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Foreign-Exchange Rate Dynamics: An Empirical Study Using Maximum Entropy Spectral Analysis

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This study examines whether foreign-exchange rates evolve as a random walk by directly comparing the predictive ability of autoregressive (AR) models of spot rates with that of the random walk. To reduce the influence of model specifications on test results, we neither specify the order of the AR process a priori nor assume that the order is necessarily the same over the entire sample period. For each subperiod, the AR model is estimated by maximum entropy spectral analysis, using Akaike's criterion of final prediction error for optimal order selection. In contrast to standard Box–Jenkins techniques, this analysis neither arbitrarily truncates the data in the time domain beyond the sample period nor imposes periodic extension in the frequency domain, and thus it mitigates against potential structural change in the time series. It is shown that for six currencies, relative to the U.S. dollar, past spot rates are irrelevant for predicting future spot rates, or in other words, spot rates behave as a random walk.

KEY WORDS: Spot-exchange rates; Random walk; Autoregressive models; Time series; Final prediction error.

There is mounting evidence that foreign-exchange spot rates evolve as a random walk. Mussa (1979) and Frenkel (1981) found exchange rates to be largely unpredictable. Mussa declared that "the natural logarithm of the spot exchange rate follows approximately a random walk" (p. 10). More recently Fama (1983), Hodrick and Srivastava (1983), and Korajczyk (1983) showed the current spot rate to be a better predictor of future spot rates than is the forward rate. This too suggests that spot rates act as a random walk. Such an interpretation, however, is weakened by the empirical evidence that the forward rate is a biased predictor of the future spot rate. More direct evidence was provided by Meese and Rogoff (1983). They compared the predictive ability of a number of structural models of exchange rate determination against that of the random walk. Predicting data outside of the sample period, they found that the random walk performs as well as any of the structural models from a 1-month to a 12-month horizon.

The purpose of this article is to provide additional empirical evidence on the same issue by using a methodology that attempts to reduce the influence of model specifications on test results. We compare directly the predictive ability of autoregressive (AR) models of spot rates with that of the random walk. Unlike the Cornell (1977) study, however, which also used AR models to capture past spot rate information, we do not specify the order of the AR process a priori. Nor do we assume that the same order is necessarily operative over the entire sample period. Rather, instead of arbitrarily positing a specific AR process that is presumed to be invariant over time, we let the data speak for itself. Specifically, the order m of the AR(m) model is chosen optimally, using Akaike's (1969) criterion of final prediction error (FPE). Furthermore, the optimal order is permitted to change (although it may not) over the data period. More important, we estimate the parameters of the AR process by maximum entropy spectral analysis. Unlike standard Box–Jenkins (1976) techniques, this

analysis neither arbitrarily truncates the time series beyond the sample period nor imposes periodicity of the time series beyond the sample period, and thus it mitigates against potential structural change in the time series. (See Van Den Bos 1971 and Makhoul 1976 for properties of the estimation technique using maximum entropy spectral analysis.)

Having estimated the AR model, we then compare its predictive ability with that of a simple random walk. We show that for six currencies, relative to the U.S. dollar, the random walk yields predictions that are no worse than the AR model. Thus we conclude that past spot rates are irrelevant for predicting the future spot rate, or in other words, that spot rates behave as a random walk. Section 1 describes the estimation technique via maximum entropy spectral analysis, Section 2 provides the empirical results, and Section 3 briefly presents our conclusions.

1. MAXIMUM ENTROPY SPECTRAL ANALYSIS

Consider the mean-adjusted AR(m) process

$$z_t = \sum_{i=1}^{m} \phi_i z_{t-i} + \epsilon_t. \tag{1}$$

By rewriting (1) in the autocorrelation form, the ϕ_i's can be solved theoretically via the Yule–Walker equations

$$\begin{bmatrix} \rho_0 & \rho_1 & \cdots & \rho_{m-1} \\ \rho_1 & \rho_0 & \cdots & \rho_{m-2} \\ \vdots & \vdots & & \vdots \\ \rho_{m-1} & \rho_{m-2} & \cdots & \rho_0 \end{bmatrix} \times \begin{bmatrix} \phi_1 \\ \phi_2 \\ \vdots \\ \phi_m \end{bmatrix} = \begin{bmatrix} \rho_1 \\ \rho_2 \\ \vdots \\ \rho_m \end{bmatrix}. \tag{2}$$

