnormal earnings is 1.4 million dollars. Overall, this sample is comprised of firms that are larger and more profitable than those in Myers'(1999) sample. This is not surprising since our sample firms have longer time series. Despite the obvious potential survivorship bias that limits the generality of this study, longer time series are necessary for estimating time series models with any degree of accuracy. Indeed, even 27 data points (per firm) is minimal at best.

Table 1. Distribution of sample firms by industry

SIC Codes	Industry	# of Firms with obs ≥ 27
100–999	Natural resources	3
1000–1999	Mining/construction	25
2000–3999	Manufacturing	405
4000–4999	Regulated	151
5000–5999	Wholesale/retail	65
7000–9999	Services	27
1000–9999	Total	676

Table 2. Descriptive statistics

Variable	Percentile 5%	25%	Median	75%	95%	Mean
ROE	−0.051	0.083	0.133	0.183	0.301	0.171
BV	10.4	61.9	218.1	732.0	3420.0	828.8
MV	11.1	81.8	355.1	1210.8	6654.0	1656.2
BTM	0.198	0.447	0.699	1.047	1.852	0.832
AE	−114.5	−5.1	1.4	16.6	151.3	7.435

Notes: This table contains one observation for each of the 19,789 firm years in the sample. ROE = Earnings before extraordinary items divided by beginning of the year book value of equity. BV = Book value of equity at the end of the fiscal year, in millions. MV = Market value of equity at the end of the fiscal year, in millions. BTM = Book value of equity divided by market value of equity at the end of the fiscal year. AE = Earnings before extraordinary items minus the multiplicand of the risk-adjusted discount rate and book value of equity at the beginning of the year, in millions.

4. Empirical results

4.1. The Ohlson model—AR(1) dynamic

Myers (1999) finds for his sample that the Ohlson model severely underestimates market values. The median ratio of estimated equity value to actual market value is on the order of 41% which is even less than the 63% median ratio of book value equity to market value. The cross-sectional correlation between market values and estimated equity values is virtually identical to the correlation between market values and book values. This implies that the Ohlson model provides very little information about security prices beyond book values.

We replicate the Myers' analysis on our sample. In contradistinction to Myers who uses per share data, thereby disregarding the potential bias induced by non-stationary time series on the regression coefficients, all variables in our regressions are scaled by beginning of the year total assets.[12] Unlike cross-sectional regressions, where scaling is done to minimize heteroscedasticity and to enhance cross-sectional comparability, scaling is performed in time series analysis in order to induce stationarity.[13] A recent study by Qi, Wu and Xiang (2000), using a sample of 98 firms with lengthy time series, finds in fact that non-stationarity is mitigated in the Ohlson model if the data are scaled.

The Ohlson model is estimated for each firm in our sample by first estimating the AR(1) abnormal earnings dynamic [Equation (2)] for each firm. Abnormal earnings are computed as earnings before extraordinary items less the multiplicand of a time varying industry cost of capital and beginning of period book value of equity. The time varying cost of capital is taken from the Fama and French (1997) study in which a time invariant industry risk-premium is added to a time varying treasury bill rate. The risk premium is estimated using a three-factor asset pricing model. The resulting parameter estimates obtained from the AR(1) abnormal earnings dynamic and the annual cost of capital estimates are then substituted into valuation equation (5) to yield an estimated annual equity value for each firm.

Table 3. AR(1) Ohlson model

Panel A: Abnormal earnings dynamic parameter estimates

Parameters	5%	25%	Median	75%	95%	Mean	t-Statistic
ω_0	−1.591	−0.198	−0.002	0.102	0.355	−0.239	−4.514
ω_1	0.119	0.378	0.469	0.570	0.773	0.462	62.10

Panel B: Ratios of estimated equity values to market values and book values to market values

Ratio	5%	25%	Median	75%	95%	Mean
V^1/P	−0.947	0.224	0.437	0.716	1.372	0.307
BV/P	0.128	0.319	0.518	0.743	1.326	0.591

Panel C: Implied valuation coefficients and value level regressions

Dependent Variable	Independent Variables			
	1/TA$_{t-1}$	V_t^1	x_t^a	y_t
(1) P_t	15.790	0.064	—	—
	(0.0001)	(0.0016)		
(2) P_t	7.228	—	—	1.685
	(0.0001)			(0.0001)
(3) P_t	15.015	—	7.828	—
	(0.0001)		(0.0001)	
(4) P_t	7.217	—	6.876	1.519
	(0.0001)		(0.0001)	(0.0001)
(5) Implied valuation coefficients (B_0, B_1, B_2)	−0.017	—	0.747	1.000

Notes: Panel A presents the cross-sectional distribution of the parameter estimates for the AR(1) abnormal earnings dynamic (ω_0 and ω_1) given by equation (2) for the sample of 676 firms. To mitigate against non-stationarity, the dynamic is estimated after scaling abnormal earnings by beginning of year total assets.

