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THE COST OF CAPITAL, MACAULAY'S DURATION, AND TOBIN'S q

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Abstract

It is shown empirically that the cost of equity capital estimated from the dividend discount model and Tobin's q are negatively related. The theoretical relationship between these variables is exploited to determine alternative estimates of the cost of equity capital and Macaulay's duration without having to estimate the growth rate g in the conventional manner. This new approach can readily be implemented for large firms reporting SFAS No. 33 data.

I. Introduction

The dividend discount model is probably the most widely employed quantitative tool used by professional investors. The most problematic aspect of this model is having to estimate the growth rate g or, alternatively, shareholders' expected rate of return on investment r. In this paper, a new method is suggested for estimating firm cost of equity capital k that obviates the need to estimate g using conventional techniques such as analysts' subjective estimates or historical growth rates of dividends or prices. Instead, firm cost of equity capital is estimated using Tobin's q ratio, which is defined as the market value of the firm (debt plus equity) divided by the replacement cost value of firm assets. Tobin's q and firm cost of equity capital are interrelated as Tobin's q measures the capitalized value of the firm's future growth potential. Thus, in economics literature, Tobin's q is used to measure industry concentration, the argument being that only monopolies are able to capture rents from growth opportunities whereas competitive firms cannot (e.g., [17]). Since Tobin's q does measure the capitalized value of firm growth potential, it acts as an alternative parameter to the conventional g for estimating firm cost of equity capital. Moreover, because Tobin's q is a market value measure, it provides the market's estimate of future growth rather than some historical or subjective estimate.

In addition to yielding an alternative measure of firm cost of equity capital, Tobin's q can estimate Macaulay's duration for firm equity. Given the present value of expected future common stock dividends, Macaulay's duration measures the weighted average time to receipt of these dividends. Furthermore, it can be shown that Ma-

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caulay's duration is a measure of the discount rate sensitivity of stock prices [10]. Generally, stocks with higher dividend growth have, *ceteris paribus,* longer durations than stocks with lower dividend growth. Although Macaulay's duration can be estimated via the conventional estimate of the growth rate g, Tobin's q again provides an alternative and less problematic approach.

Using Tobin's q to estimate k or Macaulay's duration presupposes that one can readily estimate the replacement cost value of firm assets. Indeed, before 1976 the benefits of the Tobin's q approach over the conventional g approach would not have been apparent because of the difficulties in estimating replacement cost value. Since 1976, Tobin's q has become relatively easy to apply, at least for large firms because, since that time, replacement or current cost data are routinely provided by large firms in their 10 K reports or in their annual statements.[1] More specifically, Statement of Financial Accounting Standards (SFAS) No. 33 mandates current cost information for those firms with at least $125 million of inventory, property, plant, and equipment, and for firms with total assets of at least $1 billion. Thus, for such firms it is relatively simple to calculate Tobin's q ratio. For smaller firms, application of this alternative cost of capital estimation procedure requires estimating the replacement cost of its assets as stated in Falkenstein and Weil [7, 8] or Lindenberg and Ross [14].

II. Tobin's q and the Cost of Equity Capital: The Empirical Relationship

Before illustrating that Tobin's q can be used to estimate firm cost of equity capital, an empirical relationship between these two variables must be shown. If no empirical relationship obtains, then the theoretical relationship would not be meaningful and the cost of equity capital estimate derived from such a relationship vacuous. To demonstrate that there is an empirical relationship, the cost of equity capital is regressed on the following independent variables:

1. Debt-equity ratio ($D.E.$)—a measure of financial risk;
2. Standard deviation of firm earnings scaled by total assets (SD)—a measure of both financial and business risk;
3. Industry dummy variables (IND_i)—a measure of business risk; and
4. Tobin's q—a measure of growth.

