Shocks to Shocks: A Theoretical Foundation for the Information Content of Earnings*

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1. Introduction

This study synthesizes and generalizes the variance decomposition approach to firm-level valuation using accounting earnings. A number of the insights offered by this study are available in piecemeal fashion in other papers (Vuolteenaho 2002; Callen and Segal 2004; Callen, Hope, and Segal 2005; Callen, Livnat, and Segal 2006; and Callen, Segal, and Hope 2009) and are brought together here in systematic fashion. Along with variance decomposition, this study also offers a rigorous derivation of Ball-Brown analysis — namely, that shocks to returns are a linear function of shocks to earnings, under fairly general conditions.¹ Proofs are based on Vuolteenaho’s 2002 parsimonious accounting return decomposition model. The theoretical form of the model is especially parsimonious insofar as it only requires firm-level dividend yields and book-to-market ratios to be covariance stationary processes and the accounting clean surplus relation to hold.² In order to relate the model directly to Ball and Brown 1968, we must further specify the time series of the firm’s earnings, the time series of the firm’s expected future discount rates (costs of capital), and the time series of other value-relevant information (hereinafter “other information”), where by value-relevant information we mean information that affects contemporaneous and/or future equity returns. Although we initially make rather restrictive assumptions about these time series in order to simplify the discussion, we subsequently show that Ball-Brown analysis holds under general time-series processes.

This study evaluates and elaborates on the Ball-Brown metric of information content from a fundamental theoretical perspective with special emphasis on the empirical implications of the theory. In addition to providing a general theoretical foundation for Ball-Brown analysis, this paper addresses a number of related issues. What are the implications of value-relevant information other than earnings (and book value) to Ball-Brown analysis? What adjustments if any must be made to Ball-Brown analysis if discount rates are dynamic? If earnings are decomposed into components (e.g., cash flows and accruals), can one determine from a fundamental theoretical perspective which earnings component has more information content? Is the Ball-Brown emphasis on the sensitivity of returns to earnings shocks — also called the earnings response coefficient (ERC) — a sufficient metric of value-relevance, or should value-relevance also be evaluated with reference to

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the volatility of unexpected returns generated by shocks to earnings? Can the generalized Ball-Brown methodology developed in this paper be used to improve extant empirical methodologies and to suggest further lines of research?

Our initial analysis addresses these and other issues using a simple time-series model — earnings, "other information", and expected future discount rates are assumed to be loglinear stationary AR(1) processes — that lends itself to reasonably straightforward intuition. Our further analysis generalizes these results by obtaining closed-form solutions for a broad class of (log) linear time-series models. It is remarkable that with elementary matrix manipulation, the results obtained for the simple loglinear stationary AR(1) time-series model extend to loglinear stationary ARMA(p, q), ARIMA(p, d, q) and vector autoregressive (VAR) processes. These generalizations are important because there is an extensive literature on the time-series properties of accounting earnings showing that, for many firms, the time series of both annual and quarterly accounting earnings are best described by higher-order time series, such as the ARIMA(p, d, q) type.

In what follows, section 2 develops the Ball-Brown metric of information content for the case where earnings and "other information" are loglinear stationary AR(1) processes and expected future discount rates are intertemporally constant, so that the emphasis is solely on the information content of earnings and "other information". Section 2 further extends this metric to the case of dynamic interest rates where expected future discount rates are also loglinear stationary AR(1). Section 3 generalizes the results of section 2 to the broad class of loglinear stationary ARMA(p, q), ARIMA(p, d, q), and VAR processes. Section 4 illustrates the application of the analysis developed in this paper to two accounting research issues where each issue illustrates a distinctive methodological application of the theory. Further applications are also suggested. Section 5 briefly concludes.

2. The Ball-Brown measure of information content

Despite some notable early attempts to derive Ball-Brown analysis analytically (e.g., Beaver, Lambert, and Morse 1980; Watts and Zimmerman 1986, 31; Kormendi and Lipe 1987), the first truly rigorous analysis from underlying primitives can be ascribed to Ohlson 1991. However, Ohlson's approach is not completely satisfactory in deriving Ball-Brown analysis because the assumptions of the model are somewhat restrictive. Inter alia, the model assumes a linear dividend policy and a very specific earnings dynamic. More recently, in a critical discussion of Ramakrishnan and Thomas 1998, Easton (1998) derives a relation between shocks to earnings and shocks to returns using a returns version of the Ohlson 1995 model. The Easton 1998 version still suffers from the limitations of the Ohlson 1995 model, including the assumption of a very specific earnings dynamic. Equally problematic, both Ohlson (1991, 1995) and Easton (1998) assume that discount rates are intertemporally constant. More recently, Gode and Ohlson (2004) show a shocks-to-shocks result allowing for changes in discount rates but under the same limited time-series dynamic as in Ohlson 1995. In addition, the analyses in all these papers focus on ex post mean return shocks (ERCs); but, as we shall show, the ex ante relationship between earnings shocks and the volatility of unexpected
returns — the Beaver 1968 type of value-relevance measure — is of equal importance in evaluating the information content of earnings.

A maintained assumption of Ball-Brown analysis generally, and the literature cited in the previous paragraph, is that period $t$ information is value-relevant if it only affects period $t$ returns. For simplicity of exposition, the initial part of this study maintains the same assumption. However, this definition of value-relevance is unnecessarily restrictive. Gonodes (1978) provides an example in which period $t$ information affects future returns but has no effect on period $t$ returns. Antle, Demski, and Ryan (1994) extend Gonodes’s analysis and show that a proportional relation between the revision in returns and unexpected earnings — that is, Ball-Brown analysis — holds if and only if the market revision of dividend expectations is proportional to the earnings revision, a most unlikely occurrence if only because accounting systems process different and more restrictive information sets than does the market. Instead, these papers make the case for a more expansive definition of value-relevance — namely, that period $t$ information is value-relevant if it affects period $t$ and/or future returns. The analysis below, in which expected future returns are dynamic, necessitates the latter, more expansive definition of value-relevance.

**The Vuolteenaho 2002 return decomposition model**

We develop the Ball-Brown measure of information content from underlying primitives using Vuolteenaho’s 2002 accounting return decomposition model, which, in turn, is an extension of the return decomposition framework of Campbell and Shiller 1988a, b, Campbell 1991, and Campbell and Ammer 1993. In order to make the intuition underlying our analysis reasonably transparent and self-contained, we begin by briefly proving the accounting return decomposition relation of Vuolteenaho 2002.

The Vuolteenaho 2002 return decomposition relation is derived from the definition of the log book-to-market ratio and the accounting clean surplus relation. Formally, define $BV_t$ to be the book value of equity at time $t$, $P_t$ the market value of equity at time $t$, $D_t$ dividends at time $t$, and $X_t$ earnings in period $t$; $\Delta$ is the differencing operator. From the book-to-market ratio definition, one obtains:

$$\frac{BV_t}{P_t} = \frac{1 + \Delta BV_t / BV_{t-1}}{1 + (\Delta P_t + D_t) / P_{t-1} - D_t / P_{t-1}} \cdot \frac{BV_{t-1}}{P_{t-1}}$$

$$= \frac{1 + (X_t - D_t) / BV_{t-1}}{1 + (\Delta P_t + D_t) / P_{t-1} - D_t / P_{t-1}} \cdot \frac{BV_{t-1}}{P_{t-1}}$$

(1)

where the second equality follows from the accounting clean surplus relation:

$$\Delta BV_t = X_t - D_t$$

(2)

Taking logs of both sides of (1) yields the log book-to-market ratio ($bm_t$):

$$\Delta BV_t = X_t - D_t$$

(3)
If $D_t = 0$, then (3) can be written in the convenient loglinear form:

\[ \text{roe}_t - r_t = \text{bm}_t - \text{bm}_t - 1 \] (4a),

where $\text{roe}_t = \log(1 + X_t/BV_t - 1)$ = the log of (one plus deflated) earnings and $r_t = \log(1 + (\Delta P_t + D_t)/P_t - 1)$ = the log of (one plus) the cum-dividend equity return.\(^8\)

If $D_t \neq 0$, then (3) can no longer be expressed directly in loglinear form. In order to maintain the loglinearity property, Vuolteenaho (2002) applies a Taylor series approximation to (3) — expanding around a convex combination of the unconditional means of the log dividend-to-book-value and the log dividend-to-market-value ratios — thereby yielding the loglinear pricing relation:

\[ \text{roe}_t - r_t = \rho \text{bm}_t - \text{bm}_t - 1 + \xi_t \] (4b),

where $\xi_t$ = approximation error and $\rho$ = discount rate coefficient. Because this convex combination of means for the United States is historically approximately 4 percent, $\rho$ is often assumed empirically to take on a value close to 1 (0.967), as in Campbell 1991, Campbell and Ammer 1993, Vuolteenaho 2002, and Callen and Segal 2004.