Panel B presents the distribution of estimated firm equity values (V_t^1)—derived from equation (5)—divided by market values (P_t) and book values (y_t) divided by market values, as of the end of fiscal year 1996. There are 623 firms with requisite data for 1996.

Panel C presents the estimated coefficients obtained by regressing market value (P_t) on V_t^1, x_t^a and/or y_t for the 623 firms with requisite data for 1996. Two tailed p-values are presented in parentheses. Also presented are the equilibrium value coefficients (B_0, B_1 and B_2) implied by the estimated parameters and valuation equation (5). These coefficients are computed by substituting the median abnormal earnings parameter (See panel A) and the cost of capital estimates into valuation equation (5).

Panel A of Table 3 presents the cross-sectional distribution of the parameters of the estimated AR(1) dynamic for our sample. The median $\omega_0 = -0.002$ and the median $\omega_1 = 0.469$. Panel A shows these parameters to be highly statistically significant on the basis of a cross-sectional t-test. Panel B shows the cross-sectional distribution of the ratio of estimated firm equity value to actual market value (V^1/P) and the ratio of book value of equity to market value (BV/P). The results for our sample are quite similar to those of Myers. Firm equity values estimated from the Ohlson model severely underestimate market values even more so than do book values. Rows 1 and 2 of Panel C regress market values on estimated equity values and market values on book values, respectively, for the year 1996.[14] The data in these regressions were first normalized by total assets to mitigate against heteroscedasticity. These regressions suggest that the Ohlson model provides little information about security prices beyond book values and, in fact, may provide less information since market values are more highly correlated with book values than with estimated equity values. Row 4 of Panel C shows the Ohlson valuation equation [equation (5)] estimated cross-sectionally (using scaled data) for the year 1996. Results for the other years are quite similar. Row 5 gives the implied coefficients for the valuation equation based on substituting the median parameter estimates of the abnormal earnings dynamic and the cost of capital into the Ohlson valuation equation (5). The regression coefficient estimates are significantly different from the implied coefficients, both individually and as a group, on the basis of standard F-tests. Overall, the results for our sample are very similar to Myers' indicating that the Ohlson AR(1) model is no better than book values in predicting security prices.

4.2. The Ohlson model—AR(2) dynamic

The Ohlson AR(2) valuation model is estimated in a manner similar to the AR(1) model. First, the AR(2) abnormal earnings dynamic [equation (7)] is estimated for each firm. The resulting parameter estimates of the AR(2) dynamic and the Fama-French cost of capital estimates are then substituted into the AR(2) valuation equation (8), thereby yielding an estimated annual equity value for each firm.

Panel A of Table 4 lists the cross-sectional distribution of the AR(2) abnormal earnings parameter estimates from equation (7) for the our 676 firm sample. The median $\omega_0 = 0.006$, the median $\omega_1 = 0.346$ and the median $\omega_2 = 0.043$. Based on the cross-sectional t-tests, the AR(2) dynamic provides a much poorer specification of the abnormal earnings dynamic by comparison to the AR(1) dynamic. (See Table 3.) The AR(2) parameter estimates for ω_0 and ω_1 are not statistically significant. ω_2 is marginally significant at the 10% level but not the 5% level. In addition, the distribution of the AR(2) parameter estimates suggests that firms in both tails of the distribution exhibit non-stationary abnormal earnings dynamics.

Panel B of Table 4 evaluates the predictive ability of the Ohlson AR(2) valuation model for 1996.[15] The results for the AR(2) model are similar to those of the AR(1) model. Equity values estimated from the AR(2) model [equation (8)] severely underestimate market values even by comparison to book values. Rows 1 and 2 of Panel C imply that the equity value estimates of the AR(2) model are even inferior to book value estimates in explaining market values. Row 5 gives the implied coefficients for the valuation equation based on substituting the median parameter estimates of the abnormal earnings dynamic and the cost of capital into the Ohlson valuation equation (8). A comparison of rows 4 and 5 of Panel C (and related F-tests) indicate that the regression estimates from the AR(2) model are significantly different from the implied price coefficients both individually and as a group.

