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Miller's Irrelevance Mechanism: A Note

VAROUJ A. AIVAZIAN and JEFFREY L. CALLEN*

THIS NOTE ATTEMPTS TO clarify Miller's [12] capital structure irrelevance mechanism using a geometric (Edgeworth box diagrams) presentation. Such an exercise is useful because Miller's proposition is founded on analytically complex equilibrium arguments that tend to mask the intuition and logic behind his result. Since Miller's original discussion unnecessarily assumed a zero tax on equity income and since Miller did not prove his clientele effects rigorously, we also focus our attention on the DeAngelo and Masulis [8, 9] (henceforth D-M) generalization of the Miller model.¹ The D-M framework is also important because it assumes a proportional tax code. As we shall see, whether the tax code is proportional or progressive is an important factor in understanding Miller's proposition.

I. Preliminaries

Miller argued that the following three results would hold in a world with personal and corporate taxes but no bankruptcy costs: (a) capital structure is irrelevant for the firm; (b) there is a unique debt-equity ratio for the aggregate corporate sector; and (c) investors partition themselves into clienteles since there is a unique relationship between the individual's marginal personal tax rate and his or her holdings of corporate securities. In their formal proof of the Miller proposition, D-M assume that there is at least one investor in each of the following tax brackets:

$$\begin{aligned} \text{T1: } (1 - t_{PB}^i) &> (1 - t_{PE}^i)(1 - t_c), \\ \text{T2: } (1 - t_{PB}^i) &= (1 - t_{PE}^i)(1 - t_c), \\ \text{T3: } (1 - t_{PB}^i) &< (1 - t_{PE}^i)(1 - t_c), \end{aligned} \quad (1)$$

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¹ DeAngelo and Masulis [8] reestablish the Miller results for a world of uncertainty where security markets are complete. Auerbach and King [3] derive conditions, in a mean-variance uncertainty framework with differential personal taxes, for Miller's results to hold. Once "tax-spanning" conditions (see Auerbach [2]) are relaxed, the Miller results no longer must hold. See DeAngelo and Masulis [9], Dammon [7], and Ross [15]. DeAngelo and Masulis [9] show that there will be an optimal internal capital structure when there are leverage-related costs, due to the inability of firms to capitalize on tax deductions in excess of earnings in bankruptcy states as well as on the existence of potential tax-shield substitutes to debt.

where t_{PB}^i and t_{PE}^i are the i th investor's personal tax rate on debt and equity incomes, respectively, and t_c is the corporate tax rate. D-M assume these tax rates to be constant and exogenously determined by the tax code. They then proceed to show that, in equilibrium, the following condition must hold at the firm level:

$$P_B(s) = P_E(s)(1 - t_c), \quad (2)$$

where $P_B(s)$ and $P_E(s)$ are the current market prices per state s dollar of before-personal-tax debt and equity income, respectively. The equilibrium condition (2) states that the ratio of these prices should equal the relative tax-minimizing effectiveness of debt versus equity to the firm, namely, $1 - t_c$.

As for the individual investor, he or she is assumed to solve the utility-maximizing program

$$\begin{aligned} \text{Max } U^i\{\{Y^i(s)\}\} \\ B^i(s), \quad E^i(s) \end{aligned} \quad (3)$$

subject to

$$P_B(s)B^i(s) + P_E(s)E^i(s) = W^i \quad (4)$$

$$Y^i(s) = (1 - t_{PB}^i)B^i(s) + (1 - t_{PE}^i)E^i(s) \quad (5)$$

$$Y^i(s), \quad B^i(s), \quad E^i(s) \geq 0, \quad (6)$$

where $B^i(s)$ denotes investor i 's debt income in state s , $E^i(s)$ the equity income in state s , W^i the initial wealth, and $Y^i(s)$ the (personal) after-tax income in state s . Solving the investor's program leads D-M to conclude that each investor's optimal portfolio holdings are a function of his or her personal tax bracket. Specifically, the after-tax yields (given equilibrium prices) for debt and equity satisfy the relationships

$$\frac{(1 - t_{PB})}{P_B(s)} > \frac{(1 - t_{PE})}{P_E(s)} \quad \text{for investors in tax bracket T1,} \quad (7a)$$

$$\frac{(1 - t_{PB})}{P_B(s)} = \frac{(1 - t_{PE})}{P_E(s)} \quad \text{for investors in tax bracket T2,} \quad (7b)$$

and

$$\frac{(1 - t_{PB})}{P_B(s)} < \frac{(1 - t_{PE})}{P_E(s)} \quad \text{for investors in tax bracket T3.} \quad (7c)$$