Data are collected for ninety-eight firms for which COMPUSTAT, CRSP, and Tobin's q data were available. This sample is limited by requiring each firm to have the relevant COMPUSTAT and CRSP data for 1966–1985, and by eliminating firms belonging to industries in which only a limited number of observations were available. A sample of firms is chosen that clusters in nine industries, a procedure that allows a manageable number of dummy variables representing business risk.

To minimize the effect of measurement errors (mainly on replacement costs), the

[1] In the absence of technological change, current cost and replacement cost are identical. Otherwise, current cost is replacement cost adjusted for the value of any operating advantages or disadvantages of the assets owned. As a practical matter, the techniques for estimating replacement and current costs are virtually identical.

cost of equity capital is estimated as of the end of 1973, just before the double-digit inflation of the 1970s. The cost of equity capital for 1973 is estimated using the standard dividend yield plus growth rate approach. Specifically, each firm's twenty-year history is used to estimate a dividend per share (*DPS*) trend line: $\ln DPS_t = a_0 + a_1 t$. This trend line produces an "expected" *DPS* value for 1974 ($\hat{D}PS$ (1974)). The ratio of $\hat{D}PS$ (1974) to the end-of-1973 share price gives the dividend yield portion of the estimated cost of equity capital of the firm. The growth term is estimated by running a trend line of the end-of-year share price. Presumably, share prices reflect expectations of both short- and long-term growth potential. Thus, for each stock, the time trend $\ln P_t = b_0 + b_1 t$ is estimated using the twenty-year time series. The growth rate is estimated by $e^{b_1} - 1$. The financial leverage variable, *D.E.*, is measured by the ratio of firm book value of 1973 debt to the market value of its equity. One may assume that as of the end of 1973 this ratio was reasonably close to the market value *D.E.* ratio with both debt and equity measured in market values. *SD* is computed as the standard deviation of firm return on book investment over the twenty years. The sample firms are further classified into nine two-digit SIC industries and dummy variables assigned for each industry.

Results of the cross-sectional step-wise regression are presented in Table 1. These results show that the Tobin's q variable is highly significant and inversely related to firm cost of equity capital.[2] In addition, signs of *D.E.* and *SD* variables correspond to one's intuition (and capital structure theory). The cost of equity capital increases with both financial and business risk, although the *D.E.* variable is apparently not significant.[3] Thus, it is concluded that it is potentially meaningful to develop the theoretical relationship between Tobin's q and firm cost of equity capital in order to derive *inter alia* an alternative method of estimating firm cost of equity capital.

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[2]The regression was also run with industry dummy slope variables on the q coefficient. Letting S_i denote the dummy slope variable for industry i, these regression results are

$$
\begin{aligned}
k = \; & \underset{(4.141)}{.1142} + \underset{(.129)}{.0009\,D.E.} + \underset{(1.456)}{.003\,SD} - \underset{(2.416)}{.0117q} + \underset{(.553)}{.0189\,IND_1} \\
& + \underset{(.183)}{.0056\,IND_2} - \underset{(.575)}{.0145\,IND_3} - \underset{(.541)}{.0292\,IND_4} + \underset{(.042)}{.0017\,IND_5} - \underset{(1.655)}{.0337\,IND_6} \\
& - \underset{(.694)}{.0511\,IND_7} - \underset{(1.655)}{.0716\,IND_8} - \underset{(.222)}{.0023\,S_1q} - \underset{(.300)}{.0034\,S_2q} - \underset{(.225)}{.0059\,S_3q} \\
& - \underset{(1.236)}{.0288\,S_4q} - \underset{(.762)}{.0269\,S_5q} - \underset{(.553)}{.0415\,S_6q} - \underset{(1.094)}{.0202\,S_7q} + \underset{(1.446)}{.0330\,S_8q}
\end{aligned}
$$

where the numbers in parentheses are t-values. On the whole, the slope coefficients are not significant. Again, Tobin's q is inversely and significantly related to k.

[3]Earlier studies tend to find an inverse relationship between k and *D.E.* See Arditti [2], who finds an inverse relationship for all of his regressions. Arditti explains this counterintuitive result as being a result of omitted variables. Results here tend to confirm his conjecture since the addition of Tobin's q yields a positive relationship.

