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SPOT AND FORWARD EXCHANGE RATES: A CAUSALITY ANALYSIS

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INTRODUCTION

The purpose of this paper is to establish empirically the causality relationships between forward and spot exchange rates using the notion of Granger (1969) causality. Intuitively, spot rates cause forward rates in the sense of Granger if past spot rate information is useful in predicting future forward rates. Similarly, forward rates cause spot rates if past forward rates have predictive content with respect to future spot rates.

The rationale for estimating the causality relationships between spot and forward exchange rates is twofold. First, there is an extensive literature in international finance which is concerned with predicting future exchange rates for exchange rates management purposes.[1] In the context of this literature, we want to see if causality analysis yields superior exchange rate predictions at least in comparison to standard univariate time-series analysis. Indeed, from the theory of exchange rate determination (see, for example, Grauer, Litzenberger and Stehle, 1976) we know that forward rates — or at least the current forward rate — contain information about future spot rates.[2] Therefore, we should expect to find that forward rates cause spot rates.[3] That is, forward rates are useful in predicting future spot rates.

Second, there exists a related literature which is concerned with the efficiency of foreign exchange markets.[4] According to this literature, in an efficient market, the information conveyed by past spot rates should be fully impounded in future spot and forward rates. In particular then, spot rates should not prove useful in predicting forward rates, or alternatively, spot rates should not cause forward rates. The remainder of this paper is devoted to using causality analysis to examine these two issues empirically, namely, do forward rates cause spot rates and do spot rates not cause forward rates.

The following section defines Granger causality more rigorously, and the third section introduces the models and procedures for determining causality relationships. The fourth section describes the data and empirical results. The final section concludes the paper.

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DEFINING CAUSALITY

The concept of Granger causality stems from the simple observation that the future cannot cause the present or the past. Consider the bivariate stochastic process $\{S_t, F_t\}$ where S_t denotes the spot rate and F_t the forward rate at time t. Suppose that at time n we are trying to predict S_{n+1}. Then forward rates are said to cause spot rates provided S_{n+1} is better predicted by adding past forward rates to past spot rates than by using the past spot rate series alone. Similarly, spot rates cause forward rates if F_{n+1} is better predicted by the bivariate series of past spot rates and forward rates than by using past forward rates alone. Nothing in this definition precludes forward and spot rates from causing each other and, in that case, the two time series are said to exhibit feedback.

Causality can be defined more rigorously in terms of information sets. Let X_t denote a stationary stochastic process with components $\{F_t, S_t\}$. Let $\bar{X}_t$, $\bar{F}_t$, $\bar{S}_t$ denote the set of past values of X_t, F_t, S_t *before* time t, respectively. Let $\bar{X}_t - \bar{F}_t$ denote the set of elements in $\bar{X}_t$ and not in $\bar{F}_t$. Define $\alpha^2(F_t/\bar{X}_t)$ to be the minimum mean square linear prediction error of F_t given the information set $\bar{X}_t$. We can now define Granger causality formally.

Definition: (Causality) If $\alpha^2(F_t/\bar{X}_t) < \alpha^2(F_t/\bar{X}_t - \bar{S}_t)$ — that is, the information set which includes past spot rate data yields a more accurate prediction of forward rates than in the same information set without past spot rate data — in the mean squared error sense, then spot rates are said to cause forward rates.

Definition: (Feedback) If $\alpha^2(F_t/\bar{X}_t) < \alpha^2(F_t/\bar{X}_t - \bar{S}_t)$ and $\alpha^2(S_t/\bar{X}_t) < \alpha^2(S_t/\bar{X}_t - \bar{F}_t)$ then feedback between forward rates and spot rates is said to occur.

The relationship between market efficiency and Granger causality is reasonably obvious from the above definition. If markets are efficient (semi-strong) then knowledge of past spot rates should have no bearing on predicting today's forward rate. In fact, if markets are efficient, $\alpha^2(F_t/\bar{X}_t)$ should be equal to $\alpha^2(F_t/\bar{X}_t - \bar{S}_t)$ since the information sets are the same except for the publicly available information contained in past spot rates. But, at the same time, according to exchange rate theory, past forward rates will incorporate relevant information about market expectations of future spot rates. Thus, one should expect that adding past forward rates to the information set will yield better predictions of future spot rates, i.e., $\alpha^2(S_t/\bar{X}_t) < \alpha^2(S_t/\bar{X}_t - \bar{F}_t)$.

