Journal of International Money and Finance (1986), **5**, 449–466

International Exchange Risk and Asset Substitutability

VAROUJ A. AIVAZIAN

Department of Economics, 150 St George Street, Toronto, Canada, M5S 1V4

JEFFREY L. CALLEN

Jerusalem School of Business Administration, The Hebrew University, Mount Scopus, Jerusalem, Israel, and Faculty of Business, McMaster University, Hamilton, Canada, L8S 4M4

ITZHAK KRINSKY

Faculty of Business, McMaster University, Hamilton, Canada, L8S 4M4

AND

CLARENCE C. Y. KWAN*

Faculty of Business, McMaster University, Hamilton, Canada, L8S 4M4

This paper estimates substitutability/complementarity relations among financial assets denominated in foreign currencies. Utilizing a representative investor and a flexible functional form methodology, a mean-variance utility function was estimated and used to determine expected return and variance elasticities between assets in the world portfolio. The hypothesis that international assets are perfect substitutes was rejected. It was also found that relative changes in variances tended to have a bigger impact on asset demand than did relative changes in expected returns. Substitutability/complementarity relationships were not strong except in specific cases where strong relationships were expected *a priori*.

'Asset-market' models of exchange-rate determination are commonly classified into two basic categories. Those belonging to the monetary approach assume that domestic and foreign assets are perfect substitutes for each other so that expected rates of return [yield + currency depreciation (appreciation)] are equalized. On the other hand, those models which belong to the portfolio balance category presuppose that foreign and domestic assets are indeed imperfect substitutes and, hence, their returns are systematically related to asset supplies.[1]

Of the latter category of models, much attention has recently been focused on the (international) portfolio optimization framework. This framework argues that

* We would like to express our deep appreciation to Professor Jeffrey A. Frankel for supplying us with the data set underlying this study. We would also like to thank Les Robb and an anonymous referee for very valuable comments on an earlier version of the paper. Financial support for this research was provided by the Social Sciences and Humanities Research Council of Canada.

non-substitutability of assets stems from the inherent differential riskiness of these assets. Following the pioneering work of Kouri (1976, 1977), a mean-variance portfolio approach has been used by Kouri and de Macedo (1978), de Macedo (1982) and Dornbusch (1980) to estimate optimal world portfolio currency holdings. In a somewhat different vein, Cornell and Dietrich (1978) and Roll and Solnik (1977) have tried to determine if world currency holdings are consistent with an International Capital Asset Pricing Model. Unfortunately, because of their contradictory findings and limitations of methodology,[2] the conclusions reached by these studies are suggestive at best. Most importantly, these studies make the implicit and unpalatable assumption that expected returns are constant over time, contradicting recent exchange rate history and the lack of persistent bias in the forward rate.

In order to rectify this problem, Frankel (1982, 1983) has suggested a mean-variance portfolio framework which incorporates asset supply data and, hence, allows for changes in expected returns.[3] The basic intent of Frankel's study is to test the hypothesis that domestic and foreign assets are perfect substitutes for each other. From his mean-variance portfolio model, Frankel derives the asset demand system:

$$i_t - \ell \cdot i_t^S - \underline{\Delta} S_{t+1} = \rho\Omega(x_t - \alpha w_t) + \varepsilon_{t+1}$$

where

i_t = vector of one-period interest rates on bonds denominated in various currencies (other than the US dollar).

$\underline{\Delta} S_{t+1}$ = vector of depreciation rates for different currencies relative to the US dollar.

ℓ = a vector of ones.

i_t^S = interest rate on US bond.

ρ = coefficient of relative risk aversion.

Ω = variance–covariance matrix of currency depreciation rates.

$$E[(\Delta S_{t+1} - E(\Delta S_{t+1}))(\Delta S_{t+1} - E(\Delta S_{t+1}))].$$

x_t = vector of portfolio shares in assets (denomination in various currencies).

w_t = vector of each country's share of world wealth.

α = vector of consumption preferences of residents of various countries.

ε_{t+1} = stochastic error term.

Frankel's test of the null hypothesis is based on the rather ingenious observation that Ω is in fact the variance–covariance matrix of the error terms. Thus, Frankel proceeds to employ a maximum likelihood technique to estimate the above equation subject to the information contined in Ω. He finds that his likelihood function takes its maximum at $\rho = 0$ so that, as he argues, one cannot reject the hypothesis that investors are risk neutral.[4] Nor can one reject the hypothesis that foreign and domestic assets are perfect substitutes. However, as Frankel himself points out, his test for not rejecting $\rho = 0$ is not at all very powerful.[5] Because of the flatness of the likelihood function, he is unable to reject as well such plausible values as $\rho = 1.0$ and $\rho = 2.0$. In addition to testing the perfect substitutes assumption, Frankel also estimates own and cross expected return elasticities for asset demands.

The purpose of this paper is identical to Frankel's; that is to see whether or not

domestic and foreign assets are perfect substitutes. However, our substitutability measures are defined differently from Frankel's. We generate several measures of substitutability depending on the alternative portfolio characteristics (expected return, variance) considered.[6] Our framework will enable us to estimate the underlying representative investor utility function and to estimate expected return, variance, and covariance elasticities in a direct fashion. Therefore, we shall be able to ascertain substitutability and complementarity relationships among the assets in terms of both risk and return characteristics.

Our technique for determining substitutability and complementarity relationships between domestic and foreign assets can be summarized as follows. We take the perspective of a 'representative' investor. We assume that his preferences over assets are represented by a utility function defined over portfolio risk and return. We assume that the investor's true utility can be approximated by a generalized Box–Cox flexible functional form. This latter function takes on the generalized Leontief, generalized square root quadratic, and translog utility function as special or limiting cases.[7] First-order conditions for risky asset demand are derived from the generalized Box–Cox utility function using a standard portfolio optimization framework. This demand system is then estimated from data on the financial asset holdings of six countries and the associated market yields. Utilizing the theory of portfolio demand and a Chi-square text based on the estimated equations, we determine which of the three specific flexible functional forms mentioned earlier best fits the data. Having determined that utility function which best fits the data, we then generate estimated mean, variance, and covariance elasticities for financial asset demands.

In what follows, Section I develops the elasticity relationships and the demand system to be estimated by utilizing a standard optimization procedure and a generalized Box–Cox utility function. Section II estimates the demand system given international financial asset holdings and market return data. After using the data to determine the 'optimal' form of the utility function, estimates of expected return, variance, and covariance elasticities are obtained and analyzed. Section III concludes this paper.