TABLE 1. Cost of Equity Capital Regression Results.[a]

$$
\begin{aligned}
k = \; & \underset{(5.515)^{b}}{.0907} + \underset{(.278)}{.0018\,D.E.} + \underset{(1.663)}{.0003\,SD} - \underset{(3.018)}{.0126q} \\
& + \underset{(1.806)}{.0361\,IND_1} + \underset{(1.205)}{.0217\,IND_2} + \underset{(.490)}{.0076\,IND_3} + \underset{(.110)}{.0023\,IND_4} \\
& - \underset{(.735)}{.0145\,IND_5} - \underset{(1.935)}{.0370\,IND_6} + \underset{(1.277)}{.0297\,IND_7} \\
& + \underset{(.092)}{.0017\,IND_8}
\end{aligned}
$$

			DF	Sum of Squares	Mean Squares
Multiple R	.51080	Analysis of Variance			
R^2	.26915				
Adjusted R^2	.17567	Regression	11	.06105	.00555
Standard error	.04390	Residual	86	.16578	.00193
		$F = 2.87918$			

[a]The variables are defined as

k = cost of equity capital;
$D.E.$ = debt-equity ratio;
SD = standard deviation of the return on investment;
q = Tobin's q ratio; and
IND_i = industry intercept dummy variables $i = 1, \ldots, 8$.

[b]Figures in parentheses are t-values.

III. Tobin's q, the Cost of Equity Capital, and Macaulay's Duration: The Theoretical Relationship

Tobin's q and the Cost of Equity

According to the constant growth rate dividend discount model, the value of the firm's stock V can be expressed as [11, 12, 13]

$$V = \frac{(1 - b)E_1}{(k > br)} \tag{1}$$

where

E_1 = expected value of accounting earnings in the coming year;[4]
b = expected value of retention rate;
r = expected return on investment; and
k = cost of equity capital.

[4]Although the literature does not specify whether earnings in this model are historical cost or economic earnings, all empirical studies have used historical cost earnings. More substantively, work by Lintner [15], Fama and Babiak [9], and Bar-Yosef and Lev [3] indicate that firms target historical cost earnings in determining their dividend payout ratio (or, equivalently, the retention rate).

This model can be generalized to incorporate stock as well as retention financing. With stock financing, equation (1) becomes

$$V = \frac{(1 - c)E_1}{(k - cr)} \tag{2}$$

Here, c is the firm's investment rate, which is equal to $b + s$ where s is the expected stock financing rate (expressed as a proportion of earnings). Assuming the firm is solely equity financed, V is also the market value of the firm.

Tobin's q is defined as the market value of the firm divided by the replacement cost of firm assets. As argued by Lindenberg and Ross [14], the market value of the firm differs from the replacement cost of its assets by the present value of monopoly and firm-specific factor rents. Specifically, the market value of the firm (MV) can be represented as

$$MV = RC + F + M \tag{3}$$

where

RC = the replacement cost of firm assets;
F = capitalized value of rents attributable to firm-specific factors; and
M = capitalized value of rents attributable to monopoly profits.

Thus, a firm that earns no such rents will be valued at the replacement cost of its assets so that its Tobin's q is one. A firm that earns monopoly or firm-specific factor rents will have a Tobin's q greater than one.

The constant growth rate valuation model incorporates monopoly and firm-specific factor rents through the expected rate of return r. Indeed, the relationship $r > k$ means that the firm is able to capture rents from its investments thereby yielding a return on investment greater than the market-required return k. The relationship $r = k$ means that the firm is unable to earn monopoly or firm-specific factor rents.