MODEL

It is well known (e.g., Masani, 1966) that a regular full rank stationary stochastic process $\{St, Ft\}$ can be modelled, under fairly general conditions, by the autoregressive representation

$$F_t = \psi_{11}(L)F_t + \psi_{12}(L)S_t + v_t, \quad (1)$$

$$S_t = \psi_{21}(L)F_t + \psi_{22}(L)S_t + u_t. \quad (2)$$

Here, L denotes the lag operator $L^kF_t = F_{t-k}$ and $\psi_{ij}(L)$ is the lag polynomial $\Sigma^{\infty}{}_{k=1}\ \psi_{ijk}L^k$. The $\{v_t, u_t\}$ are zero mean white noise innovations with constant variance-covariance matrix.

Causal relationships enter the model in a very natural way. If $\psi_{12}(L)\equiv 0$ (i.e., $\psi_{12k} = 0$ for all k), then it is clear from equation (1) that spot rates have no effect on predicting forward rates, that is, spot rates do not cause forward rates. Similarly, if $\psi_{21}(L)\equiv 0$, forward rates do not cause spot rates. Thus, in theory, one could determine the causal relationships between spot and forward rates by first of all fitting equations (1) and (2) by least squares — yielding estimates which are consistent and asymptotically normally distributed — and then testing to see if $\psi_{ij}(L) = 0$ for $i \neq j$. This approach is problematic, however, because the test of $\psi_{ij}(L) = 0$ is generally quite sensitive to the order of the lags of the $\psi_{ij}(L)$. (See Hsiao, 1979a and 1979b). Instead, following Hsiao we will use the data itself to determine the lag structure. Specifically, the optimal order to the lag for each $\psi_{ij}(L)$ in each of equations (1) and (2) is determined using Akaike's (1969a and 1969b) Final Prediction Error (FPE) criterion as described below.

The FPE Criterion

The *FPE* (of F_t) is defined to be the (asymptotic) mean squared prediction error

$$E(F_t - \hat{F}_t)^2 \quad (3)$$

where $\hat{F}_t$ is the predictor of F_t given by the least square estimates

$$\hat{F}_t = \hat{b} + \hat{\psi}^m_{11}(L)F_t + \hat{\psi}^n_{12}(L)S_t. \quad (4)$$

The superscripts m and n denote the order lags of $\psi_{11}(L)$ and $\psi_{12}(L)$ and $\psi_{12}(L)$, respectively, where m and n are bounded above by the maximum lag order investigated, say Q. $\hat{\psi}^m_{11}(L)$, $\hat{\psi}^n_{12}(L)$ and $\hat{b}$ are the least squares estimates of $\psi^m_{11}(L)$, $\psi^n_{12}(L)$, and the constant term b, respectively. Akaike estimates the Final Prediction Error by

$$FPE_F(m,n) = \frac{\mathrm{T+m+n+1}}{\mathrm{T-m-n-1}}\left[\sum_{t=1}^{\mathrm{T}} (F_t - \hat{F}_t)^2/T\right] \quad (5)$$

By choosing the lag structure with minimum *FPE,* Akaike's criterion tries to balance the bias from choosing too small a lag order against the increased variance from a higher lag order specification. More specifically, Akaike has shown that, if the dependence between ψ_{ij} and recent values of the variables (in our case spot and forward rates) decreases as the length of past history increases, the *FPE* is comprised of two components. The first component is due to the *FPE* of the best linear prediction for *given m* and *n* while the second component is due to the statistical deviations of $\hat{\psi}^{m}_{11}(L)$ and $\hat{\psi}^{n}_{12}(L)$ from $\psi^{m}_{11}(L)$ and $\psi^{n}_{12}(L)$. Generally, as m and n are increased, the first component decreases whereas the second component increases for a finite length of observations of spot and forward rates.

