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A Medieval Controversy About Profit and Loss Allocations

This paper analyses a controversy between Maimonides and Rabbi Abraham Ben David of Posquierres over the allocation of partnership profits and losses in the absence of a partnership contract. The Maimonidean allocation system depends upon whether the partnership is formed for business or non-business (consumption) activities. The controversy concerns the notion of indivisibilities in partnership capital. This early debate is analysed in light of the modern—including game-theoretic—cost allocation literature.

Key words: Cost allocation methods; Partnerships.

The issue of profit or loss allocation in partnerships can be traced to the third century AD if not to earlier times.[1] This note describes and analyses a medieval controversy on the subject between Maimonides (1135-1204) and his contemporary Rabbi Abraham Ben David of Posquierres (1125-1198) (hereafter, Rabad).[2]

The next section describes Maimonides' allocation system and then the Rabad's objections to it are discussed. The relationship of this early discussion of the allocation problem to the modern literature is then considered.

THE MAIMONIDEAN ALLOCATION SYSTEM

Although Maimonides is best known to the western world as a medieval philosopher, his fame during his lifetime stemmed primarily from the compilation of the Mishnah Torah which is a legal code of Biblical and Jewish oral laws based on the Talmud.[3] Many of the issues discussed in the Mishnah Torah deal with business contract law. For example, there are sections which deal with borrower-lender relationships, buyer-seller relationships, laws of business property, laws of rental property, laws of inheritance, agency and, most important for our purposes, partnerships.

One of the most interesting issues dealt with is the problem of allocating profits and losses accruing to partnerships or joint ventures in cases where there are *no* specific contractual agreements. The following is a translation of the relevant section of the Mishnah Torah:

[1] See, for example, the discussion of partnership allocations in the Mishnah Ketubot, Chapter 10, Section 4. The Mishnah was re-enacted in the third century AD but is based on much earlier oral traditions. On the Mishnah, see *Encyclopedia Judaica* (1972), Vol. 12, pp. 93-109.

[2] On Rabad, see Twersky (1962).

[3] On the Talmud, see *Encyclopedia Judaica* (1972), Vol. 15, pp. 750-79.

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> Partners who invest together, one [partner invests] one hundred, the other two hundred, and the third three hundred, and they use the funds jointly in a business transaction, the gain or loss is divided equally among them, and not in proportion to their [original] investment. Although, had [the partners] bought an ox for slaughter, and had they slaughtered it, each [partner] would have taken meat in proportion to his investment, nevertheless, if they sold it alive and they lost or gained [on the sale], they share the profit or loss equally. When do these rules apply? If they utilized the money which they jointly invested in the partnership. But, if the money is still around and they did not expend it and they lost or gained because of the monetary policy of the king or of the decisionmakers of the state, they divide the profit or loss in proportion to their [original] investment. (Maimonides, Chapter Four, Section Three, 1968)

The Maimonidean allocation system would appear to be dichotomous in nature. If the partnership invests in a business-related activity, then profits or losses must be divided equally. In contrast, profits or losses on capital invested in a consumption activity, that is, buying the ox for slaughter, or profits and losses on funds which were not invested altogether, are allocated in proportion to each partner's initial investment. Although Maimonides does not explain the rationale for his allocation system, it can be inferred from Rabad's commentary.

RABAD'S DISAGREEMENT

Rabad wrote a critical commentary on the Mishnah Torah in which he voiced his disagreements with many of Maimonides' legal rulings. His work is important not only because it explains Rabad's own rulings but also because it sheds light on Maimonides' position. One such criticism by Rabad is concerned directly with the issue at hand. Commenting on Maimonides' decision in the case of the live ox, Rabad says:

> These words [of Maimonides] are correct in the case where they sold the live ox, but only in so far as it concerns the profits earned *from its work* because the smaller portion [of the ox] needs the larger portion. (Ben David, Chapter Four, Section Three, 1968, emphasis added)