Iterating (4b) forward $N$ periods gives:

\[ \text{bm}_{t - 1} = \sum_{j=0}^{N} \rho^j r_{t + j} - \sum_{j=0}^{N} \rho^j \text{roe}_{t + j} + \sum_{j=0}^{N} \rho^j \xi_{t + j} + \rho^{N + 1} \text{bm}_{t + N} \] (5).

If the book-to-market ratio follows a covariance stationary process, the variance of the last term of (5) converges to zero as the horizon $N$ increases to infinity, yielding:

\[ \text{bm}_{t - 1} = \sum_{j=0}^{\infty} \rho^j r_{t + j} - \sum_{j=0}^{\infty} \rho^j \text{roe}_{t + j} + \sum_{j=0}^{\infty} \rho^j \xi_{t + j} \] (6).

Taking the change in expectations from time period $t - 1$ to $t$ and assuming that the change in expectations of the cumulative (discounted) error approximation is sufficiently small gives:\(^9\)

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\[ r_t - E_{t-1}(r_t) = \Delta E_t \sum_{j=0}^{\infty} \rho^j \text{roe}_t + j - \Delta E_t \sum_{j=1}^{\infty} \rho^j r_t + j \]  
\hspace{1cm} (7a),

where \( E_t \) = the expectations operator at time \( t \) and \( \Delta E_t(.) = E_t(.) - E_{t-1}(.) \). Equation (7a) is the Vuolteenaho 2002 return decomposition relation. It can be expressed more succinctly as:

\[ r_t - E_{t-1}(r_t) = N_e_t - N_r_t \]  
\hspace{1cm} (7b),

where \( N_e_t = \Delta E_t \sum_{j=0}^{\infty} \rho^j \text{roe}_t + j \) = earnings news and \( N_r_t = \Delta E_t \sum_{j=1}^{\infty} \rho^j r_t + j \) = expected return (discount rate) news.

Equation (7b) is essentially an identity with the following intuitive interpretation. Because the price of a security is the present value of expected future dividends, a shock to returns has only two potential sources: shocks to expected dividends — measured here by earnings news because of the clean surplus relation — and shocks to expected future returns or discount rate news. Moreover, as is to be expected, a positive shock to expected future earnings results in a positive shock to returns, whereas a positive shock to expected future discount rates results in a negative shock to returns, just as an increase in the expected yield reduces bond prices.

\textbf{Measuring information content when discount rates are intertemporally constant}

Let us assume for now, as is assumed in most accounting valuation models, that discount rates (costs of capital) are intertemporally constant so that there are no shocks to expected future discount rates. Also, for simplicity of exposition, let us assume (until the next section of the paper) that \( \rho = 1 \). Let \( v_t \) denote value-relevant information other than earnings and book value, which we have elected to call “other information” for expositional simplicity. We assume that earnings (roe\(_t\)) and “other information” (\( v_t \)) follow stationary AR(1) processes. The following proposition derives the standard earnings information content metric of Brown-Ball and the variance of unexpected returns (the Beaver-type earnings information content metric). The proof for this and the other propositions in this paper can be found in the Appendix.

\textbf{PROPOSITION 1.} Assuming that discount rates are intertemporally constant and assuming that \( \text{roe}_t \), (the log of one plus deflated) earnings, and \( v_t \), “other information”, follow stationary AR(1) processes of the form:

\[ \text{roe}_t = \beta \text{roe}_{t-1} + v_{t-1} + \epsilon_t \]  
\hspace{1cm} (8a),

\[ v_t = \alpha v_{t-1} + \epsilon_t^v \]  
\hspace{1cm} (8b),

where the persistence parameters \( \alpha \) and \( \beta \) lie between 0 and 1, and \( \epsilon_t \sim (0, \sigma^2) \) and \( \epsilon_t^v \sim (0, \sigma_v^2) \) are mean-zero independent error terms, then
Furthermore, the variance of the shock to (log) unexpected returns is:\textsuperscript{14}

\begin{equation}
\text{Var}(r_t - E_{t-1}(r_t)) = \frac{\sigma^2}{(1-\beta)^2} + \frac{\sigma_i^2}{(1-\alpha)^2(1-\beta)^2}
\end{equation}

Proposition 1 yields a rigorous derivation of Ball-Brown and Beaver-type measures of information content. A number of observations follow from this proposition. First, absent “other information” shocks, (9) indicates that shocks to returns are proportional to shocks to earnings where the proportionality constant is the ERC. Furthermore, the ERC is solely a function of the persistence of earnings, $\beta$, such that the greater is the persistence of earnings, the greater is the ERC. Thus far, what we have described is the precisely well-known Ball-Brown analysis. Second, given “other information” shocks, the shock to returns is proportional to the shock to earnings plus the shock to “other information” where the proportionality constant for earnings is again the ERC and the proportionality constant for “other information” is the product of the ERC and the “other information” response coefficient (OIRC). The OIRC is the inverse of one minus the persistence parameter for “other information”, $\alpha$. Thus, the impact of the “other information” shock on unexpected returns is a function of both its own persistence and the persistence of earnings, a result that is not overly surprising because “other information” has a direct impact on earnings as per (8a). Third, the variance of unexpected returns is proportional both to the variance of the earnings shock, where the proportionality constant is the square of the ERC, and to the variance of the “other information” shock, where the proportionality constant is the product of the square of the ERC and of the OIRC. Fourth, the variance of unexpected returns provides a different, if complementary, measure of information content to the standard Ball-Brown metric, just as the mean and variance of a distribution provide different, but complementary, pieces of information about a distribution. More specifically, abstracting from “other information”, the Ball-Brown metric measures the expected shock to returns, conditional on a 1 percent shock to earnings.\textsuperscript{15} But, because the Ball-Brown metric is an ex post metric, it fails to inform about the ex ante expected frequency of return shocks generated by earnings shocks. In contrast, the variance metric of information content measures the ex ante likelihood that earnings shocks will occur and generate return shocks. Equation (10) shows that the variance of unexpected returns is a function of the volatility of earnings shocks and the persistence of earnings. The greater the volatility of earnings shocks and the more persistent are earnings, the greater the ex ante likelihood that shocks to returns will occur.

To illustrate the complementary nature of the Ball-Brown and variance metrics, suppose that earnings are highly persistent, $\beta = 0.90$, but the variance of earnings is small — say, 0.1 percent — and there is no shock to “other information”. In this case, the ERC is 10 but the volatility of unexpected returns generated by the earnings
shock is only 1.9 percent. This means that although shocks to earnings, when they occur, have a dramatic impact on returns, shocks to earnings, and hence shocks to returns, are nevertheless expected to be relatively infrequent ex ante. Next consider the diametrically opposite case where earnings are not very persistent, \( \beta = 0.1 \), and the variance of earnings is relatively large — say, 5 percent. Here, the ERC is only 1.11 but the volatility of unexpected returns generated by the earnings shock is 6.1 percent. In contrast to the first case, shocks to earnings are expected to occur relatively frequently ex ante, but each shock that occurs ex post will have a relatively small impact on equity returns. On the basis of a typical ERC analysis, the researcher would have concluded that earnings in the first case (ERC = 10) are far more "value-relevant" than in the second case (ERC = 1.11). However, as these examples illustrate, the information contained in the Ball-Brown metric together with the variance metric paint a very different picture about the true value-relevance of earnings from that obtained from the Ball-Brown metric alone. The most important empirical implication of Proposition 1 is that value-relevance cannot be measured solely by reference to a returns—earnings regression. The variance of unexpected returns is equally important for determining value-relevance.

One of the more ubiquitous applications of Ball-Brown analysis in accounting research involves decomposing earnings into components in order to determine which earnings component is more value-relevant. Examples include decomposing net income into accruals and cash flows or decomposing net income into net income from foreign and domestic sources. What are the relationships between shocks-to-earnings components and shocks to returns? To answer this question, let us assume that earnings, \( X_t \), are decomposed into two components, \( X_{1t} \) and \( X_{2t} \), and our intent is to research the relative information content of these two earnings components. As before, we assume initially that expected future discount rates are intertemporally constant and that \( \rho = 1 \). Proposition 2 shows that one obtains results similar to those of Proposition 1, provided that each earnings component (normalized by beginning-of-period book value) and "other information" follow stationary AR(1) processes.