I. Elasticities and Budget Share Equations for Risky Assets

A. The Elasticities

To make the theory and empirical work manageable, we adopt the commonly-made assumption of homothetic separability, namely that the decision to invest in financial assets denominated in various currencies is independent both of the overall consumption-investment decision and the investment in non-financial assets. This means that the total amount of wealth to be invested in financial assets is exogenous to the model and the only issue of consequence is the proportion of wealth to be invested in each currency.[8]

The investor's preferences are assumed to be captured by a Lancaster-type utility function defined over portfolio characteristics.[9]

$$U = U(E, V)$$

where E is the expected end-of-period wealth of the portfolio and V is its standard deviation. This utility function is assumed to be continuous and twice

differentiable with $U_E > 0$ and $U_V < 0$ where the subscripts denote partial derivatives. In short, the investor is assumed to be risk averse with indifference curves in E–V space which are upward sloping and—given additional assumptions to be made further—convex from below.

The investor's financial asset choice framework is assumed to be described by the program:

$$\langle 1 \rangle \quad \underset{\{X_1, \ldots, X_m\}}{\text{Maximize}} \; U(E, V)$$

$$\text{subject to:} \quad E = W_0\left[1 + \sum_{i=1}^{m} X_i E_i\right]$$

$$V = \left[W_0^2 \sum_{i=1}^{m}\sum_{j=1}^{m} X_i X_j G_{ij}\right]^{1/2}$$

$$\sum_{i=1}^{m} X_i = 1$$

where

X_i = the proportion of wealth invested in financial assets denominated in currency i, $i = 1, 2, \ldots, m$;

E_i = the expected rate of return on assets (denominated in currency) i, $i = 1, \ldots, m$;

G_{ij} = the covariance of returns between assets (denominated in currencies) i and j, $i, j = 1, 2, \ldots, m$;

W_0 = total initial wealth invested in financial assets.

Solving the utility maximizing program ⟨1⟩ yields the first-order conditions:

$$\langle 2 \rangle \quad \text{(a)} \quad W_0 U_E E_i + W_0^2 U_V V^{-1} \sum_{j=1}^{m} X_j G_{ij} = \gamma \qquad (i, j = 1, 2, \ldots, m)$$

$$\text{(b)} \quad 1 - \sum_{i=1}^{m} X_i = 0$$

where γ is a Lagrange multiplier.

The second-order conditions for a maximum require the principal minors of the determinant D—obtained by differentiating ⟨2a⟩ and ⟨2b⟩ with respect to the X's—to alternate in sign. In particular,

$$\langle 3 \rangle \qquad D = \begin{bmatrix} Z_{11} & \ldots & Z_{1m} & 1 \\ \vdots & & \vdots & \vdots \\ Z_{m1} & \ldots & Z_{mm} & 1 \\ 1 & \ldots & 1 & 0 \end{bmatrix}$$

where

$$\langle 4 \rangle \qquad Z_{ij} = W_0^2 U_{EE} E_i E_j + V^{-1} W_0^2 U_{EV}\left(E_i \sum_{j=1}^{m} X_j G_{ij} + E_j \sum_{i=1}^{m} X_i G_{ij}\right)$$

$$+ W_0^4 (U_{VV} V^{-2} - U_V V^{-3}) \sum_{i=1}^{m} X_i G_{ij} \sum_{j=1}^{m} X_j G_{ij} + V^{-1} W_0^2 U_V G_{ij}$$

The impact of a change in the rth asset return on the demand for the kth asset is determined by differentiating the first-order conditions (equations ⟨2a⟩ and ⟨2b⟩) with respect to E_r. This procedure yields the matrix equation

$$\langle 5\rangle \qquad \begin{bmatrix} Z_{11} & \dots & Z_{1m} & 1 \\ \vdots & & \vdots & \vdots \\ Z_{m1} & \dots & Z_{mm} & 1 \\ 1 & \dots & 1 & 0 \end{bmatrix} \begin{bmatrix} \dfrac{\partial X_1}{\partial E_r} \\ \vdots \\ \dfrac{\partial X_m}{\partial E_r} \\ \dfrac{\partial \lambda}{\partial E_r} \end{bmatrix} = \begin{bmatrix} T_1 \\ \vdots \\ T_m \\ 0 \end{bmatrix}$$

where $\quad T_i = -W_0 U_E \delta_{ir} - X_r \left(W_0^2 U_{EE} E_i + V^{-1} W_0^3 U_{EI} \sum_{j=1}^{m} X_j G_{ij} \right)$

and $\quad \delta_{ir} = 1 \quad$ for $i = r$

$\qquad\qquad 0 \quad$ for $i \neq r$

Solving for $\partial X_k / \partial E_r$ yields:

$$\langle 6\rangle \qquad \frac{\partial X_k}{\partial E_r} = -W_0 U_E \frac{D_{rk}}{|D|} - X_r \left[W_0^2 U_{EE} \sum_{i=1}^{m} E_i \frac{D_{ik}}{|D|} + V^{-1} W_0^3 U_{EI} \sum_{i=1}^{m} \sum_{j=1}^{m} X_j G_{ij} \frac{D_{ik}}{|D|} \right]$$

where D_{rk} is the rkth cofactor of D. The demand elasticity of asset k with respect to the expected return on asset r is easily calculated from equation ⟨6⟩ as

$$\langle 7\rangle \qquad \eta(X_k, E_r) = \frac{\partial X_k}{\partial E_r} \cdot \left| \frac{E_r}{X_k} \right|$$

In an analogous fashion, one can derive the impact of a change in the covariance between assets r and f on the demand for asset k, namely,

$$\langle 8\rangle \qquad \frac{\partial X_k}{\partial G_{rf}} = -V^{-1} U_E W_0^2 \left(X_r \frac{D_{fk}}{|D|} + X_f \frac{D_{rk}}{|D|} \right) a_{rf} - V^{-1} X_r X_f W_0^3 \times \left[U_{EI} \sum_{i=1}^{m} E_i \frac{D_{ik}}{|D|} + W_0 V^{-1} (U_{II} - U_I V^{-1}) \sum_{i=1}^{m} \sum_{j=1}^{m} X_i G_{ij} \frac{D_{ik}}{|D|} \right]$$

where $a_{rf} = 1$ if $r \neq f$ and $a_{rr} = \frac{1}{2}$. Similarly, the demand elasticity of asset k with respect to a change in the covariance between assets r and f is given by

$$\langle 9\rangle \qquad \eta(X_k, G_{rf}) = \frac{\partial X_k}{\partial G_{rf}} \cdot \left| \frac{G_{rf}}{X_k} \right|$$