Thus, Rabad disagrees with Maimonides in the case of the live ox bought for speculative purposes. According to Rabad, speculative gains and losses should be allocated in proportion to initial capital investment. An equal allocation of gains and losses—the Maimonidean solution—is sensible *only* in so far as it concerns any income generated from the animal's work because then 'the smaller portion [of the ox] needs the larger portion'. In effect, Rabad argues that operating profits—profits from the ox's work—are generated by an indivisible input, namely the ox. Just as one cannot say that the larger portion of the ox (e.g., the body) is more important than a smaller portion (e.g., the heart) in generating operating profits, both are equally necessary, so one cannot rationally argue that operating profits be allocated in proportion to the initial investment. Rather, it is the entire indivisible partnership investment, namely, the ox, which generates operating profits so that only an equal allocation of operating profits, independent of initial investment, is 'rational'.

It would appear, therefore, that the only real point of contention between Maimonides and Rabad is whether or not speculative investments in an ox are divisible or indivisible. Maimonides implicitly argues that such investments are indivisible so that speculative profits, like operating profits, are allocated equally, independently of initial investment. On the other hand, the Rabad suggests that speculative

investment, unlike investment for operating purposes, is divisible so that resultant profits or losses should be allocated in proportion to each partner's investment, not equally. Both Maimonides and Rabad agree that where the investment is for a non-business activity such as consumption (or the capital has yet to be invested), the indivisibility problem does not arise, and the liquidation of the partnership profits or losses is proportional to the initial investment.

THE CONTROVERSY IN LIGHT OF THE MODERN ALLOCATION LITERATURE

Although 800 years separate the Maimonides-Rabad controversy from the work of Thomas, the latter is relevant for analysing the former. In particular, Thomas (1969, 1974) has shown that, unless the inputs into a production technology are additively separable, no unique allocation of costs or profits is possible, and all allocations are necessarily arbitrary. The same conclusion, albeit of far more limited scope, appears in Rabad's analysis. Specifically, Rabad argues that non-separability of the ox's constituent parts in generating profits means that no unique allocation, such as in proportion to initial investment, is reasonable. However, since some allocation must be specified by the courts in the absence of a prior partnership agreement, an equal allocation is mandated. An equal allocation, it would appear, reflects the concept that it is total investment in the ox which is responsible for generating operating earnings.

Maimonides further broadens the scope of the equal allocation ruling to all business ventures, speculative or operating. Thus, Maimonides seems to be arguing that the non-separability concept applies not only to the ox's action in generating output but also to the partnership capital as well. That is, the minute partnership funds are invested in an indivisible input, the ox, no unique allocation of the ox's profit is possible whether the profit is operating or speculative. This is in contradistinction to the case where the money has yet to be invested, so that the inputs are separable, or the case of consumption where the input itself is physically separable.[4]

The equal allocation solution of Maimonides, and of Rabad in the case of operating profits, has its parallel in the game-theoretic allocation literature.[5] Much of this literature, which is concerned with the problem of allocating profits (or costs) to indivisible inputs, that is, inputs which interact to generate profits, argues for the centrality of the Shapley value as the premier allocation mechanism. Indeed, Shapley allocations have been suggested for joint cost and transfer pricing issues (Shubik, 1962; Hamlen, Hamlen and Tschirhart, 1977, 1980), public utility pricing (Littlechild, 1970; Loehman and Whinston, 1971, 1974), allocating merger benefits (Mossin, 1968), and financial reporting (Callen, 1978). To see how the Shapley value applies to our specific context, let the function v – called the characteristic function – denote the value

[4] However, the modern literature is unlikely to agree with the notion that the consumption case is different from the investment scenario. For example, consider two partners: one wants to consume one-quarter and the other three-quarters of the ox. Now, suppose if they act independently, the former would pay $25 and the latter $75 for their consumption activities. However, if they pool their resources, they can acquire an ox for $90. Clearly, there is no uniquely rational manner in which to allocate the $10 of savings.

