Auctions in Markets: Common Outside Options and the Continuation Value Effect*

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Abstract

We study auctions with outside options determined through bargaining in an outside market. We show that auctions with less information revelation to bidders may yield higher revenues. It is never optimal for the auctioneer to reveal information after the auction, while it may be optimal to do so before. These results differ from the case where the value of the outside option arises exogenously in the post-auction market. Non-transparent auctions are preferred by the auctioneer if: (i) the value of the outside option depends less on market conditions per se than on whether the action chosen in the external market reflects those conditions adequately; (ii) bidders have imprecise signals about conditions prevailing in the outside market. Consequently, the auctioneer prefers to hide information about aggregate market conditions exactly when the value of information in the outside market is high for the bidders.

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1 Introduction

Auctions usually do not take place in isolation. In most real-world auctions losing bidders have the option of purchasing substitute goods somewhere else. This is true for auctions on eBay as well as for auctions for timber, off-shore drilling, municipality bonds and many more examples of auctions and auction-like trading institutions. The current article is motivated by the need for a simple and tractable framework to study auctions that are embedded in markets; we investigate whether and to what extent insights from the analysis of isolated static auctions extend once the availability of substitutes is explicitly accounted for.

The abundance of trading institutions and a wide variety of channels through which outside options might affect bidding behavior require careful modeling choices to maintain tractability. We choose a highly stylized "reduced from" approach to keep our model lean and transparent. We concentrate on a particular channel through which the outside option affects bidding behavior. Specifically, (i) losing bidders have to take actions when pursuing the outside option and (ii) the value of the outside option as well as the optimal course of action might be unknown to the losing bidders. For example, on eBay, losing bidders might be uncertain about the distribution of ending prices in future auctions and, when participating in other auctions, bidders have to actively choose bids.[1]

The contribution of our work is, first, to provide a tractable framework to study auctions that are embedded in a market where the losing bidders *actively* pursue outside options. Second, we apply the model to gain insights into the endogenous choice of the trading institutions in markets. In particular, we identify conditions under which an auctioneer might have incentives to run "transparent" auctions and when not.

Our model consists of two stages. In the first stage a fixed number of buyers participate in an auction in which a single, indivisible good is for sale. In the second stage losing bidders take an action in a decision problem under uncertainty where each losing bidder's payoff depends on this bidder's action and a common unknown state of nature.[2] Before entering the auction, bidders privately observe noisy signals about this common state of nature. In addition, the auctioneer might choose to reveal information before or after the auction and bidders might learn about other bidders' signals or directly about the state, depending on the transparency of the auction format. Therefore, the auctioneer's choice of information

[1] Sailer (2005) documents that on eBay losing bidders bid more aggressively in subsequent auctions. She argues that this stylized fact is broadly consistent with the idea that bidders are uncertain about market conditions, that optimal bids depend on bidders' beliefs about these conditions, and that bidders learn and update after losing an auction.

[2] For concreteness, we specialize the second stage decision problem in our base model to a bargaining situation in which the buyers' actions correspond to price offers to (other) sellers of substitute goods. We discuss other interpretations as well.

policy affects the bidders' beliefs about the payoffs achievable in the decision problem, and thereby influences their bids. The reason is that bidders shade their bids by the forgone expected payoff from the decision problem (the opportunity cost of winning the auction), and a bidder's beliefs about the state influence the bidder's estimate of the expected payoff from the decision problem.

Our main application is in auction design where we study the trade-offs that a revenue maximizing auctioneer faces when losing bidders can actively pursue their outside options. Among other questions, we ask whether the auctioneer has an incentive to (ex ante) commit to revealing a perfectly informative signal about the state before the auction. We show that one can decompose the revenue impact of revealing an informative signal about the state into two opposing effects. This decomposition allows a clean analysis of the auctioneer's choice between information policies. First, we identify an effect that is akin to the *linkage principle*. Milgrom and Weber (1982) observed that the auctioneer prefers to reveal information if bidders observe affiliated signals about the object. The fact that the linkage principle is active in our environment is due to some similarities of our dynamic setting with common outside options to a standard interdependent value auction as explained later. Second, our analysis also identifies the new *continuation value effect.* The continuation effect favors opaque auctions. The continuation value effect works roughly as follows: A transparent auction would reveal information about outside market conditions. Additional information improves payoffs of losing bidders in the decision problem in expectation because bidders can make "better" choices. The improved outside option leads to more bid shading and decreases the auctioneer's revenue.[3]

We use this decomposition to provide comparative static results of the revenue impact of information revelation with respect to the informativeness of the bidders' signals and also with respect to properties of the decision problem. Let us explain the findings by describing how the continuation value effect and the linkage effect respond to changes in the parameters of the model. The continuation value effect is strong when bidders enter the auction with imprecise signals, since in this case revealing the state has a larger impact on losing bidders' information about market conditions. Consequently, the quality of the decisions made in the post-auction market improves substantially. The continuation effect is also strong when the optimal price offer is sensitive to the state. In this case improving one's information about the market conditions has a larger impact on one's decision, and

[3] Technically, the continuation value effect is related to the convexity of the continuation payoffs in buyers' posteriors. Revealing information leads to a mean-preserving spread in the distribution of posteriors. Therefore, when continuation payoffs are (strictly) convex, revealing information increases expected continuation payoffs. Convexity of continuation values in beliefs is likely to be a robust feature of many environments; see our discussion in Section 5.2.

hence on one's continuation utility. Since in both of these cases the continuation value effect is strong, revealing information about the state reduces expected revenues. When the two states provide intrinsically different continuation payoffs regardless of the action taken in the post-auction decision problem, the linkage effect is strong. The intuition is immediate from the literature on common value auctions: revealing information to all bidders makes the knowledge of all bidders more symmetric, which reduces bidders' rents and increases the auctioneer's revenues.

Our reduced-form approach to modeling the outside option calls for further elaboration. Our second-stage decision problem is best interpreted as a reduced form of a dynamic matching and bargaining game similar to the one studied by Satterthwaite and Shneyerov (2007):[4] After the auction has ended, losing bidders return to a large stock of potential buyers. Bidders from the stock are then matched into new auctions in the future. The assumption that the stock is large ensures that no two losing bidders are matched again into the same auction and no bidder interacts again with the same seller. Thus, our reduced form decision problem is intended to capture an environment in which there is no strategic interaction among the participants in the future.[5] We return to this interpretation in the Discussion (Section 5.2), and explain precisely how our two-stage model is equivalent to the strategic problem of buyers and sellers in a large dynamic matching and bargaining market. In that section, we also comment briefly on potential welfare implications of the fact that sellers prefer opaque auctions when buyers have the least precise information, and relate our findings to the recent literature on information percolation initiated by Duffie and Manso (2007).

Our article is a contribution to the literature on auction design, focusing on optimal information policy. It is well known that the linkage principle alone is not sufficient to determine the optimal information policy because the principle may fail outside symmetric single unit auctions. Perry and Reny (1999) show that the linkage principle does not extend to multi-unit auctions. Board (2009) shows that the linkage principle does not apply if new information can change the *ranking* of bidders' valuations (allocation effect); see Krishna (2008) for further discussions of the limitations of the scope of the linkage principle. Some other works on disclosure policies in auctions are Bergemann and Pesendorfer (2007), Bergemann and Horner (2010), and Forand (2010).[6]

[4] See also Duffie and Manso (2007), Golosov, Lorenzoni, and Tsyvinski (2011) and Majumdar, Shneyerov and Xie (2011).

[5] The absence of repeated strategic interaction distinguishes our analysis from the literature on multi-unit auctions, see, for example Mezzetti et. al. (2008). The absence of strategic interaction and the presence of substitute goods distinguishes our work from the literature on auctions with resale; see, for example, Garratt and Tröger (2006), Hafalir and Krishna (2008) and Haile (2001).

[6] As in their models, the auctioneer cannot charge fees to buyers for information in ours either. Among others, Gershkov (2009), and Eső and Szentes (2007) consider the case where information can be sold.

The remainder is organized as follows. Section 2 describes the model, Section 3 provides the results for the second-price auction format, and Section 4 contains revenue comparisons between the first-price, second-price and English auctions. Section 5 provides discussion and robustness checks. Section 6 concludes. Most of the proofs are in the Appendix.

2 Model and preliminary analysis

2.1 Setup

The interaction unfolds as follows. First, the auctioneer and the N bidders receive signals about the state of the world. Second, the auctioneer runs an auction for an indivisible object. Third, each losing bidder chooses a price offer in a bilateral bargaining problem. The final stage is simply a single-person decision problem under uncertainty.

Information. There are two states of the world, $w \in \{H, L\}$, and the realization is not observed by the bidders. The probability of the high state is ρ_0. The state of the world is interpreted as the aggregate market condition. The bidders receive private signals that are correlated with the state, and these signals are denoted by $s_1, s_2, ..., s_N$. In state w, the bidders' signals are distributed independently and identically according to G_w on support $[\underline{s}, \overline{s}]$. We assume that G_w admits a differentiable density function g_w. With a signal s, the Bayesian posterior probability of the high state is denoted by $\rho(s) = \rho_0 g_H(s) / ((1-\rho_0) g_L(s) + \rho_0 g_H(s))$. We assume that the likelihood ratio of the signal g_H/g_L is strictly increasing, and thus the posterior is strictly increasing in s. To simplify exposition, we assume that $\rho(\overline{s}) = 1$ and $\rho(\underline{s}) = 0$, that is, there are some perfectly informative signals.[7]

Auction. All bidders participate in an auction where a single indivisible object is for sale. We analyze bidding in standard auction formats, including the first-price, the second-price, and the ascending (English) auction.

Preferences and payoffs. The winning bidder receives the object and pays a price p, while the losing bidders do not make payments in any of the auctions studied. The valuation for the object, v, is the same for all bidders and publicly known. The utility of the winner is equal to $v - p$, while that of the losers' is equal to their continuation payoffs defined below.

Outside Option After the auction each losing bidder proceeds to a "market" which is modeled as follows. After losing, the bidder is matched with another seller with probability μ_ω. If matched, the buyer can make a take-it-or-leave-it offer to the seller. To ease exposition we assume that when making the offer the losing bidder does not observe whether he is

[7] Later, in a discussion we show that this assumption does not drive our results.

matched yet.[8] The seller accepts the offer whenever it is below his costs. The buyer does not know the cost of the seller but believes that, conditional on state ω, the costs are distributed on some interval $[\underline{c}, \overline{c}]$ according to a distribution function F_ω which is atomless and has a differentiable density. The expected utility from an offer x is

$$u_w(x) = \mu_w(v - x) F_w(x).$$

Let $a(\rho) = \arg\max \rho u_H(x) + (1-\rho) u_L(x)$ be the optimal price offer given belief ρ and let $V(\rho) = \rho u_H(\alpha(\rho)) + (1-\rho) u_L(\alpha(\rho))$ be the value function. Let $U_w(\rho)$ be the payoff in state w of a buyer who takes action $a(\rho)$.[9] It is important to stress that both V and U_w are exogenous for the auction stage as they embody information about the optimal action *after* the auction given any belief.

When only the matching probabilities—but not the distribution of the seller's costs—differ across states, the optimal action is independent of the state. In this case, $V(\rho)$ is simply a linear function of the probability of the high state, $V(\rho) = \rho V(1) + (1-\rho) V(0)$. If the distribution does depend on the state, the optimal action depends on the belief ρ. In the latter case, it follows from standard arguments from the economics of information that the value function V is convex in beliefs. The curvature of the value function plays an important role in our analysis.

The following example illustrates the continuation problem we are considering:

Example 1: A continuation value problem. Let $\mu_H = \mu_L = 1$ and let $v \geq 1$, $k \geq 0$ with $F_L(x) = \frac{d-0.5kx^2+t}{v-x}$ and $F_H(x) = \frac{k(x-0.5x^2-0.5)+t}{v-x}$.[10] The example is constructed so that the optimal price offer is identical to the belief, that is, $\alpha(\rho) = \rho$. The value function is

$$V(\rho) = t + (1-\rho)(d - 0.5k) + 0.5k\rho^2.$$

The value function is strictly convex in ρ if $k > 0$, highlighting the value of information. If $k = 0$, the value function is linear and payoffs depend only on the state, not on the action.

[8] This assumption can be relaxed if one imposes conditions on the relative values of μ_H and μ_L so that a monotone equilibrium exists. Alternatively, instead of matching probabilities, one can think of μ_H and μ_L as probabilities with which the offer gets communicated to the potential seller

[9] The above notation (and indeed all of the main text) implicitly assumes that $a(\rho)$ is the unique optimal price when the belief is ρ. Whenever there are multiple actions that are optimal (set $\alpha(\rho)$) and that deliver different payoffs conditional on the realized state, let $U_H(\rho) = \max_{x\in\alpha(\rho)} u_H(x)$ and let $U_L(\rho) = \min_{x\in\alpha(\rho)} u_L(x)$. In the Appendix we show that for almost all ρ all optimal actions provide the same payoffs in the two states, that is, $U_w(\rho) = u_w(x)$ for all $x \in \alpha(\rho)$.

[10] To ensure that F_L, F_H are proper distribution functions, we assume that $k \leq 1$, $t \geq k/2$, and that $0.5 + \frac{d}{k} + \frac{t}{k} \geq v \geq d + t - 0.5k + 1$. This region is non-empty if $k \leq 1$. To ensure (strict) monotonicity of V we assume that $d \geq k/2$.

3 Second-price Auction

To start our analysis of the effects of the information policy on revenues we consider a sealed-bid second-price auction. This auction format lends itself to a very tractable analysis, since bidding incentives are relatively simple. We characterize equilibrium bidding behavior and compare revenues for three different information policies. In Section 3.1 we consider the case where no information (other than who has won) is released, in Section 3.2 we consider the case where the auctioneer has perfect information about the state and reveals his information, and in Section 3.3 the case where the winning bid is revealed.

3.1 Equilibrium without information revelation

To simplify analysis and aid understanding we connect our dynamic auction setting with that of the standard static interdependent values setup of Milgrom and Weber (1982). The effective valuation of a bidder for the object given state w net of the continuation payoff is given by

$$v - U_w(\rho),$$

where ρ is the belief of the bidder that determines his optimal action in the decision problem.

In the special case where the decision problem is degenerate—optimal actions are independent of beliefs—the effective valuation of a bidder is $v - U_L(\hat{\rho})$ and $v - U_H(\hat{\rho})$ in the low and in the high state, respectively, for arbitrary $\hat{\rho}$. In this special case, the bidding problem in our dynamic setting is equivalent to the bidding problem in a static auction with pure common values $v - U_L(\hat{\rho})$ and $v - U_H(\hat{\rho})$.

Equilibrium bidding can be easily characterized in this special case. With y denoting the highest signal among the $n-1$ competitors of any given bidder and with

$$\rho(s,x) = \Pr(H \mid s, y = x),$$

in the second-price auction undominated equilibrium bids are

$$b(s) = v - [\rho(s,s)U_H(\hat{\rho}) + (1 - \rho(s,s))U_L(\hat{\rho})],$$

for arbitrary $\hat{\rho}$, that is, the bid is equal to the valuation v net of the expected continuation value conditional on signal s and conditional on being tied with the highest competing bidder having signal s. This characterization is familiar from standard auction theory and is proven as a special case of the proposition below. Note that, in the special case where the decision problem is degenerate, our dynamic auction is equivalent to a common value auction in the framework of Milgrom and Weber (1982) with appropriately defined values that take the continuation payoffs into account.