The constant growth model can be reformulated to correspond to equation (3). Specifically, the market value of the firm can be decomposed into two components

$$V = \frac{E_1}{r} + \frac{(r - k)E_1}{r(k - cr)} \tag{4}$$

The first component is the economic replacement cost value of the assets:

$$RC = \frac{E_1}{r} \tag{5}$$

and the second component is the capitalized value of the firm's monopoly and firm-specific factor rents

$$F + M = \frac{(r - k)E_1}{r(k - cr)} \tag{6}$$

Equations (2) and (5) can be used to determine Tobin's q, which takes on the simple form

$$q = \frac{V}{RC} = \frac{(r - cr)}{(k - cr)} \tag{7}$$

Three aspects of equation (7) are worth noting. First, as argued by Lindenberg and Ross [14], $q > 1$ when the firm earns monopoly or firm-specific factor rents, that is, whenever $r > k$.[5] Second, Tobin's q is an increasing function of r. This accords with one's intuition since the greater the monopoly and firm-specific factor rents to be captured are, the greater r is (all other things equal). Third, Tobin's q is a decreasing function of k, the rate of return required by the firm's shareholders, which is consistent with the above empirical findings.

Rather than employing equation (7), k is best represented by the equation:[6]

$$k = [1 - c + cq]\frac{E_1}{V} \tag{8}$$

The importance of equation (8) is that it relates the firm's cost of capital to Tobin's q rather than the conventional growth rate g.[7] The cost of equity capital can be readily estimated from equation (8) since all the variables on the right side are available from annual statements and market data sources.[8]

If the firm has debt (and/or preferred shares) in its capital structure, the calculation of k is only slightly more complicated. Assuming that monopoly and factor rents accrue only to the firm's common shareholders, Tobin's q with debt is defined by

$$q \equiv \frac{\text{market value of the firm}}{\text{replacement cost value of the firm's assets}}$$

$$= \frac{V + D}{E_1/r + D} \tag{9}$$

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[5]To see this, note that $k > cr$; otherwise the model will not converge.

[6]Equation (8) is derived by noting that

$$q = V/RC = rV/E_1$$

so that

$$r = qE_1/V$$

Substituting this latter equation for r in equation (2), yields equation (8).

[7]Equation (8) still has the usual interpretation, namely, $(1 - c)E_1/V$ is the dividend yield and cqE_1/V is the growth rate yield where growth is now defined in terms of Tobin's q.

[8]There is a literature that argues that k is investors' required yield, not the cost of equity capital [13, 1]. However, even according to this literature, k has to be estimated to compute the cost of equity capital and the approach given here does that.

where D is the market value of the firm's debt (and/or preferred shares).[9] Equation (9) can be rewritten as

$$r = \frac{qE_1}{V + (1 - q)D} \tag{10}$$

Substituting equation (9) into equation (2) to eliminate r yields

$$\begin{aligned} k &= \frac{(1 - c)E_1}{V} + \frac{cqE_1}{V + (1 - q)D} \\ &= \left[1 - c + \frac{cqV}{V + (1 - q)D}\right]\frac{E_1}{V} \end{aligned} \tag{11}$$

Again, the variables on the right side of equation (11) are readily determinable so that equation (11) can be used to estimate a levered firm's cost of equity capital. For an unlevered firm, equation (11) is identical to equation (8).

Since Tobin's q incorporates growth, it should be possible to get an estimate of g from Tobin's q.[10] Rewriting equation (11) in the form

$$k = \text{Dividend Yield} + g \tag{12}$$

yields

$$k = \frac{(1 - c)E_1}{V} + \frac{cqE_1}{[V + (1 - q)D]} \tag{13}$$

so that

$$g = \frac{cqE_1}{[V + (1 - q)D]} \tag{14}$$

Tobin's q and Macaulay's Duration

Using the conventional approach, it is shown in the Appendix that Macaulay's duration (MD) can be estimated by [4]:

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[9]The assumption that firm-specific factors and monopoly rents accrue only to (existing) common shareholders is not completely innocuous. If the firm has risky noncallable debt in its capital structure before new investments are discovered, then existing bondholders also benefit from the firm's ability to capture rents from these investments since their bankruptcy region is thereby reduced [16].