The variance cross elasticities are obtained from ⟨9⟩ by setting $r = f$.[10]

B. Asset Demand Relations From Flexible Functional Form Utility Functions

To operationalize the theory developed in the previous section, we assume that the utility function $U(E, I^c)$ can be specified as a generalized Box–Cox function of the form[11]

$$\langle 10\rangle \qquad \ell(U;\delta) = \alpha_0 + \alpha_1 f(E;\lambda) + \alpha_2 g(I^c;\lambda) + \tfrac{1}{2}\alpha_3[f(E;\lambda)]^2 + \tfrac{1}{2}\alpha_4[g(I^c;\lambda)]^2 + \alpha_5 f(E;\lambda)g(I^c;\lambda)$$

where $\ell(U;\lambda)$, $f(E;\lambda)$ and $g(I^c;\lambda)$ are the Box–Cox transformations

$$\langle 11\rangle \quad \text{(a)} \qquad \ell(U;\lambda) = (U^{2\delta} - 1)/2\delta$$

$$\text{(b)} \qquad f(E;\lambda) = (E^{\lambda} - 1)/\lambda$$

$$\text{(c)} \qquad g(I^c;\lambda) = (I^{c\lambda} - 1)/\lambda$$

As the parameters δ and λ take on different values, one obtains the following alternative flexible functional forms:

Case (a): $\delta, \lambda \to 0$: $\quad \ell(U;\lambda) = \ln U; \quad f(E;\lambda) = \ln E; \quad g(I^c;\lambda) = \ln I^c$

This case yields the translog utility function

$$\langle 12\rangle \quad \ln U = \alpha_0 + \alpha_1 \ln E + \alpha_2 \ln I^c + \tfrac{1}{2}\alpha_3(\ln E)^2 + \tfrac{1}{2}\alpha_4(\ln I^c)^2 + \alpha_5(\ln E)(\ln I^c)$$

Case (b): $\quad \delta, \lambda = \tfrac{1}{2}$: $\quad \ell(U;\lambda) = U - 1; \quad f(E;\lambda) = 2(E^{1/2} - 1);$

$$g(I^c;\lambda) = 2(I^{c1/2} - 1)$$

This case gives the generalized Leontief utility function

$$\langle 13\rangle \qquad U = 2\alpha_3 E + 2\alpha_4 I^c + 4\alpha_5 E^{1/2} I^{c1/2} + (2\alpha_1 - 4\alpha_3 - 4\alpha_5)E^{1/2} + (2\alpha_2 - 4\alpha_4 - 4\alpha_5)I^{c1/2} + 2\alpha_3 + 2\alpha_4 + 4\alpha_5 - 2\alpha_1 - 2\alpha_2 + 1$$

Case (c): $\quad \delta, \lambda = 1$: $\quad \ell(U;\lambda) = (U^2 - 1)/2;$

$$f(E;\lambda) = E - 1; \quad g(I^c;\lambda) = I^c - 1$$

This case results in the square root quadratic utility function

$$\langle 14\rangle \quad U = [\alpha_3 E^2 + \alpha_4 I^{c2} + 2\alpha_5 E \cdot I^c + 2(\alpha_1 - \alpha_3 - \alpha_5)E + 2(\alpha_2 - \alpha_4 - \alpha_5)I^c + 2\alpha_5 + \alpha_3 + \alpha_4 - 2\alpha_1 - 2\alpha_2 + 1]^{1/2}$$

It is worth noting that the ordinary quadratic can be obtained by setting $\delta = \tfrac{1}{2}$ and $\lambda = 1$. However, the ordinary quadratic yields the same budget share equations as the square root quadratic, so that they are empirically indistinguishable.

A system of asset demand relations for the generalized Box–Cox utility function can be obtained from the first-order conditions (equations ⟨2a⟩ and ⟨2b⟩):

$$W_0 U_E(E_i - E_1) + W_0^2 U_{I^c} I^{c-1}\left(\sum_j X_j G_{ij} - \sum_j X_j G_{1j}\right) = 0, \qquad i = 2, \ldots, 6$$

which yields,

$$\langle 15\rangle$$

$$E_i - E_1 = -\frac{I^c}{W_0 E^{c-1}} \cdot \frac{\alpha_2 + \alpha_4 g(I^c;\lambda) + \alpha_5 f(E;\lambda)}{1 + \alpha_3 f(E;\lambda) + \alpha_5 \lambda(I^c;\lambda)}(\beta_i - \beta_1), \qquad \text{for } i = 2, 3, \ldots, 6$$

where
$$\beta_i = \frac{\sum_{j=1}^{m} X_j G_{ij}}{\sum_{j=1}^{m}\sum_{k=1}^{m} X_j X_k G_{jk}},$$
$$U_E = [\alpha_1 + \alpha_3 f(E;\lambda) + \alpha_5 g(\Gamma;\lambda)]^{\lambda-1},$$
$$U_{\Gamma} = [\alpha_2 + \alpha_4 g(\Gamma;\lambda) + \alpha_5 f(E;\lambda)]\,\Gamma^{\lambda-1}.$$

By adding to equation ⟨15⟩ a serially uncorrelated multivariate normal disturbance term v_i, we obtain the demand system to be estimated:

$$\langle 16\rangle \qquad E - E_1 = -\frac{V^{\lambda}}{W_0 E^{\lambda-1}} \cdot \frac{\alpha_2 + \alpha_4 g(\Gamma;\lambda) + \alpha_5 f(E;\lambda)}{1 + \alpha_3 f(E;\lambda) + \alpha_5 g(\Gamma;\lambda)} (\beta_i - \beta_1) + v_i,$$
$$i = 2, 3, \ldots, 6;$$

Note that ⟨16⟩ is very similar to the demand system in Frankel. However, we make no assumptions about the coefficient of relative risk aversion. Our formulation expresses risk aversion directly in terms of parameters of the utility function (αs) estimated in our system.[12,13]

The demand system corresponding to the translog, square root quadratic (and quadratic), and generalized Leontief utility functions can be obtained from equation ⟨16⟩ by setting λ equal to zero, one-half, and one, respectively.