[5] See, for example, Moriarity (1975), Hamlen, Hamlen, and Tschirhart (1977, 1980), Callen (1978), and Gangolly (1981).

of the partnership under alternative partnership structures. For example, v(1) denotes the value of the partnership comprised only of partner 1, in essence a single proprietorship, v(1,2) is the value of the partnership comprised of partners 1 and 2, whereas v(1,2,3) is the value of the partnership constituting the partners 1, 2, and 3.

Intuitively, the Shapley value allocates profits to each input, or equivalently to each partner, on the basis of the partner's incremental benefit to the partnership. In general, however, the incremental value of a specific partner will depend upon the order in which the partner is presumed to join the partnership. Consider the partnership comprised of partners 1 and 2. If partner 1 starts the partnership off, his incremental value to the partnership is v(1). If partner 2 is first, then partner 1's incremental value to the partnership is [v(1,2) − v(2)]. The Shapley solution assumes that each possibility is equi-probable. Therefore, the Shapley allocation to partner 1, x_1, takes on the values:

$$x_1 = 1/2\ v(1) + 1/2\ [v(1,2) - v(2)]$$
$$x_2 = 1/2\ v(2) + 1/2\ [v(1,2) - v(1)]$$

Total partnership profits are allocated by this approach since $x_1 + x_2 = v(1,2)$.

With three partners, the concept is identical although the formula is more complex. Focusing again on partner 1, there is 1/3 probability that he will be first in the partnership so that his incremental value to the partnership is v(1). There is a 1/3 probability that he will be second, in which case there are two possibilities. Either partner 2 was first so 1's incremental value is [v(1,2) − v(2)] or partner 3 was first and then 1's incremental value is [v(1,3) − v(3)]. Finally, there is a 1/3 probability that partner 1 is last in which case his incremental value to the partnership is [v(1,2,3) − v(2,3)]. Thus, in total, the Shapley allocation to partner 1 is:

$$x_1 = 1/3\ v(1) + 1/6\ [v(1,2) - v(2)] + 1/6\ [v(1,3) - v(3)] + 1/3\ [v(1,2,3) - v(2,3)]$$

Similarly, the Shapley allocations to partners 1 and 2 are:

$$x_2 = 1/3\ v(2) + 1/6\ [v(1,2) - v(1)] + 1/6\ [v(2,3) - v(3)] + 1/3\ [v(1,2,3) - v(1,3)]$$
$$x_3 = 1/3\ v(3) + 1/6\ [v(1,3) - v(1)] + 1/6\ [v(2,3) - v(2)] + 1/3\ [v(1,2,3) - v(1,2)]$$

Again, total profits are allocated, since $x_1 + x_2 + x_3 = v(1,2,3)$.

Let us now reconsider Maimonides' example in which partners 1, 2, and 3 invest 100, 200, and 300, respectively. Assume that the partnership capital is in fact expended on an indivisible business (non-consumption) venture, like the ox, and that all three partners are needed to finance the venture. Maimonides' example will then take on the characteristic function form:

$$v(1) = 100; \qquad v(2) = 200; \qquad v(3) = 300$$
$$v(1,2) = 300; \qquad v(1,3) = 400; \qquad v(2,3) = 500$$
$$v(1,2,3) = Y$$

where Y is the liquidation value of the partnership. Solving for the Shapley value of the game yields the allocations:

$$x_1 = 100 + (1/3)\ (Y - 600)$$
$$x_2 = 200 + (1/3)\ (Y - 600)$$
$$x_3 = 300 + (1/3)\ (Y - 600)$$

But these Shapley allocations are identical to Maimonides' equal allocation solution in which partnership profits (losses) Y − 600 are allocated equally. In other words, given the inherent indivisibility of the input, Maimonides' solution is to allocate profits and losses generated by this input on the same basis as that obtained by a Shapley allocation.