Solving for the ϕ_i's, however, requires complete knowledge of the autocorrelations ρ_k, $k = 0, 1, \ldots, m$. But with the exception of $\rho_0 = 1$, all other autocorrelations are unknown and can only be estimated. Conventionally, in estimating ρ_k, $k = 1, 2, \ldots, m$, the time series beyond the sample period is implicitly treated as if it were composed of zeros. Alternatively, (2) can be transformed to the frequency domain, and the ϕ_i's can then be estimated spectrally. But this alternative approach imposes periodicity of the time series beyond the sample period. Thus relatively long time series data are required to reduce the undesirable consequences of truncation in the time domain or periodic extension in the frequency domain. Unfortunately, the potential gain may be offset by possible structural changes in the time series over a long sample period.

To avoid this problem, Burg (1967, 1968, 1975) proposed that the estimator of autocorrelations ought to maximize the randomness of the time series outside the sample period yet remain consistent with the autocorrelations based on the observed time series data. Since he measured randomness by the entropy of information, Burg's approach is called the Maximum Entropy Method (MEM).

The essence of the MEM is to find the power spectrum $P(f)$ that maximizes entropy subject to the constraint that $P(f)$ agree with the $N + 1$ measured values of the autocorrelation function. Finally, the problem is to

$$\underset{P(f)}{\text{maximize}} \int_0^w \log P(f)\, df \tag{3}$$

$$\text{subject to} \int_0^w P(f)\cos(2\pi f \tau t')\, df = \rho(\tau),$$

$$\tau = 0, 1, \ldots, N, \tag{4}$$

where $\rho(\tau)$ is the autocorrelation function, t' is the sample period of the time series, and $w = 1/(2t')$. Burg showed that the solution to this problem can be obtained by minimizing the sum of squared residuals associated with forward and backward predictions. That is, given an n-term time series z_t and a set of parameter estimates ϕ_i^*, the fitted values $\vec{z}_t$, defined by

$$\vec{z}_t = \sum_{i=1}^{m} \phi_i^* z_{t-i}, \qquad t = m+1, m+2, \ldots, n, \tag{5}$$

are called forward predictions for z_t. The backward predictions $\overleftarrow{z}_t$ are obtained by reversing the time series so that

$$\overleftarrow{z}_t = \sum_{i=1}^{m} \phi_i^* z_{t+i}, \qquad t = 1, 2, \ldots, n-m. \tag{6}$$

The MEM parameter estimates can be obtained by minimizing the sum of squared residuals,

$$S_m = \sum_{t=m+1}^{n} (z_t - \vec{z}_t)^2 + \sum_{t=1}^{n-m} (z_t - \overleftarrow{z}_t)^2. \tag{7}$$

This is equivalent to solving for Φ^* in

$$(\vec{Z}'\vec{Z} + \overleftarrow{Z}'\overleftarrow{Z})\,\Phi^* = \vec{Z}'\vec{Y} + \overleftarrow{Z}'\overleftarrow{Y}, \tag{8}$$

where

$$\vec{Z} = \begin{bmatrix} z_m & z_{m+1} & \cdots & z_1 \\ z_{m+1} & z_m & \cdots & z_2 \\ \vdots & \vdots & & \vdots \\ z_{n-1} & z_{n-2} & \cdots & z_{n-m} \end{bmatrix},$$

$$\overleftarrow{Z} = \begin{bmatrix} z_2 & z_3 & \cdots & z_{m+1} \\ z_3 & z_4 & \cdots & z_{m+2} \\ \vdots & \vdots & & \vdots \\ z_{n-m+1} & z_{n-m+2} & \cdots & z_n \end{bmatrix},$$

