Gray markets arise when an intermediary buys a product in a lower-priced, often emerging market and resells it to compete with the product’s original manufacturer in a higher priced, more developed market. Evidence suggests that gray markets make the original manufacturer worse off globally by eroding profit margins in developed markets. Thus, it is interesting that many firms do not implement control systems to curb gray market activity. Our analysis suggests that one possible explanation lies at the intersection of two economic phenomena: firms investing to build emerging market demand, and investments conferring positive externalities (spillovers) on a rival’s demand. We find that gray markets amplify the incentives to invest in emerging markets, because investments increase both emerging market consumption and the gray market’s cost base. Moreover, when market-creating investments confer positive spillovers, each firm builds its own market more efficiently. Thus, firms can be better off with gray markets when investments confer spillovers, provided the spillover effect is sufficiently large. These results provide a perspective on why firms might not implement control systems to prevent gray market distribution in sectors where investment spillovers are common (e.g., the technology sector) and, more broadly, why gray markets persist in the economy.

Key words: gray markets; unauthorized distribution; emerging markets; investment spillovers; management control systems

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nel internal controls (33% of respondents – number one response) and a lack of monitoring/detection processes (25% of respondents – number two response). The combined evidence leads to two questions: Why are some technology firms not implementing management control systems to curb gray market activity? Given this widespread inaction, might there be an upside to gray market activity for technology firms?

We argue that this inaction may relate to the nature of the technology sector itself, and in particular the existence of positive investment externalities that are conferred on a competitor’s demand when a firm invests to build its own market. Positive investment externalities can arise from various sources. For example, Lewis and Nguyen (2012) illustrate that firm investments in online advertising increase searches for not only the advertised brand but also for other brands in the advertiser’s category.

Our results suggest that a possible explanation for the pervasiveness of gray markets in certain industries lies at the intersection of these investment externalities and a growth strategy that is common amongst firms entering emerging markets: investing in market creation to build demand in the new market. These market-creating investments include building distribution networks, creating marketing partners, and promoting advertising that informs or educates potential consumers about benefits associated with the category. To provide a real world example of the interactions we seek to document, we consider a technology product with the following characteristics: (1) an active gray market, (2) a manufacturer that invests in market-creating investments to increase demand in an emerging market, (3) positive externalities to investment that enhance an emerging market opportunity for not only the firm but also its rivals, and (4) a manufacturer that has a muted response to gray market activity.

We choose the example of Apple’s iPhone. First, the iPhone’s gray market is extremely active. Second, Apple invests significantly in building emerging markets. Third, Apple’s market-creating investments for the iPhone arguably created positive externalities for other products in the category. For example, as the first touch screen smartphone to gain global prominence, Apple spent considerable amounts educating consumers on how to use the iPhone. These investments shifted global consumer preferences away from keyboard interfaces to touch screen interfaces, and helped build the touch screen market for not only the iPhone but also for its touch screen competitors like the Samsung Galaxy and Google Android. Importantly, we also observe gray market activity across the entire range of iPhone competitors. Finally, we note that popular press articles have commented on Apple’s lack of interference with gray market iPhone sales. For example, “Apple doesn’t appear to be doing much to combat its gray market success; in fact, if anything, it has taken steps to encourage off-contract business in recent years, by selling factory unlocked devices directly through its own store” (Etherington 2011).

To assess the impact of gray markets on firm profits when a firm’s investments confer positive externalities on its rivals, we present a model of Cournot competition where two firms choose production quantities in both a larger domestic market (where retail prices are higher) and a smaller emerging market (where retail prices are lower). Additionally, we incorporate a gray market competitor—a Stackelberg follower in the game—that purchases volume from the smaller, less expensive emerging market, and resells this volume in the higher priced domestic market. Finally, we allow each firm to make market-creating investments in the first stage of the game. These investments increase emerging market demand for the investing firm and, when investment spillovers exist, the investing firm’s rival. We find that both firms can be better off with gray markets when a firm’s investments confer positive externalities on a rival’s demand. This result arises from several forces that affect each firm’s profits in the two markets.

First, because of lost domestic sales resulting from the gray market’s diversion, firm profits in the domestic market are always lower when gray markets exist. This finding echoes the anecdotal and empirical evidence of domestic manufacturers observing reduced profits due to increased gray market cannibalization (e.g., KPMG 2008). However, each firm’s investment in the emerging market is strictly larger when gray markets exist. This outcome arises because gray markets provide each firm with an added incentive to increase emerging market demand. Specifically, by investing more to increase demand, not only does each firm increase emerging market consumption for its product, it also limits the gray market’s arbitrage opportunity by increasing emerging market prices. These larger investments have both positive and negative impacts on each firm’s profits. On one hand, investments are costly and the costs are increasing in the investment’s size. It follows that, as investments are strictly larger in a gray market setting, so too are their costs, and these higher costs lead to lower firm profits when gray markets exist. On the other hand, the larger the investment in the emerging market, the greater the emerging market consumption, retail prices, and profits for each firm. Thus, larger investments also have a positive impact on firm profits when gray markets exist.

In a setting where firms make investments with no spillovers, the aforementioned costs outweigh the benefits, and firms are better off without gray markets. Conversely, in a setting where firms make investments that do have spillover effects on a rival’s
demand, either the negative or positive forces discussed above can dominate. The positive forces may dominate because a firm increases demand for its product in the emerging market more efficiently when investments confer positive spillovers. Said differently, the return on investment, measured as the ratio of increased demand to the cost of the investment, increases in the magnitude of the spillover effect. Because gray markets induce larger investments, and investments are more efficient when they confer spillovers, firms can be better off with gray markets than without them provided the spillover effect is sufficiently large.

This inference helps to explain why industries where positive investment externalities are common, like the technology sector, appear to allow gray market activity to thrive. Conversely, it also explains why some industries vigorously crack down on gray market activity. For example, relative to the technology sector, investments in the North American auto industry tend to be very asset specific. In turn, these investments have either a negligible or little positive effect on a competitor’s demand. Interestingly, in order to reduce (or eliminate) the gray market distribution of American cars to Canadian consumers, auto manufacturers have recently begun to not honor US warranties when cars are purchased in the United States and exported to Canada. This strategy seems to be consistent with the model’s prediction of a strict preference for firms to control gray markets when investments yield muted demand spillovers on a firm’s rivals.

