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The Coase Theorem and Coalitional Stability

By VAROUJ A. AIVAZIAN,[1] JEFFREY L. CALLEN[2] and IRWIN LIPNOWSKI[3]

[1] *University of Toronto*; [2] *Hebrew University of Jerusalem and McMaster University*; [3] *University of Manitoba*

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It is the contention of this paper that the Coase theorem has yet to be formulated and proved in a rigorous manner. The problem addressed is that of strategic coalitional behaviour when more than two agents are involved in the externality. It is shown that there are examples for which non-efficient allocations are stable in the sense of Aumann-Maschler bargaining sets, thereby vitiating the Coase theorem. Hence a proper formulation of the Coase theorem must exclude bargaining set stability.

It is our contention that the Coase theorem—which asserts that, with zero transaction costs and zero income effects, the identical Pareto-efficient allocation of resources will emerge regardless of the initial assignment of property rights—has yet to be formulated and proved in a rigorous manner.[1] The problem addressed here is that of strategic coalitional behaviour when more than two agents are involved in the externality. For the Coase theorem to hold in such situations, two conditions are important. First, assuming that the maximum joint payoff can be effected only by the formation of the grand coalition (e.g. by the merger of all parties involved in the externality), the grand coalition must be stable. Second, *only* the grand coalition must be stable; for if some other coalition is also stable, then the latter may form instead of the grand coalition so that a Pareto-optimal allocation need not obtain.

The point of this note is to show that a rigorous statement of the Coase theorem must exclude bargaining set notions of coalitional stability because, for both $M_1^{(i)}$ and M_1 bargaining set stability, the Coase theorem is violated.[2]

I. BARGAINING SETS AND THE COASE THEOREM

Following Aivazian and Callen (1981), we consider two factories (firms 1 and 2) polluting a neighbourhood laundry (firm 3). Let the characteristic function V denote profits per day where it is assumed that $V(1)=1$, $V(2)=2$, $V(3)=3$, $V(1,2)=8$, $V(1,3)=9$, $V(2,3)=10$, $V(1,2,3)=12$. The characteristic function is strictly superadditive, since only the grand coalition can maximize joint profits. Thus, with no transaction costs all three firms have an incentive to negotiate (i.e. to form coalitions) and internalize the externality. For example, if firms 1 and 3 were to merge, they could realize a net gain of 5.[3] However, if the grand coalition were to form, thereby internalizing the externality completely, the joint gain would be one more than for any other potential coalition structure.

As originally noted by Aivazian and Callen in a similar example, if the polluters are liable, then the grand coalition would result, the polluters would discontinue operations, and a Pareto-optimal allocation of resources would obtain. However, suppose there is no liability for polluting. Then the allocation of resources depends on bargaining among the three firms, and thus on coalitional stability.[4]

Let us first adopt the $M_1^{(i)}$ definition of bargaining set stability. The superscript (i) denotes that any allocation (payoff configuration) must be individually rational. That is, if x_i defines the allocation of profits to firm i in a specific coalition structure, then $x_i \geq V(i)$, all i. The bargaining set is based on the notions of 'objections' and 'counter-objections'. Specifically, suppose two firms k, l are in the same coalition. Then k has an objection against l if k can form another coalition excluding l in which k and all other members of the new coalition earn more than in the original coalition. On the other hand, l has a counter-objection against k if l can form another coalition excluding k in which: (1) l and the other members of the new coalition are no worse off than in the original coalition and (2) any member of l's (counter-objection) coalition who is also a member of k's potential (objection) coalition can do as well in l's coalition as in k's. The bargaining set $M_1^{(i)}$ is the set of all individually rational allocations which are coalitionally stable in the sense that every objection by an individual has a counter-objection.