[10]Equation (14) is unnecessary for estimating the cost of equity capital (equation (11)) except perhaps to estimate next year's earnings E_1 as is done further below in the paper.

$$MD = \frac{1 + k}{k - cr} \tag{15}$$

To obtain the relationship between Macaulay's duration and Tobin's q, substitute equation (2) into equation (15) by eliminating $k - cr$ to obtain

$$MD = \frac{(1 + k)V}{(1 - c)E_1} \tag{16}$$

If the firm is all equity, equation (8) can be substituted into equation (16) to yield the desired relationship

$$MD = \frac{1}{1 - c}\left[\frac{V}{E_1} + 1 - c + cq\right] \tag{17}$$

If the firm has debt or preferred stock, equation (11) can be substituted into (16) to yield

$$MD = \frac{1}{1 - c}\left[\frac{V}{E_1} + 1 - c + \frac{cqV}{V + (1 - q)D}\right] \tag{18}$$

Clearly, if $D = 0$, equations (17) and (18) are identical.

IV. An Example

The 1983 year-end cost of equity capital and Macaulay's duration are estimated for Abbott Laboratories, a large supplier of health-care products and services, using Tobin's q ratio.[11] With one minor exception, namely the average year-end interest rate for industrial *Aa* bonds, all of the data are obtainable from Abbott Laboratories' annual reports.

Replacement Cost of the Assets

Current assets excepting inventory are valued at book value. Investments are valued at market value. Inventory and property, plant, and equipment are valued at current cost. Deferred charges and other assets are valued at book value. This yields a replacement cost value for the firm's assets of $3,186,621,000.

Market Value of the Firm

Shareholders' equity is valued at the year-end market price of the stock at $5,387,650,000. Short-term liabilities are valued at book value, as are other liabilities,

[11] Relatively detailed calculations are provided to allow readers to make their own adjustments.

excluding deferred taxes. Long-term debt is broken down into listed securities, industrial development revenue bonds, and other debt, principally term borrowings by foreign subsidiaries guaranteed by the parent. Each listed bond is valued at its market price as of the last trade during 1983. All industrial development revenue bonds are valued at the approximate average year-end interest rate for industrial *Aa* bonds, as computed by Moody's, of 12 percent. Other debt is valued at par. These calculations yield a total market value for the firm's debt of $1,186,327,000. Thus, the total market value of the firm (debt plus equity) is $6,573,977,000.

Cost of Equity Capital

Given the previous calculation, Tobin's q is:

$$q = 6{,}573{,}997/3{,}186{,}621 = 2.06$$

This is reasonably close to the 2.35 value calculated by Lindenberg and Ross [14] for Abbott Laboratories' average Tobin's q for 1960–1977. Except for 1976–77, Lindenberg and Ross's replacement cost calculations are based on their own estimation techniques rather than on published accounting data, which were unavailable before 1976. They also estimated bond values without recourse to market prices.

E_1, next year's earnings, can be estimated by multiplying this year's earnings of $347,617,000 by $(1 + g)$, where g is the Tobin's q estimate of growth given by equation (14). This yields an estimated $g/(1 + g) = 0.1127$, or $g = 12.7$ percent.

Abbott Laboratories' cost of equity capital for December 31, 1983 can now be estimated from equation (11), where[12]

$$\begin{aligned} q &= 2.06 \\ E_1 &= 391{,}764{,}360 \\ V &= 5{,}387{,}650{,}000 \\ D &= 1{,}186{,}327{,}000 \\ c &= b = 0.65 \end{aligned}$$

This yields an estimated cost of equity capital of 15.25 percent.[13] Using Lindenberg and Ross's q value of 2.35 yields an estimate of 18.94 percent. Using equation (18) and a Tobin's q of 2.06 results in a Macaulay's duration for Abbott of forty-five years.