Our results are also robust to a number of extensions. In our first extension, we assess a different source of investment externality: complementary goods. When products are complements, any investment to build demand for one product also increases demand for the other. We find that both firms can be better off with gray markets when products are complements, even if there is no other source of investment externality aside from the complementarity of the products. In a second extension, we allow each firm to stop its own gray market without cost. Consistent with the findings in the main model, we find a unique, pure strategy equilibrium where both firms are better off allowing their respective gray market to operate, provided the emerging market is large enough. Finally, we confirm that our results are robust to making the gray market more competitive and to changing the nature of competition in the market.

The study proceeds as follows. We review the literature on firm outcomes and gray markets in section 2. Section 3 introduces the main model and analyzes the results. In section 4, we present several extensions for robustness. In section 5, we conclude.

2. Literature Review

The literature assessing gray markets and their impact on firms finds the nearly unanimous result that gray markets make original manufacturers less profitable. For example, Antia et al. (2006), Assmus and Wiese (1995), Autrey and Bova (2012), Cavusgil and Sikora (1988), Cespedes et al. (1988), Li and Robles (2007), Ahmadi and Yang (2000), Maskus and Chen (2004) and Weigand (1991) take the position that gray markets are a problem for manufacturers for reasons that include the following: increased competition which leads to narrower manufacturer profit margins, losing control of distribution, a decreased ability to price discriminate, a mitigated incentive to invest in research and development and the erosion of brand equity.

We note that our model shares some common features with a dual distribution setup (see Arya and Mittendorf 2013a, b), in which a focal firm in essence divides into two firms and competes as two franchises (e.g., a traditional retailer and an online store). Both a gray market competitor and an extra authorized channel (e.g., an online store) increase competition in the market. For example, if a firm originally competes in a duopoly, the inclusion of either a gray market competitor or an additional authorized distributor increases the number of competitors in the market from two to three. However, in the dual distribution setup, the initiating firm can often be better off, even if dual distribution does not lead to an increase in the size of the market. For example, in a Cournot model of dual distribution, although a firm dividing itself into two competitors makes competition more intense and thus lowers total industry profits, the focal firm now enjoys a larger share of the industry profits, and is ultimately better off enjoying a larger share of a smaller pie (see Baye et al. 1996).

Unlike the dual distribution setting, the focal firm is strictly worse off if the new competitor in the market is the gray market. While a focal firm earns higher profit in the emerging market on sales to the gray marketer, these additional sales are not enough to offset the impact that the gray market has on both eroding total industry profits and reducing the firm’s share of total industry profits. Specifically, the focal firm now obtains less than half of a smaller domestic profit pie (with a gray marketer) vs. half of a larger domestic profit pie (without a gray marketer). The conventional wisdom, then, is that dual distribution leads to firms being better off while gray markets lead to firms being worse off.

In this light, it is interesting that gray markets persist even when firms have the ability to eliminate them by implementing comparatively inexpensive control mechanisms. These mechanisms may include...
the systematic testing of all incentivized transactions, monitoring the Internet and other sources for price anomalies, and insisting that channel partners assess their own internal control environments (Deloitte 2011). The observation that firms are not implementing these sorts of control mechanisms is especially puzzling, given evidence that management control systems are typically effective at preventing undesired behavior (see Abernethy et al. 2004, Abernethy and Lillis 1995, Chenhall 2003, Ittner and Larcker 1997, Langfield-Smith 1997). We believe that our model of market-creating investments with positive externalities provides a plausible explanation for this unexpected behavior.

3. Main Model

Our model consists of a setting with two distinct markets; we refer to the larger market as the domestic market, and the smaller as the emerging market. In our analysis, we model the gray market product as a perfect substitute for the authorized product (i.e., the size of the market, the slope for demand, and the substitutability of the two products are identical). While it is plausible that consumers may have a lower willingness to pay for some gray market products, we also believe that, increasingly, gray market goods are viewed as perfect substitutes for their authorized counterparts. For example, gray markets provide well-known retailers such as Wal-Mart, Target, and Costco the opportunity to offer a perfect substitute for the manufacturer’s authorized product (Stohr 2012). Further, a common justification for the inferiority of gray market products is that they lack warranty protection (e.g., Ahmadi and Yang 2000). However, increasingly retailers such as Costco offer their own warranties for products sourced through the gray market when the manufacturer does not offer a warranty for these goods. Finally, by assuming that the gray market product is a perfect substitute for the authorized product, we allow for the gray market to have its maximum impact on cannibalizing the original manufacturer’s domestic sales and, accordingly, on eroding its global profits. This modeling choice should, in turn, bias our results away from finding evidence that original manufacturers prefer gray markets to not preferring them.

The timeline is shown in Figure 1. In the first stage of the model, each firm chooses its level of market-creating investments, \( x_i \geq 0 \), at a cost of \( x_i^2 \), following the standard quadratic cost function. Examples of market-creating investments include building distribution networks, creating marketing partners, and advertising that informs or educates potential consumers about benefits associated with the category. These investments increase the size of the emerging market by \( x_i \) for the investing firm but also increase the size of the emerging market by \( \beta x_i \), where \( \beta \in (0, 1) \), for its rival. The parameter \( \beta \) represents the extent of investment spillovers, or the increase in the rival’s emerging market demand that results from the investing firm’s market-creating investments.

In the second stage, the two firms engage in differentiated Cournot quantity competition simultaneously in both markets. Demand curves in each market are generated from quadratic consumer utility preferences as in Singh and Vives (1984). We define \( \gamma \in (0, 1) \) as the degree of substitutability between the firms’ products. As \( \gamma \to 0 \), the products become perfectly differentiated. As \( \gamma \to 1 \), the products become perfect substitutes.