Applying the $M_1^{(i)}$ concept to the above example yields the following bargaining set:[5]

(i) $\{1, 2, 3; (1), (2), (3)\}$ (ii) $\{3{\cdot}5, 4{\cdot}5, 3; (1, 2), (3)\}$

(iii) $\{3{\cdot}5, 2, 5{\cdot}5; (1, 3), (2)\}$ (iv) $\{1, 4{\cdot}5, 5{\cdot}5; (2, 3), (1)\}$

(v) $\{3, 4, 5; (1, 2, 3)\}$

Each element denotes the allocation and the coalition structure that is in the bargaining set. For example, element (iv) states that the coalition of firms 2 and 3 will be in the bargaining set provided firm 1 receives 1, firm 2 receives 4·5 and firm 3 receives 5·5 in profits, respectively. To see that element (iv) is in the bargaining set, suppose firm 2 proposes an objection against firm 3 of $\{3{\cdot}5-\varepsilon, 4{\cdot}5+\varepsilon, 3; (1, 2), (3)\}$ where $\varepsilon > 0$. In other words, firm 2 threatens to form a coalition with firm 1 which excludes firm 3 and which in the process makes 1 and 2 better off and 3 worse off by comparison with the original coalition structure (iv). Note that firm 2's objection is reasonably strong in the sense that 2 is only $\varepsilon > 0$ better off under the new coalition as compared with (iv). Nevertheless, the original coalition in (iv) is stable because firm 3 has a counter-objection to 2, namely, $\{3{\cdot}5, 2, 5{\cdot}5; (1, 3), (2)\}$. Hence, firm 3 can respond to firm 2's objection by forming a coalition with firm 1 in which both members of the coalition are better off by comparison with the objection and no worse off by comparison with (iv). Similarly, if firm 3 should threaten the objection $\{3{\cdot}5-\varepsilon, 2, 5{\cdot}5+\varepsilon; (1, 3), (2)\}$ to firm 2, the latter can respond by the counter-objection $\{3{\cdot}5, 4{\cdot}5, 3; (1, 2), (3)\}$. Therefore, the payoff configuration (iv) is stable in the sense that, if firms 2 and 3 form a coalition with the appropriate allocation given by (iv), neither firm in the coalition has a credible threat against the other that could lead to the dissolution of the coalition.[6]

From the perspective of the Coase theorem, the bargaining set $M_1^{(i)}$ definition of coalitional stability is problematic. Every coalition structure, including no coalition at all, is potentially stable.[7] This means that the attainment of the grand coalition is not assured. Far from it. In our example, there are four other possible stable allocations and coalition structures, any one of which will *not* result in a Pareto-optimal allocation of resources.[8] A proper formulation of the Coase theorem should exclude by assumption the bargaining set $M_1^{(i)}$.

Even more problematic to the Coase theorem is the bargaining set concept M_1, which differs from $M_1^{(i)}$ by the imposition of an additional requirement, namely, coalitional rationality. Coalitional rationality here means that no proper subset of any coalition is able to obtain more for itself than its allocation in the coalition. It turns out that the bargaining set M_1 for the above example is the same as $M_1^{(i)}$ except that it excludes the grand coalition. That the grand coalition does not belong to M_1 is easily seen by noting that the sub-coalition of firms 1 and 2 earn only 7 in the grand coalition. This is less than the 8 they would have earned by forming their own coalition. Indeed, every two-firm sub-coalition earns less in the grand coalition allocation (v) than it would by forming a separate coalition, thereby violating coalitional rationality. In the case of M_1, not only is the grand coalition not stable, but every other coalition arrangement is potentially stable. Therefore, if the polluters are not liable for damages, bargaining will not yield a Pareto-optimal allocation of resources. Again, a proper formulation of the Coase theorem must preclude the bargaining set M_1.

II. Conclusion

We showed that there are examples for which non-efficient allocations are stable in the sense of Aumann–Maschler bargaining sets, thereby vitiating the Coase theorem. Although there may be alternative negotiating mechanisms, such as that considered by Harsanyi (1977) in his construction of the Harsanyi–Nash–Shapley value, which yield a Pareto-optimal allocation of resources, nevertheless, we have shown that a proper formulation of the Coase theorem must exclude bargaining set stability.