**Proposition 2.** Assuming that discount rates are intertemporally constant and assuming that each earnings component (deflated by beginning-of-period book value) and "other information" follow stationary AR(1) processes of the form:

\[
X_{1t}/BV_t - 1 = \beta^1 X_{1t-1}/BV_t - 2 + \nu_t - 1 + \varepsilon^1_t
\]

\[
X_{2t}/BV_t - 1 = \beta^2 X_{2t-1}/BV_t - 2 + \nu_t - 1 + \varepsilon^2_t
\]

\[
\nu_t = \alpha \nu_{t-1} + \varepsilon^\nu_t
\]

where \( 0 < \alpha, \beta^1, \beta^2 < 1 \) and \( \varepsilon^1_t \sim (0, \sigma^2_1) \) and \( \varepsilon^2_t \sim (0, \sigma^2_2) \) are mean-zero error terms that are independent of \( \varepsilon^\nu_t \sim (0, \sigma^\nu_2) \), then
Proposition 2 generalizes the Ball-Brown relation of Proposition 1 to earnings components in a straightforward fashion. In this proposition, (12) shows that the shock to returns, abstracting from other value-relevant information, is the sum of the shocks to each earnings component weighted by its respective ERC = 1/(1 − β^i) \ i = 1, 2. The impact of the “other information” shock on returns is a function not only of its own OIRC but also the ERCs of both earnings components.

Until recently (see Callen and Segal 2004 and Callen et al. 2005, 2006), no similar decomposition of the Beaver-type metric — namely, of the variance of unexpected returns — was found in the accounting literature. The variance decomposition of (13) generalizes the variance metric of the information content of earnings to the information content of earnings components. This decomposition is somewhat less elegant than the standard Ball-Brown decomposition of (12) but is equally informative. From (13), we see that the variance of unexpected returns varies with four factors: the variances of the shocks of each earnings component weighted by the square of its ERC, the covariance between the earnings components shocks weighted by the product of the two ERCs, and the variance of the “other information” shock weighted by a function of the component ERCs and the OIRC. Only if the earnings components shocks are uncorrelated is there a unique decomposition of the variance of the return shock into the sum of the weighted variances of the earnings components shocks (and the variance of the “other information” shock). In the latter case, one can ask which component is the most significant in driving security returns. If earnings components shocks are correlated, then there is an additional covariance factor that drives security returns, and one can ask to what extent does the interaction between the shocks of the earnings components drive security returns relative to the individual components. In addition to the computational aspects, the primary empirical implication of Proposition 2 is that the relative value-relevance of earnings components cannot be addressed solely by reference to

\[ r_t - E_{t-1}(r_t) = \frac{\varepsilon_t^1}{(1 - \beta^1)} + \frac{\varepsilon_t^2}{(1 - \beta^2)} + \frac{(2 - \beta^1 - \beta^2)}{(1 - \alpha)(1 - \beta^1)(1 - \beta^2)} \sigma_t^u \]  

and

\[ \text{Var}(r_t - E_{t-1}(r_t)) = \frac{\sigma_t^2}{(1 - \beta^1)^2} + \frac{\sigma_t^2}{(1 - \beta^2)^2} + \frac{(2 - \beta^1 - \beta^2)^2}{(1 - \alpha)^2(1 - \beta^1)^2(1 - \beta^2)^2} \sigma_t^2 + \frac{2\text{Cov}(\varepsilon_t^1, \varepsilon_t^2)}{(1 - \beta^1)(1 - \beta^2)} \]
a regression of returns on earnings components (even if the regression incorporates “other information” regressors). Equally importantly, the relative value-relevance of earnings components requires a variance decomposition of unexpected returns in order to determine which earnings component, if not both in tandem, mostly drives security returns.

One limitation of Proposition 2 is its reliance on the approximation \( \log(1 + z) \approx z \). (See the proof in the Appendix.) It is well known that the larger is \( z \) in absolute value, the greater is the approximation error. Therefore, the empiricist applying Proposition 2 would do well to temper his or her conclusions whenever shocks to (deflated) earnings are large in absolute value.

**Measuring information content with dynamic discount rates**

Regarding the importance of incorporating time-varying discount rates in accounting valuation models, one can do no better than cite Beaver’s criticism of the accounting literature (1999, 37): “Thirty plus years ago, Miller and Modigliani (1960) spent considerable effort to estimate the cost of capital for one industry for three years. It is remarkable that the assumption of a constant [discount rate] across firms and time is the best we can do.” Although there are accounting valuation models today that incorporate stochastic interest rates (e.g., Feltham and Ohlson 1999; Ang and Liu 2001; Gode and Ohlson 2004), these models tend to be rather complex and do not readily lend themselves to defining empirically implementable measures of information content. Instead, we employ the Vuolteenaho 2002 return decomposition model developed above and assume that expected future returns (discount rates) are stochastic. Even when incorporating time-varying stochastic interest rates, the Vuolteenaho 2002 model yields a simple empirically implementable functional relationship between unexpected returns and earnings (components). Specifically, following Campbell, Lo, and MacKinlay 1997 and Cochrane 2001, we assume that the expected future return \( \mathbb{E}_t(r_{t+1}) \) is made up of a constant return \( \alpha^r \) plus a time-varying return \( r^h_t \):

\[
\mathbb{E}_t(r_{t+1}) = \alpha^r + r^h_t \tag{14a}
\]

Furthermore, \( r^h_t \) is assumed to follow a stationary AR(1) process:

\[
r^h_t = \beta^r r^h_{t-1} + \varepsilon^r_t \tag{14b}
\]

Combining (14a) and (14b) yields

\[
\mathbb{E}_t(r_{t+1}) = \alpha^r + \beta^r r^h_{t-1} + \varepsilon^r_t \tag{14c}
\]

We further assume that (deflated) earnings and “other information” are stationary AR(1) processes as in (8a) and (8b), respectively, and \( \rho = 1 \). Proposition 3 relates shocks in returns to shocks in (deflated) earnings, discount rates, and “other information”.

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PROPOSITION 3. Assuming that $\text{roe}_t$, (the log of one plus deflated) earnings, $v_t$, “other information”, and $E_t(r_{t+1})$, (the log of) expected future discount rates follow stationary AR(1) processes of the form: \(^{20}\)

\begin{align*}
\text{roe}_t &= \beta \text{roe}_{t-1} + v_{t-1} + \varepsilon_t \tag{15a} \\
E_t(r_{t+1}) &= \alpha^r + \beta^r r_{t-1}^h + \varepsilon^r_t \tag{15b} \\
v_t &= \alpha v_{t-1} + \varepsilon^v_t \tag{15c},
\end{align*}

where $0 < \alpha, \beta, \beta^r < 1$ and $\varepsilon_t \sim (0, \sigma^2)$ and $\varepsilon^r_t \sim (0, \sigma^2_r)$ are mean-zero error terms independent of $\varepsilon^v_t \sim (0, \sigma^2_v)$, then\(^ {21}\)

\begin{align*}
 r_t - E_{t-1}(r_t) &= \frac{\varepsilon_t}{(1 - \beta)} - \frac{\varepsilon^r_t}{(1 - \beta^r)} + \frac{\varepsilon^v_t}{(1 - \alpha)(1 - \beta)} \tag{16}
\end{align*}

and

\begin{align*}
\text{Var}(r_t - E_{t-1}(r_t)) &= \frac{\sigma^2}{(1 - \beta)^2} + \frac{\sigma^2_r}{(1 - \beta^r)^2} + \frac{\sigma^2_v}{(1 - \alpha)^2(1 - \beta)} - \frac{2\text{Cov}(\varepsilon_t, \varepsilon^r_t)}{(1 - \beta)(1 - \beta^r)} \tag{17}.
\end{align*}

Proposition 3 shows that with stochastic discount rates, the shock to returns increases with positive shocks to (deflated) earnings and “other information” and decreases with positive shocks to interest rates. Thus, when discount rates are dynamic, the standard Ball-Brown measure of earnings information content, which assumes intertemporally constant discount rates, overstates (understates) the impact of the earnings shocks on current returns if shocks to expected future discount rates are positive (negative). This result implies that many, if not most, earnings event studies to date are potentially biased because event studies typically abstract from shocks to discount rates (costs of capital). This is especially problematic if the earnings event itself increases (or decreases) firm risk, thereby affecting the firm’s cost of capital, as found empirically by Callen et al. 2006. Because discount rates are much more likely to change over time, Proposition 3 has even stronger implications for long-window return–earnings regressions than it does for event studies. In particular, the value-relevance of earnings in long-window studies is likely either overstated or understated depending on the direction of interest rate shocks. Proposition 3 also implies that value-relevance must be evaluated with reference to a variance decomposition that includes earnings, “other information”, and discount rates, as in (17).