II. Estimation and Empirical Results

A. *The Data*

The database employed to estimate the demand system ⟨16⟩ was developed by Frankel (1982). It includes data on the financial asset holdings and yields of six countries: Germany, the United Kingdom, Japan, France, Canada and the USA. Since the data are complex and calculations are involved, the reader is referred to Appendix 4 of Frankel for an in-depth description of the database. In brief, the X_is are measured as the world supply of assets denominated in currency i as a proportion of total world assets. All assets are translated into US dollars to obtain a common base. The world supply of assets denominated in currency i is defined to be the sum of the government debt issued in currency i and the cumulative central bank sales of assets in currency i less reserve holdings (if any) of currency i assets by foreign central banks.[14] The (real) asset yields are defined to be one-period bond rates (adjusted for the exchange rate against the US dollar) and the rate of inflation (expressed in terms of US dollars) for the appropriate basket of goods. More formally, the currency j one period asset yield r_t^j is derived by:

$$\langle 17\rangle \qquad 1 + r_{t+1}^j = \frac{1 + i_t^j}{(1 + \pi_{t+1}^{\$})(1 + \Delta S_{t-1}^j)}$$

Here $\pi\$$ is the dollar inflation index which is computed as:

$$\langle 18\rangle \qquad \pi_t^{\$} = \alpha'(\bar{\pi}_t - \Delta St) + (1 - \alpha'/\pi_{USA}^{\$}$$

where $\bar{\pi}_t$ represents the vector of inflation rates in the goods of the six countries ($\pi_{USA}^{\$}$ is the inflation rate in the USA). In other words, $\pi_t^{\$}$ is a weighted average of

the six country inflation rates where the elements in vector α' represent the share of world consumption allocated to each country's goods. Finally, wealth is defined as the sum of the net wealth of the citizens of each of the six countries. Again, for its computation, we refer the reader to Frankel.

Our sample consisted of 87 monthly observations from June 1973 to August 1980. A rolling sample technique was used to estimate the mean and standard deviation for the international financial asset portfolio. Specifically, the first twelve months of yield data, June 1973 to May 1974 (12 data points for each asset category) were employed to calculate sample covariances between asset yields.[15] These sample estimates were then used to calculate the (sample) expected return and variance for the portfolio for June 1974. The Xs for that month were the actual proportions of each asset while W_0 was total world holdings of financial assets. Therefore, the calculated June 1974 sample mean returns, variances and covariances for the separate assets as well as the portfolio mean and standard deviation represent one data point to be utilized in estimating the demand system. The second data point (for July 1974) was calculated by an updating or rolling sample technique. Sample means, variances and covariances were recalculated after dropping the June 1973 monthly yield data and substituting the June 1974 data. Again, twelve months of data were used to generate the asset portfolio expected return and standard deviation. These new estimates together with the asset proportions held in July 1974 provide another data point. By means of this procedure, a time series of 76 data points (June 1974–August 1980) was generated and utilized to estimate the demand system (equation ⟨16⟩).[16] We estimated asset demand elasticities for this period using average values (over the whole period) of the expected returns, variances, covariances, and wealth calculated from the rolling sample.

The reliability of our statistical estimates depends on the range of variation of observations on the input data. Table 1 provides information on the sample properties of the input data. While the use of moving averages does reduce the period-to-period variability of the data, the table shows that variability over the whole period (76 data points) is still large.

B. *The Estimation Procedure and the 'Optimal' Utility Function*

The demand system (equation ⟨16⟩) is non-linear in the parameters and was estimated by a maximum likelihood technique. Since each equation in the system ⟨16⟩ is homogeneous of degree zero in the α parameters, these parameters were normalized with respect to α_1.

In our previous discussion, we said that we would discriminate among three specific flexible functional form utility functions. However, *a priori* it is impossible to discriminate between the three forms on purely economic grounds since each represents arbitrary well-behaved preferences in the neighbourhood of a given point with second-order accuracy. *A priori*, we are also unable to choose among the forms on econometric grounds. The estimation of each one of the forms involves the same dependent variable, the same number of free parameters and the maximization of a similar likelihood function. In order to use traditional tests, a fourth form is estimated, namely the unrestricted system where λ is a free parameter. Thus, the three 'original' forms are nested (*i.e.*, they are special cases of the unrestricted case). Therefore, four different budget share models were

TABLE 1. Summary statistics of input data.

Variable[Δ]	Mean	Standard deviation	Range
W	866.93	241.62	511.15–1303.96
X_1	0.4853	0.0603	0.3825–0.5607
X_2	0.1218	0.0094	0.1061–0.1416
X_3	0.0427	0.0044	0.0312–0.0546
X_4	0.0561	0.0077	0.0449–0.0733
X_5	0.1522	0.0603	0.0747–0.2404
X_6	0.1419	0.0203	0.1110–0.1805
E_1	−0.0011	0.0123	−0.0361–0.0566
E_2	0.0030	0.0628	−0.3538–0.2654
E_3	0.0037	0.0232	−0.0554–0.1104
E_4	−0.0024	0.0219	−0.1253–0.0480
E_5	0.0007	0.0236	−0.0672–0.0634
E_6	0.0016	0.0229	−0.0539–0.0575
G_{11}	0.0001585	0.0001596	0.0000223–0.0005309
G_{12}	−0.0003843	0.0003408	−0.0010340–0.0000070
G_{13}	−0.0001594	0.0001624	−0.0004955–0.0000356
G_{14}	0.0001187	0.0001471	−0.0000267–0.0004939
G_{15}	−0.0001600	0.0002147	−0.0007394–0.0000322
G_{16}	−0.0000841	0.0000809	−0.0002785–0.0000134
G_{22}	0.0038263	0.0048559	0.0001217–0.0115970
G_{23}	0.0004597	0.0004168	−0.0000606–0.0015270
G_{24}	−0.0001973	0.0002946	−0.0009249–0.0002057
G_{25}	0.0002657	0.0003367	−0.0001923–0.0010329
G_{26}	0.0003974	0.0005260	−0.0002030–0.0016873
G_{33}	0.0005694	0.0003052	0.0001092–0.0010850
G_{34}	−0.0001392	0.0001567	−0.0004544–0.0001146
G_{35}	0.0000138	0.0002309	−0.0003072–0.0005745
G_{36}	0.0000946	0.0000928	−0.0000905–0.0003353
G_{44}	0.0002824	0.0002230	0.0000544–0.0015043
G_{45}	−0.0001912	0.0002331	−0.0008612–0.0000468
G_{46}	−0.0000738	0.0001172	−0.0003439–0.0002867
G_{55}	0.0004388	0.0004240	0.0000332–0.0015001
G_{56}	0.0000309	0.0000996	−0.0001716–0.0002683
G_{66}	0.0004754	0.0002565	0.0000615–0.0010961
β_1	−0.1834	1.8340	−4.3478–3.1420
β_2	3.1802	4.1123	−3.4186–9.1003
β_3	0.7748	3.0494	−5.0093–6.2453
β_4	−0.7013	1.9426	−6.0440–2.4424
β_5	1.2217	2.3034	−1.9951–9.1051
β_6	2.6811	2.1315	−0.9219–7.1506