Table 4. AR(2) model-abnormal earnings dynamic: Parameter estimates

Panel A: Abnormal earnings dynamic—parameter estimates: ($n = 676$)

Parameters	5%	25%	Median	75%	95%	Mean	t-Statistic
ω_0	−1.024	−0.137	0.006	0.123	0.537	−0.043	−1.011
ω_1	−8.157	0.026	0.346	0.791	8.360	−0.295	−0.686
ω_2	−8.220	−0.320	0.043	0.489	9.381	0.805	1.716

Panel B: Ratios of estimated equity values to market values and book values to market values

Ratio	5%	25%	Median	75%	95%	Mean
V^2/P	−0.555	0.245	0.477	0.773	1.986	0.522
BV/P	0.128	0.319	0.518	0.743	1.326	0.591

Panel C: Implied valuation coefficients and value level regressions

Dependent Variable	Independent Variables				
	1/TA$_{t-1}$	V_t^2	x_t^a	x_{t-1}^a	y_t
(1) P_t	15.667	0.043	—	—	—
	(0.0001)	(0.106)			
(2) P_t	7.228	—	—	—	1.685
	(0.0001)				(0.0001)
(3) P_t	14.306	—	4.554	5.022	—
	(0.0001)		(0.0001)	(0.0001)	
(4) P_t	6.993	—	4.126	4.289	1.445
	(0.0001)		(0.0001)	(0.0001)	(0.0001)
(5) Implied valuation coefficients (C_0, C_1, C_2, C_3)	0.028	—	0.566	0.340	1.000

Notes: Panel A presents the cross-sectional distribution of the parameter estimates for the AR(2) abnormal earnings dynamic (ω_0, ω_1 and ω_2) given by equation (7) for the sample of 676 firms. To mitigate against non-stationarity, the dynamic is estimated by scaling abnormal earnings by beginning of year total assets.

Panel B presents the distribution of estimated values (V_t^2)—derived from equation (8)—divided by market values (P_t) and book values (y_t) divided by market values, as of the end of fiscal year 1996. There are 623 firms with requisite data for 1996.

Panel C presents the estimated coefficients obtained by regressing market value (P_t) on V_t^2, x_t^a, x_{t-1}^a and/or y_t for the 623 firms with requisite data for 1996. Two tailed p-values are presented in parentheses. Also presented are the equilibrium value coefficients (C_0, C_1, C_2, and C_3) implied by the estimated parameters and valuation equation (8). These coefficients are computed by substituting the median abnormal earnings parameters (see panel A) and the cost of capital estimates into valuation equation (8).

Since the AR(2) earnings dynamic is unlikely to fit all firms in the sample, the sample was further partitioned into firms for which the ω_2 parameter is significant at the 5% level and firms for which the ω_2 parameter is not significant at the 5% level. The AR(2) model was evaluated separately for both sub-samples (not shown). Again, for both sub-samples, the AR(2) model severely underpriced equity values.

5. Conclusion

Overall, these results imply that the AR(2) version of the Ohlson model is no better at predicting security prices than the AR(1) model. Even firms whose abnormal earnings dynamic is ostensibly AR(2) rather than AR(1) yielded valuation estimates that were no better overall than book values alone.

There are a number of potential explanations for these results. It could well be that the true dynamic is neither AR(1) nor AR(2) but some alternative stochastic process. Alternatively, the true lag structure is highly firm specific and no one structure can be used to model an entire sample. Another possibility is that the Ohlson (1995) model is mis-specified and that alternative models such as Garman and Ohlson (1980) or Feltham and Ohlson (1995) hold more promise. Finally, it is well known that *all* linear valuation accounting models to date are based on the MM assumptions. These assumptions are problematic in a world of corporate taxes, bankruptcy costs, debt tax-shield substitutes, asymmetric information and signaling. The latter criticism implies that none of the linear accounting valuation models to date are likely to yield meaningful value estimates. These conflicting possibilities leave much scope for future research.