$$\vec{Y} = [z_{m+1} \quad z_{m+2} \quad \cdots \quad z_n]',$$

$$\overleftarrow{Y} = [z_1 \quad z_2 \quad \cdots \quad z_{n-m}]',$$

and

$$\Phi^* = [\phi_1^* \quad \phi_2^* \quad \cdots \quad \phi_m^*]'.$$

Regardless of whether a conventional method or the

MEM is used to estimate the parameters in the AR(m) process, the order m must first be specified. The usual procedure for order selection is an iterative one involving model identification, parameter estimation, and fairly subjective diagnostic checking of the residual autocorrelations. Instead, the optimal order selection process used here is based on Akaike's (1969) FPE criterion. For an n-point time series fitted to an AR(m) process, the FPE is defined by

$$\mathrm{FPE}_m = P_m(n + m + 1)/(n - m - 1), \tag{9}$$

where P_m is the mean squared residual, which in the case of the MEM is given by

$$P_m = S_m^*/[2(n - m)]. \tag{10}$$

Here S_m^* is the minimum sum of squared residuals defined by (7). The optimal order m is the one that minimizes the FPE. Barrodale and Erickson (1980a,b) recently developed an algorithm for determining m and estimating ϕ_i, $i = 1, 2, \ldots, m$, of the AR(m) model. (The numerical shortcomings of Burg's algorithm were first reported by Chen and Stegen 1974.) Barrodale and Erickson's algorithm, which is efficient and numerically stable, is adopted for this study.

2. EMPIRICAL RESULTS

The data used in this study are month's-end spot exchange rates collected from the *Wall Street Journal.* The time period for the data was primarily April 1973–June 1983; for the Canadian dollar and the French franc, the period began in October 1973 and April 1974, respectively. (In all cases, we used the 3 p.m. eastern standard time quote for the last business day of the month.) The period was characterized by flexible exchange-rate regimes for the currencies under consideration—the British pound, the Canadian dollar, the French franc, the Japanese yen, the Swiss franc, and the West German mark, all relative to the U.S. dollar. Plots of these time series data are shown in Figure 1.

Our test of the random walk hypothesis for exchange rates involves comparing the conditional expectations $E_{t-1}(X_t \mid X_{t-1})$ and $E_{t-1}(X_t \mid X_{t-1}, X_{t-2}, X_{t-3}, \ldots, X_{t-m-1})$ with X_t. Here $E_{t-1}(X_t \mid X_{t-1}, \ldots, X_{t-m-1})$ is the period $t - 1$ prediction of period t's spot rate X_t given that the market uses past spot-rate information in making its prediction. In contrast, $E_{t-1}(X_t \mid X_{t-1})$ is the period $t - 1$ prediction of period t's spot rate when the market chooses to ignore the information in past spot rates.