In general, our dynamic setting cannot be reduced to an equivalent static auction, however. The reason is that payoffs depend not only on the state w but also on the belief ρ of a losing bidder. Because the beliefs of losing bidders in general depend on the (equilibrium) action profile and the information policy of the auctioneer, there does not exist a transformation of bidders' payoffs that allows reducing the dynamic auction to an equivalent static auction.

The following proposition characterizes symmetric equilibrium where each bidder's bid b is strictly monotone in his signal in the general case where the decision problem is not degenerate. The proposition also provides sufficient and necessary conditions for a monotone equilibrium to exist. For the statement of the proposition, we denote

$$\rho_{lose}(s,x) = \Pr(H|s, y \geq x);$$

the probability of the high state conditional on losing with a signal s, and the highest other signal being at least x.

Proposition 1 *(i) If $V(\rho) = \rho U_H(\rho) + (1-\rho) U_L(\rho)$ is strictly decreasing in ρ, then there exist monotone and symmetric equilibria where bidders bid according to (the unique) bid function*

$$b(s) = v - [\rho(s,s) U_H(\rho_{lose}(s,s)) + (1-\rho(s,s)) U_L(\rho_{lose}(s,s))]. \tag{1}$$

for almost all s.
(ii) If V is not strictly decreasing, then a monotone and symmetric equilibrium does not exist.
(iii) The value function V is strictly decreasing if and only if $U_H(\rho) < U_L(\rho)$ for all $\rho < 1$.

It is well known that in such a static second-price auction the symmetric equilibrium bid function is equal to the expected value conditional on being tied at the top. To explain (1), note that the relevant continuation value must be assessed conditional on tieing ($\rho(s,s)$) as in Milgrom and Weber (1982). However, in our setup the continuation value also depends on the action taken in the continuation problem, and this action is optimal given belief upon losing. Given symmetric and monotone bidding strategies, the posterior conditional on losing is $\rho_{lose}(s,s)$.

For the remainder of the article we assume that V is decreasing (except when it is stated otherwise, see Example 3), and we concentrate on the monotone equilibrium in the game with no information revelation.

Remark: Relation to Static Auctions with Belief Dependent Payoffs.

Purely formally one might argue that our dynamic setting is equivalent to a static auction with *belief-dependent payoffs* $v - U_w(\rho)$, where ρ is the posterior belief of bidders at the terminal nodes of the auction game. In general, the fact that the effective valuations depend on the posterior beliefs of bidders is indeed the feature that distinguishes our dynamic auction from Milgrom and Webers' static auction. In this formulation, the seller's information policy as well as the equilibrium itself affects bidders' valuations directly through their impact on bidders' beliefs. While illuminating, we chose not to analyze a static auction game with belief-dependent payoffs because such a game seemed somewhat artificial to us.

Remark: A Static Auction that Yields Equivalent Bidding

In the following we argue that there exists valuations that depend only on signals and not on beliefs such that an auction with such valuations has an equilibrium that is equivalent to the symmetric and monotone equilibrium of the dynamic auction. We will then argue why this equivalence is not helpful for our purposes.

Let

$$v(s,y) = v - (\rho(s,y)U_H(\rho_{lose}(s,y)) + (1-\rho(s,y))U_L(\rho_{lose}(s,y)). \tag{2}$$

Consider a static auction in which payoffs are defined as in (2). In this static auction, payoffs are interdependent and depend on a bidder's own signal s and on the other bidders' signal profile through the first order statistic y. It is straightforward to verify that the (undominated) equilibrium bidding strategy of that static auction is equivalent to the bidding function (1). The details are provided in the Appendix in the proof of Proposition 1.

Note, however, that the constructed static interdependent value auction is not strategically equivalent to our original auction. To illustrate this point, note that we have used the monotonicity of the bidding functions to construct the payoffs $v(s,y)$. For example, one can easily see that a bidder's best response to a profile of non-monotone bidding strategies would be different in the two auctions. The reason is that $\rho_{lose}(s,y)$ would no longer correctly capture the posterior of a losing bidder. Similarly, across the information policies considered below, equilibria of the constructed static auction would be different from the equilibria of the dynamic auction. Thus, the fact that there is a particular static auction that yields the same symmetric monotone equilibrium in the case with no information revelation is not helpful for our purposes to characterize equilibria across information policies and, later, across auction formats.[11]

[11] The only exception is the revenue comparison of the first-price and second-price formats where the constructed value functions are the same. For details, see Section 4 where the two formats are compared.

3.2 Revenue comparison when the state may be revealed

3.2.1 Main result

We now derive the ex-ante expected revenue when the auctioneer who knows the state precisely[12] reveals the state before the auction (ER^{Before}), and compare it to the ex-ante expected revenue when the state is not revealed (ER^{None}). In order to obtain insights into the main trade-offs involved we calculate the difference in revenues ($ER^{Before} - ER^{None}$) as the difference of two opposing effects.

To formally introduce those two effects, let ER^{After} denote the ex-ante expected revenue in the second-price auction when the state is revealed *after* the auction. We decompose the change in revenue from revealing the state before the auction:

$$ER^{Before} - ER^{None} = \underbrace{\left(ER^{Before} - ER^{After}\right)}_{\text{Linkage Effect}} - \underbrace{\left(ER^{None} - ER^{After}\right)}_{\text{Continuation Value Effect}}.$$

The first of the two opposing effects is the *linkage effect*. This effect measures the increase in revenues when the state is revealed as it reduces bidders' rents as in Milgrom and Weber (1982).[13] The second effect, the *continuation value effect* measures the decrease in revenue when the state is revealed as the result of improving outside options, and lower willingness to pay in the current auction. We show that the two revenue differences work in the opposite direction, that is, they are both positive.

The ex ante expected revenues of the auctioneer when the state is revealed before or after the auction are derived next. When the state is revealed *before* the auction takes place, all bidders bid $v - V(1)$ in the high state, and all bidders bid $v - V(0)$ in the low state. The ex-ante expected revenue is

$$ER^{Before} = v - (1 - \rho_0) V(0) - \rho_0 V(1) = v - V(0) + (V(0) - V(1)) \rho_0. \tag{3}$$

When the state is revealed *after* the auction, the full information optimal action is taken in both states, yielding utilities $V(1)$ and $V(0)$. In this case, our model can be reduced to Milgrom and Weber (1982) as discussed before. From their analysis, it follows that the equilibrium bid function is $v-(\rho_{tie}(s)V(1) + (1 - \rho_{tie}(s))V(0))$. Let $g^{(2)}(s)$ denote the density function of the second largest signal of the N signals from an ex-ante perspective.[14] The

[12] At the end of this Section we discuss the case where the auctioneer's information about the state is not fully precise, and show that our results still hold qualitatively.

[13] This effect was introduced in Milgrom and Weber (1982) in the context of common value auctions. See also Perry and Reny (1999) for a discussion about how the linkage principle operates.

[14] Formally, $g^{(2)}(s) = \rho_0 N g_H(s)(1 - G_H(s))G_H^{N-2}(s) + (1 - \rho_0) N g_L(s)(1 - G_L(s))G_L^{N-2}(s)$.

bidder with the second highest signal determines the revenue in the second-price auction, and the expected revenue is

$$ER^{After} = v - V(0) + (V(0) - V(1)) \int_{\underline{s}}^{\overline{s}} \rho_{tie}(s) g^{(2)}(s) ds. \tag{4}$$

The following result collects our findings:

Proposition 2 *The linkage effect and the continuation value effect are given by:*

$$LP = ER^{Before} - ER^{After} = (V(0) - V(1)) \int_{\underline{s}}^{\overline{s}} g^{(2)}(s) \left(\rho_0 - \rho_{tie}(s)\right) ds, \tag{5}$$

$$CV = ER^{None} - ER^{After} = \int_{\underline{s}}^{\overline{s}} g^{(2)}[\rho_{tie}(V(1) - U_H(\rho_{lose})) + (1 - \rho_{tie})(V(0) - U_L(\rho_{lose}))] ds. \tag{6}$$

The revenue of the auctioneer decreases when the state is revealed before the auction rather than not revealed at all if and only if $CV > LP$.

Proof. Using (1), the ex-ante expected revenue of the auctioneer from not revealing the state is

$$ER^{None} = \int_{\underline{s}}^{\overline{s}} g^{(2)}(s) b(s) ds = v - V(0) +$$

$$+ (V(0) - V(1)) \int_{\underline{s}}^{\overline{s}} g^{(2)} \rho_{tie} ds + \int_{\underline{s}}^{\overline{s}} g^{(2)} [\rho_{tie}(V(1) - U_H(\rho_{lose})) + (1 - \rho_{tie})(V(0) - U_L(\rho_{lose}))] ds.$$

Then the result follows directly from formulas (3) and (4). **Q. E. D.**

Let us interpret the two effects. First, the linkage effect measure the difference of equilibrium rents the bidders make under the two information policies. This difference is captured by the auctioneer just as in Milgrom and Weber (1982).[15] Second, the continuation value effect measures how much the relevant bidder's (the one with the second highest signal) continuation value increases when the auctioneer reveals the state. To see this, note that $V(1) - U_H(\rho_{lose}(s))$ and $V(0) - U_L(\rho_{lose}(s))$ measure the increase in the continuation value of a bidder with type s when the state is revealed.

Another way to think about the two effects is to rewrite the bid function as $b(s) = v + \rho(s,s)[U_L(\rho_{lose}(s,s)) - U_H(\rho_{lose}(s,s))] - U_L(\rho_{lose}(s,s))$. As Perry and Reny (1999) show, the term $\rho(s,s)$ is increased in expectation when more information is revealed. Therefore, when the auctioneer reveals more information, then his revenue should increase through this channel, which is the linkage effect in our model. On the other hand, note that $b(s) =$

[15] See the first paragraph and the corresponding footnote in Section 4.

$v - [\rho(s,s)U_H(\rho_{lose}(s,s)) + (1-\rho(s,s))U_L(\rho_{lose}(s,s))]$. Therefore, when the state is revealed the term in the bracket increases to $\rho(s,s)U_H(1) + (1-\rho(s,s))U_L(0)$, so the bid decreases, which is captured by our continuation value effect.

Now, suppose that information has no value, that is, the utility in the continuation problem depends only on the state, but not on the action. In this case for all beliefs $\rho \in [0,1]$ it holds that $V(1) - U_H(\rho) = V(0) - U_L(\rho) = 0$. Inspection of the continuation value effect shows that it is zero. Since only the linkage effect remains active, revealing information is profitable. This is not surprising, since (as we argued before Proposition 1) in this case our model reduces to the Milgrom and Weber (1982) setup.

3.2.2 Comparative statics of information revelation

We now discuss conditions under which revealing or hiding information may be profitable. We consider an example to show that revealing the state is profitable if the value of information is low, if the payoff differences between the states are high and if bidders have precise signals. General comparative statics results follow after the example.

Example 2. There are two bidders and the two states are equally likely ex-ante. The signal distribution function is $G_L(x) = 1 - (1-x)^\beta$ in the low state, and $G_H(x) = x^\beta$ in the high state for some $\beta \geq 1$. Importantly, the beliefs ρ_{lose} and ρ_{tie}[16] depend only on β, but not on the other parameters of the example (d and k). To measure the precision of signals, introduce $a = 1 - 1/\beta$ and note that when $a = 0$ the signals are uninformative as they have the same distribution in the two states. As a increases the signals become more precise, and in the limit when $a \to 1$ the signals in the low state converge to 0 in probability, and the signals in the high state converge to 1 in probability, that is, signals are perfectly informative in the limit. The value function is the same as in Example 1, that is $V = t + k\rho(\rho - 0.5\rho^2 - 0.5) + (1-\rho)(d - 0.5k\rho^2)$. The utility from taking the action that is optimal with belief ρ is $U_H(\rho) = k(\rho - 0.5\rho^2 - 0.5) + t$ if the state is high, and $U_L(\rho) = d - 0.5k\rho^2 + t$ if the state is low. The difference of the expected utility in the two states is $U_H(\rho) - U_L(\rho) = d + k\rho - 0.5$, so d can be interpreted as a payoff difference between the two states. A higher value of k means that information is more valuable, because the utility losses from taking a suboptimal action in the low state ($U_L(0) - U_L(\rho) = \frac{k\rho^2}{2}$) and in the high state ($U_H(1) - U_H(\rho) = k(\frac{1}{2} - \rho + \frac{\rho^2}{2})$) are both increasing in k. Using that $V(0) = U_L(0)$ and $V(1) = U_H(1)$, straightforward algebra shows that

$$CV = k \int_{\underline{s}}^{\overline{s}} g^{(2)}(s)(0.5\rho_{lose}^2(s) + 0.5\rho_{tie}(s) - \rho_{tie}(s)\rho_{lose}(s))ds. \tag{7}$$

[16] Formally, $\rho_{tie} = \frac{(g_H)^2}{(g_H)^2 + (g_L)^2}$ and $\rho_{lose} = \frac{g_H(1-G_H)}{g_H(1-G_H) + g_L(1-G_L)}$.

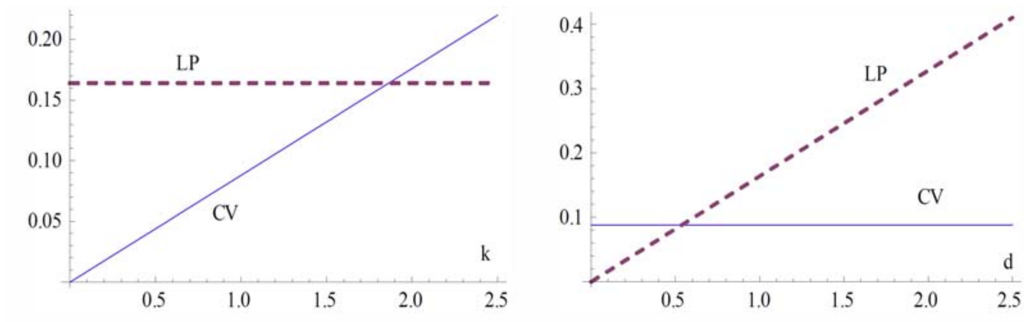

Figure 1: CV and LP as functions of the value of information k and the payoff difference between states d (with signal precision $\alpha = 0.4$, $d = 1$ and $\alpha = 0.4$, $k = 1$) in Example 2.

The fact that $d = V(0) - V(1)$ implies that the linkage effect can be written as

$$LP = d(0.5 - \int_{\underline{s}}^{\overline{s}} g^{(2)}(s)\rho_{tie}(s)ds). \tag{8}$$

The continuation value effect is increasing in k, since the larger the value of information is, the more it helps the losing bidders in the continuation problem. The linkage effect is increasing in d, since the greater the payoff differences between the states are, the more information helps to reveal the state in reducing bidders' rents. Figure 1 captures these comparative statics results by depicting the continuation value effect and linkage effect as a function of k and d.