Hence, investors in tax bracket T1 demand only debt, those in brackets T2—the so-called marginal investors—are indifferent between debt and equity, and the investors in bracket T3 demand only equity. These are, in short, Miller's clientele.

II. The Geometrics of Equal Access and Progressive Taxes

To understand Miller's results, it is useful to consider first a tax code that allows investors unrestricted scope for tax minimization.² Specifically, we shall initially assume that the tax code has the following characteristics:

1. The tax code allows investors unrestricted scope for tax minimization. There are no constraints on the purchase or short sale of personal or corporate securities.
2. Interest on personal or corporate debt is tax deductible.
3. Personal tax rates are continuously progressive and convex in income.
4. The corporate tax rate is constant and identical for all firms.

This tax system, together with the assumptions of competitive and frictionless markets, provides a tax-world analogue to the Modigliani and Miller [13, 14], Stiglitz [16], and Fama [10] equal-access assumption. In addition to the above, we shall also assume a world of perfect certainty; this assumption simplifies the geometrics but can be modified.

To see the implications of the above assumptions to Miller's proposition, let us consider tax-minimizing trades between two representative investors i and j , who are initially in tax brackets T_1 and T_3 , respectively. In Figure 1, the dimensions of the Edgeworth box $O; T_0; R$ are determined by the total pretax debt and equity incomes available to individuals i and j given their initial wealth levels and the market prices of debt and equity. (See equations (4) and (5).) Let the lines denoted by IAI depict each individual's iso-after-tax income curves; i.e., each IAI _{i} curve defines the locus of pretax debt and equity incomes that provide a constant level of after-tax income Y^i . Movements within the box in the northeasterly direction represent increases in after-tax income (and hence utility) for i , while movement in the southwesterly direction represent increases in after-

² The government plays a limited role in the Miller model. Specifically, the government issues a tax code resulting in a heterogeneous tax rate system. After introducing the tax code, the government disappears, and firms and individuals engage in tax-minimizing exchanges (of securities). The extent to which tax-minimizing exchanges take place is a function of the restrictions imposed by the tax code. On one end of the continuum is a tax code that allows for all exchanges. This tax code tends to a homogeneous one. On the other extreme, the tax code could remove all scope for tax-minimization activity by dictating each corporation's debt-equity mix and each individual's holdings. Corporate capital structure matters trivially here, and there is a clientele effect. The Miller model lies between these extremes by precluding investors (to some extent) from undertaking other sorts of tax-minimizing activities so that the corporate sector becomes a viable tax intermediary. Note also that, while tax-minimizing activity is a positive-sum game among members of the private sector, it is a zero-sum game once the government is incorporated into the analysis. It is unlikely that, if the private sector could circumvent a progressive tax system and reduce its tax obligations, government would simply not react. On the contrary, one might expect the government to enter into a game with the private sector by restructuring the tax system. This brings in dynamic game-theoretic considerations involving the interaction of government policy variations and Le Chatelier-like reactions from the private sector, issues that are beyond the scope of this paper. For one study that is concerned with the interaction of government policy variations and private-sector responses in a dynamic setting, see Fischer [11].