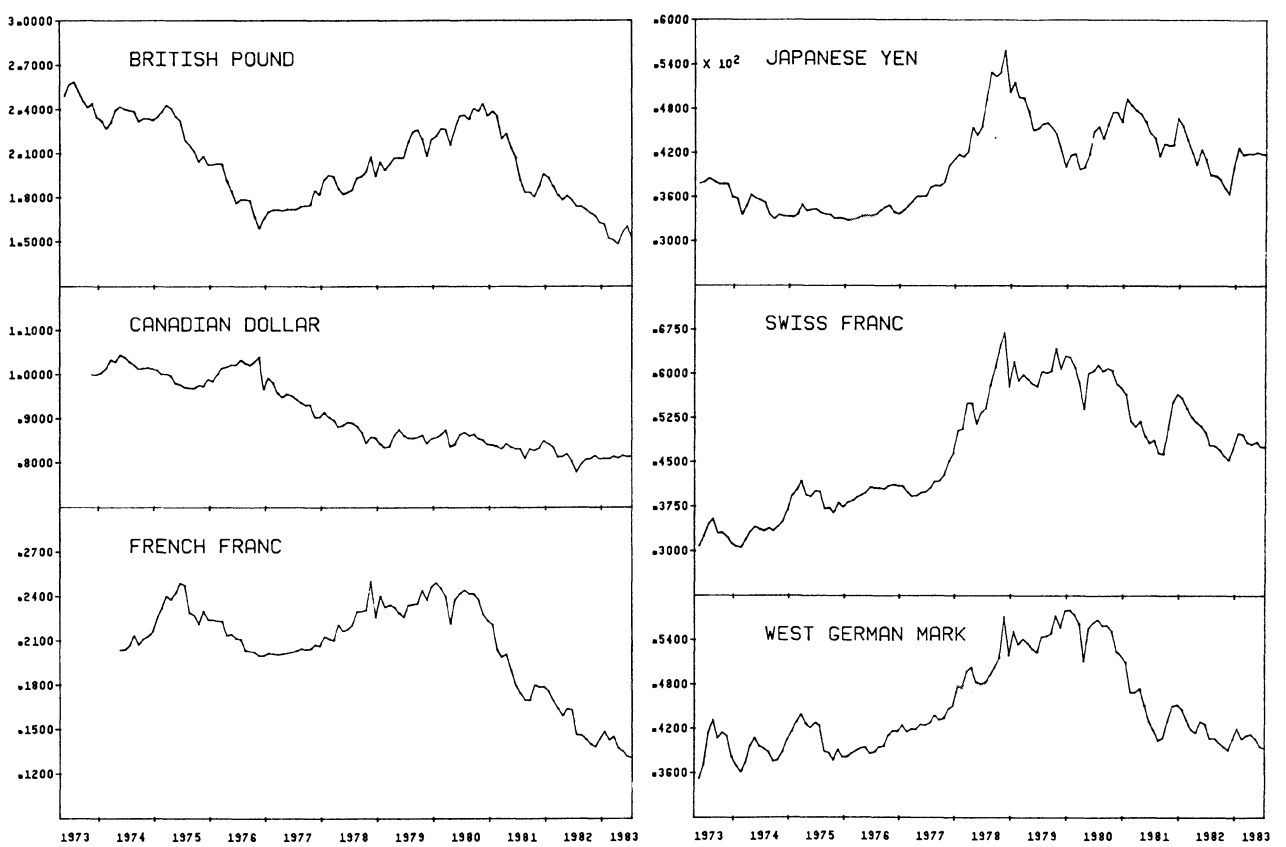

Figure 1. Plots of month's-end spot exchange rates for six currencies, relative to the U.S. dollar.

Although these conditional forecasts are unobservable, they can be estimated. In particular, $E_{t-1}(X_t \mid X_{t-1})$ is assumed to be the current spot rate X_{t-1} and $E_{t-1}(X_t \mid X_{t-1}, \ldots, X_{t-m-1})$ is estimated to be $\hat{X}_t$, where

$$\hat{X}_t = X_{t-1} + \hat{a}_0 + \sum_{i=1}^{m} \hat{a}_i(X_{t-1} - X_{t-i-1}). \qquad (11)$$

The order m and the parameter estimates $\hat{a}_i$ in (11) are determined by Akaike's criterion and the MEM, as described in Section 1.

To facilitate a comparison of the predictive ability of the AR(m) models with that of the random walk, we group the original time series of N monthly observations of spot rates, labeled $X_1, X_2, \ldots, X_N$, into $N - h + 1$ overlapping subseries of h terms each:

$$\begin{array}{l} X_1, X_2, \ldots, X_h; \\ X_2, X_3, \ldots, X_{h+1}; \\ \vdots \\ X_{N-h+1}, X_{N-h+2}, \ldots, X_N. \end{array}$$

Each of these $N - h + 1$ series is fitted to the AR(m) process

$$Z_t = a_0 + \sum_{i=1}^{m} a_i Z_{t-i} + \epsilon_t, \qquad (12)$$

by using the MEM, where $Z_t = X_t - X_{t-1}$. In doing so, we allow the model to be specified by the properties of the h terms of the particular subseries. The model specification could differ across all $n - h + 1$ subseries for each currency, so instead of searching for an optimal model specification for the entire sample period, we permit the model to be specified by the subseries data. Then for each series ending at X_L, where $L = h$ to N, we calculate $\hat{X}_{L+1}$ by using (11). The deviations of $\hat{X}_{L+1}$ and X_L from the observed spot rate X_{L+1} are expressed as proportional deviations,

$$U_L = (X_{L+1} - \hat{X}_{L+1})/X_{L+1} \qquad (13)$$

and

$$V_L = (X_{L+1} - X_L)/X_{L+1}, \qquad (14)$$

to standardize the results for the different currencies.