The Ball-Brown and variance measures of the information content of earnings components generalize in a fairly obvious way when discount rates are stochastic, as shown in Proposition 4.
PROPOSITION 4. Assuming that deflated earnings components $X_{it}/BV_{it-1} (i = 1, 2)$, (the log of) expected future discount rates $E_t(r_{t+1})$, and “other information” $v_t$ follow stationary AR(1) processes of the form:

$$X_{1t}/BV_{t-1} = \beta^1 X_{1t-1}/BV_{t-2} + v_{t-1} + \varepsilon^1_t \quad (18a),$$

$$X_{2t}/BV_{t-1} = \beta^2 X_{2t-1}/BV_{t-2} + v_{t-1} + \varepsilon^2_t \quad (18b),$$

$$E_t(r_{t+1}) = \alpha^r + \beta^r r^h_{t-1} + \varepsilon^r_t \quad (18c),$$

$$v_t = \alpha v_{t-1} + \varepsilon^v_t \quad (18d),$$

where $0 < \alpha, \beta^1, \beta^2, \beta^r < 1$ and where $\varepsilon^1_t \sim (0, \sigma^1_\varepsilon), \varepsilon^2_t \sim (0, \sigma^2_\varepsilon), \varepsilon^r_t \sim (0, \sigma^r_\varepsilon)$ are mean-zero error terms independent of $\varepsilon^v_t \sim (0, \sigma^v_\varepsilon)$, then

$$r_t - E_{t-1}(r_t) = \frac{\varepsilon^1_t}{(1 - \beta^1)} + \frac{\varepsilon^2_t}{(1 - \beta^2)} - \frac{\varepsilon^r_t}{(1 - \beta^r)}$$

$$+ \frac{(2 - \beta^1 - \beta^2)}{(1 - \alpha)(1 - \beta^1)(1 - \beta^2)} \varepsilon^v_t \quad (19),$$

and

$$\text{Var}(r_t - E_{t-1}(r_t)) = \frac{\sigma^1_\varepsilon}{(1 - \beta^1)^2} + \frac{\sigma^2_\varepsilon}{(1 - \beta^2)^2} + \frac{\sigma^r_\varepsilon}{(1 - \beta^r)^2}$$

$$+ \frac{(2 - \beta^1 - \beta^2)^2 \sigma^v_\varepsilon}{(1 - \alpha)^2(1 - \beta^1)^2(1 - \beta^2)^2}$$

$$+ \frac{2 \text{Cov}(\varepsilon^1_t, \varepsilon^2_t)}{(1 - \beta^1)(1 - \beta^2)} - \frac{2 \text{Cov}(\varepsilon^1_t, \varepsilon^r_t)}{(1 - \beta^1)(1 - \beta^r)}$$

$$- \frac{2 \text{Cov}(\varepsilon^2_t, \varepsilon^r_t)}{(1 - \beta^2)(1 - \beta^r)} \quad (20).$$

3. Generalizing the Ball-Brown and variance measures of information content

The case of ARMA($p, q$) and ARIMA($p, d, q$) processes

The relation between shocks to returns and shocks to earnings depends crucially on the time series of earnings, discount rates, and “other information”. In this section, we emphasize that the results obtained above generalize to more complex time-series processes. It is important to generalize these results first because the accounting literature on the time series of earnings teaches us that the simple AR(1) earnings model is untenable for many firms. More specifically, although
some of the older literature on the time series of annual earnings concluded that annual earnings are a random walk for the average firm, nevertheless, many firms in their samples were better described by higher-order processes.22 Indeed, it would be most surprising if the time-series properties of all firms fit into one restrictive mold like an AR(1) process. Second, the more recent accounting literature shows that higher-order models are more descriptive of the time series of annual earnings, even for the average firm.23 Third, we know from the accounting literature on the time-series properties of interim earnings that quarterly earnings are better modeled as more complex time-series processes than the simple AR(1).24 Fourth, higher-order autoregressive processes and ARMA processes capture time-series movements well beyond the AR(1). In fact, the flexibility of an AR(2) process relative to an AR(1), in describing various time-series movements, is quite extraordinary, and the increase in order from 1 to 2 fails to do adequate descriptive justice to the generalization.

We initially generalize our results from the previous section to the broad class of stationary (log) linear autoregressive moving average time-series models of the ARMA($p, q$) type. Autoregressive (AR) terms have long-term permanent consequences, and we allow for $p$ such terms, where $p$ is any finite number. Moving average (MA) terms are one-time transitory shocks to earnings that have no long-term consequences. Here we allow for $q$ such transitory shocks, where $q$ is any finite number. The generalization to an ARIMA($p, d, q$) model is immediate if we assume that differencing ($d \geq 1$) eventually yields a stationary ARMA($p, q$) model. Formal statements and proofs of the generalizations of the four propositions above to ARMA($p, q$) processes are available from the author.

It is worth noting that the generalization to ARIMA processes relative even to higher-order AR processes is important in accounting research. There are accounting shocks that are permanent, such as changes to standards that are best described by permanent time-series processes such as those of the AR variety. There are accounting shocks that are transitory, such as special items that are best described by MA processes.25 More generally, the ARIMA process is of singular importance to accounting research because it encapsulates both types of shocks that firms are subject to in a fairly parsimonious fashion. Examples of accounting research that recognize the flexibility of ARIMA processes and employ different varieties of ARIMA processes in their empirical analyses include Beaver et al. 1980, Ali and Zarowin 1992, and Kumar and Visvanathan 2003.

To appreciate the nature of these generalizations, consider the generalization of Proposition 1. Absent shocks to “other information” and to expected future discount rates, we are able to show that the Ball-Brown and variance measures of information content for an ARMA($p, q$) earnings process also result in a proportional relationship between (the variance of) shocks to returns and (the variance of) shocks to earnings, except that the ERC is now a function of multiple time-series parameters. In particular, the ERC is an increasing function of the autoregressive (persistence) parameters and a decreasing function of the moving-average parameters. In addition, although each autoregressive parameter has the same impact on the ERC no matter the time period, moving-average terms have less of an impact
on the ERC the further back they are in time. As a consequence, the returns of firms with identical sums of persistence parameters will react identically to permanent earnings shocks but not necessarily to transitory earnings shocks. This implies, for example, that one cannot assume a homoscedastic error structure for returns when analyzing, say, special items, which tend to be transitory. Note that the proportionality constant for “other information” is a function of the autoregressive (persistence) parameters of both earnings and “other information”, as well as being a function of the “other information” moving-average parameters.

The case of VAR processes

Up to now we have assumed that all of the time series are independent of each other. In other words, the time series of the firm’s earnings is independent of the time series of the firm’s expected returns and independent of the time series of “other information”. However, the independence assumption comes at some cost. Consider, for example, the issue of persistence. Accounting researchers have long recognized the importance of controlling for the impact of current shocks on future earnings by controlling for earnings persistence and the persistence of “other information”. However, what if the various persistences are interrelated? For example, what if the return—earnings relation depends not only on the persistence of earnings and the persistence of growth, but also on the interaction of these two persistences? In other words, what if firms with more persistent earnings are also firms with more persistent growth opportunities? To take another example, what if the persistence of foreign earnings interacts with the persistence of domestic earnings in generating firm-level returns, perhaps because of cost synergies or economies of scope arising out of multiple markets? Although one could potentially control for all persistences and their interactions in a return—earnings regression, in point of fact this is never done. In contrast, as we shall see below, a multivariate time-series process of the vector autoregressive (VAR) type with returns, earnings, and growth state variables incorporates by construction controls for the persistence of expected returns (discount rates), the persistence of earnings, the persistence of growth, and their interactions. Persistences are only one time-series characteristic for which VAR processes are superior to univariate processes. More generally, because VARs involve current and lagged values of multiple time series, they capture co-movements among state variables that cannot be detected by univariate AR and ARMA processes.

Interestingly, there are a number of early empirical studies indicating that the time series of firm-level earnings are better described as part of VAR processes than as univariate time-series processes: see Bar-Yosef, Callen, and Livnat 1987, 1996; and Finger 1994. More recent work includes Morel 1999; Vuolteenaho 2000, 2002; Callen and Segal 2004; and Callen et al. 2005, 2006.

The generalization of the Ball-Brown and variance measures of information content to (log) linear stationary VAR processes is due to Vuolteenaho 2002, although he neither refers to metrics of earnings information content nor to Ball-Brown analysis, nor does he address the issue of earnings components. Formally, define \( z_t \) to be a vector of firm-specific state variables and \( \eta_t \), a vector of zero-mean
error terms. Without loss of generality, let the first-state variable be returns \((r_t)\) and the second-state variable earnings \((roe_t)\). The remaining variables are generic and undefined and can readily be conceptualized as other value-relevant information ("other information"). We assume that the state vector follows the multivariate log-linear dynamic:

\[
  z_t = \Gamma z_{t-1} + \eta_t
\]  

(21).

The VAR transition coefficient matrix \(\Gamma\) is assumed to be constant over time. The error terms \(\eta_t\) are assumed to have a variance-covariance matrix \(\Sigma = E(\eta_t, \eta_t')\) and to be independent of everything known at \(t-1\).