The *FPE* approach yields a number of distinct benefits in terms of identifying the model. Firstly, as we already pointed out, the data itself is used to determine the lag structure rather than presupposing some arbitrary lag order specification. Secondly, the *FPE* criterion does not constrain the lag structure of each variable to be identical, i,e,. in general, $m \neq n$. Thirdly, it has been shown that the *FPE* criterion is equivalent to choosing the model specification on the basis of an F test with varying significance levels.[5] Thus, rather than specifying an *ad hoc* significance level of 5 or 10 percent, the choice of whether to include a variable is determined by an explicit optimality criterion, namely, minimizing the mean square prediction error.[6] Specifically, suppose that $FPE_F(m,n) < FPE_F(m+p,\ n+q)$ so that (m,n) is chosen as the optimal lag structure for $\psi_{11}(L)$ and $\psi_{12}(L)$.[7] This is equivalent to choosing (m,n) on the basis of an approximate F statistic. A formal discussion of this point can be found in Hsiao (1981, pp. 89–90).

Hsiao's Approach

In addition to identifying the model, the *FPE* criterion can be used to determine causal relationships directly. Suppose we are to test if spot rates cause forward rates. Then Hsiao (1979a) has suggested the following sequential procedure. First, the *FPE* criterion is used to determine the optimal order of the univariate autoregressive process for forward rates alone. Call this order q, so that the resulting *FPE* is $FPE_F(q,0)$. Second, fix the lag structure for the forward rate at q and use the *FPE* criterion to specify equation (1). Let r be the resulting order for the lag operator $\psi_{12}(L)$ so that the *FPE* is $FPE_F(q,r)$. Third, holding the order of the lag operator $\psi_{12}(L)$ at r, let the order of lag operator $\psi_{11}(L)$ vary from 0 to q. Choose the order of $\psi_{11}(L)$ that gives the smallest *FPE*, say s – s is not necessarily equal to q – thereby yielding $FPE_F(s,r)$. This third step is a check to see whether the lag structure for forward rates (which was orginally q) is sensitive to the lag structure of the manipulated variable, that is, the spot rates.[8] Finally, if $FPE_F(q,0) \leq FPE_F(s,r)$ then forward rates are best represented by a one-dimensional autoregressive process so that spot rates do not cause forward rates. Conversely, if $FPE_F(q,0) > FPE_F(s,r)$ then we would conclude that spot rates cause forward rates. A similar approach would be used on equation (2) to test if forward rates cause spot rates.

It is worth noting that this FPE procedure for testing causality directly is very much consistent with the Granger definition of causality. In both cases, the criterion for causality is to minimize the mean square prediction error. Furthermore, as we have already noted, minimizing the FPE involves a tradeoff between the size and power of the test. The usual alternative approach, involving standard hypothesis testing, arbitrarily specifies the size (i.e., the significance level) of the test. The latter need bear no relationship to the underlying Granger criterion of minimizing the mean square prediction error. However, if one is interested primarily in the true model structure of the relationships between spot and forward exchange rates, an alternative criterion based on determining and identifying which of the models has the highest probability of being the true model structure would be more appropriate. Yet for testing causality in the Granger sense, the minimization of the forecast error should be the yardstick in choosing the most appropriate model. Therefore, the FPE criterion which is based explicitly on minimizing the forecast error is an appropriate choice here.

THE EMPIRICAL RESULTS

Causality Relationships

The data base is comprised of month-end non-overlapping spot rates and one month forward rates for six foreign currencies against the US dollar. All data were collected from the Wall Street Journal. The data period for model estimation is from September 1977, to August 1981. An additional six data points for each currency, from September 1981, to February 1982, were held back for testing forecasting accuracy. To reduce the problem of serial correlation in the time series, first differences in the logarithms of both spot and forward data were taken. The model [Equations (1) and (2)] was then estimated for each foreign currency. $Q = 14$ was the *a priori* specified maximum possible lag for each variable.

Table 1 lists the *FPE* for spot (S) and forward (F) rates of the pound, the yen, the French franc, the Canadian dollar, the Swiss franc, and the West German marc, where each variable is treated as a one-dimensional autoregressive process. The smallest *FPE* for each currency and each variable is underlined. Thus, for example, a univariate autoregressive model of lag 2 in the spot rates yields the smallest *FPE* of 0.2448×10^{-3} in the case of the Canadian dollar.