Appendix: Solving the Ohlson valuation equation when abnormal earnings are AR(2)

Assumptions 1 and 3 of the Ohlson model yield the valuation equation (See Ohlson (1995, p. 667))

$$P_t = y_t + \sum R_f^{-k} E_t \left[x_{t+k}^a\right] \tag{A1}$$

We solve for x_{t+k}^a under the assumption that abnormal earnings follow an AR(2) process:

$$x_{t+1}^a = \omega_0 + \omega_1 x_t^a + \omega_2 x_{t-1}^a + \epsilon_{t+1} \tag{A2}$$

This abnormal earnings dynamic—without the zero mean error term—can be written trivially as a dynamic matrix equation of the form:

$$\begin{pmatrix} x_{t+1}^a \\ x_t^a \\ \omega_0 \end{pmatrix} = V \begin{pmatrix} x_t^a \\ x_{t-1}^a \\ \omega_0 \end{pmatrix} \tag{A3}$$

where

$$V = \begin{pmatrix} \omega_1 & \omega_2 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Given the recursive nature of the abnormal earnings dynamic, it follows that:

$$\begin{pmatrix} x^a_{t+k} \\ x^a_{t+k-1} \\ \omega_0 \end{pmatrix} = V^k \begin{pmatrix} x^a_t \\ x^a_{t-1} \\ \omega_0 \end{pmatrix}. \tag{A4}$$

Diagonalizing the matrix V, it is straightforward to show that

$$V^k = \begin{pmatrix} -\omega_2 & -\omega_2 & -1 \\ \lambda_2 & \lambda_1 & -1 \\ 0 & 0 & \omega_2 + \omega_1 - 1 \end{pmatrix} \begin{pmatrix} \lambda_1^k & 0 & 0 \\ 0 & \lambda_2^k & 0 \\ 0 & 0 & 1 \end{pmatrix}$$
$$\times \begin{pmatrix} \frac{-\lambda_1}{\omega_2(\lambda_1-\lambda_2)} & \frac{-1}{(\lambda_1-\lambda_2)} & \frac{-\lambda_1-\omega_2}{(\lambda_1-\lambda_2)\omega_2(\omega_2+\omega_1-1)} \\ \frac{\lambda_2}{\omega_2(\lambda_1-\lambda_2)} & \frac{1}{(\lambda_1-\lambda_2)} & \frac{\lambda_2+\omega_2}{(\lambda_1-\lambda_2)\omega_2(\omega_2+\omega_1-1)} \\ 0 & 0 & \frac{1}{\omega_2+\omega_1-1} \end{pmatrix}$$

where the λ's are the eigenvalues of V, namely, $\lambda = 1$, $\lambda_1 = (\omega_1 + \sqrt{\omega_1^2 + 4\omega_2})/2$ and $\lambda_2 = (\omega_1 - \sqrt{\omega_1^2 + 4\omega_2})/2$. Note that $\lambda_1 + \lambda_2 = \omega_1$ and $\lambda_1\lambda_2 = -\omega_2$.

Substituting the diagonalized form of V^k into (A4) and solving the matrix equation yields:

$$\begin{aligned} E_t\left[x^a_{t+k}\right] &= \left[\frac{\lambda_1^k(\lambda_1+\omega_2) - \lambda_2^k(\lambda_2+\omega_2) - (\lambda_1-\lambda_2)}{(\lambda_1-\lambda_2)(\omega_2+\omega_1-1)}\right]\omega_0 \\ &\quad + \left[\frac{\omega_2\left(\lambda_1^k - \lambda_2^k\right)}{(\lambda_1-\lambda_2)}\right] x^a_{t-1} + \left[\frac{\left(\lambda_1^{k+1} - \lambda_2^{k+1}\right)}{(\lambda_1-\lambda_2)}\right] x^a_t \end{aligned} \tag{A5}$$