Δ The subscripts denote:
1 = US $
2 = Mark
3 = Franc
4 = Canadian $
5 = Yen
6 = Pound

estimated; the translog ($\lambda = 0$), the generalized Leontief ($\lambda = \frac{1}{2}$), the square root quadratic ($\lambda = 1$), and the unrestricted system where λ is a free parameter. The unrestricted model involves non-linear estimation of five free parameters, the α (normalized) and λ. In all other versions of the model only four free (normalized) α parameters need to be estimated.

Table 2 summarizes the results for each of the four estimated systems. As a first attempt to discriminate among the three specific functional forms, we utilize a Chi-square test. In particular, it can be shown that $-2/nL$ is asymptotically distributed $\chi^2(1)$ where L is the ratio of the value of the unrestricted likelihood function (*i.e.*, when λ is a free parameter) to the value of the restricted likelihood function (where λ is constrained to a specific value).[17] Using the Chi-square test, we cannot reject the generalized Leontief ($\lambda = \frac{1}{2}$) but we can reject the other two utility functions at both 1 per cent and 5 per cent significance levels.

In addition to the above test, we should be able to discriminate among the various functional forms on the basis of the theory of asset demand. From the theory of asset demand, we expect our 'optimal' utility function to satisfy the conditions that the sign of U_E should be positive and the sign of U_{EE} should be negative. As can be seen from Table 2, based on the signs of marginal utilities, the square root quadratic can be rejected.

Tables 3, 4, and 5 list expected return, variance, and covariance elasticities, respectively, for the selected functional form—the Leontief—over the whole period. The boxed-in numbers in the tables show the own expected return and variance elasticities. The own expected return elasticities are in all cases with positive sign, while the own variance elasticities, are uniformly negative. Most elasticities in these tables were significant at the 1 per cent level.

The elasticities in Tables 3 and 4 are point estimates since they were calculated from the point estimates of the α parameters. The question naturally arises as to the significance of these elasticity estimates. To answer this question, the following procedure was adopted. A multivariate normal distribution was created for the estimated α parameters from equation ⟨16⟩. The point estimates of the α parameters were designated the mean of the distribution and the asymptotic variance-covariance matrix of these parameters was designated the variance-covariance matrix of the distribution. Five hundred random draws of the α parameters were then obtained from this distribution. For each random draw, the expected return and the variance elasticities were calculated so that for each expected return and variance elasticity, a sample of 500 estimates was obtained. The bracketed numbers in these tables under each elasticity estimate are the mean and standard deviation of the sample. In almost all cases in these tables, the standard deviations are small relative to the magnitudes of the means. The sample elasticities were found to be, for expected return elasticities, significant at the 1 per cent level in all cases, and for variance elasticities significant at the 1 per cent level in most cases.

C. The Elasticities and Their Implications

We now evaluate the expected return (Table 3), variance (Table 4), and covariance (Table 5) elasticities to see what they imply about the demand for international financial assets. Consider first the expected return elasticities. Own asset expected return elasticities are all positive. Cross elasticities are in general quite small with

TABLE 2. Parameter estimates and marginal utilities for different functional forms.

Functional form	Specified value of λ	Estimated parameters (standard errors)	Value of log likelihood	Marginal utilities
Unrestricted maximum	—	$\lambda = 0.86125\ (0.91567 \times 10^{-1})$	1358.426	$U_E = 0.37632 \times 10^{-1}$
		$\alpha_2/\alpha_1 = -0.72887 \times 10^{-1}\ (0.36126 \times 10^{-1})$		$U_1 = -0.14227 \times 10^{-1}$
		$\alpha_3/\alpha_1 = -0.23041 \times 10^{-2}\ (0.12107 \times 10^{-2})$		$U_{EE} = -0.35855 \times 10^{-3}$
		$\alpha_4/\alpha_1 = -0.40735 \times 10^{-2}\ (0.13374 \times 10^{-2})$		$U_{11} = -0.21131 \times 10^{-2}$
		$\alpha_5/\alpha_1 = 0.18779 \times 10^{-3}\ (0.29228 \times 10^{-4})$		$U_{E1} = 0.56518 \times 10^{-4}$
Translog	0	$\alpha_2/\alpha_1 = -0.85339 \times 10^{-3}\ (0.14485 \times 10^{-3})$	1352.373	$U_E = 0.21495 \times 10^{-4}$
		$\alpha_3/\alpha_1 = -0.14510\ (0.12978 \times 10^{-3})$		$U_1 = -0.10700 \times 10^{-4}$
		$\alpha_4/\alpha_1 = -0.77835 \times 10^{-4}\ (0.79588 \times 10^{-5})$		$U_{EE} = -0.21778 \times 10^{-6}$
		$\alpha_5/\alpha_1 = 0.13742 \times 10^{-3}\ (0.21765 \times 10^{-4})$		$U_{11} = -0.16173 \times 10^{-6}$
				$U_{E1} = 0.23964 \times 10^{-7}$
Generalized Leontief	$\frac{1}{2}$	$\alpha_2/\alpha_1 = -0.11093 \times 10^{-1}\ (0.19066 \times 10^{-2})$	1357.043	$U_E = 0.20319 \times 10^{-2}$
		$\alpha_3/\alpha_1 = -0.16537 \times 10^{-1}\ (0.40238 \times 10^{-4})$		$U_1 = -0.88720 \times 10^{-1}$
		$\alpha_4/\alpha_1 = -0.10263 \times 10^{-2}\ (0.86187 \times 10^{-4})$		$U_{EE} = -0.20244 \times 10^{-4}$
		$\alpha_5/\alpha_1 = 0.21158 \times 10^{-3}\ (0.32339 \times 10^{-4})$		$U_{11} = -0.88109 \times 10^{-4}$
				$U_{E1} = 0.27940 \times 10^{-5}$
Square root quadratic	1	$\alpha_2/\alpha_1 = 0.13691 \times 10^{5}\ (0.10156 \times 10^{3})$	1350.749	$U_E = -0.21693 \times 10^{5}$
		$\alpha_3/\alpha_1 = -0.24979 \times 10^{2}\ (0.21458 \times 10)$		$U_1 = 0.16364 \times 10^{4}$
		$\alpha_4/\alpha_1 = -0.48686 \times 10^{3}\ (0.13166 \times 10^{3})$		$U_{EE} = -0.24979 \times 10^{2}$
		$\alpha_5/\alpha_1 = -0.10763 \times 10^{2}\ (0.16186 \times 10)$		$U_{11} = -0.48686 \times 10^{3}$
				$U_{E1} = -0.10763 \times 10^{2}$