A simple example of a three-variable VAR process is:

\[
  \begin{pmatrix}
    r_t \\
    roe_t \\
    g_t
  \end{pmatrix}
  = \begin{pmatrix}
    \alpha_1 & \alpha_2 & \alpha_3 \\
    \beta_1 & \beta_2 & \beta_3 \\
    \gamma_1 & \gamma_2 & \gamma_3
  \end{pmatrix}
  \begin{pmatrix}
    r_{t-1} \\
    roe_{t-1} \\
    g_{t-1}
  \end{pmatrix}
  + \begin{pmatrix}
    \eta_{1t} \\
    \eta_{2t} \\
    \eta_{3t}
  \end{pmatrix}
\]

yielding the VAR specification:

\[
  r_t = \alpha_1 r_{t-1} + \alpha_2 roe_{t-1} + \alpha_3 g_{t-1} + \eta_{1t}
\]

(22a)

\[
  roe_t = \beta_1 r_{t-1} + \beta_2 roe_{t-1} + \beta_3 g_{t-1} + \eta_{2t}
\]

(22b)

\[
  g_t = \gamma_1 r_{t-1} + \gamma_2 roe_{t-1} + \gamma_3 g_{t-1} + \eta_{3t}
\]

(22c)

where all variables are mean-adjusted. The variable \(g_t\) represents the "other information" variable.

The following proposition derives the Ball-Brown and variance measures of information content for the general VAR process of (21):

**Proposition 5.** Assuming that the state vector \(z_t\) follows a multivariate log-linear stationary VAR process \(z_t = \Gamma z_{t-1} + \eta_t\) (with \(r_t\) and \(roe_t\) the first two state variables, respectively) then

\[
  r_t - E_{t-1}(r_t) = \Delta E_t \sum_{j=0}^{\infty} \rho^j roe_{t+j} - \Delta E_t \sum_{j=1}^{\infty} \rho^j r_{t+j}
  = (e_2' + \lambda_2') \eta_t - \lambda_1' \eta_t
\]

(23),

where

\[
  \lambda_k' = e_k' \rho \Gamma (I - \rho \Gamma)^{-1}
\]

(24),

and \(e'_k = (0, \ldots, 1, \ldots, 0)\) is a row vector with 1 as the kth element and 0 elsewhere. Furthermore,
Equation (23) indicates that the shock to returns is a linear function of the shocks \((\eta_t)\) to the other variables in the VAR system. More specifically, assuming the VAR is made up of, say, \(s\) variables, then (23) can be expressed in the form:

\[
r_t - E_{t-1}(r_t) = \sum_{j=1}^{s} f_j \eta_{jt}
\]

where each of the \(f_j\) are (generally) nonlinear functions of the parameters of the VAR system \(\Gamma\) (and \(\rho\)). To better understand the implications of Proposition 5 and how (23) through (25) relate to the univariate case ((12) and (13)), we consider a number of special cases. To simplify the discussion, let us assume that discount rate news is zero so that only earnings news drives security returns and \(\rho = 1\). Furthermore, assume that (22a), (22b), and (22c) make up the VAR system.

First, consider the case where all off-diagonal parameters (of the matrix \(\Gamma\)) are zero. It is straightforward to show that \(f_1 = f_3 = 0\) and \(f_2 = (1 - \beta_2)^{-1}\). In this case, only earnings shocks drive security returns and the ERC is the same as in the simple AR(1) case with no “other information” shocks. Second, assume that \(\beta_3\) is positive, but all other off-diagonal parameters are zero. \(\gamma_3\) positive implies that “other information” is a factor in predicting future earnings (see (22b)). Here it turns out that \(f_1 = 0, f_2 = (1 - \beta_2)^{-1}\), and \(f_3 = [(1 - \alpha_1)(\beta_3)] / [1 - \gamma_3(1 - \beta_2)]\). This case is similar (but not equivalent) to the univariate AR(1) case where both earnings and “other information” affect security returns. As in the univariate case, the ERC is only a function of the earnings persistence despite the fact that “other information” affects earnings. However, the coefficient for “other information” depends not only on earnings persistence \(\beta_2\) and own persistence \(\gamma_3\), but also on the impact of “other information” on earnings \(\beta_3\) and, more surprisingly, return (discount rate) persistence \(\alpha_1\). This result did not obtain in the univariate case because, in the univariate case, returns (discount rates), “other information”, and earnings are independent of each other. Third, let us further extend the analysis to the case where there is feedback between earnings and “other information” so that both \(\beta_3\) and \(\gamma_2\) are positive, but the other off-diagonal parameters are still zero. In this case, \(f_1 = 0, f_2 = (1 - \gamma_3) / Y\), and \(f_3 = [(1 - \alpha_1)(\beta_3)] / Y\), where \(Y = (1 - \gamma_3)(1 - \beta_2) - (\gamma_2\beta_3)\). Here, the ERC is a function of its own (earnings) persistence, the persistence of “other information”, the impact of “other information” on future earnings, and the impact of earnings on future “other information”. In addition to the latter parameters, the “other information” coefficient is also a function of return persistence. Feedback among the various interrelated time series yields an ERC that is more than simply a function of “own” persistence, a result that could not obtain in a univariate analysis.

The examples until now yielded \(f_1 = 0\) but that need not be the case. In general, unexpected returns are functions not only of the persistence of earnings and
“other information” but also of the persistence of returns and the interactions between these persistences. (See Y, above, for interactions.) Furthermore, if discount rate news is not zero, then (23), or equivalently (26), incorporates the impact of discount rate shocks on current unexpected returns, including the effect of earnings, “other information”, and return persistences and their interactions.

Equation (25) is the equivalent of (13) in a multivariate world. Following on (26), (25) can be rewritten in the quadratic form:

\[
\text{Var}(r_t - E_{t-1}(r_t)) = \sum_{j=1}^{s} f_j^2 \sigma_j^2 + \sum_{i=1}^{s} \sum_{j \neq i}^{s} f_i f_j \text{Cov}(\eta_{it}, \eta_{jt}) \tag{27}
\]

In words, the variance of current unexpected returns is a quadratic function of the variances of the individual shocks — earnings, expected returns, and “other information” shocks — and their covariances. Here too the VAR approach automatically accounts for all variable persistences and their interactions when computing the impact of earnings, “other information”, and expected future returns shocks in driving security returns.

One can use also use the VAR system to further decompose earnings into earnings components X_{1t} and X_{2t}, with the intent of determining which earnings component is more value-relevant, both in terms of mean ex post effects and in terms of ex ante variance effects. Such a VAR system automatically incorporates the persistences of the earnings components and their interactions together with the persistences of “other information” and returns. The following proposition shows how to compute the impact of shocks to earnings components on unexpected returns in a VAR context.

**Proposition 6.** Assuming that the state vector z_t follows a multivariate loglinear stationary VAR process (as in (21) with r_t, X_{1t}/BV_t - 1, and X_{2t}/BV_t - 1 the first three state vectors, respectively), then

\[
\begin{align*}
  r_t - E_{t-1}(r_t) &= \Delta E_i \sum_{j=0}^{\infty} \rho^j (X_{1t+j}/BV_t+j-1) \\
  &\quad + \Delta E_i \sum_{j=0}^{\infty} \rho^j (X_{2t+j}/BV_t+j-1) \\
  &\quad - \Delta E_i \sum_{j=1}^{\infty} \rho^j r_{t+j} \\
  &= (e'_2 + \lambda'_2) \eta_t + (e'_3 + \lambda'_3) \eta_t - \lambda'_1 \eta_t \tag{28a}
\end{align*}
\]

Furthermore,
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\[ \text{Var}(r_t - E_{t-1}(r_t)) = (\epsilon'_2 + \lambda'_2)\Sigma(e_2 + \lambda_2) + (\epsilon'_3 + \lambda'_3)\Sigma(e_3 + \lambda_3) \]
\[ + \lambda'_1 \Sigma \lambda_1 - 2\lambda'_1 \Sigma(e_2 + \lambda_2) - 2\lambda'_1 \Sigma(e'_3 + \lambda'_3) \]
\[ + 2(\epsilon'_2 + \lambda'_2)\Sigma(e_3 + \lambda_3) \] (29).

Some empirical issues

The theory above assumes that each of the variables that drive returns (and returns themselves) is stationary. Of course, stationarity cannot be guaranteed and some accounting variables are likely to be nonstationary. Still, as was mentioned with regard to ARIMA models, often nonstationary variables can be made stationary by differencing. Also, the propositions will go through in the absence of stationarity provided that the nonstationary variables are cointegrated — that is, a linear combination of them is stationary. Although cointegration cannot be guaranteed either, there are well-developed tests of cointegration of VAR systems — for example, the Johansen 1988 procedure. If the VAR state variables are neither stationary nor cointegrated, then the estimated VAR parameters are potentially biased. But this problem applies to standard ordinary least squares (OLS) regressions as well. Any time the data have a time-series component, nonstationarity (and noncointegration) will yield potentially biased coefficients. This is so whether the data are pooled or panel data techniques are employed. For example, to the extent that variables are nonstationary and noncointegrated, an OLS return-earnings regression (with controls) using time-series cross-sectional data will potentially yield biased ERC estimates.

