Management Bonus Plans in a Multiple-agent Environment

JEFFREY L. CALLEN
Jerusalem School of Business Administration, The Hebrew University, Israel and Faculty of Business, McMaster University, Canada

This paper analyses optimal linear incentive contracts in a single-principal multiple-agent setting. It is proved that the second-best share ratios are function of the other agents' effort levels as well their degrees of risk aversion and the risk aversion of the principal. It is also shown that the total bonus pool to all agents in a difficult to monitor (decentralized) firm is greater than the total bonus pool in a easier to monitor (centralized) firm.

INTRODUCTION

The use of incentive contracts (bonus plans) to motivate senior managers of highly decentralized firms is ubiquitous. It has been estimated that 'more than 90 per cent of the top managers of decentralized profit centres in large corporations are eligible for an annual bonus' (Kaplan, 1982, p. 566). It is argued that these bonus plans serve to motivate the manager (the agent) to operate as efficiently as possible in the firm's (the principal) overall interest, especially in circumstances where it is very costly to monitor the manager. One of the most common of these bonus plans is the linear incentive contract. In a linear contract, the agent is compensated by a fixed fee, that is, a wage plus a percentage—called the 'sharing ratio'—of the firm's overall profits, where profits are usually defined in terms of an accounting number. In general, the greater the sharing ratio, the greater the agent's incentive to reduce costs. On the other hand, a larger ratio increases the agent's risks and, hence, generally necessitates a concomitant increase in fixed wages. The linear incentive contract has been studied extensively by Stiglitz (1974), Demski and Feltham (1978) and Weitzman (1980), among others.

The purpose of this article is to extend the analysis of the linear incentive contract to a single-principal multiple-agent framework. In general, the incentive contract in a large corporation applies to all senior managers. For want of a better example, we will assume that these are the divisional managers of the firm. However, then it is quite clear that the efforts of one divisional manager will generally impact on the returns obtained by his colleagues. This raises the interesting issue as to the effect of such interactions on the optimal form of the compensation function. This issue can, of course, only be studied in a multiple-agent framework. Most surprisingly, we will show that, even in those circumstances in which there is no direct interaction between one division and another, it is still true in general that each divisional manager's sharing ratio will depend upon the efforts of the other divisional managers. Furthermore, we will show that the total allocation of profits to the divisional managers—basically, the bonus pool—in a diversified company is greater than for a centralized firm in which it is possible to directly monitor the managers.

In what follows, the next section introduces the single-principal multiple-agent model utilized in this paper. The third section solves for and characterizes the optimal first-best sharing ratios for a firm in which the agent's actions can be monitored directly. These sharing ratios are the benchmark against which the optimal second-best sharing ratios are to be compared. The fourth section solves for and characterizes the optimal second-best sharing ratios for a decentralized operation. This section also compares first- and second-best ratios. The final section briefly concludes the paper.

THE MODEL

We will utilize the single-principal multiple agent model introduced by Baiman and Demski (1980). This is a straightforward extension of the Holmstrom (1979) single-agent model. However, unlike either of these two articles, we will focus our analysis on linear incentive contracts in the spirit of Weitzman (1980). Formally, we assume that the firm comprises a principal and two agents. The firm's profits are assumed to be a function of both of the agents' efforts and the state of nature. More specifically, we assume that the firm's profits, denoted by X, are normally distributed.
with the mean of the distribution a function of the agents’ efforts and the variance, a constant. Formally,

\[ X \sim N(F(e_1, e_2), \sigma^2) \]

where

\[ e_i = \text{effort of agent } i \]

\[ F = \text{mean profits, a concave function of } e_1 \text{ and } e_2 \]

\[ F(e_1, e_2) = F(e_1, 0) = 0, \quad \frac{\partial^2 F}{\partial e_1 \partial e_2} \geq 0 \]

\[ \sigma^2 = \text{variance of profits.} \]

In addition, the principal and the agents are assumed to have utility functions which exhibit constant absolute risk aversion.