In the domestic market, each firm chooses the quantity \( q_i \geq 0 \) to produce for sale in its domestic market at price \( p_j \geq 0 \), given the following inverse demand curve \((i \neq j \in \{1, 2\})\):

\[
p_i = 1 - (q_i + q_{Gi}) - \gamma(q_j + q_{Gi}).
\]

Each firm’s inverse demand curve in the emerging market is as follows \((i \neq j \in \{1, 2\})\):

\[
p_{Ei} = E + x_i + \beta x_j - (Q_{Ei} - q_{Gi}) - \gamma(Q_{Ej} - q_{Gi}).
\]

The emerging market is smaller as reflected by the demand intercept, \( E \), which we assume to be less than 1. In the emerging market, each firm chooses quantity \( Q_{Ei} \geq 0 \) to sell in the emerging market at price \( p_{Ei} \geq 0 \). This \( Q_{Ei} \) includes both sales to the gray marketer, \( q_{Gi} \geq 0 \), and sales to consumers in the emerging market, \( q_{Ei} = Q_{Ei} - q_{Gi} \geq 0 \). Additionally, we assume that emerging market consumers and the gray marketer pay the same retail price, \( p_{Ei} \), for product purchased in the emerging market.

We also note that the \( q_{Gi} \) units diverted to the domestic market do not satisfy demand in the
emerging market. Only the $q_{EI}$ units that are not siphoned off by the gray marketer are available to satisfy the demand of the local end users. As $q_{EI}$ determines the retail price, $p_{EI}$, that emerging market consumers are willing to pay in a Cournot market, and because we assume that the gray market and local consumers pay the same price, $q_{EI}$ in turn determines the market price that clears the emerging market. Importantly, the quantity consumed by emerging market consumers, $q_{EI}$, is strictly lower when a gray marketer is present, ceteris paribus (a point we expand on below). In turn, the emerging market retail price that clears that market is higher when a gray marketer exists or not, respectively.

We solve the model by backward induction. The gray marketer’s objective function is:

$$\text{Max}_{q_G, q_{G2}} \pi_G = q_{G1}(p_1 - p_{EI}) + q_{G2}(p_2 - p_{E2}), \quad (3)$$

subject to Equations (1) and (2).

Substituting these constraints into Equation (3), taking first order conditions with respect to $q_{G1}$ and $q_{G2}$ and solving, we obtain the following best response function ($i \neq j \in \{1,2\}$):

$$q_{Gi}(q_i, Q_{Ei}) = \frac{1 - \gamma - (q_i - Q_{Ei})(1 - \gamma^2) - [E(1 - \gamma) + x_i(1 - \beta \gamma) + x_j(\beta - \gamma)]}{4(1 - \gamma^2)}. \quad (4)$$

We also assess settings where no gray market exists. In these settings, $q_{Gi} = 0$ ($i = 1, 2$).

Anticipating the gray market demand, in the second stage each firm’s objective is as follows ($i = 1, 2$):

$$\text{Max}_{q_i, q_{EI}} \pi_i = q_i(p_i) + Q_{EI}(p_{EI}) - x_i^2, \quad (5)$$

subject to Equations (1) and (2), and either Equation (4) or $q_{Gi} = 0$, depending on whether the gray market exists or not, respectively.

To solve, we take the first order conditions with respect to $q_1$, $q_2$, $Q_{E1}$ and $Q_{E2}$, which creates a system of four equations and four unknowns. The resulting equilibrium quantities are as follows ($i \neq j \in \{1,2\}$):

$$q_i^* = \frac{1}{2 + \gamma}; \quad Q_{Ei}^* = \frac{E(2 - \gamma) + x_i(2 - 3\beta \gamma) + x_j(2\beta - \gamma)}{4 - \gamma^2}. \quad (6)$$

Interestingly, we note that for a given set of investment levels, $x_i$ and $x_j$, the equilibrium quantities in the domestic and emerging markets, $q_i^*$ and $Q_{Ei}^*$, are identical and strictly positive whether gray markets are active or not. An explanation for this outcome can be gleaned from an example in Tirole (1988, p. 316). In the example, a Stackelberg leader sets the same quantity in a Cournot economy irrespective of whether a Stackelberg follower exists or not, provided (a) demand is linear, (b) the Stackelberg leader and Stackelberg follower produce perfect substitutes, and (c) the Stackelberg leader commits to its quantity decision. All of these provisions are also assumed in our model (where the Stackelberg leaders and the Stackelberg follower are represented by the competing firms and the gray market, respectively). These model assumptions provide some insight as to why the firms’ quantity choices are invariant to the gray market’s existence.

Thus, when no gray market exists, $Q_{Ei}^* = q_{Ei}$ and all of the product produced in the emerging market is consumed locally. When gray markets are active, $Q_{Ei}^* - q_{Gi} = q_{Ei}$, and part of the emerging market quantity is siphoned off by the gray market leaving less product available for emerging market consumers. As we note above, the emerging market pricing function in Equation (2) is decreasing in the amount of product consumed locally by emerging market consumers, $q_{Gi}$. Thus, we should expect strictly higher emerging market prices, $p_{EI}$, for a given set of investment levels $x_i$ and $x_j$ when gray markets are active, given that the product consumed locally, $q_{Ei}$ is strictly lower in a gray market setting.

We obtain the equilibrium gray market quantities by substituting the second-stage solutions into the best response functions in Equation (4), which yields ($i \neq j \in \{1, 2\}$):

$$q_{Gi} = \frac{(1 - E)(2 - 3\gamma + \gamma^2) - x_i(2(2 - 3\beta \gamma + \gamma^2) - x_j(2\beta + \beta \gamma^2 - 3\gamma)}{4(4 - 5\gamma^2 + \gamma^4)}. \quad (7)$$

Our main analysis proceeds as follows. We begin with the following two benchmark cases. (1) Firms cannot invest to increase emerging market demand (i.e., $x_i = 0$, $i = 1, 2$). In this setting, we calculate and compare firm profits when gray markets exist and when gray markets do not exist. (2) Market-creating investments are possible, but there is no spillover (i.e., $\beta = 0$). In this setting, we complete the backward induction by calculating optimal investment levels in the first stage of the game. We then calculate and compare firm profits when gray markets exist and when gray markets do not exist.