Let us now provide general counterparts to the comparative statics result depicted in Figure 1. Consider an auction where the continuation value function is indexed by parameters δ and κ, while the signal distributions are indexed by a parameter α. The interpretation of these variables is the same as those of d, k and a in Example 2: δ measures the payoff differences between states, κ measures the value of information and α measures the precision of the signals of the bidders. In what follows we formalize these three measures for a general environment.

First, to measure the value of information in the post-auction market, let us introduce functions $W_L^\kappa(\rho), W_H^\kappa(\rho)$ that measure the baseline utility of an agent (in states L and H) who take the action optimal for belief ρ. Assume that $W_L^\kappa(\rho), W_H^\kappa(\rho)$ are decreasing in κ for all $\rho \in (0,1)$ and $W_H^\kappa(1) = W_L^\kappa(0) = t$, that is, full information payoffs are not affected by κ. Then the utility loss from not taking the optimal action in the high state, that is, $V(1) - W_H^\kappa(\rho) = W_H^\kappa(1) - W_H^\kappa(\rho)$, is increasing in κ for any $\rho < 1$, and a symmetric statement is true for the loss in the low state. Consequently, the value of information is increasing in κ. Second, to measure intrinsic payoff differences between states, let $U_L^\kappa(\rho) = \delta + W_L^\kappa(\rho)$, and $U_H^\kappa(\rho) = W_H^\kappa(\rho)$; that is, an increase in δ implies a boost of the utility in the low state that does not depend on the action taken, and leaves the utility in the high state unchanged. Finally, note that the value function induced by the above specification is

$$V_\delta^\kappa(\rho) = \rho U_H^\kappa(\rho) + (1-\rho)U_L^\kappa(\rho) = \delta(1-\rho) + \rho W_H^\kappa(\rho) + (1-\rho)W_L^\kappa(\rho), \tag{9}$$

where functions $W_H^\kappa(\rho)$, $W_L^\kappa(\rho)$ do not depend on δ.

We are ready to state our general comparative statics result as the value of information (κ) and the payoff differences between states (δ) change:

Proposition 3 ***(Incentives for Information-Revelation and the Shape of V.)*** *The continuation value effect is strictly increasing in the value of information κ, and is independent of the payoff difference between states δ. The linkage effect is strictly increasing in the payoff difference between states, and is independent of the value of information.*

Proof. First, (9) and the fact that $W_H^\kappa(1) = W_L^\kappa(0) = t$ imply that for any fixed κ it holds that $V_\delta^\kappa(0) - V_\delta^\kappa(1) = \delta + W_L^\kappa(0) - W_H^\kappa(1) = \delta$ and thus the linkage effect can be written as

$$LP = \delta \int_{\underline{s}}^{\overline{s}} g^{(2)}(s)\left(\rho_0 - \rho_{tie}(s)\right) ds, \tag{10}$$

which is clearly increasing in δ and is independent of κ. Second, by construction for all ρ, $V_\delta^\kappa(1) - U_H^\kappa(\rho_{lose}) = t - W_H^\kappa(\rho_{lose})$ and $V_\delta^\kappa(0) - U_L^\kappa(\rho_{lose}) = \delta + t - U_L^\kappa(\rho_{lose}) = t - W_L^\kappa(\rho)$ are increasing in κ and constant in δ, since functions W_H^κ and W_L^κ are constant in δ. Therefore, the continuation value effect

$$CV = \int_{\underline{s}}^{\overline{s}} g^{(2)}[\rho_{tie}(V_\delta^\kappa((1) - U_H^\kappa(\rho_{lose})) + (1-\rho_{tie})(V_\delta^\kappa(0) - U_L^\kappa(\rho_{lose}))]ds \tag{11}$$

is increasing in κ and independent in δ. **Q. E. D.**

The intuition for this result can also be gained from directly studying the bid functions. Note, that $b(s) = v + \rho(s,s)[U_L(\rho_{lose}(s,s)) - U_H(\rho_{lose}(s,s))] - U_L(\rho_{lose}(s,s))$. As we already observed, the linkage effect comes through the (expected) increase in $\rho(s,s)$ when a more transparent auction is used. Therefore, when the payoff difference between states $(U_L - U_H)$ is increased this effect is strengthened. Next, note that $b(s) = v - [\rho(s,s)U_H(\rho_{lose}(s,s)) + (1 - \rho(s,s))U_L(\rho_{lose}(s,s))]$, and that when the state is revealed the term in the bracket increases to $\rho(s,s)U_H(0) + (1-\rho(s,s))U_L(0)$. The increase of the term in the bracket is greater if information has greater value, therefore our continuation value effect is greater in this case.

Let us return to Example 2 and study the effect of signal precision (a) on revenues. Taking any d and k such that a monotone equilibrium exists (that is, $d \geq k/2$), let us vary

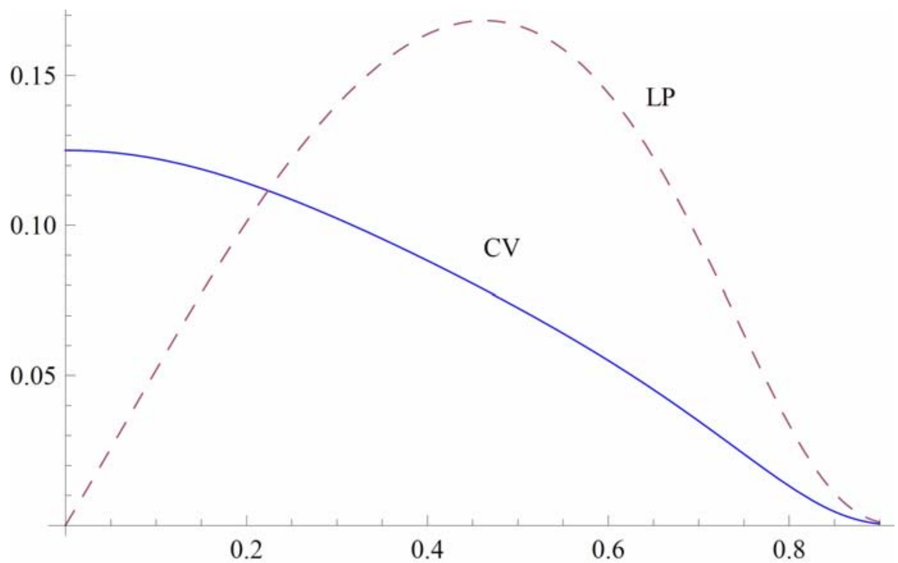

Figure 2: CV and LP as functions of signal precision α (with payoff difference $d = 1$, and value of information $k = 1$) in Example 2.

a ranging from full precision ($a = 1$) to being completely uninformative ($a = 0$). We depict the resulting graph in Figure 2.

If signals are uninformative, then the linkage principle effects is absent, because bidders know the value of the object, and thus their rents are competed away already. However, the continuation value effect is strong, because bidders value any extra information about the state a lot when they do not have precise information to begin with. When signals are perfectly informative both effects disappear, because the bidders know the state, so it does not make a difference whether the auctioneer reveals his information or not. More interestingly, when signals are *almost* perfectly informative the linkage principle dominates.[17] The figure shows that for some a^*

$$LP \geq CV \Longleftrightarrow a \geq a^* \in (0, 1),$$

that is, the linkage effect wins over if and only if signals are precise enough. Consequently, the auctioneer has incentives to reveal his information only if the bidders have precise information already when entering the auction.

Since the linkage effect tends to dominate when signals are precise, the auctioneer has more incentives to reveal the state when the bidders have fairly good signals on their own. On the flip side, when bidders are less well informed, the auctioneer prefers to hide information

[17] The reason is that when the losing bidders have very precise information about the state, any increased precision has only a second order effect on their continuation utilities.

to avoid increasing the continuation utilities of the bidders. This outcome is unfavorable to bidders, since the auctioneer hides his information exactly when the bidders need it the most to make better decision on the market unfolding after the auction. The implications of this effect for a fully dynamic model of trade are discussed in Section 5.2 where we also discuss its significance for the current literature on information percolation.

We now show that the above results hold more generally. Let the signal distribution functions G_H^α, G_L^α be parameterized by α and let $\rho^\alpha(s) = \frac{\rho_0 g_H^\alpha(s)}{\rho_0 g_H^\alpha(s)+(1-\rho_0)g_L^\alpha(s)}$ denote the posterior upon receiving signal s. We adopt the convention that signals are uninformative when $\alpha = 0$ holds, and perfectly informative when $\alpha = 1$. In particular, we assume that for all s it holds that $G_H^\alpha(s) = G_L^\alpha(s)$ when $\alpha = 0$, so each signal realization s has the same meaning. To capture that signals become perfectly informative when α is close to 1 we assume that for any $y > 0$ it holds that

$$\lim_{\alpha\to 1} \Pr(\rho^\alpha(s) \geq y \mid \omega = L) = \lim_{\alpha\to 0} \Pr(\rho^\alpha(s) \leq 1 - y \mid \omega = H) = 0.$$

Our next result shows that it remains true in the general case that hiding the state is revenue enhancing when bidders have uninformative signals:

Proposition 4 ***(Incentives for Information-Revelation when Signals are Uninformative.)*** *The continuation value effect is stronger than the linkage effect if signals are uninformative ($\alpha = 0$), and information has value, that is, V is not linear.*

Proof. When $\alpha = 0$ for all s it holds that $g_H^\alpha(s) = g_L^\alpha(s)$ and thus $\rho_{tie}^\alpha(s) = \rho_0$ holds[18]. Therefore, $\int_{\underline{s}}^{\overline{s}} g^{(2)}(s)\,(\rho_0 - \rho_{tie}(s))\,ds = 0$. This then implies that when signals are not informative then the linkage effect disappears, that is,

$$\alpha = 0 \Rightarrow LP = 0.$$

However, as long as V is not linear it holds for a set of beliefs $\rho \in (\underline{\rho}, \overline{\rho})$ that $V(1) - U_H(\rho)$, $V(0) - U_L(\rho) > 0$, and the continuation value effect is strictly positive (see formula (6). **Q.E.D.**

Again, this result can also be explained through inspection of the bid functions. When bidders do not have any informative signals, then the bid function without information revelation becomes $b(s) = v - [\rho_0 U_H(\rho_0) + (1 - \rho_0)U_L(\rho_0)]$, while with state revelation the expected value of the bid is $Eb = v - [\rho_0 U_H(1) + (1 - \rho_0)U_L(1)]$, which is smaller by construction.

[18] To see this, note that in this case $\rho^\alpha(s) = \frac{\rho_0 g_H^\alpha(s)}{\rho_0 g_H^\alpha(s)+(1-\rho_0)g_L^\alpha(s)} = \rho_0$, and $\rho_{tie}^\alpha(s) = \frac{\rho^\alpha(s)g_H^\alpha(s)(1-G_H^\alpha)^{N-2}}{\rho^\alpha(s)g_H^\alpha(s)(1-G_H^\alpha)^{N-2}+(1-\rho^\alpha(s))g_L^\alpha(s)(1-G_L^\alpha)^{N-2}} = \rho_0$ for all s, since $G_H^\alpha = G_L^\alpha$ also holds.

Finally, we would like to confirm that the linkage effect dominates the continuation value effect when signals become almost perfectly informative in the general environment:
Assumption CR: *There exists* $T > 0$ *and* $\widehat{\varepsilon} > 0$ *such that for all* $0 < \varepsilon \leq \widehat{\varepsilon}$

$$\frac{\lim\limits_{\alpha \to 1} \int_{s:\rho(s) \geq 1-\varepsilon} g^{(2)}(s)(1-\rho(s))^2 ds}{\lim\limits_{\alpha \to 1} \int_{s:\rho(s) \leq \varepsilon} g^{(2)}(s)\rho(s) ds} \leq T.$$

To interpret this assumption, imagine that for any fixed α the signals are distributed symmetrically in the sense that $\Pr(\rho(s) \leq t \mid L) = \Pr(\rho(s) \geq 1 - t \mid H)$ for any $t \in [0, 1]$. In this case the relevant ratio converges to 0, as it can be shown.[19] Indeed, the only way to violate Assumption CR is to assume that signals are much more precise in the low state than in the high state in the limit. At the end of the online Appendix we consider such an example, and show that the conclusion of our Proposition below fails.

Proposition 5 ***(Incentives for Information-Revelation when Signals are very Informative.)*** *Under Assumption CR the continuation value effect is weaker than the linkage effect if the signals are informative, that is when* $\alpha \in (\widehat{\alpha}, 1)$ *for some* $\widehat{\alpha} < 1$.

Proof. See the online Appendix.

As the signals become almost perfectly informative the (integrand of the) linkage effect (see formula (5)) dominates the (integrand of the) continuation value effect (see formula (6)) except for signals which indicate that the high state is very likely. Moreover, the continuation value effect is only relevant even for such signals if the precision of such signals are not too high in the limit, otherwise upon receiving a high signal, one is almost sure that it is the high state and thus the state revelation by the auctioneer would not change actions in the continuation problem by much. Therefore, if the signal precision converges not much slower in the high state than in the low state (an equal rate is more than sufficient), then the linkage effect is stronger than the continuation value effect.

To show that our results do not depend on our assumption that the auctioneer's signal is fully informative, we study the case where the auctioneer's signal is noisy. This also allows studying the case where the auctioneer can choose the precision of the signal he wishes to reveal. In the online Appendix we provide the general framework, analysis, and conduct

[19] The key idea is that the ratio $\frac{\lim_{\alpha \to 1} \int_{s:\rho(s) \geq 1-\varepsilon} g^{(2)}(s)(1-\rho(s)) ds}{\lim_{\alpha \to 1} \int_{s:\rho(s) \leq \varepsilon} g^{(2)}(s)\rho(s) ds}$ (that is, omitting the square sign from the numerator) is equal to 1 for any α, as the two states are completely symmetric. On the other hand, $\lim\limits_{\alpha \to 1} \frac{(1-\rho(s))^2}{1-\rho(s)} = 0$ if it is known that $\rho(s) \geq 1 - \varepsilon > 0$, since in this case the high state is very likely and thus the posterior converges to 1 in probability.

some numerical calculations. Here, we report our findings for the special case where the auctioneer receives one of two possible signals s_H or s_L. Moreover, let $\Pr[s_H \mid H] = \Pr[s_L \mid L] = z \in [0.5, 1]$, that is, z measures the precision of the auctioneer's signal, which is common knowledge. While few general results are available about the behavior of the two effects (CV and LP) when signals are imperfectly precise, two observations still hold. First, when bidders have uninformative signals ($\alpha = 0$), the LP effect is still zero, while $CV > 0$ so revealing information (even not fully precise information) hurts the revenues of the auctioneer. Second, when the bidders are fully informed ($\alpha = 1$), by construction $LP = CV = 0$.

To obtain additional insight, we study Example 2 in the online Appendix with $k = d = 1$ and allow precision z to vary between 0.5 and 1. In Figure 3 we depict the two effects for a fixed level of precision z, and varying α between 0 and 1. The picture is qualitatively similar to the full precision case: the auctioneer reveals information if and only if the bidders have precise private signals. We obtain several further observations. First, for any fixed level of the signal precision of the bidders (α) if an auctioneer wants to reveal a signal with precision z, then he also wants to reveal any signal with higher precision. Consequently, the auctioneer has no incentive to garble his signal. Suppose that he observes the state fully ($z = 1$), but can commit to announce a signal with an arbitrary precision. We show the auctioneer would always choose to reveal his fully informative signal or no signal at all, but it is never revenue maximizing to choose a signal with intermediate precision.[20] Second, for some parameter values, revealing the fully informative signal is revenue enhancing compared to not revealing any signal, but revealing a partially informative signal hurts the auctioneer. Investigating the non-monotonicity of the revenue impact in the precision of the auctioneer's signal is an interesting question left to future research.