TABLE 3. Expected return elasticities[Δ] $\eta(X_i, E_j)$.

i \ j	1	2	3	4	5	6
1	0.2743 (0.2785, 0.0283)*	0.0588 (0.0596, 0.0053)*	−0.0548 (−0.0555, 0.0049)*	−0.4419 (−0.4487, 0.0455)*	−0.0264 (−0.0267, 0.0026)*	−0.0446 (−0.0454, 0.0051)*
2	0.0887 (0.0898, 0.0078)*	0.1952 (0.1969, 0.0184)*	−0.1485 (−0.1492, 0.0179)*	−0.1106 (−0.1117, 0.0098)*	−0.0221 (−0.0223, 0.0024)*	−0.0367 (−0.0378, 0.0091)*
3	−0.2029 (−0.2054, 0.0176)*	−0.3582 (−0.3600, 0.0421)*	2.5695 (2.6048, 0.2417)*	−0.0644 (−0.0667, 0.0231)*	−0.1080 (−0.1102, 0.0161)*	−0.3911 (−0.3958, 0.0344)*
4	−1.7855 (−1.8127, 0.1832)*	−0.2994 (−0.3025, 0.0264)*	−0.0690 (−0.0716, 0.0267)*	3.9255 (3.9851, 0.4028)*	0.0861 (0.0872, 0.0078)*	−0.0599 (−0.0602, 0.0066)*
5	−0.1314 (−0.1333, 0.0128)*	−0.0763 (−0.0768, 0.0083)*	−0.1522 (−0.1533, 0.0231)*	0.1033 (0.1046, 0.0093)*	0.1434 (0.1455, 0.0142)*	−0.0967 (−0.0979, 0.0087)*
6	−0.1063 (−0.1081, 0.0122)*	−0.0607 (−0.0624, 0.0149)*	−0.2672 (−0.2704, 0.0237)*	−0.0376 (−0.0379, 0.0039)*	−0.0463 (−0.0469, 0.0042)*	0.4292 (0.4355, 0.0426)*

* Significant at the 1 per cent level.

Δ The subscripts denote:

1 = US $ 4 = Canadian $
2 = Mark 5 = Yen
3 = Franc 6 = Pound

TABLE 4. Variance elasticities[Δ] $\eta(X_i, G_{ij})$.

i \ j	1	2	3	4	5	6
1	−1.0491 (−1.0493, 0.0107)*	−0.4745 (−0.4745, 0.0020)*	0.0214 (0.0214, 0.0011)*	0.1686 (0.1686, 0.0006)*	0.1483 (0.1481, 0.0047)*	0.1147 (0.1147, 0.0014)*
2	−0.0743 (−0.0769, 0.0993)	−1.3013 (−1.3018, 0.0182)*	0.0634 (0.0631, 0.0104)*	0.0485 (0.0484, 0.0052)*	0.1928 (0.1916, 0.0435)*	0.1586 (0.1583, 0.0135)*
3	0.0297 (0.0368, 0.2689)	1.9855 (1.9868, 0.0493)*	−0.9871 (−0.9863, 0.0283)*	0.0062 (0.0065, 0.0140)	0.3673 (0.3703, 0.1177)*	0.7451 (0.7461, 0.0367)*
4	6.7332 (6.7361, 0.1117)*	2.2126 (2.2132, 0.0206)*	0.0182 (0.0185, 0.0115)	−1.4982 (−1.4980, 0.0062)*	−0.5350 −0.5336, 0.0493)*	0.0731 (0.0735, 0.0151)*
5	0.3775 (0.3788, 0.0508)*	0.4610 (0.4612, 0.0093)*	0.0534 (0.0535, 0.0053)*	−0.0425 (−0.0425, 0.0027)*	−0.8029 (−0.8023, 0.0223)*	0.1974 (0.1976, 0.0069)*
6	0.5749 (0.5734, 0.0579)*	0.7732 (0.7729, 0.0106)*	0.1048 (0.1046, 0.0060)*	0.0180 (0.0180, 0.0031)*	0.2899 (0.2892, 0.0254)*	−0.9931 (−0.9933, 0.0079)*

* Significant at the 1 per cent level.

Δ The subscripts denote:

1 = US $ 4 = Canadian $
2 = Mark 5 = Yen
3 = Franc 6 = Pound

TABLE 5. Covariance elasticities[Δ] $\eta(X, G_{jk})$

jk \ i	1	2	3	4	5	6
12	−0.8461	−0.7285	1.2454	5.1751	0.4980	0.5638
13	−0.0273	0.1718	−3.0778	0.6813	0.2155	0.3707
14	0.5195	0.1469	0.0867	−4.8357	−0.1101	0.1018
15	−0.1691	0.1160	0.6614	1.6097	−0.7702	0.4693
16	−0.0978	0.0430	0.5654	1.1370	0.1992	−0.5357
23	0.0274	0.0743	−2.1458	0.1553	0.1512	0.2640
24	0.2433	0.0330	0.0823	−2.2080	−0.0486	0.0401
25	0.0268	−0.0548	0.4441	−0.0255	−0.3308	0.1869
26	0.0195	−0.0927	0.9056	0.3782	0.2227	−0.6479
34	0.0698	0.0362	−0.3088	−0.5531	0.0024	0.0391
35	0.0031	0.0066	−0.0805	−0.0024	−0.0021	0.0113
36	0.0182	0.0404	−0.4896	0.0193	0.0434	−0.0039
45	0.3322	0.1084	0.1015	−2.8241	−0.2011	0.0728
46	0.1181	0.0376	0.0611	−0.9811	−0.0138	−0.0515
56	0.0172	0.0189	0.0886	−0.0243	−0.0364	−0.0529