Finally, in our focal case, we analyze a situation with both investments and spillovers. As in the previous setting, we complete the backward induction by calculating optimal investment levels in the first stage of the game, and then calculate and compare firm profits across different settings. We first demonstrate that firms are strictly better off without gray markets.
unless investment spillovers exist. We then characterize the conditions under which both firms are better off with gray markets when investments confer positive spillovers.

3.1. No Market-Creating Investments and No Spillovers

In the first benchmark scenario, firms cannot make market-creating investments. To determine the impact of gray markets in this setting, we derive the equilibrium profits when gray markets are active and when they are not. To facilitate comparisons, we use the superscript GB to designate the benchmark setting with gray markets and the superscript B to designate the benchmark setting with no gray markets.

We obtain the equilibrium quantities for each firm by substituting \( x_1 = x_2 = 0 \) into Equations (6) and (7). We then substitute these equilibrium quantities into Equation (5) to yield the equilibrium profit:

\[
\pi^G_B = \frac{3 + 2E + 3E^2}{4(2 + \gamma)^2}, \quad i = 1, 2.
\]  

When there is no investment or gray markets (i.e., \( q_{1i} = 0, i = 1, 2 \)), the model is simply the standard differentiated Cournot model, with the following well-known equilibrium profits:

\[
\pi^B = \frac{1 + E^2}{(2 + \gamma)^2}, \quad i = 1, 2.
\]  

It is straightforward to show that \( \pi^B > \pi^G_B \) for all \( \gamma \in (0, 1) \) and \( E \in (0, 1) \). Therefore, in the absence of market-creating investments, firms are strictly better off without gray markets.

3.2. Market-Creating Investments and No Spillovers

For the next benchmark scenario, firms have the option to invest in market-creating activities such as developing distribution networks, listings, relationships with foreign partners or perhaps simply advertising. However, here, a firm’s market-creating investments have no effect on the competitor’s demand (i.e., \( \beta = 0 \)). We denote the setting with market-creating investments but no spillovers with the superscripts GX and X, respectively, to indicate active gray markets or no gray markets.

To obtain the profit-maximizing level of market-creating investment, we continue with backward induction. Anticipating the optimal quantities obtained in the second stage of the game with active gray markets, each firm now maximizes Equation (5) with respect to \( x_i \), subject to the equilibrium quantities from Equations (6) and (7). Substituting in these constraints and taking first-order conditions with respect to \( x_1 \) and \( x_2 \), we solve to obtain the following optimal investment level (\( i = 1, 2 \)):

\[
x^G_i = \frac{2(1 + 3E)}{26 + 4\gamma(4 - 2\gamma - \gamma^2)} > 0.
\]  

The equilibrium quantities for each firm are found by substituting \( x^G_1 \) and \( x^G_2 \) into the second-stage solutions in Equations (6) and (7). Although \( q^G_1 \) and \( Q^G_{E_1} \) are strictly positive, for some parameter values \( q^G_1 \) can be negative; these parameter values correspond to the situation in which \( E + x^G_i > 1 \). In other words, for these parameter values the market-creating investment causes the emerging market to become larger than the domestic market. In that situation, the available arbitrage opportunity would be for the gray marketer to divert product from the domestic market to the emerging market. To rule out this possibility and ensure that \( q^G_1 \geq 0 \), we make the technical assumption that \( E < E^G_B = \frac{6 + 4\gamma - 2\gamma^2}{(2 - \gamma)(2 + \gamma)^2} \).

At the other end of the parameter continuum, the arbitrage opportunity might be so appealing that the gray marketer would like to divert more units than are available in the emerging markets. To represent the upper bound of quantity available for diversion (i.e., \( q^G_1 \leq Q^G_{E_2} \)), we make the additional technical assumption that \( E > E^G_X = \frac{2 - \gamma^2}{(3 + 4\gamma)(4 - 2\gamma)} \). Another way to interpret this assumption is that the firm will not enter an emerging market that supplies only a gray marketer.

Finally, we obtain the equilibrium profits for each firm by substituting in the optimal investment from Equation (10) and all equilibrium quantities into Equation (5):

\[
\pi^G_i = \frac{2}{3(2 + \gamma)^2} + \frac{(1 + 3E)^2(13 - 8\gamma^2 + \gamma^4)}{3(13 + 8\gamma - 4\gamma^2 - 2\gamma^3)^2}, \quad i = 1, 2.
\]  

When there is no gray market diversion, each firm solves the first-stage game for optimal investment as above, but using \( q_{1i} = 0 \) (\( i = 1, 2 \)) in place of Equation (7). This yields the optimal market-creating investments and profits without gray markets (\( i = 1, 2 \)):

\[
x^X_i = \frac{2E}{6 + 4\gamma - 2\gamma^2 - \gamma^3} > 0,
\]  

\[
\pi^X_i = \frac{1}{(2 + \gamma)^2} + \frac{E^2(12 - 8\gamma^2 + \gamma^4)}{6 + 4\gamma - 2\gamma^2 - \gamma^3}.
\]  

Our first result compares the equilibrium levels of market-creating investments and the equilibrium firm profits, with and without active gray markets. The feasible region in this setting consists of all \( E \in (E^G_X, E^G_B), \gamma \in (0, 1) \), and \( \beta = 0 \).
PROPOSITION 1. In a setting without investment spillovers:

1. Each firm spends more on market-creating investments when gray markets are active (i.e., \( x_i^{GX} > x_i^X \), \( i = 1, 2 \)).
2. Each firm is strictly worse off when gray markets are active (i.e., \( \pi_i^{GX} < \pi_i^X \), \( i = 1, 2 \)).

PROOF. All proofs are in the Online Appendix.