3.3 Revenue when the winning bid is revealed

Another important question in auction design is whether any *bid* information should be revealed by the auctioneer. In settings in which the auctioneer does not possess information superior to the bidder's information, one may ask whether the auctioneer prefers information exchange between bidders by revealing information about the bids. In the previous Section we showed that revealing the state after the auction decreases revenues. The argument there relied on the fact that he linkage effect is absent when information is revealed after the auction, while the continuation value effect is present. This suggests that revealing any

[20] In this paper we compare the information structures of complete revelation and no revelation at all. We also show that the seller should never garble his information. However, we do not consider asymmetric policies that reveal signals with different precision for different bidders. The usefulness of such signals was shown by Mares and Harstad (2003).

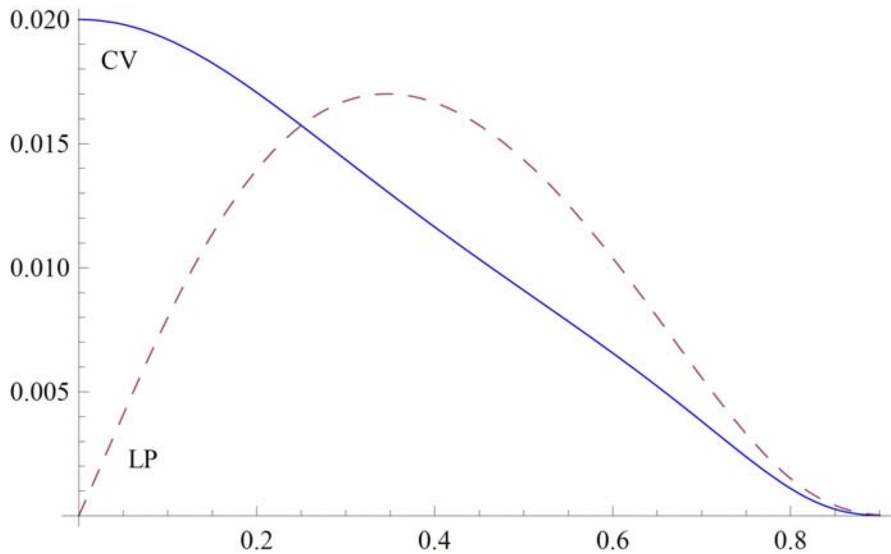

Figure 3: CV and LP as signal precision α varies and $z = 0.7$ in Example 2.

information after the auction should hurt the revenues of the auctioneer. We show that this is indeed the case when the winning bid is revealed by the auctioneer. Let ER^{Bid} denote the expected revenue when the winning bid is revealed, assuming that V is decreasing.

Proposition 6 *Revealing the winning bid after the auction decreases revenue relative to not revealing any information,* $ER^{Bid} < ER^{None}$.

Proof. First, we prove that the (static) interdependent value auction with value function $\widetilde{v}(s, y) = v - [\rho(s, y)U_H(\rho(s, y)) + (1 - \rho(s, y))U_L(\rho(s, y))]$ yields the same equilibrium bid function as our auction with bid revelation. The profit of a bidder with signal s who bids $b(y)$ can be written as

$$\pi_{bid}(s, y) = \int_0^y (v - b(t))h_s(t)dt + \int_y^1 (\rho(s, t)U_H(\rho(s, t)) + (1 - \rho(s, t))U_L(\rho(s, t))h_s(t)dt,$$

because now if a bidder loses against someone with signal t, then he in fact learns that, and behaves accordingly in the secondary market. Taking now the derivative with respect to y yields

$$\pi_{bid}^{(2)}(s, y) = h_s(y)[v - (\rho(s, y)U_H(\rho(s, t)) + (1 - \rho(s, y))U_L(\rho(s, y)) - b(y)] = h_s(y)[\widetilde{v}(s, y) - b(y)].$$

The profit function in the standard interdependent values auction with value function $\widetilde{v}(s, y)$ is written as

$$\widetilde{\pi}_{bid}(s, y) = \int_0^y (\widetilde{v}(s, y) - b(t))h_s(t)dt,$$

which has the same derivative with respect to y as is the same as $\pi_{bid}(s, y)$. Therefore, the two profit functions give rise to the same equilibrium bid functions.

Now, let us compare the equilibrium bid functions with and without bid revelation using that they are the same as the equilibrium bid functions in the corresponding interdependent value auctions. The incentive condition on the post-auction market implies that $\widetilde{v}(s, y) < v(s, y)$, since $x \in \arg\max_z xU_H(z) + (1-x)U_L(z)$. Therefore, the bid when the winning bid is revealed is

$$b_b(s) = \widetilde{v}(s, s) < v(s, s) = b(s),$$

the bid without such revelation. So, revealing the bids after the auction decreases revenues. **Q.E.D.**

This result is similar to that of Perry and Reny (1999) who show that in a multi-unit (Vickrey) auction revealing information can decrease the bid of all bidders. We obtain the same result as them, although for different reasons.

4 First-price, second-price and English auctions

In the auction design literature, the linkage principle implies that the more an auction format links the payments to the types of the other agents, the higher the expected revenue is. In a first-price auction the expected payment conditional on winning does not depend on the types of the other bidders, while in a second-price auction and in an English auction it does. Therefore, the linkage principle implies that the second-price and English auctions yield higher expected revenues than the first-price auction. Similarly, an English auction links payments to others' bids (types) even more, since all the types except for the two highest are revealed by the bids at which those bidders are dropping out. Therefore, the English auction yields higher expected revenue than the second-price auction.[21]

In our model with endogenous outside options the linkage effect is counteracted by the continuation value effect, when we analyze whether the auctioneer should reveal his exogenous information about the state of the world. It is natural to ask whether the same is true when one compares the three standard auction formats fixing the information policy. Assume that the auctioneer does not reveal any information and runs a first-price, second-price or English auction. First, we observe that the second-price auction still revenue dominates the first-price auction. The key is that in both auctions the losers learn the same information;

[21] This argument relies on the effect of information revelation on equilibrium rents, as the equilibrium allocation in the standard setup is not affected by the information policy. This is not the case in Board (2009), where the information policy's effect on equilibrium allocation modifies the insights of the linkage principle.

they only learn that there was a bidder with a higher signal than theirs. This implies that they take the same actions in the continuation decision problems, and therefore the presence of endogenous outside options does not change the comparison between the two formats. Formally, in the online Appendix we show that our first-price auction has the same equilibrium as a standard interdependent value first-price auction with value function $v(s, y)$ (as defined in (2)). But we already observed before that the same holds true for our second-price auction with the same constructed value function $v(s, y)$. Therefore, since it is known that the in such interdependent auction the second-price format yields higher revenue, it must be the case for our auctions too.

The important novelty when comparing second-price and English auctions is that the English auction reveals more information, so it allows the losers to take better decisions in the aftermarket, and thus the continuation value effect favors the second-price auction over the English auction. Since the continuation value and the linkage effects work in opposite directions, one needs to assess whether the second-price or the English auction raises higher revenues. To present our result, we concentrate on a three bidder example, where a monotone equilibrium exists and the auctioneers' revenue is higher in the second-price auction than in the ascending auction.

Proposition 7 *The expected revenue of the English auction is sometimes less than the expected revenue of the second-price auction.*[22]

In this specification the continuation value effect is stronger than the linkage effect, and revealing information via holding a more open auction decreases revenues. If one considers variations in the parameter values, then the revenue comparison has the same qualitative features as in the case of state revelation. For example, if the two states provide very different utility values (that is d is high), then the linkage effect dominates, and the auctioneer prefers the English auction. If α_H decreases or α_L increases, then information is more valuable, which favors the less transparent second-price auction.

[22] As proven in the online Appendix, this happens if $\rho_0 = 1/2$, $N = 3$, $g_L = 2(1-s)$ and $g_H = 2s$, and

$$U_H(\rho) = \rho^{n-1} - \alpha_H \rho^n (n-1),$$

and

$$U_L(\rho) = d - \alpha_L \rho^n (n-1)$$

with $\alpha_H = \alpha_L = 1/n$, $n = 1.1$, $d = 1$.

5 Discussion

5.1 Some robustness considerations

A monotone equilibrium does not always exist in our model. To discuss how our revenue comparison results change when a monotone equilibrium does not exist, let us revisit Example 2 and assume that the signals of the bidders are precise, that is, $a \geq \overline{a}$ for some appropriate value of $\overline{a}$. Then any monotone equilibrium is such that the linkage effect is stronger than the continuation value effect, regardless of the values of k and d,[23] which suggests that the linkage effect tends to be stronger. However, as the next Example shows, this is an artifact of the restrictions imposed on k and d needed for a monotone equilibrium to exist. In other words, one can expect that when a monotone equilibrium does not exist, then the continuation value effect tends to be strong relative to the linkage effect.

Example 3:

Consider an example where the intrinsic payoff differences between the two states are completely absent, but it is very important to *know* the state when making the price offer. Let $v = 2$, $\mu_H = \mu_L = 1$ and $F_H(x) = \frac{1-a(1-x)^2}{2-x}$ and $F_L(x) = \frac{1-ax^2}{2-x}$ with $a \in (0, 0.5]$. Sellers have mostly intermediate costs in the high state and more extreme costs in the low state; as a result, the revenue maximizing prices are very different in the two states.[24] Since the cost distributions F_H, F_L are not ranked by first order stochastic dominance, the induced value function may not be monotone in the belief. In fact, the two states are symmetric in that $V(\rho) = V(1-\rho)$ for all $\rho \in [0,1]$, and the value function is U-shaped. The resulting utilities in the two states are $U_H(\rho) = 1-a(1-\rho)^2$, and $U_L(\rho) = 1-a\rho^2$. To fully specify the example, assume that $\rho_0 = 1/2$, $N = 2$, and $G_H(s) = s^2$, $G_L(s) = 2s - s^2$. Since the two states are symmetric, we concentrate on a "state-symmetric" equilibrium where $b(s) = b(1-s)$ for all s. The posterior upon tieing is $\widetilde{\rho}_{tie}(s) = \Pr(H \mid s_1 = s, s_2 = s \text{ or } s_2 = 1-s) = s$, and the posterior upon losing is $\widetilde{\rho}_{lose}(s) = \Pr(H \mid s_1 = s, s_2 \in (s, 1-s)) = s$. When the state is not revealed the equilibrium bid function is $b = v - [\widetilde{\rho}_{tie} U_H(\widetilde{\rho}_{lose}) + (1-\widetilde{\rho}_{tie}) U_L(\widetilde{\rho}_{lose})] = v - 1 + as(1-s)$. The bid with state revelation is $b = v - V(0) = v - 1$ in the low state, and $b = v - V(1) = v - 1$ in the high state as well. Consequently, the revenue comparison favors

[23] To see this, note that a monotone equilibrium exists if $d \geq k/2$, so to be able to choose d, k such that a monotone equilibrium exists *and* $CV > LP$ it has to hold by (7) and (8) that $r = \frac{\int_0^1 g^{(2)}(0.5\rho_{lose}^2 + 0.5\rho_{tie} - \rho_{tie}\rho_{lose})ds}{0.5 - \int_0^1 g^{(2)}\rho_{tie} ds} > \frac{1}{2}$. However, numerical calculations show that $r \leq 1/2$ when bidders have precise signals (that is, $a \geq \overline{a}$).

[24] The cost is 0 with probability $\frac{1}{2}$ ($\frac{1-a}{2}$) in the low (high) state, while the cost is less than 1 with probability 1 $(1-a)$ in the high (low) state. In the low state the cost is prohibitively high with probability a. In the high state a high price ($p = 1$) is optimal to capture all the sellers, but in the low state the most profitable offer is $p = 0$, since that already captures a large portion of the market.

not revealing the state. In general, if the two states are similar (that is $V(0) = V(1)$), then the linkage principle loses its bite, and although a monotone equilibrium does not exist, it follows that revealing the state decreases revenues.

The assumption of two states can be relaxed without changing the results as we discuss next. We focus on comparing the revenues from the second-price auction with and without the revelation of the winning bid, the question addressed in Section 3.3 for the case of two states. Let $t \in [0, 1]$ denote the state of the world, and let $u_t(a)$ denote the continuation value when action a is taken in state t. Let g_t denote the conditional distribution of signals in state t, and let h the density function for the state of the world. Assuming that for all $u_t(a)$ is decreasing in t implies that there is an equilibrium with monotone bidding. Let $\rho^t_{tie}(s)$ be the density of state t if one ties at the top with signal s, and $\rho^t_{lose}(s)$ be the density of state t if one lost with signal s.[25] Then the optimal action after losing with signal s satisfies $a(s) = \arg\max_{x \in X} \int_{\underline{s}}^{\overline{s}} \rho^t_{lose}(s) u_t(x) dt$. Without bid revelation the equilibrium bid is $b_n(s) = v - \int_{\underline{s}}^{\overline{s}} \rho^t_{tie}(s) u_t(a(s)) dt$. With bid revelation the tieing loser learns that he in fact tied with the winner and takes an action

$$a^*(s) = \arg\max_{x \in X} \int_{\underline{s}}^{\overline{s}} \rho^t_{tie}(s) u_t(x) dt. \tag{12}$$

Therefore, the equilibrium bid becomes $b_y(s) = v - \int_{\underline{s}}^{\overline{s}} \rho^t_{tie}(s) u_t(a^*(s)) dt$. By (12) it follows that $b_n(s) > b_y(s)$, so the revenue comparison is as in the two-state model.

The winning bidder may take an action in many situations of interest. Suppose that the winning bidder's continuation utility functions are U^w_H, U^w_L, which have similar properties to U_H, U_L, the continuation utility of the losers. We keep the assumption that the winner obtains a utility v from the object and that there are two states. Let us now concentrate on the question whether in the second-price format state revelation enhances or reduces revenues; the other questions can be studied similarly. Following similar argument as in the benchmark case, the equilibrium bid function without state revelation is

$$v + [\rho_{tie}(s)(U^w_H(\rho_{lose}(s)) - U_H(\rho_{lose}(s))) + (1 - \rho_{tie}(s))(U^w_L(\rho_{lose}(s) - U_L(\rho_{lose}(s))].$$

When the state is revealed, in the high state all bidders bid $v + V^w(1) - V(1)$, and in the low state all bid $v + V^w(0) - V(0)$. From these observations it is obvious that if the winner's continuation values are not very sensitive to the state of the world (that is $U^w_H - U^w_L$ is uniformly close to zero and thus V^w is close to being a constant function), then the revenue

[25] Formally, $\rho^t_{tie}(s) = \frac{h(t) g^2_t(s) G^{N-2}_t(s)}{\int_0^1 h(z) g^2_z(s) G^{N-2}_z(s) dz}$ and $\rho^t_{lose}(s) = \frac{h(t) g_t(s)(1 - G^{N-1}_t(s))}{\int_0^1 h(z) g_z(s)(1 - G^{N-1}_z(s)) dz}$.

comparison is similar to the benchmark case where the winner's continuation problem was omitted. However, if the winner's continuation problem is important (compared to the losers'), then the continuation value effect also *favors* information revelation[26], and thus transparent auctions are always revenue enhancing.