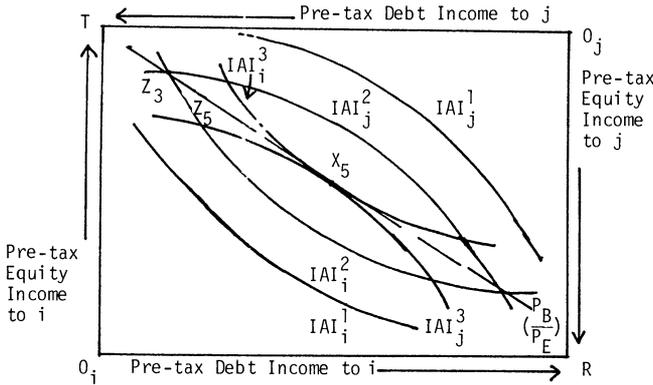


Figure 1. Gains from Exchanging Securities with Nonlinear and Convex Iso-after-Tax Income Curves:

$$IAI_i^1 < IAI_i^2 < IAI_i^3$$

$$IAI_j^1 < IAI_j^2 < IAI_j^3$$

tax income (and utility) for j . The slope of an IAI_i curve is defined by the equation

$$MRS_{BE}^i = \frac{\partial Y^i}{\partial B^i} / \frac{\partial Y^i}{\partial E^i} = \frac{1 - t_{PB}^i - B^i t_{PB}^{i'}}{1 - t_{PE}^i - E^i t_{PE}^{i'}} \tag{8}$$

which represents investor i 's marginal rate of substitution of pretax debt for pretax equity income where $t_{PB}^i = \frac{dt_{PB}^i}{dB^i}$ and $t_{PE}^i = \frac{dt_{PE}^i}{dE^i}$. Note that t_{PB}^i and t_{PE}^i are assumed to be continuously progressive (convex) functions of B^i and E^i , respectively.³

³ Equation (8) can be obtained from a framework similar to D-M's, where individual i 's choice problem (given certainty) becomes equations (3), (4), and (5) (without the state variables), with the additional requirements that

$$Y^i \geq 0, \quad E^i \geq 0, \quad B^i \geq 0,$$

$$t_{PB}^i = t_{PB}^i(B^i), \quad t_{PE}^i = t_{PE}^i(E^i),$$

and

$$t_{PB}^{i'} > 0, \quad t_{PB}^{i''} > 0, \quad t_{PE}^i > 0, \quad t_{PE}^{i''} > 0.$$

This formulation acknowledges the fact that the tax code is complex. Not all taxable characteristics are tradable so that the tax functions t_{PB}^i and t_{PE}^i may differ across individuals. On the other hand, one cannot ignore the fact that tax rates are a function of some marketable characteristics, e.g., capital market variables. The analysis could be extended to include other taxable income.

Dammon [7] analyzes the implications to capital structure relevance of the interaction between progressive personal taxes and uncertainty.

Strict convexity of an IAI curve requires that $\frac{d(MRS_{BE}^i)}{dB} < 0$ or, equivalently,

$$-(1 - t_{PE} - t_{PE}^i E)(2t_{PB}^i + t_{PB}^{i''} B) + (1 - t_{PB} - t_{PB}^i B)(2t_{PE}^i + t_{PE}^{i''} E) \frac{dE}{dB} < 0.$$

One (of many) relatively reasonably sufficient conditions for the latter inequality to be satisfied is

$$t_{PB}^i > 0, \quad t_{PB}^{i''} > 0, \quad t_{PE}^i > 0, \quad t_{PE}^{i''} > 0.$$

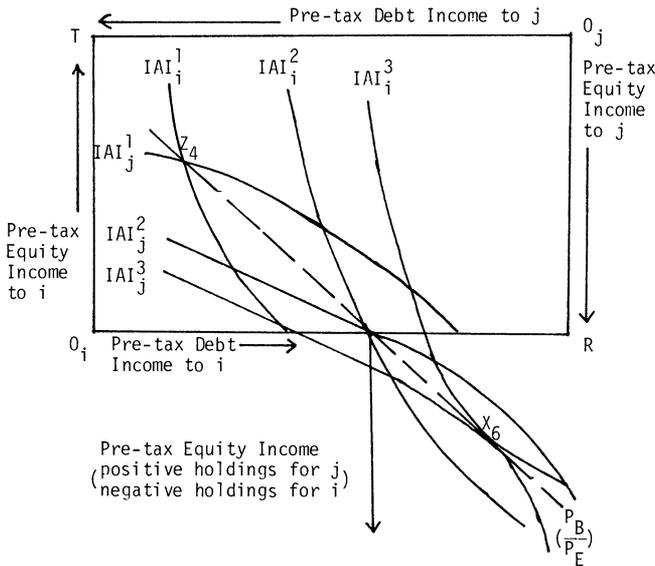


Figure 2. Gains from Exchanging Securities with Nonlinear and Convex Iso-after-Tax Income Curves:

$$IAI_i^1 < IAI_i^2 < IAI_i^3$$

$$IAI_j^1 < IAI_j^2 < IAI_j^3$$

Whether an individual is initially in a relatively high debt- or equity-tax bracket is a function of the individual's initial endowment of debt and equity. As these endowments change, relative tax brackets, given by the slopes of the IAI curves, adjust also. Thus, at Z_3 in Figure 1, individual i is in a relatively high equity-tax bracket compared with individual j ; i.e., $MRS_{BE}^i > MRS_{BE}^j$. However, as securities are traded, relative marginal tax rates adjust.

Since marginal rates of substitution are not equal at Z_3 , i and j can increase their prospective after-tax incomes by trading. Thus, i will acquire more debt in exchange for equity to j . They will trade along some equilibrium price ray (P_B/P_E) until they reach X_5 , at which point both i and j have attained higher iso-after-tax income curves, IAI_i^3 and IAI_j^3 , respectively. At X_5 , marginal rates of substitution are equalized and (tax-minimizing) trading opportunities exhausted. Thus, financial instruments are reallocated to provide equalization of marginal after-tax yields for all assets across all individuals.

Of course, point X_5 does not have to be internal to the Edgeworth box $0_i T_0_j R$. Suppose that i has a much lower initial wealth level than j , so that the initial disequilibrium point is at Z_4 in Figure 2. Investor i can issue equity to j (or sell existing equity short) to buy more debt. There will be an equilibrium (at X_6 in Figure 2) with a positive tax bill.⁴ With short selling, taxes are minimized as (debt and equity) incomes adjust and relative tax rates are equalized.

In equilibrium, potential gains from tax-minimizing trades are exhausted.

⁴ With short selling, the box, in effect, gets extended outside the positive quadrants.

Formally,

$$\text{MRS}_{BE}^i = \frac{P_B}{P_E} = 1 - t_c = \text{MRS}_{BE}^{\text{firm}K} \quad (9)$$

for all i and K , where i stands for any individual and K stands for any firm. Marginal personal tax rates as well as security prices adjust—as capital market instruments are exchanged—to satisfy these conditions.⁵ This equilibrium can come about via trades of either personal or corporate securities. Indeed, there is no reason to borrow or lend through the firm if private investors themselves can costlessly engage in such activities, so corporate leverage is irrelevant.⁶ Note, however, that there is a uniquely optimal total (corporate plus personal) debt-equity mix for each investor.

Thus, with equal access and a progressive tax code, corporate leverage is irrelevant, but, unlike Miller's proposition, there are no corporate tax clienteles. However, there exists an equilibrium aggregate (corporate plus personal) tax-minimizing debt-equity ratio.

III. The Geometrics of Proportional Taxes

The previous discussion highlighted the importance of two strong assumptions, namely of equal access and a progressive tax code. We will now relax the latter by assuming a proportional tax code with heterogeneous (across individuals) but constant (for any individual) tax rates.

Consider two representative inframarginal investors, i in tax bracket T1 and j in tax bracket T3, who, in view of (7a) and (7c), have different marginal rates of substitution and can increase their prospective after-tax incomes by trading securities. Consider the Edgeworth box in Figure 3. The slope of an IAI_i curve is now defined by $\text{MRS}_{BE}^i = \frac{(1 - t_{PB}^i)}{(1 - t_{PE}^i)}$; the IAI_i (and IAI_j) are now straight lines since tax rates are constant and we are holding current consumption fixed. Suppose i and j are initially at Z_1 . They can increase their prospective after-tax incomes by trading securities until they reach the corner point X_0 , at which point both have attained higher iso-after-tax income curves, IAI_i^3 and IAI_j^3 , respectively. At X_0 , the individuals specialize, with i holding debt and j holding equity.