A VAR system analysis of security returns using the Vuolteenaho 2002 approach must, at a minimum, contain two state variables, returns, and (deflated) earnings. Where the topic at hand is the relative value-relevance of earnings components, the VAR system must contain, at a minimum, returns and the (deflated) earnings components as state variables.

An important issue regarding VAR systems concerns the number of “other information” variables to include in the analysis. Unfortunately, there is no a priori method to determine the number of “information variables” in a VAR system, just as there is no a priori method to determine the appropriate number of control variables to include in a standard (return-earnings) regression. In both cases, given a fixed number of observations with which to estimate the parameters, one wants to include the minimum number of “other information” control variables without leaving out important correlated variables. Nevertheless, the problem is somewhat more acute in a VAR system than in a standard regression because every additional variable in a VAR system requires the estimation of many additional parameters. For example, adding another “other information” variable to the VAR system of (22a), (22b), or (22c) requires the estimation of an additional 7 parameters. Adding two “other information” variables to this VAR system requires the estimation of 16 additional parameters. If the VAR system contains a “large” number of parameters, then estimation at the firm level with typically few time-series observations leads to unstable parameter estimates. Rather, accounting and finance researchers estimate the VAR system parameters at the system-wide or, if possible, industry-wide level and then assume that the same parameter estimates hold for all firms in
These industry VAR parameter estimates, together with firm-level shocks based on cross-sectional time-series data, yield Ball-Brown and variance decomposition metrics for the firm. Ultimately, deciding what “other information” variables are relevant to the analysis is the researcher’s art, and parsimony is a desideratum.

A similar but less crucial problem arises regarding the number of lags to include in a VAR system. The simple VAR system above ((22a), (22b), (22c)) is made up of only one lag per variable. A two-lag system would require the estimation of nine additional parameters. The most common method to determine lag structure involves the use of an information criterion such as Akaike’s information criterion (AIC). These criteria trade off the better fit from additional lags against the cost of having to estimate additional parameters using a fixed sample size. Different criteria weigh the trade-off between (over) fitting and estimation differently in determining the “optimal” lag structure. On the basis of simulations and advances in statistical theory, recent research in the time-series statistics literature suggests criteria that more heavily penalize the use of additional parameters, such as the AICc criterion. All in all, parsimony is important when considering the number of lags to include in a VAR system, and it is rare to find more than two lags per variable to be optimal.

An important issue regarding the empirical estimation of variance decompositions has recently surfaced in the finance literature. Typically, with the exception of one variable, the variance contribution of each state variable to the total variance of unexpected returns is estimated directly. The variance contribution of the one remaining variable is estimated residually so as to guarantee the equality of the variance decomposition (e.g., (25)). By residual estimation, we mean that the variance contribution of the remaining variable is computed as the difference between the variance of unexpected returns and the variance contributions of all other variables.

Chen and Zhao (2009) have recently pointed out that a comparison of the relative variance contribution of different variables to unexpected returns may depend critically on which variable is estimated residually. There are two ways around this conundrum. One is to estimate all variables directly and live with the fact that the variance decomposition may not satisfy the equality exactly. Another is to make sure not to estimate residually those variables whose relative variance contributions are the focus of the study. For example, in their analysis of whether accruals or cash flows are the primary driver of security returns at the Securities and Exchange Commission (SEC) filing date, Callen et al. (2006) are careful to estimate the variance contributions of both accrual and cash flow news directly and expected return news residually so as to ensure a meaningful comparison between the earnings components.

4. Applications

This section shows that the theory developed above improves on extant empirical methodologies and suggests further lines of research. Two different accounting issues are analyzed: each illustrates a distinctive methodological application of the
theory. The first application emphasizes the complementary nature of the Ball-Brown and variance metrics of value-relevance in the context of domestic and foreign earnings. The second application shows the usefulness of the variance decomposition methodology in cases where value-relevance needs to be measured nondirectionally, as in the case of SEC filings. Three additional potential studies for future research are also suggested.

**Complementarity of Ball-Brown and variance metrics of value-relevance: The case of foreign versus domestic earnings**

There is a fairly extensive, if contradictory, literature regarding the relative value-relevance of foreign and domestic earnings of U.S. multinationals. Bodnar and Weintrop (1997) document that both foreign and domestic earnings changes are significantly positively associated with annual excess stock returns and that the coefficient on foreign earnings is significantly larger than the coefficient on domestic earnings. They also find that the incremental impact of foreign earnings is positive when foreign sales growth exceeds domestic sales growth. These results lead them to argue that the larger coefficient on foreign earnings is evidence of greater growth opportunities for foreign than for domestic operations and that foreign earnings are more value-relevant than domestic earnings.

Christophe (2002) replicates Bodnar and Weintrop 1997 and finds similar results prior to sample partition. However, upon splitting the sample into observations with positive and negative earnings changes, he observes that although investors do not value domestic and foreign earnings changes differently for positive earnings changes, the estimated coefficient on foreign earnings changes is significantly larger than the estimated coefficient on domestic earnings changes when earnings changes are negative. In addition, he finds that the larger ERC associated with negative foreign earnings changes is most pronounced for firms with substantial amounts of free cash flows and high anticipated growth opportunities. The latter findings are consistent with investors revising their future growth expectations downward when the firm discloses poor foreign operating performance. On the basis of these findings, Christophe (2002) concludes that investors do not generally value foreign operations more highly than domestic operations.

These studies focus on value-relevance as measured by the ERCs of foreign and domestic earnings. In contrast, Callen et al. (2005) analyze the relative value-relevance of foreign and domestic earnings using a variance decomposition analysis. Although they too find that foreign earnings are more persistent than domestic earnings, suggesting that foreign earnings are more value-relevant from an ERC point of view, they also find, on the basis of a variance decomposition analysis, that domestic earnings, rather than foreign earnings, are the primary driver of unexpected equity returns. They conclude that domestic earnings are more value-relevant from the perspective of the variance metric. Intuitively, the variance metric involves a trade-off between the persistences of the earnings components and the variances of the earnings components. (See (13), for example.) Although foreign earnings are more persistent than domestic earnings, domestic earnings are far more volatile than foreign earnings so that the variance of unexpected equity returns — the
value-relevant metric in a variance decomposition framework — is higher for domestic earnings than for foreign earnings.

In tandem, both metrics are quite informative. Specifically, they indicate that while a 1 percent shock to foreign earnings has a larger impact on equity returns than a 1 percent shock to domestic earnings, conditioned on these shocks occurring, the likelihood of shocks occurring to domestic earnings is far larger than that of foreign earnings. Thus, evaluating the relative value-relevance of earnings components based on mean ex post effects, such as ERCs alone, is potentially distorting because ex ante value-relevance based on variances may give a different perspective. In fact, as this case illustrates, both metrics are required (at a minimum) in order to make adequate value-relevant statements.

Nondirectional value-relevance of informational components: The case of SEC filings

A large number of papers have examined the value-relevance of SEC filings for firms that provide preliminary earnings. Although most early studies tend to find little if any value-relevance on SEC filing dates, possibly because of sample size limitations and the difficulty in pinpointing the timing of SEC information prior to the SEC's Electronic Data Gathering, Analysis, and Retrieval system (EDGAR), more recent studies tend to conclude that SEC filings are in fact value-relevant beyond preliminary earnings. Missing from this literature is an evaluation of the informational components that are value-relevant on SEC filing dates. The theoretical framework developed above provides a framework for determining the components of information that are potentially value-relevant on SEC filing dates. Three informational components are of particular interest: news about the firm's risk as reflected in expected future returns (discount rates), news about the firm's cash flows, and news about the firm's accruals. Which of these informational components is the primary driver of equity returns around SEC filing dates? Is it cash flow news, or accrual news, or risk (discount rate) news, or all of these in combination?

The variance decomposition framework is a natural framework for determining the value-relevance of these informational news components around SEC filings. Unlike preliminary earnings announcements, which can reasonably be estimated and classified as good or bad news, SEC filings provide information that (potentially) revises the information conveyed by preliminary earnings announcements and, hence, the researcher is usually unable to estimate whether the information conveyed by SEC filings is good news or bad news. Thus, the value-relevant metric for SEC filings has to be nondirectional (unsigned). The variance decomposition methodology provides such an unsigned metric — namely, the variance of unexpected returns. Although other nondirectional metrics have been employed by this literature, the relationship between these metrics and the underlying informational components (such as accruals or discount rate news) is unclear. By contrast, the variance decomposition methodology provides an empirically estimable relation between the nondirectional value-relevant metric and the variances of the informational news components and expected return news. The economic and statistical significance of the estimated variances of the informational news
components and expected returns news then indicate which of the components are value-relevant.