Table 2 presents the *FPE* of the spot and forward rates using the bivariate processes. The term controlled variable refers to the variables on the left hand side of equations (1) and (2). These variables are controlled in the sense that their lag structure is fixed at the optimum lag determined in Table 1 (Step 2 of the Hsiao procedure). That optimum lag is indicated in parentheses in column 2 beside each controlled variable. The *FPE* of the controlled variable is computed by varying the order of the manipulated (independent) variable from one to fourteen. Column 4 in Table 2 specifies the lag for the manipulated

Table 1

FPE of Fitting a One-Dimensional Autoregressive Process for Spot and Forward Rates

The Order Of Lags	Pound FPE of $Sx10^2$	Pound FPE of $Fx10^2$	Yen FPE of $Sx10^2$	Yen FPE of $Fx10^2$	French Franc FPE of $Sx10^2$	French Franc FPE of $Fx10^2$
1	0.1455	0.1385	0.1507	0.1477	0.1347	0.1314
2	0.1503	0.1422	0.1607	0.1576	0.1375	0.1332
3	0.1597	0.1509	0.1715	0.1681	0.1428	0.1376
4	0.1695	0.1600	0.1742	0.1698	0.1513	0.1447
5	0.1428	0.1361	0.1804	0.1769	0.1551	0.1474
6	0.1491	0.1426	0.1877	0.1838	0.1652	0.1573
7	0.1581	0.1506	0.1920	0.1866	0.1768	0.1681
8	0.1695	0.1614	0.2054	0.1996	0.1893	0.1801
9	0.1812	0.1722	0.2176	0.2116	0.2025	0.1927
10	0.1888	0.1796	0.2293	0.2225	0.2177	0.2071
11	0.1650	0.1588	0.2454	0.2385	0.1942	0.1801
12	0.1742	0.1680	0.2568	0.2468	0.2074	0.1907
13	0.1842	0.1761	0.2121	0.2035	0.2162	0.1987
14	0.1982	0.1903	0.2193	0.2097	0.2326	0.2149

The Order Of Lags	Canadian Dollar FPE of $Sx10^3$	Canadian Dollar FPE of $Fx10^2$	Swiss Franc FPE of $Fx10^2$	Swiss Franc FPE of $Fx10^2$	Mark FPE of $Sx10^2$	Mark FPE of $Fx10^2$
1	0.2571	0.2514	0.1705	0.1652	0.1510	0.1452
2	0.2448	0.2363	0.1818	0.1761	0.1574	0.1507
3	0.2505	0.2431	0.1919	0.1848	0.1646	0.1568
4	0.2482	0.2394	0.2048	0.1974	0.1746	0.1657
5	0.2465	0.2344	0.2180	0.2098	0.1818	0.1726
6	0.2590	0.2469	0.2320	0.2237	0.1944	0.1846
7	0.2631	0.2547	0.2484	0.2394	0.2081	0.1977
8	0.2779	0.2659	0.2628	0.2519	0.2220	0.2111
9	0.2983	0.2955	0.2809	0.2689	0.2373	0.2255
10	0.3058	0.2917	0.3019	0.2890	0.2461	0.2347
11	0.3290	0.3136	0.2772	0.2664	0.2054	0.1934
12	0.3268	0.3171	0.2866	0.2725	0.2149	0.1982
13	0.3517	0.3382	0.3103	0.2951	0.2304	0.2122
14	0.3673	0.3464	0.3230	0.3079	0.2334	0.2176

variable which yields the minimum *FPE* for the controlled variable. The minimum *FPE* is given in the last column of Table 2.