Risk averse bidders in our framework lead to qualitatively similar results as our risk neutral benchmark. With risk averse bidders the revenue comparisons between the three formats (first-price -, second-price -, and English auctions) are ambiguous even in the standard interdependent value setup of Milgrom and Weber (1982). Therefore, we only compare revenues between the second-price auction with and without bid revelation.[27] Using similar considerations, as in the case of risk neutral bidders, one can calculate the equilibrium bid functions in a straightforward manner. Letting m denote the concave utility function, one can show that the equilibrium bid function without bid revelation b_n solves

$$\rho_{tie}U_H(\rho_{lose}) + (1-\rho_{tie})U_L(\rho_{lose}) = m(v-b_n),$$

while if the winning bid is revealed, then the equilibrium b_b bid function becomes

$$\rho_{tie}U_H(\rho_{tie}) + (1-\rho_{tie})U_L(\rho_{tie}) = m(v-b_b).$$

For the same reason as when bidders were risk neutral it still holds that $\rho_{tie}U_H(\rho_{lose}) + (1-\rho_{tie})U_L(\rho_{lose}) < \rho_{tie}U_H(\rho_{tie}) + (1-\rho_{tie})U_L(\rho_{tie})$, and thus $b_b < b_n$ follows, which implies that revealing the winning bid decreases revenues.

5.2 Relationship to dynamic models of trade

Let us describe the fully structural infinite horizon model of Lauermann, Merzyn and Virag (2009), whose continuation value function fits in our framework in a straightforward way. There are a continuum of buyers and a continuum of sellers present in the market. In periods $t \in \{..., -1, 0, 1, ...\}$, traders exchange an indivisible, homogeneous good. Similar to Wolinsky (1990), there are two equally likely states of nature, a high state and a low state $\omega \in \{H, L\}$. The realized state of nature is fixed throughout and unknown to the traders. The state of nature determines the constant and exogenous number of new traders born each period (the flow). We consider steady state equilibria which induce for each state a constant and endogenous number of traders in the market (the stock).

[26] If more information is available after the auction, then the winner can make a better decision, which then makes bidders more aggressive since the winning prize has become more valuable.

[27] If the seller considers revealing the *state*, then similar trade-offs apply as when the bidders were risk neutral, but the calculations become more involved.

Each period consists of several steps:

1. Entry occurs (the *"inflow"*): A mass one of sellers and a mass d^w of buyers is born, with buyers observing signals as described before. The mass of sellers who are born each period is the same in both states and is equal to one. Each buyer enters the market with a signal $s \in [\underline{s}, \overline{s}]$. The structure of the signals is the same as in the main text.

2. Each buyer from the market (the *"stock"*) is randomly matched with one seller. A seller is matched with a random number of buyers. The probability that a seller is matched with $k = 0, 1, 2, ...$ buyers is Poisson distributed and equal to $e^{-\mu}\mu^k/k!$, where μ is the endogenous ratio of the mass of buyers to the mass of sellers in the stock.

3. Buyers do not observe how many other buyers are matched with the same seller. Each seller runs a sealed bid second price auction with no reserve price. The bids are not revealed ex post, so bidders learn only whether or not they have won with their submitted bid. Trading at price p yields payoffs $v - p$ and $p - c$ for a buyer or a seller, respectively (where c is the common cost of selling).

4. A seller leaves the market if he sold his good. Otherwise, he stays with probability $\delta \in [0, 1)$ in the stock to offer his good in the next period. A winning buyer pays the second highest bid, obtains the good and leaves the market. A losing buyer stays in the stock with probability δ and is matched with another seller in the next period. Those who do not stay, exit the market permanently. A trader who exits the market without trading has zero payoffs.

5. Upon losing, the remaining buyers update their beliefs, based on the information gained from losing with their submitted bid. The remaining agents who did neither trade nor exit stay in the market. Together with the inflow, these traders make up the stock for the next period.

Lauermann, Merzyn and Virag (2009) study the unique stationary equilibrium of the above model. The mechanics of the equilibrium is simple: losing signals that there were many rivals, and thus it is more likely that the state is high. Therefore, upon losing, a bidder realizes that the continuation values are lower than he anticipated, and thus he needs to increase his bid next period. Formally, let $V(\theta)$ denote the lifetime utility of a buyer in a stationary equilibrium who has belief θ in a given period. The article proves that V is strictly decreasing in θ, that is, as the probability attached to high state (θ) increases the bidder becomes more pessimistic about the continuation payoffs.

Crucially, the continuation value function V is derived from an infinite horizon economy, but it has all the properties we assumed in our two-stage reduced form framework. Most importantly, V is convex, and thus similar considerations apply to it as to the value function of our two-stage model. Consequently, one can show that similar considerations apply to revenues as well. In particular, Lauermann, Merzyn and Virag (2009) show that an individual seller would not like to deviate to a policy where he announces the winning bid. This is similar to our result that revealing information after the auction decreases revenues. Also, numerical simulations show that revealing information about the state before the auction may decrease or increase revenues depending on model parameters.

The previous discussion shows that many of our insights could have been presented in a fully structural, dynamic model of trade. However, our current reduced form approach has marked advantages. Most importantly, the analysis in the current article is simpler and more transparent because the given (exogenous) value function and the distribution of beliefs contain all the relevant information for the auctioneer's design problem, and we can do direct comparative statics with respect to these objects. On the other hand, it would be interesting to map conditions on the precision of signals and on the convexity of the value function into conditions on the physical environment. However, providing comparative static results that are similar to the ones presented here are unlikely to be analytically tractable in a dynamic equilibrium model. A particular problem is that the distribution of beliefs would become an equilibrium object and characterizing how the endogenous distribution of beliefs varies with exogenous parameters is a difficult task. For this reason we had to resort to numerical simulations in the dynamic model.

Our results indicate directions for future research on "information percolation". That literature was initiated by Duffie and Manso (2007), and Duffie et. al. (2009) and studies a model where traders learn about aggregate market conditions in a dynamic matching market, which is interpreted as a decentralized over-the-counter asset market. Agents meet in small groups and exchange information each period. They study how fast information spreads under the assumption that a particular auction mechanism is used in which buyers can perfectly infer each others' beliefs. A natural question is whether or not such transparent mechanisms would arise endogenously in a decentralized market place. Our model provides a starting point for analyzing this question.[28] Our results indicate (see Proposition 5) that unless bidders arrive at the transaction place with precise information about the market already, they may not be able to learn the state quickly, since the auctioneer has no incentive to reveal it to them. In other words, information might "percolate" fast in a market with

[28] Our model is only a reduced form description of dynamic trade, so the qualifications mentioned in the previous paragraph still apply.

endogenously determined trading institutions only if it was learned reasonably well already before bidders arrive on the market. Interestingly, real world over-the-counter asset markets do not always favor information percolation. In fact, the opacity of such markets raised regulatory concerns, motivating recent regulatory interventions to increase their transparency.[29] Our analysis suggests a possible explanation why opaque trading institutions may develop in decentralized markets. Moreover, the auctioneer hides information exactly when the value of information in the continuation problem is highest for buyers; see Proposition 3. Therefore, in a dynamic interaction information may spread very slowly exactly when information is most valuable. According to the empirical evidence of Besseminder and Maxwell (2008) such opacity may come at a significant welfare cost for the traders.

6 Conclusion

We study auctions with endogenous outside options determined through actions taken in the aftermarket. In contrast to the case of exogenous outside options, auctions with less information revelation may yield higher revenues. Opaque auctions decrease the information available to losing bidders, which leads to worse decisions in the aftermarket. This leads to worse outside options, and thus more aggressive bidding in the original auction. Effects that favor non-transparent auctions include a small payoff difference between the two states, a great value of information in the continuation problem, and imprecise signals of the bidders. The timing of information revelation is important: it is never optimal to reveal information after the auction, while it may be optimal to reveal information before the auction. We also show that a less transparent auction format, the second-price auction can yield higher revenues than an English auction, as the English auction fosters less learning and provides lower continuation values for the bidders. The model is robust to introducing several states, and with respect to the winner having state dependent continuation values.

[29] The over-the-counter market for corporate bond was traditionally opaque. In 2002, the market underwent a fundamental change when the Transaction Reporting and Compliance Engine (TRACE) was introduced; see Bessembinder and Maxwell (2008).

7 Appendix

First, we prove Proposition 1, and several useful results about the continuation problem. We start by establishing a monotonicity result:

Lemma 1 *The Lemma contains several statements:*

i) For all $\rho' > \rho$ and $u_H(a') - u_L(a') > u_H(a) - u_L(a)$ it holds that if type ρ weakly prefers a' over a, then type ρ' strictly prefers a' over a. Therefore, for all $b' \in \alpha(\rho')$ and $b \in \alpha(\rho)$ it holds that $u_H(b') - u_L(b') \geq u_H(b) - u_L(b)$.

ii) If for some $\rho' > \rho$ it holds that $e \in \alpha(\rho')$ and $f \in \alpha(\rho)$, then $u_H(e) \geq u_H(f)$ and $u_L(e) \leq u_L(f)$; and for almost all $\rho \in [0,1]$ if $c, d \in \alpha(\rho)$ then $u_H(c) = u_H(d)$ and $u_L(c) = u_L(d)$.

iii) Function $U_H(\rho) = \max\limits_{x \in \alpha(\rho)} u_H(x)$ is monotone increasing in ρ, while $U_L(\rho) = \min\limits_{x \in \alpha(\rho)} u_L(x)$ is monotone decreasing in ρ.

Proof of Lemma 1:

Proof. Suppose that $\rho' > \rho$ and $u_H(a') - u_L(a') > u_H(a) - u_L(a)$, and type ρ weakly prefers a' over a, that is

$$\rho u_H(a') + (1-\rho) u_L(a') \geq \rho u_H(a) + (1-\rho) u_L(a). \tag{13}$$

Then $u_H(a') - u_L(a') \geq u_H(a) - u_L(a)$ implies that

$$(\rho' - \rho)\left(u_H(a') - u_L(a')\right) > (\rho' - \rho)\left(u_H(a) - u_L(a)\right).$$

Adding the last inequality to (13) implies that

$$\rho' u_H(a') + (1-\rho') u_L(a') > \rho' u_H(a) + (1-\rho') u_L(a),$$

which establishes the first claim. To prove the second part of i), suppose that $u_H(b) - u_L(b) > u_H(b') - u_L(b')$. Then the first part of i) implies that type ρ' strictly prefers b over b', which contradicts with the assumption that $b' \in \alpha(\rho')$. The second part of i) implies that $u_H(e) - u_L(e) \geq u_H(f) - u_L(f)$. Then $u_H(e) < u_H(f) \Longrightarrow u_L(e) < u_L(f)$, which implies that e is worse than f for any beliefs, and thus $e \in \alpha(\rho')$ could not hold. This contradiction establishes the first claim in ii). To prove the second claim in ii), let $\tau(\rho) = \max\limits_{x \in \alpha(\rho)} u_H(x) - u_L(x)$. The second claim in i) implies that τ is weakly increasing, and thus it is almost everywhere continuous. Moreover, at every continuity point ρ of τ

it holds that for all $c, d \in \alpha(\rho)$, $u_H(c) - u_L(c) = u_H(d) - u_L(d)$.[30] Then suppose that $u_H(c) > u_H(d)$. In this case, it would follow that $u_L(c) > u_L(d)$, implying that c dominates d and contradicting $d \in \alpha(\rho)$. This contradiction establishes the second result in ii). The monotonicity claim (result iii)) about U_H, U_L then follows by construction. **Q.E.D.**

Next we prove that a useful envelope condition holds for almost all ρ. In particular they hold at every continuity point of U_H, U_L. Let us formally state our claim first:

Lemma 2 *For almost all ρ it holds that*

$$a \in \alpha(\rho) \Longrightarrow V'(\rho) = u_H(a) - u_L(a) \tag{14}$$

and

$$V'(\rho) = U_H(\rho) - U_L(\rho). \tag{15}$$

Proof. Take any ρ and let $a \in \arg\max_{x\in\alpha(\rho)} u_H(x)$, $b \in \arg\min_{x\in\alpha(\rho)} u_H(x)$. Note, that by definition of $\alpha(\rho)$ it must hold that $a \in \arg\min_{x\in\alpha(\rho)} u_L(x)$, $b \in \arg\max_{x\in\alpha(\rho)} u_L(x)$ and thus for all $x \in \alpha(\rho)$

$$u_H(a) - u_L(a) \geq u_H(x) - u_L(x) \geq u_H(b) - u_L(b).$$

Next, note that for all $\rho' > p$ it holds that $V(\rho') \geq U(\rho', a)$. Therefore,

$$V(\rho') - V(\rho) \geq (\rho' - \rho)(u_H(a) - u_L(a)).$$

Also, the right hand derivative of a convex function exists everywhere, therefore the right hand derivative at ρ satisfies

$$V'_+(\rho) \geq u_H(a) - u_L(a).$$

A similar argument yields that the left hand derivative satisfies

$$V'_-(\rho) \leq u_H(b) - u_L(b).$$

Since $V'(\rho)$ exists almost everywhere, therefore for almost all ρ it must hold for all $a, b \in \alpha(\rho)$ that $u_H(a) - u_L(a) = u_H(b) - u_L(b)$. Therefore, wherever a derivative exists (which is almost everywhere) it holds that $V'(\rho) = u_H(a) - u_L(a)$ for all $a \in \alpha(\rho)$, which establishes that

[30] Suppose that $c, d \in \alpha(\rho)$, and $U_H(c) - U_L(c) > U_H(d) - U_L(d)$. Then for all $\rho' > \rho$ it holds by the second claim that for any $c' \in \alpha(\rho')$, $U_H(c') - U_L(c') \geq U_H(c) - U_L(c)$, and thus $\tau(\rho') \geq U_H(c) - U_L(c)$. Similarly, for any ρ'' and $d' \in \alpha(\rho'')$, $U_H(d') - U_L(d') \leq U_H(d) - U_L(d)$, and thus $\tau(\rho'') \leq U_H(d) - U_L(d)$. Therefore, the function τ must have a jump at such a ρ.

(14) for almost all ρ. To establish that (*(15)* holds for all continuity points of U_H, U_L (which is almost everywhere) note that the proof of Lemma 1 implies that for any such continuity point ρ it holds that if $c, d \in \alpha(\rho)$ then $u_H(c) = u_H(d)$ and $u_L(c) = u_L(d)$. Therefore, the argument establishing (*14*) applies to show that $V'(\rho) = u_H(c) - u_L(c) = U_H(\rho) - U_L(\rho)$, concluding the proof. **Q.E.D.**

To relate our work to static interdependent value auctions, we choose an indirect proof of Proposition 1 that relies on constructing an appropriate interdependent value auction first.