Given the above data, we can now describe the principal’s optimization program, namely,

\[
\begin{align*}
\text{maximize} & \quad E \{ (1 - S_1 - S_2) X - W_1 - W_2 \} \\
\text{subject to} & \quad U_i[S_1 X + W_1 + r_i(e_i - e_i)] \geq U_i[r_i e_i] \\
& \quad U_j[S_2 X + W_2 + r_j(e_j - e_j)] \geq U_j[r_j e_j] \\
& \quad e_1 \text{argmax } E U_1[S_1 X + W_1 + r_i(e_i - e_i)] \\
& \quad e_2 \text{argmax } E U_2[S_2 X + W_2 + r_j(e_j - e_j)]
\end{align*}
\]

where

\[ E = \text{expectations operator} \]

\[ G = \text{principal’s utility function} \]

\[ U_i = \text{agent } i's \text{ utility function} \]

\[ S_i = \text{agent } i's \text{ sharing ratio} \]

\[ W_i = \text{agent } i's \text{ wage} \]

\[ e_i = \text{agent } i's \text{ maximum working capacity} \]

\[ r_i = \text{agent } i's \text{ opportunity cost for work outside of the firm.} \]

In the absence of constraints (4) and (5), the above program determines the first-best Pareto-optimal sharing ratios and effort levels. Such a solution would be obtained provided the agents’ efforts could be monitored (costlessly) by the principal, as, for example, in a highly centralized firm. Since the essence of decentralization is the inability of the principal to monitor his agents, the first-best solution only serves as a benchmark against which the second-best solution, that is, the solution of the above program, can be compared. Constraints (4) and (5) simply state that each agent will choose that effort level which maximizes his own objective rather than the one which maximizes the firm’s objective.

The assumptions that each participant has constant absolute risk-aversion and that \( X \) is normally distributed allows us to transform the above program into:

\[
\begin{align*}
\text{maximize} & \quad (1 - S_1 - S_2) F(e_1, e_2) \\
& \quad - 1/2C(1 - S_1 - S_2) \sigma^2 - W_1 - W_2 \\
\text{subject to:} & \quad S_1 F(e_1, e_2) - 1/2C_1 S_1^2 \sigma^2 + W_1 + r_1(e_1 - e_1) \geq r_1 \bar{e}_1 \\
& \quad S_2 F(e_1, e_2) - 1/2C_2 S_2^2 \sigma^2 + W_2 + r_2(e_2 - e_2) \geq r_2 \bar{e}_2
\end{align*}
\]

where

\[ C = \text{the principal’s absolute risk-aversion coefficient} \]

\[ C_i = \text{the agent } i's \text{ absolute risk-aversion coefficient.} \]

### The First-Best Solution

Before we solve the above program we shall analyze our benchmark first-best solution which is determined by deleting constraints (4a) and (5a). Solving constraints (2a) and (3a) as equalities yields:

\[
\begin{align*}
S_1 F(e_1, e_2) - 1/2C_1 S_1^2 \sigma^2 + W_1 - r_1 e_1 = 0 \quad (6) \\
S_2 F(e_1, e_2) - 1/2C_2 S_2^2 \sigma^2 + W_2 - r_2 e_2 = 0 \quad (7)
\end{align*}
\]

Substituting Eqs (6) and (7) into (1a) gives the optimization problem:

\[
\begin{align*}
\text{maximize } & \quad F(e_1, e_2) - 1/2C(1 - S_1 - S_2)^2 \sigma^2 \\
& \quad - 1/2C_1 S_1^2 \sigma^2 - 1/2C_2 S_2^2 \sigma^2 - r_1 e_1 - r_2 e_2 \quad (8)
\end{align*}
\]

Solving (8) yields the first-order conditions:

\[
\begin{align*}
C(1 - S_1 - S_2) \sigma^2 - C_1 S_1 \sigma^2 = 0 \quad (9) \\
C(1 - S_1 - S_2) \sigma^2 - C_2 S_2 \sigma^2 = 0 \quad (10)
\end{align*}
\]

which in turn yield the first-best sharing ratios

\[
\begin{align*}
S_1^* = \frac{C C_2}{C C_2 + C C_1 + C_1 C_2} \quad (11) \\
S_2^* = \frac{C C_1}{C C_2 + C C_1 + C_1 C_2} \quad (12)
\end{align*}
\]

We can characterize the first-best optimal sharing ratios by the following proposition:

**Proposition I**

The optimal first-best sharing ratios are:

1. Both separately and in total less than one;
2. Decreasing in the agent’s own level of risk-aversion;
3. Increasing in the degree of risk-aversion of the principal and the other agent;
4. Independent of each agent’s efforts.