Tables 1 and 2 correspond to the first two steps of the Hsiao sequential procedure for testing causality. The third step — determining if the lag structure

Table 2

The Optimum Lags of the Manipulated Variable and the *FPE* of the Controlled Variable

Currency (1)	*Controlled Variable* (2)	*Manipulated Variable* (3)	*The Optimum Lag of Manipulated Variable* (4)	$FPE \times 10^2$ (5)
Pound	S(5)	F	1	0.1464
	F(5)	S	1	0.1402
Yen	S(1)	F	1	0.1594
	F(1)	S	1	0.1565
French Franc	S(1)	F	1	0.1437
	F(1)	S	1	0.1402
Canadian Dollar	S(2)	F	2	0.02284
	F(2)	S	2	0.02316
Swiss Franc	S(1)	F	1	0.1620
	F(1)	S	1	0.1575
Mark	S(1)	F	1	0.1375
	F(1)	S	1	0.1339

of the controlled variable is sensitive to the lag structure of the manipulated variable — changed the minimum *FPE* only in the case of the Canadian dollar. Specifically, when *F* is the controlled variable and *S* is fixed at a lag of 2, the optimum lag structure for *F* is reduced from 5 to 2.

Causal relationships are determined by comparing the minimum *FPE* of the univariate process (Table 1) with the appropriate minimum *FPE* for the bivariate process which, with the exception of the Canadian dollar noted above, is listed in Table 2. For example, in the case of the pound, the minimum *FPE* for the univariate spot process is 0.1428×10^{-2}. However, by adding the forward rate (which enters optimally with a one period lag) to the spot rate process, the minimum *FPE* of the spot rate is 0.1464×10^{-2}. Since the univariate process yields a lower *FPE*, we conclude that forward rates do not cause spot rates in the case of the pound. Taking another example, consider the Swiss franc. The univariate process for the forward rate yields a minimum *FPE* of 0.1652×10^{-2} at a lag of one. The bivariate process for the forward rate, on the other hand, gives a minimum *FPE* of 0.1575×10^{-2} where spot and forward rates enter the equation with a lag of one each. Since the overall minimum FPE is obtained by the bivariate process, we conclude that spot rates cause forward rates in the case of the Swiss franc.

A similar analysis for each currency provides the following overall results.

For three currencies, the pound, the yen, and the French franc there are no causal relationships, i.e., forward and spot rates are unrelated. This is in contradistinction to the remaining three currencies, the Canadian dollar, Swiss franc, and mark for which feedback is exhibited, i.e., forward and spot rates cause each other.

The upshot of these results is that foreign exchange markets do not appear to be efficient in general. This conclusion is clear at least for the Canadian dollar, Swiss franc, and mark since in these cases spot rates cause forward rates. In an efficient market, the information conveyed by past spot rates should already be incorporated in the forward rate so that no causality from spot rates to forward rates should be observed.

While foreign exchange markets do not appear to be inefficient in the case of the pound, yen and French franc, they do not yield expected results. We have expected from the theory of forward exchange that forward rates should cause spot rates. Perhaps, we obtain these anomalous results because forward rates are unsystematically biased estimates of future spot rates so that causality from forward rates to spot rates may be difficult to observe.[9]

It is also worth noting that, for the yen, the French franc, the Swiss franc and the mark, the univariate spot and univariate forward rate processes appear to be random walks.[10] But, in at least two cases — the Swiss franc and the mark — the random walk characteristic of spot rates is deceptive. Rather, the underlying model is basically bivariate in nature with spot and forward rates entering with a one period lag each.[11] Therefore, the commonly found result that exchange rates are random walks may simply reflect, at least for some currencies, misspecification of the underlying model which is bivariate and not univariate.

Forecast Accuracy

As we noted earlier, some of the data — from September 1981 to February 1982 — were held back to test the forecasting accuracy of the causality models. The models are described more fully in Table 3 for the pound, yen, and French franc and in Tables 4 and 5 for the Canadian dollar, Swiss franc and mark. Since the currencies in Table 3 did not exhibit any causality relationships, each equation for spot and forward rates represents a univariate autoregressive model which was estimated by ordinary least squares.