Consider in particular the issue of expected returns news. Other nondirectional metrics employed by the literature fail to control for changes in costs of capital (expected returns) that may be generated by the news event. A variance decomposition explicitly estimates the impact of changes in expected returns (costs of capital) on security returns during the news event. Callen et al. (2006) show in their study that expected returns news increases significantly on SEC filing dates, and in fact are able to validate their finding by reference to alternative cost-of-capital measures.

**Other potential studies**

As well as the two applications mentioned above, many potential studies could benefit from the analysis suggested in this paper. Almost all research involving the (relative) value-relevance of earnings components would likely benefit from a variance decomposition to complement a standard ERC analysis. One immediate example involves the value-relevance of fair value adjustments relative to historical cost earnings. Although the ERCs of historical earnings tend to be greater than those of most fair value adjustments for banking firms, fair value adjustments tend to be more volatile than historical cost earnings. Whether fair value income adjustments drive unexpected security returns more than do historical cost earnings is still an open empirical question that could be addressed by the approach suggested in this paper. A similar issue involves joint ventures. Joint ventures are inherently risky because they involve trust and control issues between companies that are often competitors. This raises the question whether a firm’s joint venture earnings are more likely to drive security prices than its single venture earnings. Such an issue could be researched in Canada, where local generally accepted accounting principles (GAAP) for joint ventures mandate proportional consolidation.

Another potential application involves the term structure of costs of capital. Finance and accounting techniques for estimating costs of capital typically yield one cost of capital estimate for each firm which is assumed to be intertemporally constant. However, in a variance decomposition analysis, the firm’s costs of capital (expected future returns) are assumed to vary over time. One could, in fact, try to estimate a term structure of costs of capital at earnings announcements. First, the firm’s costs of capital are estimated by conventional means (for example, by an asset-pricing model), prior to the earnings announcement, to get a baseline cost-of-capital estimate. Subsequently, the time-wise components of expected returns news are estimated using the ARMA or VAR methodology outlined in this study. Each estimated time component is then added to the baseline cost of capital component to obtain a term structure of expected returns (costs of capital). The estimated term structure of costs of capital can be used, for example, to determine whether management is myopic — the horizon problem — or to estimate the extent of management’s risk aversion over time.
5. Conclusion

This study synthesizes and generalizes the variance decomposition approach to firm-level valuation using accounting earnings. In particular, this study offers a rigorous development of the information content of earnings using Vuolteenaho's 2002 return decomposition model. Standard Ball-Brown and variance (Beaver-type) measures of information content are obtained by assuming that the time series of (log-deflated) earnings and "other information" are AR(1), and expected future discount rates are intertemporally constant. These standard measures are generalized to the case of dynamic discount rates by assuming initially that (log) expected future discount rates are also AR(1) with drift. Further generalized closed-form measures of information content are obtained for cases where (a) the time series of (log-deflated) earnings, "other information", and (the log of) expected future discount rates are stationary ARMA(p, q) or ARIMA(p, d, q) processes; and (b) the time series of (log-deflated) earnings, "other information", and (log) expected future discount rates follow a loglinear stationary VAR process. Furthermore, closed-form measures of information content are obtained where earnings are decomposed into earnings components such as accruals and cash flows or domestic and foreign earnings, thereby allowing empirical researchers to determine which earnings component has more information content.

This study has a number of messages, but three bear emphasis. The first major implication of this study is that Ball-Brown and variance metrics provide complementary information about the value-relevance of earnings and earnings components. Whereas the Ball-Brown metric measures the ex post impact of earnings shocks on security returns, the variance measure informs about the ex ante likelihood that earnings shocks will occur. Past empirical accounting research studies that fail to use both measures are potentially misleading. Abstracting from exceptional circumstances, both measures are necessary (but not necessarily sufficient) for a full understanding of earnings relevance. Our rigorous development of these metrics provides accounting researchers with the tools to undertake a comprehensive value-relevance analysis.

The second major implication of this study relates to the potential bias inherent in the standard Ball-Brown and variance measures of earnings relevance when discount rates are dynamic. The standard measures are based on the assumptions that earnings are an AR(1) process and that discount rates (costs of capital) are intertemporally constant. Generalizing these metrics to more complex earnings time-series models further buttresses the empirical validity of the basic model; that is, shocks to returns are still proportional to shocks to earnings, although the constant of proportionality depends on the specific earnings time series. In contrast, failure to account for stochastic discount rates results in potentially biased measures of earnings relevance. When discount rates are dynamic, the standard Ball-Brown measure of information content overstates (understates) the impact of the earnings shocks on current returns if shocks to expected future discount rates are positive (negative). The bias is especially crucial in long-window studies and in event studies for which the event itself affects firm risk and, hence, the firm's cost of capital.
The third major implication of this study is that Vuolteenhao's 2002 return decomposition model and the related variance decomposition methodology provide a new empirical approach for addressing many issues in accounting research. This paper has emphasized two accounting issues where this approach provides new insights — namely, in the contexts of foreign and domestic earnings and SEC filings — and suggested others.

The underlying Vuolteenaho 2002 model can be extended in a number of directions. This study assumes that earnings and discount rate error terms are mean zero with constant variance so that returns have no memory; current shocks to earnings and discount rates cause revisions to contemporaneous and/or future returns. An alternative, more sophisticated approach is to allow error terms to have stochastic variances so that perhaps prior as well as current shocks to earnings (or to discount rates) affect revisions to current returns. Measuring the information content of earnings in the presence of stochastic error term variances is one potential extension of the model. Another extension is to incorporate one-sided error terms into the analysis. The impact of conditional conservatism on security returns could be analyzed by incorporating two error terms into the earnings equation of the VAR system: (a) a standard symmetric error term and (b) a one-sided error term. This formulation would allow for negative and positive earnings shocks to have a different and possibly nonlinear impact on returns.

The theory developed in this paper has a number of limitations, three of which raise some rather deep modeling issues. In Ohlson and Sims fashion, we assumed throughout the paper that the underlying information dynamics have an autoregressive reduced-form structure, which leaves open the question what underlying information structure and economic setting would produce this structure. In other words, what structural model of the firm would yield an autoregressive reduced-form representation? A related issue is that in the absence of underlying incentives, actors, and decisions, it could be argued that the theoretical measures of information content developed in this and related studies are somewhat ad hoc. Another vexing problem is the well-known issue of aggregation. The theory developed above relates to the individual firm. But empirical implementation is typically based on multiple and heterogeneous firm-level data. Extending the models developed here to address these issues is part of a future research agenda.

Appendix: Proofs of propositions

Proof of Proposition 1

The proofs of the propositions to follow are simplified substantively by the use of matrix notation. The expectation of the linear dynamic of (8a) and (8b) can be rewritten in the autoregressive matrix form:

\[
E_t \left( \begin{array}{c}
roe_{t+1} \\
v_{t+1}
\end{array} \right) = \left( \begin{array}{c}
\beta \\
0
\end{array} \right) \left( \begin{array}{c}
roe_t \\
v_t
\end{array} \right) \tag{A1}
\]
Solving for the $t + j$ period expected values of the two variables in (A1) by standard eigenvalue methods yields:

$$E_t(\text{roe}_{t+j}) = \beta_j \text{roe}_t + \left(\frac{\alpha^j - \beta^j}{\alpha - \beta}\right) u_t$$
$$E_t(u_{t+j}) = \alpha j u_t$$ (A2).

By assumption

$$\Delta E_t \sum_{j=1}^{\infty} \rho^j r_{t+j} = 0$$

and $\rho = 1$ so that Vuolteenaho's return decomposition relation (7a) becomes

$$r_t - E_{t-1}(r_t) = \Delta E_t \sum_{j=0}^{\infty} \text{roe}_{t+j}$$ (A3).

Substituting (A2) into (A3) yields (9). Taking variances of both sides of (9) yields (10).

**Proof of Proposition 2**

The expectation of the linear dynamic of (11a), (11b), and (11c) can be rewritten in the autoregressive matrix form:

$$
E_t \begin{pmatrix}
X_{1t+j}/BV_{t+j-1} \\
X_{2t+j}/BV_{t+j-1} \\
u_{t+1}
\end{pmatrix} = \begin{pmatrix}
\beta^1 & 0 & 1 \\
0 & \beta^2 & 1 \\
0 & 0 & \alpha
\end{pmatrix} \begin{pmatrix}
X_{1t+j-1}/BV_{t+j-2} \\
X_{2t+j-1}/BV_{t+j-2} \\
u_t
\end{pmatrix}
$$ (A4).

Solving for the $t + j$ period expected values of the three variables in (A4) by standard eigenvalue methods yields:

$$E_t(X_{1t+j}/BV_{t+j-1}) = \beta^1_j X_{1t}/BV_{t-1} + \left(\frac{\alpha^j - \beta^1_j}{\alpha - \beta^1}\right) u_t$$
$$E_t(X_{2t+j}/BV_{t+j-1}) = \beta^2_j X_{2t}/BV_{t-1} + \left(\frac{\alpha^j - \beta^2_j}{\alpha - \beta^2}\right) u_t$$
$$E_t(u_{t+j}) = \alpha_j u_t$$ (A5).