For a given N, the choice of the length of the base period h involves a trade-off between having sufficient data to fit the AR(m) processes for each subseries and having sufficient numbers of U_L's and V_L's ($L = h$ to N) for hypothesis testing. Different feasible values of h were tried, and we found the empirical evidence to be robust with respect to the choice of h. Thus there is no need to report our results for all attempted values of h. Instead, we will report only the results for $h = 36$. (We have also performed the same tests with logarithmic transformed spot-rate data. The results, which are very similar, are omitted here.)

The analysis described in Section 1 was used to determine the optimal AR(m) process (according to the criterion of minimum FPE) for each of the $N - h + 1$ subseries for each currency. Before presenting our results, it is worth noting that the analysis in Section 1 actually precludes the random walk—or in somewhat informal notation, the AR(0) process—from consideration. The procedure requires the solutions of the Yale–Walker equations in (2). Such equations do not exist for $m = 0$, and thus the optimal AR(m) processes obtained there are limited to $m \geq 1$. The computation of the AR(0) process is nevertheless a simple exercise via (7), (9), and (10) with $m = 0$. By comparing the value of this FPE and the FPE of the optimal AR(m) process for $m \geq 1$, we determined the optimal order m for $m \geq 0$.

Should the minimum FPE for each subseries of the data indeed correspond to the random walk, the prediction errors U_L and V_L (for $L = h$ to N), as defined by (13) and (14), respectively, would be the same. Such evidence would support the hypothesis that spot rates evolve as a random walk. The empirical results in Table 1, however, which show the distributions of the optimal order m of the AR(m) processes for $m \geq 0$ for all six currencies, indicate that the AR(m) processes with $m \geq 1$ actually fit each subseries better than did the random walk. Over the entire sample period, the time series of spot-rate changes in the majority of cases are best described as some AR(1) or AR(2) processes. But this evidence should not be interpreted as implying the rejection of the random walk hypothesis. This evidence merely indicates that an AR model provides a better description of the autocorrelation structure of each subseries of spot-rate changes than does the random walk. Of course, if the autocorrelation structure estimated from a subseries of data did indeed enable a better prediction of the future spot rate, the random walk hypothesis would be rejected. Otherwise, the hy-

Table 1. The Distribution of the Optimal Order m for the AR(m) Process for h = 36

Currency	AR(0)*	AR(1)	AR(2)	AR(3)	AR(4)	AR(5)	AR(6)	AR(7)	AR(8)	AR(9)	Total
British pound	0	71	11	1	4	0	0	0	0	0	87
Canadian dollar	0	39	35	3	3	1	0	0	0	0	81
French franc	0	60	8	2	1	2	1	0	0	1	75
Japanese yen	0	73	6	5	0	1	2	0	0	0	87
Swiss franc	0	67	12	3	4	1	0	0	0	0	87
West German mark	0	77	6	1	3	0	0	0	0	0	87

* Random walk

pothesis would not be rejected. This could be because the estimated autocorrelation structure according to some AR models was overshadowed by the inherent noise of the time series, or such a structure failed to persist beyond the estimation period, or both. To test the random walk hypothesis, we compared the prediction errors U_L and V_L for $L = h$ to N; our results are as follows.