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NOTES

1. See Coase (1960, 1981). Implicit in discussions of the Coase theorem is the assumption that agents have complete information about one another's production opportunities or preference functions. We shall retain this assumption.
2. See Aumann and Maschler (1964) on bargaining set theory.
3. In this example, the two factories also have an incentive to negotiate with each other since $V(1,2) > V(1) + V(2)$. This is either because one factory also pollutes the other, or because of any type of synergy in their production processes; e.g. they can economize on fixed costs after they merge.
4. In Aivazian and Callen (1981), the core is empty when there is no liability for polluting and thus the Coase theorem is violated.
5. The following set of linear inequalities, involving x_1, x_2 and x_3 as unknowns in the example, determines (see Aumann and Maschler, 1964, Theorem 2.1) the bargaining set $M_1^{(i)}$ imputation for the grand coalition. For k not to have a justified objection against l, at least one of the three conditions must obtain in each row k/l ($k, l = 1, 2, 3$):

   1/2: $x_1 + x_3 \geq 9$ or $x_2 = 2$ or $10 - x_2 \geq 9 - x_1$

1/3: $x_1+x_2 \geq 8$ or $x_3=3$ or $10-x_3 \geq 8-x_1$
2/1: $x_2+x_3 \geq 10$ or $x_1=1$ or $9-x_1 \geq 10-x_2$
2/3: $x_1+x_2 \geq 8$ or $x_3=3$ or $9-x_3 \geq 8-x_2$
3/1: $x_2+x_3 \geq 10$ or $x_1=1$ or $8-x_1 \geq 10-x_3$
3/2: $x_1+x_3 \geq 9$ or $x_2=2$ or $8-x_2 \geq 9-x_3$

Solution: $x=(x_1, x_2, x_3)=(3, 4, 5)$.

6. One might object to this process by arguing that the excluded party might make simultaneous offers to both parties in the coalition to enter. This will not change the stability relationships. For example, suppose element (ii) obtains and firm 3 offers 0·25 to both members of the coalition to enter. Suppose in fact that firm 3 reaps all the benefits from the grand coalition yielding the allocation $\{3{\cdot}75, 4{\cdot}75, 3{\cdot}5; (1, 2, 3)\}$. Nevertheless, the latter allocation is not stable for two reasons. First, two potential breakway coalitions—namely $\{2, 3\}$ and $\{1, 3\}$—could do better than under the terms proposed by player 3, since under the proposed imputation $(3{\cdot}75, 4{\cdot}75, 3{\cdot}5)$ we have $x_1+x_3=7{\cdot}25<V(1,3)=9$ and $x_2+x_3=8{\cdot}25<V(2,3)=10$. Thus player 3 can do better in either of these two-player coalitions, as in elements (iii) or (iv) of the bargaining set. The potential gain for a breakway coalition involving either $\{1, 3\}$ or $\{2, 3\}$ creates instability in the proposed imputation, which plagues any imputation in this example because of the empty core. A second source of instability for proposed imputation is that, unlike the payoff configurations in the bargaining set, there is no constraining factor neutralizing a threatened breakaway ('objection') by a 'counter-objections'. See also note 8.
7. This is a general result for the bargaining set $M_1^{(i)}$ and does not depend on the specific example. See Peleg (1963).
8. Note that the example was constructed so that every coalition structure containing two parties provides a return to all participants of 11 whereas the grand coalition obtains 12. Thus, there is a large number of grand coalition structures that are Pareto-superior to a stable two-party coalition structure; e.g. $\{3{\cdot}75, 4{\cdot}75, 3{\cdot}5; (1, 2, 3)\}$ is Pareto-superior to $\{3{\cdot}5, 4{\cdot}5, 3; (1, 2)\ 3\}$. The point of our example is that, unless the Coase theorem is to be a tautology, stability and Pareto optimality must be defined independently of each other. Given the Aumann-Maschler definitions of stability, an imputation such as (ii), namely $\{3{\cdot}5, 4{\cdot}5, 3; (1, 2)\ 3\}$, is stable despite the fact that it is Pareto-dominated by many grand coalition structures. Intuitively, although 3 may offer more to both 1 and 2 than they get in (ii), 1 and 2 may still prefer to have their own coalition and exclude 3, provided neither has a credible threat to break away and form a *separate* coalition with 3. Notice that the proposed grand coalition allocation is unstable since two potential breakaway coalitions, $\{2, 3\}$ and $\{1, 3\}$, could do better (as explained in note 6) *and* that the player left out of a potential breakaway coalition with 3 would be worse off than in (ii).

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