Note that each firm’s investment in the emerging market is strictly larger when gray markets exist.
This outcome arises because gray markets provide each firm with an added incentive to increase emerging market demand. By investing more to increase demand, not only does each firm increase the emerging market consumption for its product, it also limits the gray market’s arbitrage opportunity by increasing its cost base. While these larger investments result in greater equilibrium profits for each firm, these increased profits are not enough to outweigh the costs related to gray market cannibalization in the domestic market. This leads to both firms being strictly worse off in an economy with gray markets when investments confer no spillovers on a rival’s demand.

3.3. Market-Creating Investments and Spillovers
In this subsection, we present the main model. Here, both firms can invest in market-creating activities in the emerging market, and investments increase not only the focal firm’s demand but also the competitor’s demand. Specifically, we define the spillover effect for Firm \( i \) as an increase in the emerging market demand intercept of \( \beta \in (0, 1) \) times the investment choice of its rival, \( x_j \). We solve the first-stage game exactly as in the previous subsection, except that we do not restrict the spillover parameter \( \beta \) to zero. We denote the settings with market-creating investments and spillovers with the superscripts \( G \beta \) and \( \beta \), respectively, to indicate active gray markets or no gray markets.

When gray markets are active, the optimal investment level is as follows (\( i = 1, 2 \)):

\[
x_i^{G\beta} = \frac{(1 + 3E)(2 - \beta \gamma)}{26 - 3\beta(2 - \gamma) + 3\beta^2 \gamma + 4\gamma(4 - 2 \gamma - \gamma^2)} > 0.
\]

(14)

Similar to the previous subsection, we again impose two technical assumptions to ensure that equilibrium gray market quantities are non-negative (i.e., \( q_{Gi} \geq 0 \)), and that they do not exceed the available supply (i.e., \( q_{Gi} \leq Q_{Gi}^E \)). Accordingly, we restrict

\[
E < E = \frac{2(3 - \beta) + (1 + \beta)\beta \gamma + \gamma(4 - 2 \gamma - \gamma^2)}{(2 - \gamma)(2 + \gamma)^2}
\]

and

\[
E > E = \frac{2 - 2\beta + \beta \gamma(1 + \beta) - \gamma^2}{(5 + 4\gamma)(4 - \gamma^2)}
\]

respectively.

Finally, we obtain the equilibrium profits for each firm by substituting in the equilibrium investment and all equilibrium quantities into Equation (5):

\[
\pi_i^{G\beta} = \frac{2}{3(2 + \gamma)^2} + \frac{(1 + 3E)[52 + 12\beta \gamma - \gamma^2(32 + 3\beta^2) + 4\gamma^4]}{3[26 - 3\beta(2 - \gamma) + 3\beta^2 \gamma + 4\gamma(4 - 2 \gamma - \gamma^2)]}, \quad i = 1, 2.
\]

(15)

When there is no gray market diversion (i.e., \( q_{Gi} = 0 \), \( i = 1, 2 \)), the optimal market-creating investments and profits are as follows (\( i = 1, 2 \)):

\[
x_i^\beta = \frac{E(2 - \beta \gamma)}{6 - \beta(2 - \gamma) + \beta^2 \gamma + \gamma(4 - 2 \gamma - \gamma^2)} > 0,
\]

(16)

\[
\pi_i^\beta = \frac{1}{(2 + \gamma)^2} + \frac{E^2[12 + 4\beta \gamma - \gamma^2(8 + \beta^2) + 4\gamma^4]}{(6 - \beta(2 - \gamma) + \beta^2 \gamma + \gamma(4 - 2 \gamma - \gamma^2))^2}.
\]

(17)

Comparing the equilibrium levels of market-creating investments with and without active gray markets, we again find that each firm invests more when facing active gray markets. However, in contrast to subsection 3.2, profits can be higher or lower when there are gray markets. We characterize the parameter region in which firms are better off with gray markets in Proposition 2. The feasible region in this setting consists of all \( E \in (E^L, E^R) \), \( \gamma \in (0, 1) \), and \( \beta \in (0, 1) \).

PROPOSITION 2. Let \( \beta^* = \gamma/2 \). In a setting with market-creating investments and spillovers:

1. Each firm spends more on market-creating investments when gray markets are active (i.e., \( x_i^{G\beta} > x_i^\beta \), \( i = 1, 2 \)).
2. Provided \( \beta > \beta^* \), there exists a boundary \( E^* \) such that each firm is strictly better off with gray markets (i.e., \( \pi_i^{G\beta} > \pi_i^\beta \), \( i = 1, 2 \)) for all feasible \( E > E^* \). When \( \beta \leq \beta^* \), the firms are weakly worse off with gray markets (i.e., \( \pi_i^{G\beta} \leq \pi_i^\beta \), \( i = 1, 2 \)) for all feasible \( E \).

Proposition 2 presents the overall effect of several competing forces. On one hand, the gray market
increases the number of competitors in the domestic market, which lowers each firm’s profits in the domestic market. In addition, each firm invests more in the emerging market when gray markets are present, and those market-creating investments are both costly and increasing in the investment size. On the other hand, market-creating investments help the firm in two ways. First, firm investments increase the market-clearing price in the emerging market, which increases profit in the emerging market. Second, firm investments also increase the gray market’s cost base which reduces its competitiveness in the domestic market and leads to less cannibalization of domestic profits.

When investment spillovers are non-existent, as in Proposition 1, the negative forces dominate, and firms are strictly better off in an economy without gray markets. Conversely, in a setting where each firm’s market-creating investments have spillover effects, either the negative or positive forces can dominate, as shown in Figure 2. This outcome is driven by the positive externality conferred on each competitor’s demand in the emerging market. This positive investment externality not only makes the emerging market demand larger for each firm than in a setting with no investment spillovers (e.g., \( x_i^{G} + \beta x_i^{G} > x_i^{C} \)), for all feasible \( \gamma, \beta, E \) but, importantly, allows each firm to more efficiently build its own market. In other words, the return on investment, measured as the ratio of the increase in market demand relative to the cost to increase demand, is higher when investments confer spillovers than when investments confer no spillovers (e.g., \( (x_i^{C} + \beta x_i^{G})/x_i^{C} > (x_i^{C})^2/(x_i^{C})^2 \)), for all feasible \( \gamma, \beta, E \). As \( \beta \) increases, so does the efficiency of the firm’s investment, and the benefits of gray markets (i.e., higher emerging market consumption and gray market cost base) eventually outweigh the costs of gray markets (i.e., cannibalization of domestic product and larger investments costs). When the spillover effect is sufficiently large (i.e., \( \beta > \beta^* = \gamma/2 \)), each firm may be better off with gray markets than without them.