**Proof:** Clearly, \( 0 < S_1^* < 1 \) and \( 0 < S_1^* + S_2^* < 1 \). In addition,

\[
\frac{\partial S_i^*}{\partial C_i} < 0 \quad \text{and} \quad \frac{\partial S_i^*}{\partial C_j} > 0 \quad i \neq j
\]

The first-best sharing ratios must obviously be independent of the agents’ efforts since the latter are determined, in a first-best case, by the principal. QED.

The first-best effort levels are obtained by optimiz-
ing over total profits to all participants, that is:

$$\text{maximize } F(e_i, e_j) - 1/2C(1 - S_1 - S_2)\sigma^2 - 1/2C_1S_1^2\sigma^2 - 1/2C_2S_2^2\sigma^2 - r_1e_1 - r_2e_2$$

(13)

This yields the first-order conditions

$$F_1(e_i, e_j) = r_1$$

(14)

$$F_2(e_i, e_j) = r_2$$

(15)

where $F_i = \partial F/\partial e_i$. Equations (14) and (15) can be solved for the first-best effort levels $e_1^*, e_2^*$.

**THE SECOND-BEST SOLUTION**

The second-best solution can be obtained by solving Eqns (4a) and (5a) for the optimal effort levels as (implicit) functions of the sharing rules, that is, $e_1 = e_1(S_1, S_2)$ and $e_2 = e_2(S_1, S_2)$. These effort levels can then be substituted into Eqns (1a), (2a) and (3a), yielding an optimization program which is a function of the sharing rules.

Proceeding with this approach, Eqns (4a) and (5a) are solved to yield:

$$S_1F_1(e_i, e_j) - r_1 = 0$$

(16)

$$S_2F_2(e_i, e_j) - r_2 = 0$$

(17)

Equations (16) and (17) provide the optimal effort levels as functions of $S_1$ and $S_2$ i.e. $e_i(S_1, S_2)$ and $e_2(S_1, S_2)$.

Implicit differentiation of Eqns (16) and (17) yield the results that

$$e_{11} > 0, \quad e_{22} > 0, \quad e_{12} \geq 0 \quad \text{and} \quad e_{21} \geq 0$$

where

$$e_{ij} = \frac{\partial e_i}{\partial S_j}$$

We can now determine the optimal second-best sharing rules $\hat{S}_1$ and $\hat{S}_2$ by substituting $e_i(S_1, S_2)$ and $e_2(S_1, S_2)$ into Eqn (8), giving the optimization program:

$$\text{maximize } F(e_i(S_1, S_2), e_2(S_1, S_2))$$

$$- 1/2C(1 - S_1 - S_2)\sigma^2 - 1/2C_1S_1^2\sigma^2 - 1/2C_2S_2^2\sigma^2 - r_1e_1(S_1, S_2) - r_2e_2(S_1, S_2)$$

(18)

The first-order conditions are:

$$Z_1 + C(1 - S_1 - S_2) - C_1S_1 = 0$$

(19)

$$Z_2 + C(1 - S_1 - S_2) - C_2S_2 = 0$$

(20)

where

$$Z_1 = [(F_1 - r_1)e_{11} + (F_2 - r_2)e_{21}]/\sigma^2$$

(21)

$$Z_2 = [(F_1 - r_1)e_{12} + (F_2 - r_2)e_{22}]/\sigma^2$$

(22)

Solving Eqns (19) and (20) yields the optimal second-best sharing rules:

$$\hat{S}_1 = \frac{CC_2 + (C + C_2)Z_1 - C_2Z_2}{CC_2 + CC_1 + C_1C_2}$$

(23)

$$\hat{S}_2 = \frac{CC_1 + (C + C_1)Z_2 - C_1Z_1}{CC_2 + CC_1 + C_1C_2}$$

(24)

**Proposition II**

Assuming $Z_1 \neq Z_2$, each agent's optimal second-best sharing ratio is a function of the other agent's efforts.