The currencies in Table 4 are far more interesting because they exhibited feedback. Therefore, two potential models could be estimated for each of these latter currencies. The first model [equations (12)–(17)] as shown in Table 4 is simply the univariate autoregressive scheme determined in Table 1 and estimated by ordinary least squares. The second model [equations (18)–(23)] as shown in Table 5 is based on the causality relationships described in Table 2 (except for the Canadian dollar which required the additional step mentioned above). Given the causality relationships, this second model is simultaneous

Table 3

Univariate Autoregressive Time Series Models Estimated by OLS for the Pound, Yen and French Franc

Pound

$$S_t = \underset{(0.0060)}{-0.00532} \underset{(0.1627)}{-0.03022\, S_{t-1}} \underset{(0.1613)}{+0.13659\, S_{t-2}} \quad (6)$$

$$\underset{(0.1767)}{+0.08447\, S_{t-3}} \underset{(0.1841)}{+0.21865\, S_{t-4}} \underset{(0.1808)}{+0.48641\, S_{t-5}}$$

$SER = 0.03329 \qquad DW = 2.0003$

$$F_t = \underset{(0.0059)}{-0.00527} \underset{(0.1627)}{-0.026111\, F_{t-1}} \underset{(0.1610)}{+0.14666\, F_{t-2}} \quad (7)$$

$$\underset{(0.1772)}{+0.09033\, F_{t-3}} \underset{(0.1850)}{+0.22515\, F_{t-4}} \underset{(0.1805)}{+0.47487\, F_{t-5}}$$

$SER = 0.03249 \qquad DW = 1.9754$

Yen

$$S_t = \underset{(0.0065)}{-0.0045} \underset{(0.1644)}{-0.00274\, S_{t-1}} \quad (8)$$

$SER = 0.03649 \qquad DW = 1.7774$

$$F_t = \underset{(0.0070)}{-0.004668} \underset{(0.1700)}{-0.01389\, F_{t-1}} \quad (9)$$

$SER = 0.03603 \qquad DW = 1.7822$

French Franc

$$S_t = \underset{(0.0063)}{-0.01030} \underset{(0.1611)}{-0.13981\, S_{t-1}} \quad (10)$$

$SER = 0.03441 \qquad DW = 1.5979$

$$F_t = \underset{(0.0062)}{-0.01046} \underset{(0.1613)}{-0.1188\, F_{t-1}} \quad (11)$$

$SER = 0.03398 \qquad DW = 1.5923$

Note: Numbers in parentheses are standard errors of the estimate;
SER = Standard Error of the Regression;
DW = Durbin-Watson

in nature so that the parameters were re-estimated by a full information maximum likelihood technique.[12]

Table 6 presents summary statistics of the forecast accuracy of each of the models described in Tables 3, 4 and 5. Forecast accuracy is measured by both

Table 4

Univariate Autoregressive Time Series Models Estimated By OLS for the Canadian Dollar, Swiss Franc, and Mark

Canadian Dollar

$$S_t = \underset{(0.0025)}{0.00134} \; \underset{(0.1802)}{-0.19185} S_{t-1} \; \underset{(0.1863)}{-0.35461} S_{t-2} \qquad (12)$$

$SER = 0.01443 \qquad DW = 2.0709$

$$F_t = \underset{(0.0025)}{-0.00164} \; \underset{(0.1863)}{-0.22643} F_{t-1} \; \underset{(0.2009)}{-0.45284} F_{t-2}$$

$$\underset{(0.1969)}{-0.10816} F_{t-3} \; \underset{(0.1822)}{-0.23605} F_{t-4} \; \underset{(0.1807)}{+0.28404} F_{t-5} \qquad (13)$$

$SER = 0.01349 \qquad DW = 1.9198$

Swiss Franc

$$S_t = \underset{(0.0060)}{-0.01026} \; \underset{(0.1400)}{-0.32821} S_{t-1} \qquad (14)$$

$SER = 0.03933 \qquad DW = 0.7973$

$$F_t = \underset{(0.0068)}{-0.01050} \; \underset{(0.1409)}{-0.34617} F_{t-1} \qquad (15)$$

$SER = 0.03811 \qquad DW = 1.7719$

Mark

$$S_t = \underset{(0.0066)}{-0.00955} \; \underset{(0.1614)}{-0.21299} S_{t-1} \qquad (16)$$