Further substituting (A5) into (A1), it can be shown after tedious calculation that

$$\begin{aligned} P_t &= y_t + \sum R_f^{-k} E_t\left[x^a_{t+k}\right] \\ &= y_t + \left[\frac{(R_f-1)[R_f(\omega_1+\omega_2)+\omega_2] - (R_f-\lambda_1)(R_f-\lambda_2)}{(\omega_1+\omega_2-1)(R_f-\lambda_1)(R_f-\lambda_2)(R_f-1)}\right]\omega_0 \\ &\quad + \left[\frac{\omega_2 R_f}{(R_f-\lambda_1)(R_f-\lambda_2)}\right] x^a_{t-1} + \left[\frac{R_f\omega_1+\omega_2}{(R_f-\lambda_1)(R_f-\lambda_2)}\right] x^a_t \\ &= y_t + \left[\frac{R_f^2}{(R_f^2-\omega_1 R_f-\omega_2)(R_f-1)}\right]\omega_0 + \left[\frac{\omega_2 R_f}{(R_f^2-\omega_1 R_f-\omega_2)}\right] x^a_{t-1} \\ &\quad + \left[\frac{R_f\omega_1+\omega_2}{(R_f^2-\omega_1 R_f-\omega_2)}\right] x^a_t \end{aligned} \tag{A6}$$

Equation (A6) is the valuation equation estimated in the text. Note than when $\omega_2 = 0$, the abnormal earnings process is AR(1) and (A6) simplifies to the standard Ohlson valuation equation

$$P_t = y_t + \left[\frac{R_f}{(R_f - \omega_1)(R_f - 1)}\right]\omega_0 + \left[\frac{\omega_1}{(R_f - \omega_1)}\right]x_t^a \tag{A7}$$

Notes

1. See Burgstahler and Dichev (1997), Nwaeze (1998), and Collins, Pincus and Xie (1999) among many others.
2. The ERC approach of Beaver, Lambert and Morse (1980) is an example of the former. The Garman-Ohlson (1980) approach is an example of the latter.
3. See Rubinstein (1976).
4. For the properties of AR(1) and AR(2) processes, see, for example, the textbooks by Wei (1989) and Hamilton (1994).
5. Not surprisingly, the Ohlson model also assumes away any frictions such as transactions costs, taxes and asymmetric information.
6. Whether the "other information" variable has empirical content is currently a subject of debate. See Dechow, Hutton and Sloan (1999) for one potential measure of "other information."
7. The Clean Surplus relationship is usually written as $y_t = y_{t-1} + x_t - d_t$. Substituting the definition for abnormal earnings—$x_t^a = x_t - (R_f - 1)y_{t-l}$—and eliminating x_t, yields equation (3).
8. Since Ohlson assumes a risk neutral probability measure, he discounts by the risk-free rate. Empirically, risk is accounted for by grossing up the risk-free rate by a risk premium in which case R_f refers to (one plus) the firm's cost of capital.
9. To derive equation (4), one also has to assume the transversality condition: $\lim \tau \to \infty\ R_f^{-\tau} E_t[y_{t+\tau}] = 0$.
10. With a maximum of two lags, this leaves a minimum of 25 years of data per firm.
11. Annual earnings are defined as earnings per share before extraordinary items and discontinued operations (Annual Compustat item 58), book value is defined as common equity (Annual Compustat item 60), price is defined as the closing price at the end of the third month of the first quarter after the firm's fiscal year-end (Quarterly Compustat item 14), all items which are listed in per share form are scaled by the adjustment factor (Annual Compustat item 27), Total assets are defined as book value of equity (60) plus total liabilities (181). The latter definition was necessary because Compustat contained many missing values for total assets, especially in the early sample years. Since this study investigates whether annual earnings and book values are reflected in market values and since annual earnings are generally released towards the end of the first quarter, market values at the end of the first quarter are used in the analysis.
12. Myers argues that the short time series in his sample mitigates the non-stationarity issue. This is doubly problematic. A shorter time series implies more error in the coefficient estimates and, moreover, it has no effect on stationarity. If a series is non-stationary, cutting it does not make the sub-series stationary. It just makes it more difficult to test for non-stationarity. Presumably the non-stationarity that Myers is thinking of is structural change. However, the ubiquitous non-stationarity problem that appears to affect financial data is the upward trend in the time series induced by inflation, growth and other macroeconomic effects. The latter problem is not solved by shortening the time series. In any case, the issue is empirical. In point of fact, Qi, Wu and Xiang (2000) show that while cutting the data to 20 data points a la Myers does not mitigate the non-stationarity issue for their sample, scaling does mitigate non-stationarity.
13. We normalized by total assets rather than book value in the cross-sectional regressions below, because total assets are a better measure of firm size for levered firms. To be consistent, we normalize by total assets in the time series regressions as well.
14. 1996 was chosen to eliminate hindsight bias.
15. The results for the other years are similar.

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