D-M constrain the system—see inequalities (6)—to prevent further trading (short selling) from X_0 even though marginal rates of substitution are not equalized at X_0 . With no constraints, however, i and j can engage in short selling (of corporate or personal securities), moving from X_0 to higher iso-after-tax income curves at X_1 , X_2 , etc., into the bottom right-hand box in Figure 3. In moving from X_0 to X_1 , i acquires more debt (issued or sold short by j), which is

⁵ Note that, since all firms have the same constant marginal rate of substitution between debt and equity, $1 - t_c$, they determine the price ratio of debt to equity.

⁶ Each investor's opportunity set for after-tax income is unaffected by alternative corporate debt-equity ratios. In fact, under equal access, no rents are generated by corporate debt so that the Miller supply and demand curves for corporate debt would be horizontal and coincident.

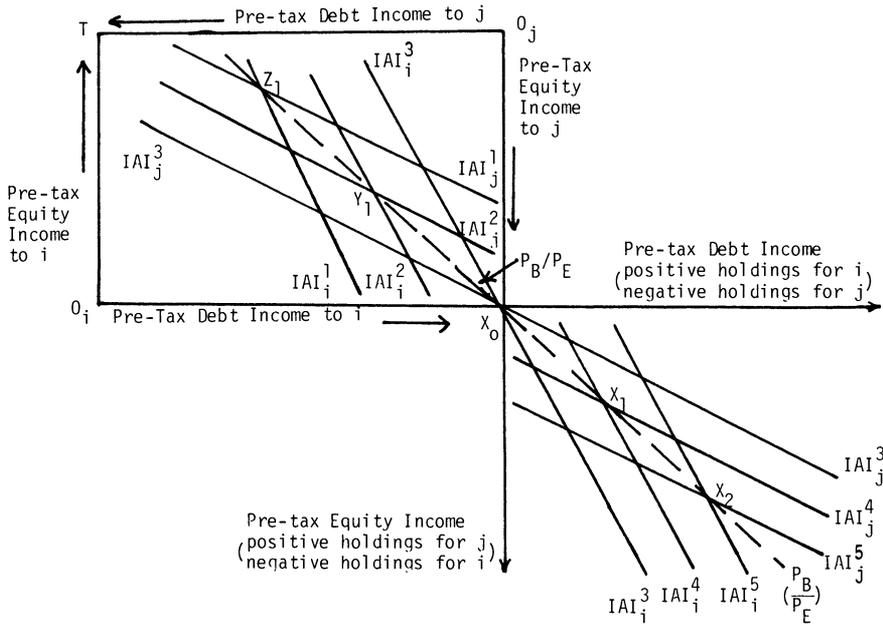


Figure 3. Gains from Exchanging Securities (at P_B/P_E Ratio) with Linear Iso-after-Tax Income Curves:

$$IAI_i^1 < IAI_i^2 < \dots < IAI_i^5$$

$$IAI_j^1 < IAI_j^2 < \dots < IAI_j^5$$

financed by issuing or short selling equity to j . Unless some mechanism constrains it, there is no equilibrium to this process.⁷ In terms of condition (9), marginal rates of substitution are not equalized. This points out the crucial importance of the progressive-tax-rates assumption to the previous irrelevance results or of constraints on short selling here.

Let us now briefly explore the implications of relaxing the equal-access assumption in the D-M model by assuming that the tax code only partially limits tax-minimizing activity. For example, suppose short sales by individuals are treated asymmetrically to long sales by the tax code. Let us, in fact, take the extreme position that individual short sales are precluded entirely. If initial endowment is at Z_2 in Figure 4, trade will take place to X_3 . At X_3 , i has no more equity and cannot issue any by assumption. However, j can still buy equity from a firm, trading in his or her debt until he or she reaches X_4 (and IAI_j^4), where his or her debt holdings are exhausted. At this point, trade stops. Corporate equity matters, and there is an obvious clientele effect. This trade in "endow-

⁷ Thus, investors with a comparative advantage for holding debt (in relatively low income-tax brackets) would hold arbitrarily large amounts of debt financed by short selling equity, and those with a comparative advantage for holding equity would finance such positions by borrowing on personal account or short selling corporate debt. See Auerbach and King [3]. An obvious constraint that would need to be imposed is that of no negative aggregate tax bill.