Thought of in economic terms, each firm invests until the marginal return from investment equals the marginal cost of investment. When a gray market is introduced with no spillovers, the marginal return for a given investment level increases because the investment reduces cannibalization in the domestic market. When there are spillovers, the marginal return for a given investment level increases not only because of cannibalism reduction but also because the other firm invests more and that makes the focal firm’s investment more valuable.

We note that the threshold \( \beta^* \) increases in the substitutability parameter, \( \gamma \). The intuition behind this result is straightforward. Because higher \( \gamma \) (i.e., the level of substitutability between the competing firms’ products) leads to more cannibalized domestic sales for both the focal firm and its rival, as \( \gamma \) approaches 1 (i.e., each firm produces a near perfect substitute), the aggregate negative forces associated with gray markets increase, resulting in a correspondingly higher threshold \( \beta^* \) over which the benefits from gray markets will outweigh the costs from the gray market.

Finally, conditional on \( \beta > \beta^* \), both firms are better off with gray markets provided the emerging market is large enough (i.e., \( E > E^* \)). The smaller the emerging market, the lower the emerging market retail price and, in turn, the lower the gray market’s cost base. A lower cost base allows the gray marketer to be more competitive in the domestic market leading to more cannibalized sales from the domestic firm. For sufficiently low \( E \), the larger market-creating investments brought on by the gray market’s existence do not increase the gray market’s cost base enough to outweigh the negatives of gray market cannibalization. Once \( E > E^* \), however, the emerging market is large enough such that, following each firm’s investment, the gray market becomes sufficiently uncompetitive and both firms are better off with gray markets.

4. Model Extensions

4.1. Complementary Goods and Investments

In the previous section, our results show that firms are better off with gray markets provided there are sufficient positive externalities (i.e., spillovers) generated when firms make market-creating investments in the emerging market. In our first extension, we demonstrate the model’s robustness to a different source of positive externality, complementary goods. Specifically, we consider a setting in which the focal firm invests in the emerging market and the economy is characterized by products that are complements as
opposed to substitutes. When products are comple-
ments, any market-creating investment for one firm’s
product also increases demand for the other firm’s
product.

To analyze a setting with complementary goods
and no spillovers, we use the model from subsection
3.2 (i.e., a market where \( \beta = 0 \)) but assume that the
firms produce complements (i.e., \( \gamma \in (-1, 0) \)) instead
of substitutes. Note that all of the optimal invest-
ments, quantities, and profits are identical to those in
subsection 3.2, except that we now consider outcomes
over the range \( \gamma \in (-1, 0) \) as opposed to \( \gamma \in (0, 1) \).

The feasible region in this setting consists of all
\( E \in (\mathbb{R}, \mathbb{R}) \), \( \gamma \in (-1, 0) \), and \( \beta = 0 \).

PROPOSITION 3. In a setting with complementary prod-
ucts and market-creating investments with no investment
spillovers:

1. Each firm spends more on market-creating invest-
ments when gray markets are active (i.e.,
\( x_i^G X > x_i^X \), \( i = 1, 2 \)).
2. There exists a boundary \( E^{**} \) such that each firm is
strictly better off with gray markets for all feasible
\( E > E^{**} \) and weakly worse off with gray markets
for all feasible \( E \leq E^{**} \) (i.e., \( \pi_i^G X > \pi_i^X \), \( i = 1, 2 \) if
and only if \( E > E^{**} \)).

This result is driven by the same forces as the spill-
over results in Proposition 2. Specifically, in an econ-
omy with complementary products, when a focal firm
invests in the emerging market, there is a positive exter-
nalinity generated on its competitor’s demand. Similar
to a setting with investment spillovers, pro-
vided the emerging market is large enough, this posi-
tive externality leads to larger, more cost-efficient
investments for each firm, and each firm is better off
with gray markets than without them.

To provide additional intuition for how positive
investment externalities can lead to firms being better
off with gray markets, we contrast the results from
Propositions 1 and 3. In both settings, market-creating
investments create no spillovers (i.e., \( \beta = 0 \)). In the
first setting, firms produce substitute goods. In the
second setting, investments confer a positive extern-
alinity on a rival’s demand through the complementary
nature of the products.

As can be seen in Figure 3, in a market with no
investment spillovers, firms are strictly worse off with
gray markets if they produce substitutes. If firms pro-
duce complements, however, there exists an emerging
market size threshold above which both firms are
better off with gray markets. These results echo our find-
ings in Proposition 2, and broaden the reach of our results
from investment spillovers in particular to positive investment externalities in general.

4.2. Endogenizing the Gray Market

The original model exogenously determined whether
the gray market existed or not. We now extend the
model to consider the outcomes when each firm has
the ability to stop the gray market for its own product
without cost. To analyze this scenario, we return to
our main model of investment spillovers from subsec-
tion 3.3, and consider whether a firm might prefer to
stop its own gray market and free ride on the spill-
over effects of a competitor’s market-creating invest-
ments.

To endogenize the gray market, we include an
additional decision at the beginning of the game as
shown in Figure 4. Specifically, each firm now
chooses whether to stop the gray market for its own
product without cost. We denote the strategy combi-
nations of Firms 1 and 2 by superscripts \( \{s_1, s_2\} \),
where \( G \) implies that the firm has allowed the gray
market for its product and \( N \) means that the firm has
decided not to allow the gray market for its product.