Figure 2 shows, for all six currencies, the difference of absolute prediction errors $|U_L| - |V_L|$ for $L = h$ to N. A casual inspection of these plots tends to indicate that on average, the random walk did marginally better than or at least as well as the AR models in predicting spot rates. To provide more formal statements about the relative predictive ability of the random walk and the AR models, the following tests were performed. For each of the six currencies, the Wilcoxon signed rank test was first performed on the $N - h + 1$ pairs of observations $(|U_h|, |V_h|)$, $(|U_{h+1}|, |V_{h+1}|)$, ..., $(|U_N|, |V_N|)$ of the bivariate random variables, say, (u, v). The null hypothesis that the median of the distribution of $u - v$ is greater than or equal to zero was tested against the alternative hypothesis that it is less than zero. In this test, the failure to reject the null hypothesis implies that past spot rates are irrelevant for predicting the future spot rate, or in other words, that

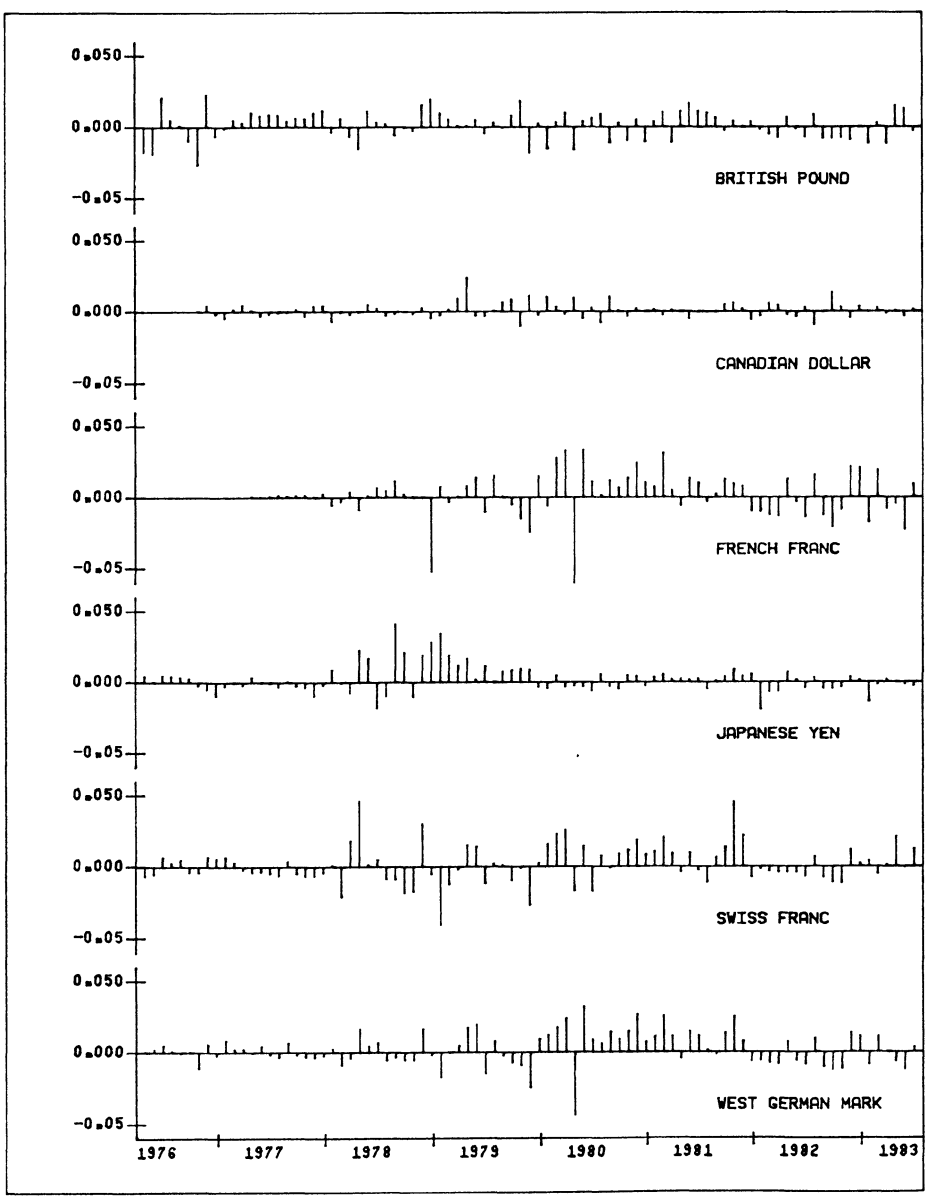

Figure 2. Plots of the difference of absolute prediction errors $|U_L| - |V_L|$ (U, time series; V, random walk) for $h = 36$.