V. Tobin's *q* Versus Conventional Estimates

The previous discussion shows by example how one might estimate the firm's cost of equity capital using Tobin's q in contradistinction to the conventional approach. This section compares the costs of equity capital for 1973 measured by Tobin's q with the conventional approach for the sample of firms described in section II. This section

[12] Abbott Laboratories did not use external equity financing for at least four years before 1973. Thus, it is assumed that $c = b$ where b is the average retention rate over this five-year period. In any case, the retention rate is fairly stable over this period.

[13] For a somewhat different calculation based on the same model, see [6].

TABLE 2. Comparison of the Cost of Equity Capital: Tobin's q Versus the Conventional Approach.

	Mean Cost of Equity Capital		
Size	Tobin's q	Conventional	t-statistic
First quartile	0.1303	0.0892	3.38
Second quartile	0.1253	0.0752	4.23
Third quartile	0.1445	0.0783	5.66
Fourth quartile	0.1205	0.0739	4.66
Overall	0.1301	0.0792	8.88

also describes how the cost of equity capital is estimated conventionally, that is, using the dividend yield plus growth rate approach.

Table 2 summarizes the results of the comparison. Specifically, the last row of Table 2 shows that the sample mean cost of equity capital is about 13 percent using the Tobin's q approach versus 8 percent using the conventional approach. The t-statistic to test whether the difference between these means is statistically significant is in fact significant at the 1 percent level. Thus, the null hypothesis that the means are not significantly different from each other can be rejected. Firms are further ranked by (1973) sales, and the mean cost of equity capitals using both methods are computed for each quartile. Table 2 shows that in each quartile, the means are significantly different from each other at the 1 percent level.

The major implication of Table 2 is that while both cost of capital estimation procedures use the same underlying model, they nevertheless yield very different results because they use different estimates of the underlying parameters. Since the cost of capital cannot be observed, it is impossible to say which estimate is "correct" in any absolute sense. However, considering that the data are 1973 vintage, the mean cost of equity capital estimate using Tobin's q looks more realistic than the estimate derived from the conventional approach. After all, the yield on *AAA* corporate bonds in 1973 was around 7 percent, making an estimated cost of equity capital of about 13 percent much more plausible than the 7.92 percent estimate yielded by the conventional method.

VI. Conclusion

Analyzed here are the empirical and theoretical relationships among Tobin's q ratio, the firm's cost of equity capital, and Macaulay's duration in the context of the constant growth rate dividend discount model. This analysis yields an alternative method for estimating a firm's cost of equity capital using Tobin's q ratio. The method is especially applicable for large firms that are required by SFAS No. 33 to publish current cost data in their annual (or 10 K) statements. The Tobin's q approach is then applied to a sample of firms and demonstrated in detail for a large firm, Abbott Laboratories, to show how this new technique is implemented.

Perhaps the most important aspect of Tobin's q approach to estimating the cost

of equity capital is that it does not depend upon the conventional estimate of g, the growth rate. Estimating g has always been a contentious issue when applying the constant growth rate valuation model. This is not surprising since estimates of a g are based either on a time series of historical cost data (e.g., [12]) or on subjective estimates [5]. On the other hand, the Tobin's q approach is a function of market value data so that the g implicit in Tobin's q is the market's own valuation of growth. In addition to the likelihood that the latter expectational data are likely to yield a more meaningful estimate of the cost of equity capital, they are also more readily available, at least for large firms.

Appendix

The Constant Growth Rate Valuation Model, Economic Replacement Cost, and Macaulay's Duration

The constant growth model assumes that a constant proportion b of earnings is reinvested each period and the remainder is paid out to shareholders. The firm earns a constant rate of return r per period on its investments and the cost of equity capital k is also a constant per period. The model can be generated by assuming the firm invests an initial I_0 dollars in some asset. With no further investment the firm will earn rI_0 per period. However, if a proportion b of earnings is reinvested each period, earnings in period t are $E_t = rI_0(1 + br)^{t-1}$ and dividends in period t are $DIV_t = (1 - b)E_t$. Hence, the market value of the firm (compare equation (1) in the text) is:

$$V = \sum_{t=1}^{\infty} \frac{DIV_t}{(1 + k)^t} = \frac{(1 - b)E_1}{(k - br)} \qquad (k > br)$$

This equation decomposes easily into the Lindenberg and Ross [14] RC factor and $(F + M)$ factor as in equations (4) through (6) in the text. The RC factor is obviously the economic replacement cost value of the asset, since $RC = E_1/r = rI_0/r = I_0$.