Proof of Proposition 1:

Proof: First, we establish that the common value auction with value function $v(s, y)$ (as introduced at the end of Section 3.1) and our auction provide the same bidding incentives for the bidders. Let y be the highest type of bidders other than i, and let $\rho(s, x) = \Pr(H \mid s, y = x)$. Also, let $\rho_{lose}(s, x) = \Pr(s, y \geq x)$ denote the probability of the high state conditional on losing with a signal s, and the highest other signal being at least x. Finally, let $h_s(y)$ denote the density of the highest type among the other bidders if one has signal s.[31] Suppose that a bidder with signal s bids $b(x)$. Given any symmetric profile of strictly increasing bidding strategies b, his expected utility is

$$\pi(s, x) = \int_0^x (v - b(y))h_s(y)dy + \int_x^1 (\rho(s, y)U_H(\rho_{lose}(s, x)) + (1 - \rho(s, y))U_L(\rho_{lose}(s, x))h_s(y)dy.$$

To explain this formula, imagine that the highest type among the other bidders is $y < x$. Then the bidder wins and obtains a utility of $v - b(y)$. If $y > x$, then the bidder loses. In this case the high state has probability $\rho(s, y)$, but the bidder knows only that he lost that is the highest other type is greater than x. Therefore, the bidder's belief is $\rho_{lose}(s, x)$, and takes his action in the post-auction market accordingly.

Next, recall that in state $w \in \{L, H\}$, $U_w(\rho)$ is the payoff of a buyer who takes the optimal action when the probability of the high state is ρ. Therefore, it holds for all ρ and ρ' that

$$\rho U_H(\rho) + (1 - \rho)U_L(\rho) \geq \rho U_H(\rho') + (1 - \rho)U_L(\rho'). \qquad (16)$$

[31] The posterior upon having signal s and knowing that the highest other type is y is $\rho(s, y) = \frac{\rho_0 g_H(s) g_H(y) G_H^{N-2}(y)}{\rho_0 g_H(s) g_H(y) G_H^{N-2}(y) + (1-\rho_0) g_L(s) g_L(y) G_L^{N-2}(y)}$. The posterior upon losing is $\rho_{lose}(s, y) = \frac{\rho_0 g_H(s)(1 - G_H^{N-1}(y))}{\rho_0 g_H(s)(1 - G_H^{N-1}(y)) + (1-\rho_0) g_L(s)(1 - G_L^{N-1}(y))}$. Finally, the distribution function of the first order statics from $n - 1$ i.i.d. draws is $H_s(y) = \rho(s)G_H^{n-1}(y) + (1 - \rho(s))G_L^{n-1}(y)$, with $h_s(y) = \partial H_s / \partial y$.

In particular, take the belief upon losing with bid $b(x)$ having received signal s, that is set $\rho = \rho_{lose}(s,x) = \int_x^1 \rho(s,t)h_s(t)dt$. Note, that for any ρ'

$$\int_x^1 (\rho(s,t)U_H(\rho') + (1-\rho(s,t))U_L(\rho')h_s(t)dt =$$

$$= \rho_{lose}(s,x)U_H(\rho') + (1-\rho_{lose}(s,x))U_L(\rho'). \tag{17}$$

By (16) and (17), $\rho_{lose}(s,x) \in \arg\max_{\rho'} \int_x^1 (\rho(s,t)U_H(\rho') + (1-\rho(s,t))U_L(\rho'))h_s(t)dt$, and thus

$$\frac{\partial}{\partial \rho'} |_{\rho' = \rho_{lose}(s,x)} [\int_x^1 (\rho(s,t)U_H(\rho') + (1-\rho(s,t))U_L(\rho'))h_s(t)dt] = 0. \tag{18}$$

Note, that (18) is a consequence of the envelope theorem applied to the learning problem in the post-auction market. Using (18), the partial derivative of π with respect to the second variable becomes

$$\pi^{(2)}(s,x) = h_s(x)[v - (\rho(s,x)U_H(\rho_{lose}(s,x)) + (1-\rho(s,x))U_L(\rho_{lose}(s,x)) - b(x)].$$

Now, let us take a static interdependent value (second-price) auction with value function (letting y denote the highest other signal)

$$v(s,y) = v - (\rho(s,y)U_H(\rho_{lose}(s,y)) + (1-\rho(s,y))U_L(\rho_{lose}(s,y)). \tag{19}$$

The profit from bidding $b(x)$ in such an auction is $\pi_{static} = \int_0^x (v(s,t) - b(t))h_s(t)dt$, which gives rise to $\pi^{(2)}_{static}(s,x) = \pi^{(2)}(s,x)$. This implies that there exists a function D such that for a given s it holds that

$$\forall x, \ \pi(s,x) = \pi_{static}(s,x) + D(s).$$

Consequently, the bidding incentives in the two auctions are identical, and thus they give rise to the same (monotone and symmetric) equilibrium bid functions.

It is well known that in such a static second-price auction the symmetric equilibrium bid function is $b(s) = v(s,s)$, that is, the bid is equal to the expected value conditional on being tied at the top. This equilibrium bid can be rewritten as $b(s) = v - [\rho(s,s)U_H(\rho_{lose}(s,s)) + (1-\rho(s,s))U_L(\rho_{lose}(s,s))]$. We summarize our findings in the next Lemma:

Lemma 3 *If $v(s,y)$ is strictly monotone in s and y, then a monotone equilibrium exists in the (static) interdependent values auction characterized by value function $v(s,y)$. In the monotone equilibrium of the static auction for almost all s each bidder bids according to*

$$b(s) = v - [\rho(s,s)U_H(\rho_{lose}(s,s)) + (1-\rho(s,s))U_L(\rho_{lose}(s,s))]. \tag{20}$$

If $v(s,y)$ is strictly monotone in s and y, then (20) also forms a (monotone) equilibrium in our second-price auction.

Proof. The first two statements follow from the analysis of Milgrom and Weber (1982). The last is an immediate consequence of our discussion above. **Q.E.D.** ■

We continue by establishing that the function $v(s,y)$ is strictly monotone when V is decreasing. First, when V is strictly decreasing then $U_H(\rho) < U_L(\rho)$ for all $\rho \in [0,1)$ as we show below. Also, from the above lemma we know that U_H is increasing, while U_L is decreasing in ρ. Let $s' > s$, and $\rho' = \rho(s',y) > \rho = \rho(s,y)$, and $\rho'_{lose} = \rho_{lose}(s',y) > \rho_{lose} = \rho_{lose}(s,y)$. Consider the following chain of inequalities:

$$\rho U_H(\rho_{lose}) + (1-\rho)U_L(\rho_{lose}) > \rho U_H(\rho'_{lose}) + (1-\rho)U_L(\rho'_{lose}) > \rho' U_H(\rho'_{lose}) + (1-\rho')U_L(\rho'_{lose}). \tag{21}$$

The second inequality is immediate from $\rho' > \rho$ and $U_H(\rho'_{lose}) < U_L(\rho'_{lose})$. For the first inequality, consider the following incentive conditions that hold by construction:

$$\rho_{lose}U_H(\rho_{lose}) + (1-\rho_{lose})U_L(\rho_{lose}) \geq \rho_{lose}U_H(\rho'_{lose}) + (1-\rho_{lose})U_L(\rho'_{lose})$$

This condition implies that

$$\frac{U_H(\rho'_{lose}) - U_H(\rho_{lose})}{U_L(\rho'_{lose}) - U_L(\rho_{lose})} \leq \frac{1-\rho_{lose}}{\rho_{lose}}.$$

The fact that $\rho_{lose} > \rho$ implies that $\frac{1-\rho_{lose}}{\rho_{lose}} < \frac{1-\rho}{\rho}$ and thus $\frac{U_H(\rho'_{lose})-U_H(\rho_{lose})}{U_L(\rho'_{lose})-U_L(\rho_{lose})} < \frac{1-\rho}{\rho}$. This inequality (after rearranging) yields the first inequality in (21), which then yields that $v(s,y)$ is monotone in s. The exact same argument implies that $v(s,y)$ is monotone in y. Uniqueness of b as in (1) follows from the above argument as well, since upon tieing indifference has to hold in an ex-post equilibrium which yields exactly (1) after taking it into account that the equilibrium is symmetric and monotone. The only caveat is that the bid function is not determined at the (at most countably many) discontinuity points of U_H, U_L. At such a belief ρ, the optimal action in the continuation problem is not unique which introduces multiple optimal bids when the belief is ρ. However, there are at most countably many such jump points, so this multiplicity arises only for a small set of types, and for all other beliefs the equilibrium bid is pinned down by formula (1).[32]

For the last result it is sufficient to prove that V is (strictly) monotone if and only if $U_H(\rho) < U_L(\rho)$ for all $\rho < 1$. First, if $U_H(\rho) < U_L(\rho)$ for all $\rho < 1$, then by Lemma 2 $V' < 0$ for all $\rho < 1$ whenever the derivative exists, which implies that V is indeed monotone decreasing. Second, suppose that for some $\rho^* < 1$ it holds that $U_H(\rho^*) \geq U_L(\rho^*)$. Then Lemma 1 implies that U_H is increasing, while U_L is decreasing in ρ, and thus for all $\rho \geq \rho^*$ it holds that $U_H(\rho) \geq U_L(\rho)$. Therefore, using the envelope formula (15) implies that for all $\rho > \rho^*$ it holds that $V(\rho) \geq V(\rho^*)$ and thus V is not (strictly) monotone decreasing.

Finally, assume that V is not (strictly) monotone, and we show that the unique candidate equilibrium bid function (1) is not strictly increasing. To see this take $s = 1$ and assess whether b is increasing there locally. To simplify exposition assume that b is differentiable at s, but the argument can be modified in a straightforward manner to cover other cases. One can write

$$b'(s) = -\rho'_{tie}(s)(U_H(\rho_{lose}) - U_L(\rho_{lose})) - \rho'_{lose}(s)U'_H(\rho_{lose})\rho_{tie}(s) - \rho'_{lose}(s)U'_L(\rho_{lose})(1-\rho_{tie}(s)).$$

Since signal $\overline{s}$ is perfectly informative, it follows that $\lim\limits_{s\to\overline{s}}\rho_{lose}(\overline{s}) = \lim\limits_{s\to\overline{s}}\rho_{tie}(\overline{s}) = 1$. Also, by the incentive condition it follows that $\lim\limits_{s\to\overline{s}}U'_H(\rho_{lose}(\overline{s})) = 0$. If V is not strictly decreasing, then as we observed $U_H(1) < U_L(1)$ must hold.[33] Then inspect $\frac{b'(s)}{\rho'_{tie}(s)} = -(U_H(\rho_{lose}(s)) - U_L(\rho_{lose}(s))) - \frac{\rho'_{lose}(s)}{\rho'_{tie}(s)}U'_H(\rho_{lose}(s))\rho_{tie}(s) - \rho'_{lose}(s)U'_L(\rho_{lose}(s))\frac{1-\rho_{tie}(s)}{\rho'_{tie}(s)}$. We show next that $\lim\limits_{s\to\overline{s}}\frac{\rho'_{lose}(s)}{\rho'_{tie}(s)} = 0$ and that $\lim\limits_{s\to\overline{s}}\frac{1-\rho_{tie}(s)}{\rho'_{tie}(s)} = 0$, which would then imply that $\lim\limits_{s\to\overline{s}}\frac{b'(\overline{s})}{\rho'_{tie}(s)} < 0$, since $U_H(1) - U_L(1) < 0$. Since ρ_{tie} is strictly increasing, therefore if s is large enough, then $b'(s) < 0$ follows contradicting monotonicity of b.

To prove those two claims, consider the following which is by l'Hospital's rule: $\lim\limits_{s\to\overline{s}}$ $\frac{1-\rho_{tie}(s)}{\rho'_{tie}(s)} = \lim\limits_{s\to\overline{s}}\frac{-\rho'_{tie}(s)}{\rho''_{tie}(s)} = 0$ as $\rho'_{tie}(s) = 0$, $\rho''_{tie}(s) < 0$. Finally, using l'Hospital's rule and the formulas for ρ_{lose} and ρ_{tie}

$$\lim_{s\to\overline{s}}\frac{\rho'_{lose}(s)}{\rho'_{tie}(s)} = \lim_{s\to\overline{s}}\frac{\frac{1-\rho_{lose}(s)}{\rho_{lose}(s)}}{\frac{1-\rho_{tie}(s)}{\rho_{tie}(s)}} = \lim_{s\to\overline{s}}\frac{g_L}{g_H}\frac{G_L^{N-2}}{G_H^{N-2}}\frac{1-G_H^{N-1}}{1-G_L^{N-1}} = \lim_{s\to\overline{s}}\frac{g_L}{g_H}\frac{1-G_H^{N-1}}{1-G_L^{N-1}} = 0,$$

because $\lim\limits_{s\to\overline{s}}\frac{g_L}{g_H} = 0$ by construction and $\lim\limits_{s\to\overline{s}}\frac{1-G_H^{N-1}}{1-G_L^{N-1}} < 1$ since $G_H > G_L$ for all s.

Q. E. D.

[32] When the value function is smooth such discontinuity of W_H, W_L cannot occur and the equilibrium bid is unique for all x. Moreover, the function b is continuous in this case.

[33] If V is not monotone decreasing, then convexity requires that V must be increasing at 1, which implies (using Lemma 2) that $U_H(1) > U_L(1)$.

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Online Appendix for "Auctions in Markets: Common Outside Options and the Continuation Value Effect"

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1 First-price and second-price auctions

In this Section we provide our revenue comparison for first and second price auctions. We start with our observation that our first price auction has the same revenue as a standard interdependent value auction with value function $v(s,y)$ as defined in the main text. But then the linkage principle directly applies as we can compare the revenue of a second-price and a first-price interdependent values auction with the same value function $v(s,y)$.

Therefore, we just need to prove that our first price auction can be written as an interdependent auction with value function $v(s,y)$. The profit function from bidding $b(y)$ when one has signal s is

$$\pi^{1st}(s,y) = (v - b(y))H_s(t) + \int_y^1 (\rho(s,t)U_H(\rho_{lose}(s,y)) + (1-\rho(s,t))U_L(\rho_{lose}(s,y))h_s(t)dt.$$

The interdependent value auction has profit function

$$\widetilde{\pi}^{1st}(s,y) = \int_0^y v(s,t)h_s(t)dt - b(y)H_s(t).$$

Similarly to the case of second price auction format it holds that $\frac{\partial \widetilde{\pi}^{1st}(s,y)}{\partial y} = \frac{\partial \pi^{1st}(s,y)}{\partial y}$, which then proves that the two auctions provide the same incentives for the bidders.