Δ The subscripts denote:
1 = US $
2 = Mark
3 = Franc
4 = Canadian $
5 = Yen
6 = Pound

few exceptions. Only Canadian dollar denominated assets and franc denominated assets are elastic with respect to own expected return. Also, Canadian dollar assets are elastic with respect to changes in the expected return on US dollar assets, a not surprising result considering the close linkage between Canadian and US capital markets.[18] Except for this latter case, most assets are fairly independent of each other. Thus, unlike Frankel, we conclude that the six alternative currency denominated assets are far from perfect substitutes for each other even along the expected return dimension alone. In fact, in terms of expected return, the mark and US assets are, if anything, complements.[19]

The variance elasticities are in general larger than the expected return elasticities. The results indicate that changes in 'own' variance have uniformly a negative impact on asset demand. The most sensitive are Canadian dollar denominated assets. Not unexpectedly, we see that changes in the variance of US dollar denominated assets have a marked impact on the demand for Canadian assets. Again, this substitutability relation (that an increase in the riskiness of US assets increases the demand for Canadian assets) is not surprising considering the close linkage between Canadian and US capital markets. Also, an increase in the variance of US dollar assets mildly increases the demand for pound denominated assets. US assets, however, are not too sensitive to changes in the variance of other assets with the possible exception of the mark. We also find that an increase in the variance of mark denominated assets increases the demand for franc and Canadian dollar

denominated assets and, to a lesser extent, of pound and yen denominated assets. An increase in the variance of mark denominated assets also mildly reduces the demand for US dollar assets. Increases in the variances of both franc and Canadian dollar denominated assets do not have much impact on other assets. An increase in the variance of yen denominated assets reduces the demand for Canadian dollar denominated assets. An increase in the variance of pound denominated assets increases the demand for franc denominated assets. The overall result that emerges from Table 4 is that while there are a few strong substitutability relationships among international assets, these assets are by no means perfect substitutes for each other. In fact, in some cases, there are complementarity relationships, while in many the elasticities are very small.

Table 5 provides estimates on the sensitivity of asset holdings to changes in the covariance of returns between alternative currencies. The results show that US dollar denominated assets are not very sensitive to changes in the covariance between *other* assets. US dollar assets are somewhat sensitive to the covariance between the US dollar and the mark and, to a lesser extent, to the covariance between the US and Canadian dollar. Non-US assets, on the other hand, seem to be generally sensitive to the covariance between US and other currencies. This effect is especially pronounced for Canadian dollar and French franc assets. Canadian assets also exhibit strong sensitivity to the covariances between the Canadian and other currencies, implying significant substitutability/complementarity relationships between Canadian and other assets. On the whole, the table indicates smaller magnitudes for other elasticities. These results do not support the hypothesis of perfect substitutability among international assets.

One cannot determine whether two assets are substitutes or complements based on just one parameter. Rather, if two assets are substitutes (complements) we would expect that the cross expected return elasticities will be negative (positive) while at the same time the cross variance elasticities will be positive (negative). Although there are ambiguities in our results, we still can make the following tentative generalizations from the above tables. We find that Canadian dollar assets and pound denominated assets are substitutes for US dollar denominated assets while mark denominated assets are complements to those denominated in US dollars. Franc, Canadian dollar and pound denominated assets are substitutes to mark denominated assets. Finally, pound and franc denominated assets are substitutes.

Frankel could not reject the hypothesis that foreign and domestic assets are perfect substitutes. Many of the elasticities in our tables, including some of those noted above, are small in absolute value so that the complement and substitute relationships are rather tentative and weak. Certainly, one cannot discern any perfect substitute relationships among these international assets.

III. Conclusion

The purpose of this paper has been to estimate substitutability and complementarity relationships among financial assets denominated in six currencies. Utilizing a representative investor methodology and a flexible functional form approach, we were able to estimate the mean-variance utility function which best describes the asset holdings of a representative world investor. This in turn allowed us to estimate own and cross expected return and variance

elasticities between assets within the world portfolio. We found that we could reject the hypothesis that international assets are perfect substitutes. We also found that relative changes in variances tend to have a bigger impact on asset demand than do relative changes in expected return. In general, substitute and complementarity relationships were not very strong except in specific cases where strong relationships were expected *a priori*.

Notes

1. For a review of these models, see Dornbusch (1980) and Frankel (1983).
2. Thus, Cornell and Deitrich (1978) do not find a significant positive relationship between expected return and beta while Roll and Solnik (1977) do. In addition, Kouri and De Macedo (1978) obtain optimal negative demands for the yen and the French franc although supplies are positive.
3. Changes in expected return (and variance) can also be generated by a rolling sample technique. See Section II.A below.
4. Frankel (1982) considers a whole continuous range of values for ρ upwards from 0, on past 30.
5. See Frankel (1982, p. 260).
6. Note that in addition to expected return elasticities, there are also variance elasticities to consider in a mean-variance framework. Only if expected return and variance elasticities are of opposite sign can one make unambiguous statements about such relationships.
7. This technique was first developed and used by Khaled (1977), Berndt and Khaled (1979), and Appelbaum (1979) but in a non-portfolio riskless framework. It was subsequently generalized by Aivazian, *et al.* (1983) to a portfolio framework.
8. The separation of financial asset choice from real asset choice is obviously an oversimplification of reality. Indeed, real investment activity may substitute for financial investment for the purpose of diversification and hedging against foreign exchange risk (e.g., inventories of storable commodities may be used for this purpose). Our results will fail to capture the impact of such interactions on equilibrium holdings of international financial assets.
9. See Roberts (1975).
10. Allingham and Morishima (1973), Levy (1973), and Aivazian (1976, 1977) provide detailed discussion of the above comparative static effects.
11. A direct rather than an indirect form of the utility function is used in this paper, since in a portfolio framework share equations can be easily obtained from the direct function. Furthermore, given the lack of analogy between our comparative static equations and ordinary Slutsky equations, the application of Shephard's lemma is not straightforward, and remains to be worked out in the literature. On practical grounds, one would expect the number of arguments that appear in the indirect utility function to be very large since they include the individual means and covariances among assets returns.
12. Equation ⟨16⟩ has a similar form to the Capital Asset Pricing Model (Security Market Line) with the difference of course that it is obtained from the portfolio optimization program of a representative investor rather than from market clearing conditions.
13. Our elasticity estimates were also robust with respect to another specification of the demand system developed by Krinsky (1983) where the asset shares are treated as the dependent variable.
14. We recognize that an important limitation of this database is the use of net government liabilities as a measure of wealth. The correspondence between net government liabilities and real wealth is especially tenuous if future taxes offset the value of claims against the government. To the extent that this is true, our results will fail to capture the impact of changes in real wealth of different economies on the demand for international financial assets.
15. We realize that the use of twelve-month moving averages may limit the variability of the data. But at the same time we need sufficient input data to capture risk via the variance–covariance structure. Table 1 below provides information on the sample properties of the input data and shows that variability of the whole period (76 data points) is in fact quite large. Furthermore, our overall results were similar when eight-month moving averages were tried.
16. There is of course potential aggregation bias in estimating a representative consumer utility function from aggregate data. It is therefore important that this study be replicated on panel data.