For tractability, we fix \( \beta = 1 \) in this setting. We present
the analysis for \( \beta = 1 \) because it results in the least
cumbersome expressions mathematically, but note
that our results are robust to setting \( \beta \) to other values.

Using backward induction, we solve the game with
the addition of the new first stage. Note that all of the
optimal investments, quantities, and profits when
either both firms allow or both firms do not allow the
gray market are identical to those in subsection 3.3,
and thus \( \pi_i^G | \beta = 1 = \pi_i^G \) and \( \pi_i^N | \beta = 1 = \pi_i^N \). The feasi-
ble region in this situation consists of all \( E \in (\mathbb{R}, \mathbb{R}) \),
\( \gamma \in (0, 1) \), and \( \beta = 1 \).

We also analyze the model with the firms making
asymmetric decisions in the first stage. Without loss of
generality, we assume that Firm 1 chooses not to
allow the gray market and Firm 2 chooses to allow the
gray market. We obtain \( \pi_i^{NG} \) using the same procedure

![Figure 3 With No Investment Spillovers, the Feasible Region is Between the Solid Lines. Firms are Better Off with Gray Markets Between \( E^{**} \) (dashed line) and \( E^{ox} \).](attachment:image.png)
as before, except that in stage 4, the gray market is constrained to choose $q_{G1} = 0$ and chooses only $q_{G2}$ to maximize Equation (3). The resulting equilibrium investments and quantities in the asymmetric case are as follows:

$$
q_{G1}^{NG+} = \frac{8 - \gamma - 4\gamma^2 + \gamma(1 - \gamma)(E + x_1 + x_2)}{(2 + \gamma)(8 - 5\gamma^2)},
$$

$$
q_{G2}^{NG+} = \frac{16 - 9\gamma^2 - \gamma^3 - \gamma^2(1 - \gamma)(E + x_1 + x_2)}{(2 + \gamma)(8 - 5\gamma^2)},
$$

$$
Q_{E1}^{NG+} = \frac{\gamma(1 - \gamma) + (8 - \gamma - 4\gamma^2)(E + x_1 + x_2)}{(2 + \gamma)(8 - 5\gamma^2)},
$$

$$
Q_{E2}^{NG+} = \frac{-\gamma^2(1 - \gamma) + (16 - 9\gamma^2 - \gamma^3)(E + x_1 + x_2)}{(2 + \gamma)(8 - 5\gamma^2)}.
$$

Solving the second-stage game yields the equilibrium investment levels:

$$
x_1^{NG+} = \left(\frac{16 + 32\gamma - 2\gamma^2 - 36\gamma^3 - 22\gamma^4 + 12\gamma^5 + 9\gamma^6}{2(288 + 54\gamma^2 - 244\gamma^2 - 660\gamma^3 - 49\gamma^4 + 202\gamma^5 + 54\gamma^6)}\right),
$$

$$
x_2^{NG+} = \left(\frac{48 + 32\gamma - 102\gamma^2 - 4\gamma^3 + 44\gamma^4 - 8\gamma^5 - \gamma^6}{2(288 + 54\gamma^2 - 244\gamma^2 - 660\gamma^3 - 49\gamma^4 + 202\gamma^5 + 54\gamma^6)}\right).
$$

The resulting equilibrium firm profits for asymmetric decisions, $\pi_1^{NG} = \pi_1^{NG+}$ and $\pi_2^{NG} = \pi_2^{NG+}$, are lengthy expressions and are provided in the Online Appendix. Paralleling subsection 3.3, we again impose two technical assumptions to ensure that the equilibrium gray market quantity for the firm that chooses to allow the gray market is non-negative (i.e., $q_{G2} \geq 0$), and that the firm’s gray market does not exceed the available supply in the emerging market (i.e., $q_{G2} \leq Q_{E2}$). The restrictions are

$$
E < E^{NG+} \equiv 1 - \frac{2}{(2 + \gamma)^2}
$$

and

$$
E > E^{NG} \equiv 1 + \frac{1}{2 + \gamma} + \frac{4(5 - 4\gamma)}{9(8 - 5\gamma^2)} + \frac{622 - 206\gamma - 231\gamma^2}{9(-40 + 22\gamma^2 + 3\gamma^3)}
$$

respectively. These feasibility region boundaries are somewhat more restrictive than the conditions of subsection 3.3. In particular, $E^{NG} = E^{NG+}$ and $E^{NG} < E^{NG+}$. Therefore, the feasible region for the next proposition consists of all $E \in (E^{NG}, E^{NG+})$, $\gamma \in (0, 1)$, and $\beta = 1$. Within the feasible region, we find that each firm’s equilibrium profit can be higher by allowing the gray market to exist, whether its rival chooses to stop or allow the gray market to operate (i.e., there are settings where both $\pi_1^{NG} > \pi_1^{NG+}$ and $\pi_2^{NG} > \pi_2^{NG+}$, $i = 1, 2$). Proposition 4 characterizes the parameter region in which there is a unique, pure strategy equilibrium where both firms are better off allowing gray markets.

**Proposition 4.** There exists a boundary $E^{\phi}$ in the feasible region. For all feasible $E > E^{\phi}$, there is a unique, pure strategy equilibrium in which both firms allow the gray market for their respective products to exist.

Proposition 4 states that, even if either firm had the ability to costlessly stop its own gray market, provided the emerging market is large enough, each firm has an incentive to allow its own gray market to continue. This result obtains because market-creating investments are strictly larger for both firms when gray markets exist for both products, than if gray markets exist for one product or neither product. Note that allowing the gray market to continue when it could be stopped allows the firm to credibly commit to a higher investment level. In turn, this allows for better investment coordination between a firm and its rival. Finally, this result also provides another perspective on why firms may not implement management control systems meant to mitigate gray market distribution, even if implementing such control systems were costless.
4.3. Competition in the Gray Market

To assess whether the lack of competition in the gray market institution drives the results in the main model, we extend the model to allow competing gray market firms. We return to the main setting with investment spillovers and the timeline of Figure 1. However, we now assume that two gray marketers exist, and that each gray marketer specializes in obtaining and reselling the product of a single manufacturer. The additional gray marketer intensifies competition in the domestic market, leading to increased cannibalization of domestic products compared to a setting where one gray marketer supplies both products. To simplify the expressions for a crispier presentation, we set \( \gamma = 1 \) for this subsection; however we note that the results are robust for all \( \gamma \in (0, 1) \). We denote the setting with multiple gray marketers with the superscript \( 2 \).