**Proof:** As we show in the next proposition $Z_1, Z_2 > 0$. Therefore, under the condition of this proposition, $\hat{S}_1$ is a function of $Z_2$ and $\hat{S}_2$ is a function of $Z_1$. $\hat{S}_1$ is innocuous, since the equality of $Z_1$ and $Z_2$ would occur only by happenstance.

Proposition II is interesting because it holds under fairly general conditions. Even if each agent's effort is not a direct function of his colleague's sharing ratio (i.e. $e_{12} = e_{21} = 0$) and even if the production function $F$ is separable in agents' efforts (i.e. $F_{12} = F_{21} = 0$), each agent's optimal sharing ratio will depend on the other's efforts.

The proposition which follows characterizes the optimal second-best sharing ratio.

**Proposition III**

The optimal second-best sharing ratios are:

1. Both separately and in total less than one;
2. Decreasing in the agents' own level of risk-aversion;
3. In total greater than the sum of the first-best sharing ratios;
4. In total decreasing with variance of profits.

**Proof**

(1) Suppose $\hat{S}_i \leq 0$, then $e_i = 0$ by the first-order condition (16). However, if $e_i = 0$, total firm profits are negative, contradicting the optimality of $\hat{S}_1, \hat{S}_2$. Similarly, $Z_i > 0$. It is also true that $\hat{S}_1 + \hat{S}_2 < 1$, for if $\hat{S}_1 + \hat{S}_2 > 1$, the principal earns a negative return (see Eqn (1a)), again contradicting the optimality of $\hat{S}_1, \hat{S}_2$. Since the $\hat{S}_i$ are positive and total less than one, it follows immediately that $0 < \hat{S}_i < 1, i = 1, 2$.

(2) This follows since

$$\frac{\partial \hat{S}_i}{\partial C_i} < 0, \quad i = 1, 2$$

(3)

$$\hat{S}_1 + \hat{S}_2 = \frac{CC_1 + CC_2 + C_1Z_1 + C_2Z_2}{CC_2 + CC_1 + C_1C_2}$$

Thus $\hat{S}_1 + \hat{S}_2 > S_1^* + S_2^*$, provided $Z_1, Z_2 > 0$. However, $Z_1, Z_2 > 0$, provided $F_1 - r_1 > 0$ and $F_2 - r_2 > 0$.

From the first-order condition (16),

$$F_1 - r_1 = \frac{r_1}{S_1} - r_1 > 0$$

since $0 < S_1 < 1$ and similarly for $F_2 - r_2$.

(4) This follows, since

$$\frac{\partial (\hat{S}_1 + \hat{S}_2)}{\partial \sigma^2} < 0. \quad \text{QED}$$

In addition to characterizing the optimal sharing ratios, we can also prove the following rather intuitive result about the second-best effort levels $\hat{e}_1, \hat{e}_2$. 


Proposition IV

The second-best effort levels of each agent is less than their first-best effort levels (i.e. \( \hat{e}_i < e_i^* \)).

Proof: This result is immediate by comparing the first-order conditions of the first-best solution (Eqns (14) and (15)) and the first-order conditions for the second-best solution (Eqns (16) and (17)) and noting that \( r_i/S_i > r_i \) (since \( 0 < S_i < 1 \)).

Perhaps the most important result of this section, as proved in Proposition III, is the fact that the sum of the second-best sharing ratios is greater than the sum of the first-best ones. This implies that for the decentralized firm the total bonus pool (here defined to be the ex ante profits accruing to all divisional managers) should be greater than for a centralized firm, where managers can be monitored more readily. This result is really quite intuitive. One would expect that to motivate the managers of a firm where shirking is possible, the principal would have to provide a greater bonus pool than in the first-best case, where managerial efforts are in fact dictated by the principal.