From (A3) we obtain
\[ r_t - E_{t-1}(r_t) = \Delta E_t \sum_{j=0}^{\infty} \text{roe}_t + j \]

where the second step follows from the approximation \( \log(1 + z) \approx z \). Substituting (A5) into (A6) yields (12). Taking the variance of both sides of (12) yields (13).

**Proof of Proposition 3**

Similar to the proof of Proposition 1. Note that the drift term \( \alpha' \) disappears with application of the \( \Delta E_t \) operator.

**Proof of Proposition 4**

Similar to the proof of Proposition 2.

**Proof of Proposition 5**

See Vuolteenaho 2002.

**Proof of Proposition 6**

Follows immediately from (A6) and Proposition 5.

**Endnotes**

1. “Shocks”, “revisions”, and “unexpected” are equivalent nomenclature, so, for example, instead of shocks to returns, we could equivalently refer to revisions to returns or unexpected returns. However, one should not confuse shocks to returns with changes to returns.

2. Also, the Vuolteenaho 2002 model does not allow for negative net dividends because of the log transformation. Callen and Segal (2004) finesse this limitation by using the operating assets equation from the Feltham and Ohlson 1995 breakdown of the clean surplus condition. Their approach is not costless because net financial assets must be positive in their cash flow equation. Nevertheless, we prefer to derive our results using Vuolteenaho 2002 because the formulations are simpler and more elegant.

3. The returns version of the Ohlson 1995 model has been used empirically in a number of papers (e.g., Easton and Harris 1991; Easton, Eddy, and Harris 1993). This model — equation (4) in Easton 1998 — is theoretically, but not necessarily empirically, problematic because the model holds only under fairly specific sufficient conditions: for example, dividends are intertemporally constant or prior-period dividends are zero. Because the returns equation holds for all time periods, the latter assumption leads to the reductio ad absurdum that dividends are zero for all time periods. Interestingly, this criticism does not apply to the unexpected returns version of the model — equation (6) in Easton 1998. This latter version is correct no matter what the dividend dynamic.
4. The (abnormal) earnings dynamic is AR(1) or AR(1) with drift.

5. Antle et al. (1994) show that additive and independent cash flows and nonrestrictive accounting recognition rules are sufficient for Ball-Brown analysis to hold in a Bayesian framework.

6. Because (1) is defined in terms of market value rather than price per share, the clean surplus relation of (2) is necessarily defined on a total basis rather than on a per share basis. The concerns raised by Ohlson 2005 regarding the RIV model are not relevant to the Vuolteenaho return decomposition relation because the latter is essentially an identity, not a valuation model. In particular, the proof of the Vuolteenaho return decomposition relation goes through without reference to nonarbitrage or Miller and Modigliani assumptions.

7. We follow the common convention in this literature in denoting variables by uppercase letters and their log by lowercase letters.

8. Without loss of generality, we define returns gross of the risk-free rate to simplify the notation. For empirical estimation, Vuolteenaho (2002) defines returns as net of the risk-free rate so that he has to subtract the risk-free rate from earnings news or expected returns news.

9. The maintained assumption that the change in the expectations of the cumulative (discounted) approximation error is zero cannot be demonstrated theoretically. Empirically, this term appears to be negligible. See Vuolteenaho 2002 (256), for example.

10. Campbell and Shiller (1988a, b) show that price is the present value of expected future dividends in a precursor model to Vuolteenaho’s. They obtain

\[ p_t = \frac{h}{(1 - \rho)} + E_t \left[ \sum_{j=0}^{\infty} \rho^j ((1 - \rho)d_{t+1+j} - r_{t+1+j}) \right]. \]

This valuation equation says that the current market price of equity is a constant plus the infinite sum of expected (weighted) future dividends less expected (weighted) future returns, which is similar to a standard dividend valuation model, except that the dynamic discount rates (expected returns) are in the numerator.

11. “Other information” denotes value-relevant accounting information other than earnings and book value plus value-relevant nonaccounting information.

12. All proofs in this paper go through if the various processes analyzed are assumed to have drift terms as well. The drift terms disappear from the analysis as a consequence of the “changes to expectation” operator.

13. Equation (9) is similar in form to (6) of Ohlson 1995.

14. This is also equal to the variance of realized returns given the assumption that expected returns are intertemporally constant. We assume throughout the paper that the “other information” error term is independent of all other error terms to simplify the exposition. Relaxing this assumption is a straightforward exercise that has no impact on current-period unexpected returns (as in (9), for example). However, it does affect the variance of unexpected returns. For example, relaxing the independence assumption results in a covariance term in (10).
15. The statement is conditional. Unconditionally, the ex ante expected impact of an earnings shock on returns is necessarily zero because the ex ante expected earnings shock is zero.

16. To see this, note that without “other information” shocks and because of stationarity, (10) can be written as

\[ \text{Var}(r_t - E_{t-1}(r_t)) = \frac{\sigma^2}{(1 - \beta)^2} = \frac{(1 + \beta)}{(1 - \beta)} \sigma_E^2, \]

where \( \sigma_E^2 \) is the variance of (log-deflated) earnings.

17. The literature on the relative value-relevance of domestic and foreign earnings described in section 4 further illustrates the complementary nature of the Ball-Brown and variance metrics.

18. One could combine the ERC and variance metrics into one measure using standardized regressions. However, for the many reasons cited by the statistics literature, standardized regressions are problematic relative to a separate analysis of each of the ERC and variance metrics. On this issue, see Achen 1982; Bring 1994; Darlington 1990; Greenland, Schlesselman, and Criqui 1986; Hanushek and Jackson 1978; Kim and Ferree 1981; King 1986; Kruskal and Major 1989; Pedhazur 1982; and Tukey 1954.

19. This two-step process for interest rates follows from the macroeconomics literature. Alternatively, one can assume a simpler one-step process in which expected future returns are AR(1), which yields similar, if slightly different, formulas.

20. For simplicity, we assume that “other information” does not affect interest rates directly. It is a fairly straightforward exercise to incorporate “other information” in interest rates.

21. Note that (16) can be rewritten as

\[ r_t - E_{t-1}(r_t) + \frac{1}{(1 - \beta^r)}(E_t(r_t + 1) - E_{t-1}(r_t + 1)) = \frac{\epsilon_{t-1}}{(1 - \beta)} + \frac{\epsilon_{t}^o}{(1 - \alpha)(1 - \beta)}, \]

indicating that shocks to earnings and “other information” potentially affect both contemporaneous and future returns. Thus, when expected returns are dynamic, a more expansive definition of value-relevance is required, as highlighted by Gonedes 1978 and Antle et al. 1994.


24. See Foster 1977, Griffin 1977, and Brown and Rozef 1979. Although we do not directly model seasonality, it is a fairly straightforward exercise given the generalized propositions alluded to in the text discussion below and available from the author.

25. See Dechow and Ge 2006.

26. The first part of (23) is the direct solution to
In contrast, the residual solution equals \((e'_1 + \lambda'_1)\eta_j\). On the difference between these solutions, see the discussion in the text, below.

27. For example, in the third case, if \(\beta_1\) is also positive, then \(f_1 \neq 0\).

28. Again this comes about because the VAR system captures all persistences and their interactions, whereas standard regressions do not.

29. The same approach could be used with ARIMA processes, assuming that there are not sufficient time-series data at the firm-level.

30. A well-known example of such a criterion is the adjusted \(R^2\). Including an additional lag increases the \(R^2\) but at the same time increases the penalty (the degrees of freedom) from having to estimate an additional parameter. The adjusted \(R^2\) has been shown to be a poor criterion for choosing lag structures, both on theoretical grounds and from simulations.

31. See, for example, Morel 1999, who uses an AICc criterion to determine the "optimal" lag structure for her VAR system.

32. In a follow-up study, Bodnar, Hwang and Weintrop (2003) find similar results for firms domiciled in Australia, Canada, and the United Kingdom. Similarly, Garrod and Rees (1998) find that U.K. multinational firms are valued more highly than U.K. domestic firms.


34. The accruals and cash flow news items are particularly important because the accrual/cash flow breakdown is normally only provided at the SEC filing and not at the preliminary earnings announcement date.

35. For an empirical analysis, see Callen et al. 2006.

36. See Propositions 2 and 6 and especially (13) and (29), respectively.

37. See Bhat 2008.

38. For some (contradictory) empirical results regarding the value-relevance of fair value income volatility, see Barth, Landsman, and Whalen 1995 and Hodder, Hopkins, and Whalen 2006.

39. On the issue of trust and control in joint ventures, see Emsley and Kidon 2007, for example.

40. See, for example, Hamilton 1994, regarding eigenvalue methods.

References


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