We again analyze the model of subsection 3.3, using the same procedure as before, except that in stage 3, each gray marketer simultaneously chooses the quantity \( q_{Gi} \) as follows:

\[
\text{Max}_{q_{Gi}} \pi_{Gi} = q_{Gi}(p_i - p_{Ei}), \quad i = 1, 2
\]

subject to demand conditions given in Equations (1) and (2). The resulting optimal gray market quantities are as follows (\( i \neq j \in \{1, 2\} \)):

\[
q_{Gi}^{2Gb} = \frac{1}{6} \left( 1 - (q_i - Q_{Ei}) - (q_j - Q_{Ej}) \right)
\]

Using backward induction, in the second stage game, we determine the manufacturers’ equilibrium quantities to be (\( i \neq j \in \{1, 2\} \)):

\[
q_i^{2Gb} = \frac{1}{6} (2 - 3x_i(1 - \beta) + 3x_j(1 - \beta)),
\]

\[
Q_{Ei}^{2Gb} = \frac{1}{6} (2E + x_i(7 - 5\beta) - x_j(5 - 7\beta)).
\]

Solving the first stage game yields the following equilibrium investment levels by each manufacturer (\( i = 1, 2 \)):

\[
\xi_i^{2Gb} = \frac{1 + 11E + \beta - 7E\beta}{43 - 4\beta + 7\beta^2} > 0.
\]

We again require two technical assumptions to ensure that equilibrium gray market quantities are non-negative (i.e., \( q_{Gi} \geq 0 \)), and that they do not exceed the available supply (i.e., \( q_{Gi} \leq Q_{Ei} \)). Accordingly, we compute the feasible region and restrict \( E < \frac{1}{3}(7 - \beta + \beta^2) = E^{2Gb} \) and \( E > \frac{2\beta}{7\beta} = E^{2Gb} \). The feasible region for this setting consists of all \( E \in (E^{2Gb}, E^{2Gb}), \gamma = 1, \) and \( \beta \in (0, 1] \). Proposition 5 confirms that our main model’s results do not depend on a sole gray market firm.

**Proposition 5.** Let \( \beta = \gamma/2 \). In a setting with two gray marketers, market-creating investments, and spillovers:

1. For all feasible regions, each firm spends more on market-creating investments when gray markets are active (i.e., \( x_i^{2Gb} > x_i^{0}, i = 1, 2 \)).
2. Provided \( \beta > \beta^* \), there exists a boundary \( E^* \) such that each firm is strictly better off with gray markets (i.e., \( \pi_i^{2Gb} > \pi_i^0, i = 1, 2 \)) for all feasible \( E > E^* \). When \( \beta \leq \beta^* \), the firms are weakly worse off with gray markets (i.e., \( \pi_i^{2Gb} \leq \pi_i^0, i = 1, 2 \)) for all feasible \( E \).

4.4. Price Competition

The analysis of the main model and extensions consider a standard Cournot Stackelberg game. However, there are situations (e.g., disclosure choice, transfer pricing) where recasting a Cournot model as differentiated Bertrand competition leads to different conclusions. These alternate conclusions often arise as quantities are strategic substitutes in a Cournot game whereas prices are strategic complements in a Bertrand game. Thus, it is important to assess whether inferences developed under one type of competition continue to hold under another. We denote the price competition setting with the superscripts \( G0 \) and \( \theta \), respectively, to indicate active gray markets or no gray markets.

When gray markets are active, recasting the model in a Bertrand setting presents practical challenges. Specifically, the Bertrand model breaks down because the gray market firm produces a perfect substitute to its domestic counterpart and has a higher cost base. In such a setting, the optimal solution for the original manufacturer would be to lower its prices in the domestic market to a level marginally below the gray market competitor’s cost (i.e., the retail price in the emerging market) to preclude any arbitrage opportunity. This reasoning suggests that gray markets should not be observed when firms compete in prices and the gray marketed product is a perfect substitute for the authorized product. Nevertheless, gray markets are observed in categories characterized by price setting competition.

Accordingly, we model the gray market in this context by assuming that the gray marketer is a price taker rather than a price setter, similar to the approach of Gaskins (1971). We model the gray market as a price-taking, competitive fringe player that incurs a quadratic cost to purchase stock.

Importantly, we confirm that the tenor of the Cournot results continue to hold in a Bertrand setting. Specifically, we find that: (1) for all feasible regions, each
firms and distributors may actively participate (or not) in gray market activity. Moreover, there are clearly alternate explanations for why firms do not make efforts to eliminate gray markets. For example, it is possible that the cost to implementing a control system to stop gray market activity is prohibitive. This is certainly plausible given how elusive gray markets are. These issues are not reflected in our analysis. Nevertheless, we show that there are often conditions where firms have an incentive to permit the diversion of product from the emerging market to the home market, even if there is no cost to controlling the diversion.

The collective results suggest that firms in certain sectors prone to investment spillovers (like the technology sector) may not invest in control systems meant to eliminate gray market distribution, because they may be better off investing in market creation and allowing gray markets to persist. This result may explain why some firms do not implement adequate management control systems in settings where we would expect them to be useful (i.e., the prevention of unauthorized distribution), while other firms implement controls to mitigate gray market flow. Finally, the results might also drive the observed variation in regulatory responses to gray markets. In particular, a regulator may be more apt to stop gray market activity in industries where manufacturers’ investments provide negligible spillover effects on a rival’s demand.

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## References


Supporting Information

Additional Supporting Information may be found in the online version of this article:

Appendix S1: Proofs.