It is worth noting that, while in the aggregate the second-best sharing ratios are greater than the first-best ratios, at the individual level it is quite possible that a specific divisional manager’s second best sharing ratio is less than his first-best ratio.

The first-best and second-best ratios differ on other grounds. Thus, for example, unlike the first-best ratios (see Proposition I) the second-best ratios bear no simple relationship to either the risk-aversion coefficient of the principal or the risk-aversion coefficient of the other agent. In addition, unlike the first-best ratios, the second-best ratios are functionally related to the parameters of the distribution. In particular, we have noted that the greater the variance of profits, the smaller the bonus pool (i.e. the sum of the ratios) for the divisional managers in aggregate. This latter result is also intuitive. Specifically, the greater the variance of profits, the smaller are the ex ante profits which accrue to the agents (for any given sharing ratios) and hence the less impact does the bonus pool have on motivating effort. Thus the principal will reduce the bonus pool if the firm is riskier.

CONCLUSION

The purpose of this paper has been to analyse linear incentive contracts in a single-principal multiple-agent setting. It was found that the optimal second-best share ratio of each agent is affected not only by the degree of risk-aversion of the principal and other agents but also by the effort level of other agents. In addition, it was proved that the sum of these second-best sharing ratios is larger than the sum of the first-best ratios. This result implies that in order to properly motivate managers of decentralized firms, the principal has to offer managers a larger bonus pool than would be the case for a centralized firm, in which monitoring the manager’s efforts is more easily carried out. Not surprisingly, it was also found that the effort levels of managers in decentralized firms were less than would be optimal for managers in centralized firms.

NOTES

1. See Kaplan (1982, pp. 574–5) for a brief description of some common bonus plans. The notion that such plans are linear is somewhat of a simplification. More often than not, the bonus is paid only if the firm has earned some minimum return. See Healy (1985).

2. See Weitzman (1980) for additional references.

3. To be more exact, the model presented here is a multi-agent extension of the Weitzman model as modified by Richard Khistrom—to whom we owe an intellectual debt. As mentioned in the text, all of these models are simple modifications of the Holmstrom model. For other forms of the principal agent framework, see Ross (1973) and Shavell (1979).

4. The extension to more than two agents is immediate and adds, at least in the context of this model, no new insights.

5. This model allows the agent to work outside of the firm so his return includes not only wages and bonus but also \( r_i(\hat{e}_i - e_i) \) from external opportunities.

6. Constraints (2a) and (3a) are indeed equalities, since the principal has no incentive to offer more than each agent’s opportunity cost in the external labour markets.

7. Implicit here is a standard Nash–Cournot assumption, namely, that each agent optimizes assuming that the other has chosen his optimum effort level. See also Baiman and Demski (1980, pp. 196–7).

8. The proof in Proposition III that \( Z_i, Z_j > 0 \) is predicted on the assumption that \( F(e, 0) = F(0, e_i) = 0 \), which, of course, means that the two divisions interact. However, Proposition II does not really depend on the assumption that \( Z_i, Z_j > 0 \). All that has to be shown for the latter proposition is that \( Z_i, Z_j \neq 0 \). However, the \( Z_i \neq 0, i = 1, 2 \) provided \( F_i, \#_i, i = 1, 2, \) and thus provided \( S_i \neq 1 \) (see Eqns (16) and (17)) \( i = 1, 2, \) if one of the \( S_i = 1 \), say \( S_i \), then the principal is earning a negative return unless \( S_i < 0 \). Thus for Proposition II to hold for, say, a separable effort function (i.e. \( F(e_i, e_j) = F_i(e_i) + F_j(e_j) \)) one need only assume that the bonus plan only allows for positive sharing ratios.

9. See also note 8.

10. This is so, since \( F(0, e_i) = 0 \).

REFERENCES


MANAGEMENT BONUS PLANS IN A MULTIPLE-AGENT ENVIRONMENT