2 Comparing second price auctions and English auctions

Here we provide the proof of the revenue comparison result for the English- and the second-price auctions considered in Proposition 7:

Proof. In the second price auction the middle bidder's bid is the revenue, and the bid function can be written as

$$b^{II} = v - [\widehat{\rho}_{tie}(s)U_H(\widehat{\rho}_{lose}(s)) + (1-\widehat{\rho}_{tie}(s))U_L(\widehat{\rho}_{lose}(s))],$$

where $\widehat{\rho}_{tie}, \widehat{\rho}_{lose}$ are the relevant tieing and losing posteriors. These beliefs can be written as $\widehat{\rho}_{tie}(s) = \Pr(H \mid s_1 = s_2 = s > s_3) = \frac{s^2\rho_{tie}}{s^2\rho_{tie}+(2s-s^2)(1-\rho_{tie})}$, and $\widehat{\rho}_{lose}(s) = \frac{s^2\rho_{lose}}{s^2\rho_{lose}+(2s-s^2)(1-\rho_{lose})}$

Let us now calculate the revenue in the English auction. Let z be lowest of the three types, and s be the medium one. Then the revenue is equal to

$$b^E(s,z) = v - [\widehat{\rho}_{tie}(s,z)U_H(\widehat{\rho}_{lose}(s,z)) + (1-\widehat{\rho}_{tie}(s,z))U_L(\widehat{\rho}_{lose}(s,z))],$$

where $\widehat{\rho}_{tie}(s,z) = \Pr(H \mid s_1 = s_2 = s > s_3 = z) = \frac{z\rho_{tie}}{z\rho_{tie}+(1-z)(1-\rho_{tie})}$, and $\widehat{\rho}_{lose}(s,z) = \Pr(H \mid s_1 > s_2 = s > s_3 = z) = \frac{z\rho_{lose}}{z\rho_{lose}+(1-z)(1-\rho_{lose})}$. To calculate the expected revenues let $g_{mid}(s) = 12s - 42s^2 + 60s^3 - 30s^4$ denote the density of the medium type, and let $h(z \mid s) = \frac{2\rho_{lose}(s)z+2(1-\rho_{lose}(s))(1-z)}{\rho_{lose}(s)s^2+(1-\rho_{lose}(s))(2s-s^2)}$ be the density of the low type given the medium types. Then the expected revenues from the two auctions can be written as $ER^{II} = \int_0^1 g_{mid}(s)b^{II}(s)ds$ and $ER^E = \int_0^1 g_{mid}(s)\int_0^s h(z \mid s)b^E(s,z)dzds$. After substituting in the relevant functional form assumptions about U_H, U_L, and the parameter values from the Proposition, we can use the above bid functions to show that indeed $ER^{II} > ER^E$.

3 Revenue comparison when signals are precise

In this Section we prove the revenue comparison result of the main text for the case where signals become arbitrarily precise. Take the model from the main text with two bidders an equal prior for the two states, $\rho_0 = 1/2$, and $\mu_H = \mu_L = 1$[1]. To be able to study a monotone equilibrium we assume that V is strictly decreasing, and continuously differentiable, and $V'(x) < 0$ for all $x \in [0,1]$. Each bidder observes a private signal $s \in [0,1]$ distributed according to a continuous density function $g_w^\alpha(s)$ in state $w \in \{L, H\}$, where $\alpha \in [0,1]$ parameterizes the precision of the signal. Similarly we can parameterize the posteriors by α, that is $\rho^\alpha(s) = g_H^\alpha(s)/(g_H^\alpha(s) + g_L^\alpha(s))$ denotes the probability of the high state upon observing signal s. Assume, as in the main text, that ρ^α is strictly increasing in s for all α. In what follows we suppress the α superscript to simplify notation. To model convergence to full information, assume that the probability of having the correct belief in each state converges to 1 as α converges to 1. Formally, for any $\varepsilon > 0$

$$\lim_{\alpha\to1} \Pr(\rho^\alpha(s) \geq \varepsilon \mid L) = 0 \tag{1}$$

and

$$\lim_{\alpha\to1} \Pr(\rho^\alpha(s) \leq 1-\varepsilon \mid H) = 0. \tag{2}$$

To establish our result we need to assume that the posterior upon low signals does not converge much faster than the posterior upon receiving some high signals:

Assumption CR: There exists $T > 0$ and $\widehat{\varepsilon} > 0$ *such that for all* $0 < \varepsilon \leq \widehat{\varepsilon}$

$$\frac{\lim\limits_{\alpha\to1} \int_{s:\rho^\alpha(s)\geq1-\varepsilon} g^{(2)\alpha}(s)(1-\rho^\alpha(s))^2 ds}{\lim\limits_{\alpha\to1} \int_{s:\rho^\alpha(s)\leq\varepsilon} g^{(2)\alpha}(s)\rho^\alpha(s)ds} \leq T.$$

To interpret this assumption, imagine that for any fixed α the signals are distributed symmetrically in the sense that $\Pr(\rho^\alpha(s) \leq t \mid L) = \Pr(\rho^\alpha(s) \geq 1-t \mid H)$ for any $t \in [0,1]$. In this case the relevant ratio converges to 0, as it can be shown.[2] Indeed, the only way to violate assumption CR is to assume that signals are much more precise in the low state than in the high state in the limit. At the end of the online Appendix we consider such an example, and show that the conclusion of our Proposition below fails.

Proposition: Under Assumption CR there exists $\widehat{\alpha} < 1$ *such that for all* $\alpha \in (\widehat{\alpha}, 1)$ *the linkage effect is stronger than the continuation value effect.*

Proof:

Step 1: First, we characterize the continuation value (CV) and the linkage (LP) effects. Applying the formula from the main text yields that the linkage effect can be written as[3]

$$LP = \int_0^1 g^{(2)}(s)(V(0) - V(1))(\rho_{lose}(s) - \rho_{tie}(s))ds.$$

[1] The case of unequal priors or more than two bidders could be handled without any major modification of the treatment. The same is true for the case where the matching probabilities μ_L, μ_H may be less than 1.

[2] The key idea is that the ratio $\frac{\lim_{\alpha\to1}\int_{s:\rho(s)\geq1-\varepsilon} g^{(2)}(s)(1-\rho(s))ds}{\lim_{\alpha\to1}\int_{s:\rho(s)\leq\varepsilon} g^{(2)}(s)\rho(s)ds}$ (that is, omitting the square sign from the numerator) is equal to 1 for any α, as the two states are completely symmetric. On the other hand, $\lim_{\alpha\to1}\frac{(1-\rho(s))^2}{1-\rho(s)} = 0$ if it is known that $\rho(s) \geq 1-\varepsilon > 0$, since in this case the high state is very likely and thus the posterior converges to 1 in probability.

[3] This rewriting follows, because

$$\sum_{i=1}^n g^{(2)}(s)\rho_{lose}(s) = \rho_0$$

by Bayes rule when there are two bidders. If there were more than two bidders then $\sum_{i=1}^n g^{(2)}(s)\rho_{lose}(s) < \rho_0$, and the formula underestimates the actual LP effect, so our result would still follow.

Similarly, the continuation value effect can be written as

$$CV = \int_0^1 g^{(2)}(s)[\rho_{tie}(s)(V(1) - W_H(\rho_{lose}(s))) + (1-\rho_{tie}(s))(V(0) - W_L(\rho_{lose}(s)))]ds.$$

Using that $W_H(x) = V(x) + (1-x)V'(x)$ and $W_L(x) = V(x) - xV'(x)$ we obtain that

$$CV = \int_0^1 g^{(2)}[\rho_{tie}(V(1) - V(\rho_{lose}) - (1-\rho_{lose})V'(\rho_{lose}))+$$

$$+(1-\rho_{tie})(V(0) - V(\rho_{lose}) + \rho_{lose}V'(\rho_{lose}))]ds =$$

$$= \int_0^1 g^{(2)}[\rho_{tie}V(1) + (1-\rho_{tie})V(0) - V(\rho_{lose}) + (\rho_{lose} - \rho_{tie})V'(\rho_{lose})]ds =$$

$$= \int_0^1 g^{(2)}[\rho_{lose}V(1) + (1-\rho_{lose})V(0) - V(\rho_{lose}) + (\rho_{lose} - \rho_{tie})(V'(\rho_{lose})+$$

$$+V(0) - V(1))]ds = CV = LP+$$

$$+\int_0^1 g^{(2)}[\rho_{lose}V(1) + (1-\rho_{lose})V(0) - V(\rho_{lose}) + (\rho_{lose} - \rho_{tie})V'(\rho_{lose})]ds.$$

To show that $LP > CV$ for precise information we can perform pointwise comparison to establish that the last sum (that is $CV - LP$) is negative (when signals are precise). We use the three-wise grouping we made above, and show non-positivity of $\rho_{lose}V(1) + (1-\rho_{lose})V(0) - V(\rho_{lose}) + (\rho_{lose} - \rho_{tie})V'(\rho_{lose})$ for some s, and show that although this expression can be positive for some other signals s, but it is of higher order and can be neglected.

Step 2:

In this Step we show that there exists an $\varepsilon^* > 0$ such that

$$\frac{\rho_{lose}V(1) + (1-\rho_{lose})V(0) - V(\rho_{lose}) + (\rho_{lose} - \rho_{tie})V'(\rho_{lose})}{\rho_{lose}} < \frac{V'(1)}{2} < 0$$

whenever $\rho \leq \varepsilon^*$. For this we first show that $(\rho_{lose}V(1) + (1-\rho_{lose})V(0) - V(\rho_{lose}) + \rho_{lose}V'(\rho_{lose}))/\rho_{lose} \leq V'(1) < 0$ for all $\rho_{lose} \in [0,1]$. Let $H(x) = xV(1) + (1-x)V(0) - V(x) + xV'(x)$. By the intermediate value theorem for some $0 \leq t \leq k \leq x$

$$H(x) = x(V(1) - V(0) + V'(x) - V'(t)) = x(V(1) - V(0) + V''(k)(x-t)).$$

Also,

$$H'(x) = V(1) - V(0) + V''(x)x.$$

Therefore,

$$(\frac{H}{x})' = \frac{xH' - H}{x^2} = \frac{xV''(x) - V''(k)(x-t)}{x} \geq 0,$$

because V is convex and $t \leq k \leq x$. Thus for all $x > 0$ it holds that $H/x \leq H(1)/1 = V'(1) < 0$. Next, let $\widetilde{H}(x) = xV(1) + (1-x)V(0) - V(x) + (x-\tau)V'(x)$ for some $\tau > 0$. Using standard continuity arguments it follows that for all $x \geq 0$, $\widetilde{H}(x)/x < V'(1)/2 < 0$ whenever $\tau \leq \tau^*$ for some $\tau^* > 0$.

Now, suppose that $\rho \leq \varepsilon^*$. Then as we noticed $\rho_{lose} \geq \rho$ and $\rho_{tie} = \frac{\rho^2}{\rho^2 + (1-\rho)^2}$ and thus $\frac{\rho_{tie}}{\rho_{lose}} \leq \frac{\rho}{\rho^2 + (1-\rho)^2} \leq \frac{\varepsilon^*}{1-2\varepsilon^*}$. Write

$$\rho_{lose}V(1) + (1-\rho_{lose})V(0) - V(\rho_{lose}) + (\rho_{lose} - \rho_{tie})V'(\rho_{lose}) =$$

$$\rho_{lose}V(1) + (1-\rho_{lose})V(0) - V(\rho_{lose}) + \rho_{lose}(1 - \frac{\rho_{tie}}{\rho_{lose}})V'(\rho_{lose}) = \widetilde{H}(x)$$

with the notation $\rho_{lose} = x$ and $\rho_{tie}/\rho_{lose} = \tau$. Let $\varepsilon^*/(1-2\varepsilon^*) \le \tau^*$ that is $\varepsilon^* \le \tau^*/(1+2\tau^*)$. Then $\frac{\rho_{tie}}{\rho_{lose}} \le \tau^*$ and thus it follows that if $\rho \le \varepsilon^*$ then

$$\frac{\rho_{lose}V(1) + (1-\rho_{lose})V(0) - V(\rho_{lose}) + (\rho_{lose} - \rho_{tie})V'(\rho_{lose})}{\rho_{lose}} < V'(1)/2 < 0.$$

Since $\rho_{lose} > \rho$, therefore if $\rho \le \varepsilon^*$ then

$$\frac{\rho_{lose}V(1) + (1-\rho_{lose})V(0) - V(\rho_{lose}) + (\rho_{lose} - \rho_{tie})V'(\rho_{lose})}{\rho} < V'(1)/2 < 0. \tag{3}$$

Step 3:

First, we establish important results for three carefully constructed signal groups, and then the proof concludes after putting those results together.

Low signals: Upon integration and taking limit of (3), we obtain that

$$\lim_{\alpha\to 1}\int_{s:\rho(s)\le\varepsilon^*} g^{(2)}[\rho_{lose}V(1) + (1-\rho_{lose})V(0) - V(\rho_{lose}) + (\rho_{lose} - \rho_{tie})V'(\rho_{lose})]ds \le$$

$$\le 0.5V'(1)\int_{s:\rho(s)\le\varepsilon^*} g^{(2)}(s)\rho(s)ds < 0. \tag{4}$$

Medium signals: Take any signal s such that $0 < \varepsilon^* \le \rho(s) \le 1-\varepsilon^* < 1$. Bayes rule implies that

$$\rho_{lose}(s) = \frac{g_H(s)(1-G_H(s))}{g_H(s)(1-G_H(s)) + g_L(s)(1-G_L(s))} =$$

$$= \frac{\rho(s)(1-G_H(s))}{\rho(s)(1-G_H(s)) + 1 - G_L(s)}.$$

By our convergence formulas (1), (2) the fact that signals become precise implies that $\lim_{\alpha\to 1} 1 - G_H(s) = 1$ and $\lim_{\alpha\to 1} 1 - G_L(s) = 0$.[4] Therefore, $\lim_{\alpha\to 1}\rho_{lose}(s) = 1$ for all such s. Moreover, the convergence is uniform, because ρ_{lose} is strictly increasing in s. To summarize: for all $\varepsilon > 0$ there exists an $\widehat{\alpha}((1-\varepsilon^*), \varepsilon)$ such that

if $\alpha \le \widehat{\alpha}((1-\varepsilon^*), \varepsilon)$ then for all s such that $\varepsilon^* \le \rho(s) \le 1-\varepsilon^*$ it holds that $1 - \rho_{lose}(s) \le \varepsilon$. Moreover, since $\rho_{tie} = \frac{\rho^2}{\rho^2 + (1-\rho)^2} \le \frac{(1-\varepsilon^*)^2}{(1-\varepsilon^*)^2 + \varepsilon^{*2}} < 1$. Therefore, there exists an $\widetilde{\varepsilon} < 1$ such that $\rho_{lose} \ge \widetilde{\varepsilon}$ implies that $\rho_{lose}V(1) + (1-\rho_{lose})V(0) - V(\rho_{lose}) + (\rho_{lose} - \rho_{tie})V'(\rho_{lose}) \le 0$. Thus if $\alpha \le \widehat{\alpha}(1-\varepsilon^*, \widetilde{\varepsilon}) = \widetilde{\alpha}(1-\varepsilon^*)$ then for all s such that $\varepsilon^* \le \rho(s) \le 1-\varepsilon^*$ it holds that

$$\rho_{lose}V(1) + (1-\rho_{lose})V(0) - V(\rho_{lose}) + (\rho_{lose} - \rho_{tie})V'(\rho_{lose}) \le 0. \tag{5}$$

Upon integration and taking limit of (5), we obtain that

$$\lim_{\alpha\to 1}\int_{s:\rho(s)\in(\varepsilon^*, 1-\varepsilon^*)} g^{(2)}[\rho_{lose}V(1) + (1-\rho_{lose})V(0) - V(\rho_{lose}) + (\rho_{lose} - \rho_{tie})V'(\rho_{lose})]ds \le 0. \tag{6}$$

High signals: Take any signal such that $\rho(s) \ge 1-\varepsilon^*$. First, note that for all $x \in [0,1)$ it holds that

$$\frac{xV(1) + (1-x)V(0) - V(x)}{1-x} \le V(0) - V(1). \tag{7}$$

Therefore, for all s it holds that

$$\frac{\rho_{lose}V(1) + (1-\rho_{lose})V(0) - V(\rho_{lose})}{1-\rho_{lose}} \le V(0) - V(1).$$

[4] Those formulas imply that $\lim_{\alpha\to 0}\Pr[\rho \ge \overline{\varepsilon} \mid H) = 1$ and $\lim_{\alpha\to 0}\Pr[\rho \le \varepsilon^* \mid L) = 1$. Therefore, since ρ is strictly monotone in the signals it indeed follows that $\lim_{\alpha\to 0} 1 - G_H(s) = 1$ and $\lim_{\alpha\to 0} 1 - G_L(s) = 0$.