$SER = 0.03643 \qquad DW = 1.7043$

$$F_t = \underset{(0.0064)}{-0.00975} \; \underset{(0.1602)}{-0.22756} F_{t-1} \qquad (17)$$

$SER = 0.03572 \qquad DW = 1.6927$

the root mean square error and Theil's U coefficient.[13] There are three tentative conclusions to be derived from Table 6. First, predictive accuracy does not vary substantially from one model to another or even from one currency to another. Second, on a root mean square error basis, the forecast accuracy of the forward rate is better than that of the spot rate. However, this result does not generalize to Theil's U coefficient. Third, and most importantly, the relative forecast accuracy of the two models in the case of the causally related currencies seems to be a direct function of the strength of the causal relationship. Consider spot rates first. The minimum *FPE* of the univariate model is seven percent, five percent and nine percent higher than the overall minimum

Table 5

Bivariate Autoregressive Time Series Models Estimated by FIML for the Canadian Dollar, Swiss Franc and Mark

Canadian Dollar

$$S_t = \underset{(0.0022)}{-0.00194} \underset{(1.6882)}{+3.05205}\, S_{t-1} \underset{(1.6160)}{+2.93648}\, S_{t-2} \underset{(1.7059)}{-3.40411}\, F_{t-1} \underset{(1.6345)}{-3.19665}\, F_{t-2} \quad (18)$$

$$F_t = \underset{(0.0022)}{-0.00194} \underset{(1.6938)}{+3.12845}\, S_{t-1} \underset{(1.6214)}{+2.67168}\, S_{t-2} \underset{(1.7115)}{-3.42265}\, F_{t-1} \underset{(1.6399)}{-2.91920}\, F_{t-2} \quad (19)$$

Swiss Franc

$$S_t = \underset{(0.0063)}{-0.01050} \underset{(2.0696)}{+3.83902}\, S_{t-1} \underset{(2.0814)}{-4.19928}\, F_{t-1} \quad (20)$$

$$F_t = \underset{(0.0062)}{-0.01055} \underset{(2.0408)}{+4.05326}\, S_{t-1} \underset{(2.0524)}{-4.41446}\, F_{t-1} \quad (21)$$

Mark

$$S_t = \underset{(0.0059)}{-0.01044} \underset{(3.0433)}{+7.03253}\, S_{t-1} \underset{(3.0816)}{-7.34492}\, F_{t-1} \quad (22)$$

$$F_t = \underset{(0.0058)}{-0.01043} \underset{(3.0032)}{+6.84643}\, S_{t-1} \underset{(3.0409)}{-7.15228}\, F_{t-1} \quad (23)$$

FPE for the Canadian dollar, Swiss franc, and mark, respectively.[14] Thus, since the mark exhibits the strongest causal relationship, the bivariate model predicts better for the mark spot rate than does the univariate model in terms of both error metrics. On the other hand, since causality is somewhat less evident for the dollar and Swiss franc, this could explain why the univariate model is more accurate than the bivariate model for these two currencies.

A similar result holds in the case of forward rates. For forward rates, the minimum *FPE* of the univariate model is one percent, five percent and eight percent greater than the overall minimum *FPE* for the Canadian dollar, Swiss franc, and mark, respectively. Thus, the bivariate model predicts better for the Swiss franc and mark which show the stronger causal relationships. On the other hand, the univariate model is more accurate for the Canadian dollar which exhibits the weakest causal relationship. This last conclusion is only suggestive, however, since the causality relationships are somewhat marginal even in the case of the mark.

Table 6

Summary Statistics of Forecast Accuracy

Univariate Model		*Pound*	*French Franc*	*Yen*
RMSE	*S*	0.12958	0.07734	0.10218
	F	0.12046	0.07263	0.06896
Theil's *U*	*S*	1.70747	0.99298	0.99444
	F	1.64211	1.00729	0.96837
Univariate Model		*Canadian Dollar*	*Swiss Franc*	*Mark*
RMSE	*S*	0.04223	0.14078	0.09209
	F	0.04007	0.13726	0.09010
Theil's *U*	*S*	1.17442	1.06989	1.07694
	F	1.15566	1.07162	1.07798
Bivariate Models		*Canadian Dollar*	*Swiss Franc*	*Mark*
RMSE	*S*	0.07524	0.14490	0.08948
	F	0.07313	0.13497	0.08730
Theil's *U*	*S*	2.09243	1.10120	1.04642
	F	2.10915	1.05374	1.04433