Table 2. Wilcoxon Signed Rank Test Results and MAD's for h = 36

Currency	Wilcoxon Signed Rank Test Statistic[a]	Probability of Error in Rejecting H_0[b] (%)	MAD(U)	MAD(V)
British pound	1.325	90.7	.02631	.02518
Canadian dollar	.647	74.1	.01247	.01171
French franc	1.568	94.2	.02683	.02536
Japanese yen	1.282	90.0	.02960	.02751
Swiss franc	.681	75.2	.02628	.02401
West German mark	1.913	97.2	.03058	.02903

[a] The test is based on N − h + 1 pairs of observations $(|U_h|, |V_h|), (|U_{h+1}|, |V_{h+1}|), \ldots, (|U_N|, |V_N|)$ of the bivariate variables $(\mathcal{U}, \mathcal{V})$.

[b] For H_0, the median of the distribution of $\mathcal{U} - \mathcal{V}$ is nonnegative; for H_1, the median of the distribution of $\mathcal{U} - \mathcal{V}$ is negative.

NOTE: U = time series. V = random walk.

spot rates can be described to behave as a random walk.

The Wilcoxon signed rank test results for all six currencies, as shown in Table 2, indicate that the null hypothesis—or equivalently, the random walk hypothesis—cannot be rejected at any reasonable level of significance. The probabilities of error in rejecting the hypothesis are in the mid-70% range for the Canadian dollar and the Swiss franc and are at least 90% for the four remaining currencies. Support for the hypothesis becomes even stronger when the empirical results for all six currencies are evaluated collectively. To demonstrate this, Table 2 also shows the mean absolute deviations (MAD's) of the U_L's and V_L's based on $N - h + 1$ observations, that is,

$$\mathrm{MAD}(U) = \frac{1}{N - h + 1} \sum_{L=h}^{N} |U_L|, \tag{15}$$

and

$$\mathrm{MAD}(V) = \frac{1}{N - h + 1} \sum_{L=h}^{N} |V_L|. \tag{16}$$

For all six currencies, MAD(U) is consistently greater than MAD(V). The Wilcoxon signed rank test on the six pairs of observations of MAD(U) and MAD(V) yields a test statistic $\sum_{i=1}^{n} R_i = 21$ (where $n = 6$ and $R_i = i$) based only on the positive signed ranks. To reject the null hypothesis that the random walk predicts better or as well as the AR models, as opposed to the alternative hypothesis that it predicts worse, has virtually a 100% probability of error. This empirical evidence indicates that the autocorrelation structure of spot-rate changes estimated from past data did not, on average, lead to any better prediction of spot rates. Thus spot rates can be said to evolve as a random walk. (One might also conclude that the AR model simply does not capture the true dynamics of past exchange rates. In other words, it is the model that is suspect; hence so is the random walk conclusion. But it is always the case in the literature that one is testing a joint hypothesis, namely, the adequacy of the underlying model and the conclusions derived from it. Therefore, our conclusion is really conditional on the acceptability of the AR model.)

3. CONCLUSION

The purpose of this study has been to test the random walk hypothesis for spot exchange rates. We compared the ability of the current spot rate to predict the future spot rate relative to a univariate autoregressive model of past exchange rates. This AR model was estimated by maximum entropy spectral analysis using Akaike's (1969) FPE criterion. This estimation procedure yields some nice properties in comparison to more standard estimation procedures. In particular, the order of the model is determined by the data instead of being specified a priori. The order of the model is allowed to change over the data period. The model can be estimated from a reasonably short time series, attenuating the problem of structural change in the underlying series, without the attendant difficulties of truncation or periodic extension of the data. Although these properties cannot, of course, validate the model, they enhance confidence in our conclusions.

Comparing the predictive ability of the estimated autogressive model against that of the current spot exchange rate for six currencies, all relative to the U.S. dollar, we conclude that spot exchange rates evolve as a random walk. Therefore, this study confirms, by alternative means, some of the recent findings about exchange-rate dynamics.

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