Noting, that $\rho_{lose} \geq \rho_{tie} = \frac{\rho^2}{\rho^2+(1-\rho)^2}$ implies that

$$\frac{1-\rho_{lose}}{(1-\rho)^2} \leq \frac{1-\rho_{tie}}{(1-\rho)^2} = \frac{1}{\rho^2+(1-\rho)^2} \leq 2,$$

and thus $\frac{\rho_{lose}V(1)+(1-\rho_{lose})V(0)-V(\rho_{lose})}{\rho} \leq 2(V(0)-V(1))$. Finally, since $V' < 0$ it follows that

$$\frac{\rho_{lose}V(1)+(1-\rho_{lose})V(0)-V(\rho_{lose})+(\rho_{lose}-\rho_{tie})V'(\rho_{lose})}{(1-\rho)^2} \leq 2(V(0)-V(1)). \tag{8}$$

Upon integration and taking limit of (8), we obtain that

$$\lim_{\alpha\to 1}\int_{s:\rho(s)\geq 1-\varepsilon^*} g^{(2)}[\rho_{lose}V(1)+(1-\rho_{lose})V(0)-V(\rho_{lose})+(\rho_{lose}-\rho_{tie})V'(\rho_{lose})]ds \leq$$

$$2(V(0)-V(1))\int_{s:\rho(s)\geq 1-\varepsilon^*} g^{(2)}(s)(1-\rho(s))^2 ds \tag{9}$$

Inspecting (4), (6) and (9) implies the result under assumption CR after choosing T appropriately. **Q.E.D.**

To highlight the importance of Assumption CR, let us consider an example where Assumption CR does not hold, and in this example our main revenue comparison result fails. Assume that there are two signals l, h and that $\Pr(l \mid L) = 1-\sqrt[k]{x}$, $\Pr(h \mid L) = \sqrt[k]{x}$, $\Pr(l \mid H) = x$, $\Pr(h \mid L) = 1-x$ with x being the precision parameter. It is clear that the posteriors $\rho_H = \frac{1-x}{1-x+\sqrt[k]{x}}$ and $\rho_L = \frac{x}{1-\sqrt[k]{x}+x}$ violate Assumption CR when $k > 2$, and the posterior in the high state converges to 1 much slower than the posterior in the low state converges to 0. The value function is $V = \frac{(1+d-x)^n}{n(n-1)}$ for $n = 1.5$ and $d = 0.01$. One can show that the continuation value effect is greater than the linkage effect when x is close to zero and k is chosen to be 3. As one varies k there are two possibilities: either the continuation value effect is greater than the linkage effect for all signal precision x, or it is greater if and only if signals are close to being fully informative or they are not informative at all (with the linkage effect dominating in the intermediate precision case).

4 Imprecise seller information

4.1 Setup

In this Section we consider the case of imprecise seller's signal, and show that most of our results are immune to such a modification. Let $\widetilde{s}$ be the signal of the seller, and let

$$\widetilde{\rho}_{tie}(s,\widetilde{s}) = \Pr(H \mid s_i = s_j = s \geq s_k, \widetilde{s}) = \frac{\rho_{tie}(s)\Pr(\widetilde{s} \mid H)}{\rho_{tie}(s)\Pr(\widetilde{s} \mid H) + (1-\rho_{tie}(s))\Pr(\widetilde{s} \mid L)}$$

denote the posterior upon tieing at the top if the seller's signal is $\widetilde{s}$. Similarly, let $\widetilde{\rho}_{lose}(s,\widetilde{s})$ denote the posterior upon losing if the seller's signal is $\widetilde{s}$. Let $s^{(2)}$ denote the second highest signal among all bidders, and let $\rho^{(2)}(s) = \Pr(H \mid s^{(2)} = s)$.

First, suppose that the seller reveals his signal $\widetilde{s}$ before the auction is run. The bid function becomes

$$b(s,\widetilde{s}) = v - [\widetilde{\rho}_{tie}U_H(\widetilde{\rho}_{lose}) + (1-\widetilde{\rho}_{tie})U_L(\widetilde{\rho}_{lose})].$$

The expected revenue from an ex-ante point of view can be calculated as

$$ER^{Before} = v - V(0) + (V(0)-V(1))\int_0^1 g^{(2)}\{\rho^{(2)}E_{\widetilde{s}|H}[\widetilde{\rho}_{tie}] + (1-\rho^{(2)})E_{\widetilde{s}|L}[\widetilde{\rho}_{tie}]\}ds +$$

$$+\int_0^1 g^{(2)}\{\rho^{(2)}E_{\widetilde{s}|H}[\widetilde{\rho}_{tie}(V(1)-U_H(\widetilde{\rho}_{lose})) + (1-\widetilde{\rho}_{tie})(V(0)-U_L(\widetilde{\rho}_{lose})]+$$

$$+(1-\rho^{(2)})E_{\widetilde{s}|L}[\widetilde{\rho}_{tie}(V(1)-U_H(\widetilde{\rho}_{lose})) + (1-\widetilde{\rho}_{tie})(V(0)-U_L(\widetilde{\rho}_{lose})]\}ds.$$

Let $\widetilde{\rho}(s) = \rho^{(2)} E_{\widetilde{s}|H}[\widetilde{\rho}_{tie}(s,\widetilde{s})] + (1-\rho^{(2)}) E_{\widetilde{s}|L}[\widetilde{\rho}_{tie}(s,\widetilde{s})]$ denote the expected posterior upon tieing for the bidder with the second highest signal s. Then the revenue with signal revelation (before the auction) can be written as

$$ER^{Before} = v - V(0) + (V(0) - V(1)) \int_0^1 g^{(2)} \widetilde{\rho}(s) ds +$$

$$+ \int_0^1 g^{(2)} [\widetilde{\rho}(s)(V(1) - U_H(\widetilde{\rho}_{lose})) + (1-\widetilde{\rho}(s))(V(0) - U_L(\widetilde{\rho}_{lose})].$$

A similar argument as in the main text implies that if the signal of the seller ($\widetilde{s}$) is revealed after the auction, then the bid becomes

$$b^a(s) = v - \{\rho_{tie}(s) E_{\widetilde{s}|H} U_H(\widetilde{\rho}_{lose}) + (1 - \rho_{tie}(s) E_{\widetilde{s}|L}[U_L(\widetilde{\rho}_{lose})]\},$$

and the (ex-ante) expected revenue

$$ER^{After} = v - V(0) + (V(0) - V(1)) \int_0^1 g^{(2)} \rho_{tie} ds +$$

$$+ \int_0^1 g^{(2)} \{\rho_{tie}(V(1) - E_{\widetilde{s}|H}[U_H(\widetilde{\rho}_{lose})]) + (1-\rho_{tie})(V(0) - E_{\widetilde{s}|L}[U_L(\widetilde{\rho}_{lose})])\} ds.$$

The revenue without information revelation does not depend on how precise the seller's signal is, so it is as before:

$$ER^{None} = v - V(0) + (V(0) - V(1)) \int_0^1 g^{(2)} \rho_{tie} ds +$$

$$+ \int_0^1 g^{(2)} [\rho_{tie}(V(1) - U_H(\rho_{lose}(s))) + (1-\rho_{tie})(V(0) - U_L(\rho_{lose}(s))].$$

The two effects can be written as

$$LP = ER^{Before} - ER^{After}$$

$$= (V(0) - V(1)) \int_0^1 g^{(2)} \{\widetilde{\rho}(s) - \rho_{tie}(s)\} ds +$$

$$+ \int_0^1 g^{(2)} (\widetilde{\rho} - \rho_{tie})[(V(1) - E_{\widetilde{s}|H}[U_H(\widetilde{\rho}_{lose})]) - (V(0) - E_{\widetilde{s}|L}[U_L(\widetilde{\rho}_{lose})])] ds,$$

and

$$CV = ER^{None} - ER^{After} =$$

$$= \int_0^1 g^{(2)} [\rho_{tie}(E_{\widetilde{s}|H}\{U_H(\widetilde{\rho}_{lose}\} - U_H(\rho_{lose})) + (1-\rho_{tie})(E_{\widetilde{s}|L}\{U_L(\widetilde{\rho}_{lose}\} - U_L(\rho_{lose}))] ds.$$

While few general results are available about the behavior of the two effects when signals are imperfectly precise, two observations still hold. First, when bidders have uninformative signals ($\alpha = 0$), the LP effect is still zero, while $CV > 0$ so revealing information (even not fully precise information) hurts the revenues of the seller. Second, when the bidders are fully informed ($\alpha = 1$), by construction $LP = CV = 0$, and thus the information policy does not matter.

4.2 Numerical example

Now, we conduct a numerical analysis to systematically investigate how imprecise seller signals may influence the optimal information policy. Let us suppose that the seller can receive only two signals $\widetilde{s} = s_H, s_L$ with $\Pr[s_H \mid H] = \Pr[s_L \mid L] = z \in [0.5, 1]$, that is, z measures the precision of the seller's signal, which is common knowledge. Then $\widetilde{\rho}_{tie}(s, s_H) = \frac{\rho_{tie}(s)z}{\rho_{tie}(s)z+(1-\rho_{tie}(s))(1-z)}$, $\widetilde{\rho}_{tie}(s, s_L) = \frac{\rho_{tie}(s)(1-z)}{\rho_{tie}(s)(1-z)+(1-\rho_{tie}(s))z}$ hold and $E_{\widetilde{s}|H}[\widetilde{\rho}_{tie}(s, \widetilde{s})] = z\widetilde{\rho}_{tie}(s, s_H) + (1-z)\widetilde{\rho}_{tie}(s, s_L)$, $E_{\widetilde{s}|L}[\widetilde{\rho}_{tie}(s, \widetilde{s})] = (1-z)\widetilde{\rho}_{tie}(s, s_H) + z\widetilde{\rho}_{tie}(s, s_L)$. Similar calculations can be made for the other expected values. After substituting back from Example 1, and using $d = k = 1$ one can calculate the CV and LP effects numerically for any values of z, and α (the signal precision of the bidders). We first depict the two effects for a fixed level of z (chosen $z = 0.7$) varying α between 0 and 1. The picture is qualitatively similar to the full precision case (that is, when $z = 1$): the seller reveals information if and only if the bidders have precise private signals.

Interestingly, the cutoff point of α where the two effects are equal is decreasing in z, so if a seller would like to reveal a less precise signal (like $z = 0.7$), then he would like to reveal the more precise signal as well. For example, the seller would like to reveal a signal with precision $z = 0.7$ if and only if $\alpha \geq 0.25$, while he would reveal a perfectly informative signal ($z = 1$) if and only if $\alpha \geq 0.225$. This is depicted as Figure 3 in the main text.

Our last observation concerns the question whether the seller has an incentive to reveal his signal with a noise. To study this, imagine that the seller fully observes the state, but he can choose to reveal a signal with an arbitrary precision $z \in [0.5, 1]$. Extensive numerical analysis suggests that for any fixed α, the seller maximizes his revenue by revealing the state perfectly (when $\alpha \geq 0.225$) or not revealing it at all (when $\alpha \leq 0.225$), so garbling his information is not profitable for the seller in this example. For illustration, let us study the most interesting case when α is such that the seller is close to being indifferent to revealing his fully informative signal or not revealing it at all.[5] Depicting the revenue from revealing a signal with precision z (see Figure 1) shows a non-monotonic dependence when $\alpha = 0.24$. When a signal with $z = 0.5$ is revealed the seller's revenue does no change, since such a signal is completely uninformative. Since $\alpha = 0.24 > 0.225$ then revealing a signal with precision $z = 1$ is better than not revealing the signal, as we know it from the figure in the main text. The graph provides two interesting observations. First, revealing a fully precise signal is the best policy for the seller. Second, revealing a partially precise signal ($z < 1$) may be worse than not revealing any signal at all. Future research should shed light on why such a non-monotonic dependence of revenues in the signal precision of the seller occurs when outside options are endogenous.

[5] If α is much larger than this cutoff value (0.225) then the seller's revenue is increasing in z, and when it is much smaller then it the revenue is decreasing in z.

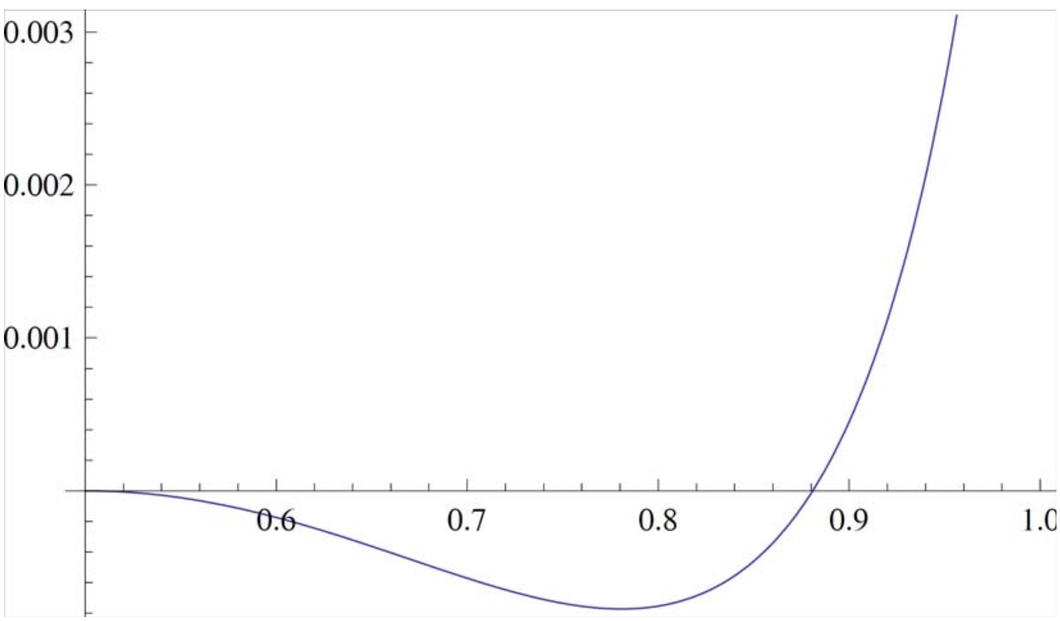

Figure 1: $ER^{Before} - ER^{None}$ as z varies and $\alpha = 0.24$