Note: RMSE = Root Mean Square Error

CONCLUSION

We argued from the theory of exchange rate determination and the theory of market efficiency that forward rates should cause spot rates but not the reverse. Utilizing monthly spot and forward rates for six foreign currencies, we were able to test for causal relationships using the Hsiao methodology. Specifically, the underlying models were assumed to be bivariate autoregressive. The lag structure of the models were estimated and causal relationships determined by Akaike's Final Prediction Error criterion. The overall results indicate that foreign exchange markets are apparently inefficient. In particular, feedback was exhibited for the Canadian dollar, Swiss franc, and mark. Yet, in an efficient market, spot rates should not cause forward rates since this implies that the publicly available information conveyed by past spot rates is not impounded in the forward rate.

Another important conclusion to be derived from the causality analysis is that exchange rates, spot and forward, do not necessarily follow a random walk. For three currencies, the Canadian dollar, the Swiss franc and the mark, both spot and forward rates are better described by bivariate rather than univariate processes.

In addition to determining the causal nature of the underlying models, we also tested these models for forecast accuracy. We found that predictive accuracy did not vary substantially from one model, or even one currency, to another. However, there was some evidence that forward rates are easier to predict than spot rates. Finally, there was some suggestive evidence that the stronger the causal relationship, the more likely that the bivariate (simultaneous) model predicts better than the univariate model as would be expected.

NOTES

1 See, for example, Goodman (1979).

2 Formally, Grauer, Litzenberger and Stehle (1976) have shown that

$$F_{t-1} = E_{t-1}(S_t) + L_t$$

where S_t is the spot rate at time t, F_{t-1} is the one period forward rate at time $t-1$, L_t is the liquidity premium at time t, and $E_{t-1}(S_t)$ is the market expectation at time $t-1$ of the future spot rate S_t.

3 In essence, we will test the joint hypothesis that exchange rate theory is correct and that market participants are correct on average in predicting future spot rates. However, as in all studies of this type, we will assume that the theory is correct and focus on the predictive ability of the market.

4 See Fama (1970) for a review of market efficiency concepts in general and Kolhagen (1978) and Levich (1979a and 1979b) for market efficiency in the context of foreign exchange markets.

5 See Hsaio (1981).

6 It could be argued that the *FPE* objective is itself arbitrary. However, as we argue below, the *FPE* criterion is consistent with the Granger definition of causality since both are based on minimizing the mean square prediction error. On the other hand, standard significance tests bear no relationship whatsoever to the Granger definition.

7 If the direction of the inequality is reversed then $(m+p, n+q)$ would be chosen.

8 As we shall see this third step is usually unnecessary. The alternative to this sequential procedure is to use all possible combinations of lags for forward and spot rates and then to choose the one combination which yields the minimum *FPE*. Clearly the sequential procedure is far more efficient.

9 Evidence of liquidity premia in foreign exchange markets was found by Jacobs (1982) and more recently by Fama (1983).

10 The random walk characteristic of exchange rates is well documented. See Kolhagen (1978) and Levich (1979a and 1979b).

11 The fact that forward rates enter only in the form of a one period lag is consistent with the theory. See note 2 above.

12 Unfortunately, a proper technique for determining causal relationships in a true simulaneous system has yet to be developed. Nevertheless, once having determined causality by OLS, better parameter estimates should be obtainable by utilizing a FIML approach.

13 Theil's coefficient is defined as

$$U = \frac{RMSE}{\left(\sum_{i=1}^{n} A_i/n\right)^{1/2}}$$

where *RMSE* is the root mean square error, A_i denotes the actual (as opposed to predicted) value, and n is the number of data points.

14 Take the mark, for example. The minimum univariate *FPE* for the spot rate is 0.1510×10^{-2}. The overall (bivariate) minimum *FPE* for the mark is 0.1375×10^{-2}, a difference of 9 percent (with the univariate spot rate as base).

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