Discounting the excess dividend stream, $(1 - b)I_0[r(1 + br)^{t-1} - k(1 + bk)^{t-1}]$, at the required rate k yields the $F + M$ factor as equation (6):

$$F + M = \frac{(r - k)E_1}{r(k - br)}$$

Incorporating the ability to issue stock simply requires the replacement of b in all of the above equations by c where $c = b + s$ and where s is the expected stock-financing rate (expressed as a proportion of earnings).

To incorporate the use of debt into the model, it is assumed that the firm maintains a constant debt/equity ratio. Thus, with each equity investment, there is a corresponding new investment that is financed by debt. Therefore, define

h = constant debt/equity ratio;
i = interest rate on debt;
I_t = investment by equity in the beginning of period t; and
D_t = debt in the beginning of year t.

Therefore, it is found:

$$D_0 = hI_0$$

$$E_1 = rI_0 + (r - i)hI_0 = [r + (r - i)h]I_0$$

$$I_1 = (1 + br)I_0 + b(r - i)hI_0$$

Generally,

$$E_t = [r + (r - i)h] \sum_{j=1}^{t} I_{j-1}$$

$$DIV_t = (1 - b)E_t$$

and the expression for the market value of the equity remains as before:

$$V = \sum_{t=1}^{\infty} \frac{DIV_t}{(1 + k)^t} = \frac{(1 - b)[r + (r - i)h]I_0}{(k - br)} = \frac{(1 - b)E_1}{(k - br)}$$

Macaulay's duration (MD) as specified by equation (15) can be obtained in the following manner. In general, duration is defined as

$$MD = \frac{\sum_{t=1}^{\infty} \frac{tD_t}{(1 + k)^t}}{V}$$

Assuming the constant dividend growth model holds, MD is written as

$$MD = \frac{\sum_{t=1}^{\infty} \frac{tDIV_0(1 + g)^t}{(1 + k)^t}}{\frac{DIV_0(1 + g)}{k - g}}$$

$$= \frac{(k - g)}{(1 + g)} \sum_{t=1}^{\infty} t\left(\frac{1 + g}{1 + k}\right)^t$$

Defining $x = (1 + g)/(1 + k)$ where $g < k$ so that $x < 1$, MD may be rewritten as:

$$MD = \frac{(k - g)}{(1 + g)} \sum_{t=1}^{\infty} tx^t$$

Now define $S(x) = \Sigma_{t=0}^{\infty} x^t$ so that $S(x) = 1/(1 - x)$ and

$$\frac{dS}{dx} = \sum_{t=1}^{\infty} tx^{t-1} = \frac{1}{(1 - x)^2}$$

Hence

$$MD = \frac{(k - g)}{(1 + g)} \cdot \frac{dS}{dx} = \frac{(k - g)}{(1 + g)} \frac{x}{(1 - x)^2}$$

Substituting $x = (1 + g)/(1 + k)$, yields

$$MD = \frac{1 + k}{k - g}$$

Macaulay's duration can also be shown to measure the discount rate sensitivity (elasticity) of equity values. Formally, if equity values are given by equation (2) in the text then

$$-\frac{\partial V}{\partial(1 + k)} \left(\frac{1 + k}{V}\right) = \frac{(1 - c)E_1}{(k - cr)^2} \frac{(1 + k)}{[(1 - c)E_1/(k - cr)]}$$

$$= \frac{1 + k}{k - cr}$$

$$= MD$$

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