This analysis does not mean to imply that Maimonides was a game theorist.[6] Rather, it suggests that Maimonides may have felt on intuitive grounds that allocating profits to each partner in terms of his average incremental value to the partnership is the most rational and defensible allocation mechanism in the presence of indivisible inputs.

The allocation in the consumption case—which is also Rabad's allocation for speculative profits and losses—corresponds to another allocation much discussed in the accounting literature, namely Moriarity's (1975) scheme. Applying Moriarity's allocation procedure to Maimonides' example yields the imputations:

$$x_1 = 100 + (1/6)\ (Y - 600) = (1/6)\ Y$$
$$x_2 = 200 + (2/6)\ (Y - 600) = (2/6)\ Y$$
$$x_3 = 300 + (3/6)\ (Y - 600) = (3/6)\ Y.$$

Unlike the Shapley allocation, the Moriarity scheme allocates profits (and losses) equally among the partners per dollar of initial investment—(Y − 600)/600 in Maimonides' example. This is quite reasonable in the case where the input is divisible. On the other hand, where the input is not divisible, so that the total cooperation of all partners is necessary, it is more reasonable to allocate profits per partner equally, which is of course the Maimonides-Shapley allocation.[7]

[6] The game theorist, in any case, is likely to derive the Shapley value on axiomatic rather than on intuitive grounds. See Shapley (1953).

[7] See also Gangolly (1981) on this.

REFERENCES

Ben David, A., *Criticisms of Rabad*, Otzar Hasefarim, 1968 [Hebrew].

Callen, J. L., 'Financial Cost Allocations: A Game-Theoretic Approach', *The Accounting Review*, 1978.

Encyclopedia Judaica, Keter Publishing, 1972.

Gangolly, J. S., 'On Joint Cost Allocation: Independent Cost Proportional Scheme (ICPS) and Its Properties', *Journal of Accounting Research*, 1981.

Hamlen, S. S., W. A. Hamlen, Jr, and J. T. Tschirhart, 'The Use of Core Theory in Evaluating Joint Cost Allocation Schemes', *The Accounting Review*, 1977.

Hamlen, S. S., W. A. Hamlen, Jr, and J. T. Tschirhart, 'Use of General Shapley Allocations in Joint Cost Allocation', *The Accounting Review*, 1980.

Littlechild, S. C., 'A Game-Theoretic Approach to Public Utility Pricing', *Western Economics Journal*, 1970.

Loehman, E. T. and A. B. Whinston, 'A New Theory of Pricing and Decision Making for Public Investment', *Bell Journal of Economics and Management Science*, 1971.

Loehman, E. T. and A. B. Whinston, 'An Axiomatic Approach to Cost Allocations for Public Investment', *Public Finance Quarterly*, 1974.

Maimonides, M., *Mishnah Torah*, Otzar Hasefarim, 1968 [Hebrew].

Moriarity, S., 'Another Approach to Allocating Joint Costs', *The Accounting Review*, 1975.

Mossin, J., 'Merger Agreements: Some Game-Theoretic Considerations', *Journal of Business*, 1968.

Shapley, L. S., 'The Value of an N-Person Game', in H. W. Kuhn and A. W. Tucker (eds), *Contributions to the Theory of Games*, Vol. 2, *Annals of Mathematical Studies*, 28, Princeton University Press, 1953.

Shubik, M., 'Incentives, Decentralized Control, the Assignment of Joint Costs and Internal Pricing', *Management Science*, 1962.

Thomas, A. L., *The Allocation Problem in Financial Accounting Theory, Studies in Accounting*, 3, American Accounting Association, 1969.

———, *The Allocation Problem: Part Two, Studies in Accounting*, 9, American Accounting Association, 1974.

Twersky, I., *Rabad of Posquierres: A Twelfth-Century Talmudist*, Harvard University Press, 1962.