However, most of the flexible functional form literature dealing with utility function estimation is based on aggregate data. See, for example, Christensen, *et al.* (1975), Christensen and Manser (1977), Donovan (1978), and Applebaum (1979).

17. See Berndt, *et al.* (1974) on this point.
18. See Miles (1978), and Bordo and Choudhry (1982).
19. Since we use returns rather than prices, the signs for substitutes and complements are opposite to the norm.

References

AIVAZIAN, V.A., 'The Demand for Assets Under Conditions of Risk: Comment,' *Journal of Finance*, June 1977, **32:** 927–929.

AIVAZIAN, V.A., 'On the Comparative Statics of Asset Demand,' Working Paper Series No. 124, McMaster University, 1976.

AIVAZIAN, V.A., J.L. CALLEN, I. KRINSKY, AND C.C.Y. KWAN, 'Mean-Variance Utility Functions and the Demand for Risky Assets: An Empirical Analysis Using Flexible Functional Forms,' *Journal of Financial and Quantitative Analysis*, December 1983, **18:** 411–424.

ALLINGHAM, M., AND M. MORISHIMA, 'Veblen Effects and Portfolio Selection,' in M. Morishima, *et al.*, eds., *Theory of Demand: Real and Monetary*, Oxford: Clarendon Press, 1973.

APPLEBAUM, E., 'On the Choice of Functional Forms,' *International Economic Review*, June 1979, **20:** 449–458.

BERNDT, E.R., B.H. HALL, R.E. HALL, AND J.A. HAUSMAN, 'Estimation and Inference in Non-Linear Structural Models,' *Annals of Economic and Social Measurement*, October 1974, **3:** 653–665.

BERNDT, E.R., AND M.S. KHALED, 'Parametric Productivity Measurement and Choice Among Flexible Functional Forms,' *Journal of Political Economy*, December 1979, **87:** 1220–1245.

BERNDT, E.R., M.N. DARROUGH, AND W.E. DIEWERT, 'Flexible Functional Forms and Expenditure Distribution: An Application to Canadian Consumer Demand Functions,' *International Economic Review*, October 1977, **18:** 651–675.

BORDO, M.D., AND E.U. CHOUDHRI, 'Currency Substitution and the Demand for Money,' *Journal of Money, Credit and Banking*, February 1982, **14:** 48–57.

CHRISTENSEN, L.R., D.W. JORGENSON, AND L.J. LAU, 'Transcendental Logarithmic Utility Functions,' *American Economic Review*, June 1975, **65:** 367–383.

CHRISTENSEN, L.R., AND M. MANSER, 'The Translog Utility Function and the Substitution of Meats in U.S. Consumption,' *Journal of Econometrics*, January 1977, **5:** 37–53.

CORNELL, B., AND J.K. DIETRICH, 'The Efficiency of the Market for Foreign Exchange Under Floating Exchange Rates,' *Review of Economics and Statistics*, February 1978, **60:** 111–120.

DE MACEDO, J.B., 'Portfolio Diversification Across Currencies,' in R. Cooper, P. Kenen, J.B. de Macedo, and J.V. Ypersele, eds., *The International Monetary System Under Flexible Exchange Rates*, Cambridge: Ballinger, 1982.

DONOVAN, D.J., 'Modeling the Demand for Liquid Assets: An Application to Canada,' *International Monetary Fund Staff Papers*, December 1978, **25:** 676–704.

DORNBUSCH, R., 'Exchange Rate Economics: Where Do We Stand?,' *Brookings Papers on Economic Activity*, 1980, **1:** 143–186.

FRANKEL, J.A., 'In Search of the Exchange Risk Premium: A Six-Currency Test Assuming Mean-Variance Optimization,' *Journal of International Money and Finance*, December 1982, **1:** 255–274.

FRANKEL, J.A., 'Monetary and Portfolio-Balance Models of Exchange Rate Determination,' in J.S. Bhandari, ed., *Economic Interdependence and Flexible Exchange Rates*, Cambridge: M.I.T. Press, 1983.

KHALED, M., 'Choice Among Functional Forms: A Parametric Approach Based on the Generalized Box-Cox Functional Form,' Discussion Paper, Department of Economics, University of British Columbia, 1977.

KOURI, P.J.K., 'The Determinants of the Forward Premium,' Seminar paper No. 62, Institute for International Economic Studies, University of Stockholm, August 1976.

KOURI, P.J.K., 'International Investment and Interest Rate Linkages Under Flexible Exchange Rates,' in R. Aliber, ed., *The Political Economy of Monetary Reform*, London: MacMillan Press, 1977.

KOURI, P.J.K., AND J.B. DE MACEDO, 'Exchange Rates and the International Adjustment Process,' *Brookings Papers on Economic Activity*, 1978, **1:** 397–407.

KRINSKY, I., 'Mean-Variance Utility Functions and the Investment Behavior of Canadian Life Insurance Companies,' Unpublished Ph.D. thesis, McMaster University, 1983.

LEVY, H., 'The Demand for Assets Under Conditions of Risk,' *Journal of Finance*, March 1973, **28:** 79–96.

MILES, M., 'Currency Substitution, Flexible Exchange Rates and Monetary Independence,' *American Economic Review*, June 1978, **68:** 428–436.

PRATT, J.W., 'Risk Aversion in the Small and in the Large,' *Econometrica*, January–April 1964, **32:** 122–136.

ROBERTS, G.S., 'Lancaster's New Demand Theory: Its Application in Portfolio Analysis,' *Journal of Economic Literature*, March 1975, **13:** 45.

ROLEY, V.W., 'A Theory of Federal Debt Management,' *American Economic Review*, December 1979, **69:** 915–926.

ROLL, R., AND B. SOLNIK, 'A Pure Foreign Exchange Asset Pricing Model,' *Journal of International Economics*, May 1977, **7:** 161–180.