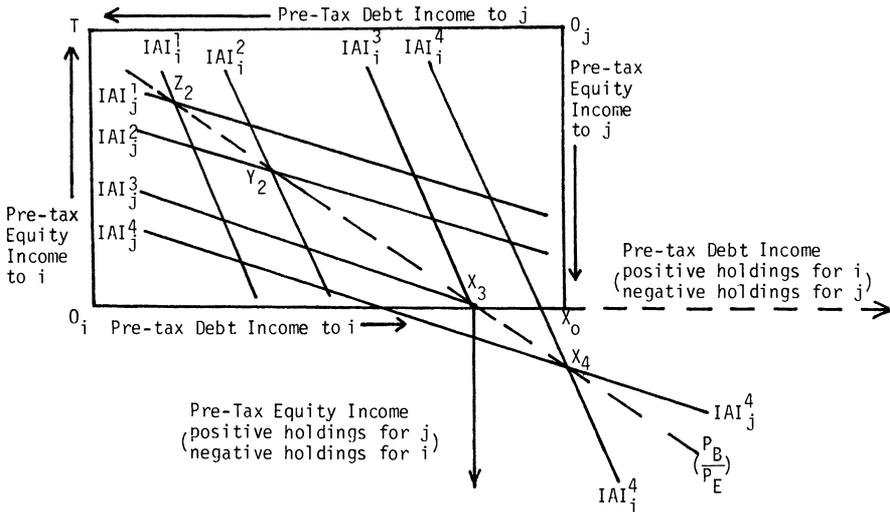


Figure 4. Gains from Exchanging Securities (at P_B/P_E) with Linear Iso-after-Tax Income Curves:

$$IAI_i^1 < IAI_i^2 < \dots < IAI_i^4$$

$$IAI_j^1 < IAI_j^2 < \dots < IAI_j^4$$

ments," made possible by corporate equity, in effect serves to enlarge the Edgeworth box. A similar argument would hold if the initial wealth distribution were at Z_6 in Figure 5 and trade took place to the vertical (rather than horizontal) side of the box to X_7 .

Suppose that we make a less drastic modification to the equal-access assumption. Specifically, suppose that individuals can issue debt on personal account, which has the same taxable characteristics as corporate debt. However, individuals cannot issue equity on personal account. Still, capital structure is relevant, and there are clienteles. To see why, reconsider Figure 4. Again, trade between individuals takes place to X_3 . At X_3 , i has no more endowed equity to trade for j 's debt, and j will have to sell his or her debt to a firm and buy corporate equity to minimize his or her tax obligations. Obviously, corporate equity matters since it enhances private tax minimization, and there is a clientele effect.

However, suppose that the initial endowment of wealth is at Z_6 in Figure 5 and trade takes place to the vertical boundary at X_7 . Here, j can issue debt on personal account to i and acquire i 's equity holdings. This process is limited, however. Trade will take place from X_7 to X_8 , at which point i 's equity holdings are exhausted. But at X_8 , j can further reduce his or her taxes by issuing even more debt to a firm and buying corporate equity. Further trade from X_8 between individuals will be impossible since further trade would require i to issue equity on personal account.⁸ Thus, the corporate sector enhances private tax minimization. Note, however, that capital structure decisions by firms to issue equity to j will result in a disequilibrium since j can keep on issuing personal debt and receive a subsidy from the government. To guarantee an equilibrium, there need to be restrictions on j 's ability to issue debt and buy corporate equity.

⁸ Again, the existence of corporate securities makes possible the expansion of the Edgeworth box.

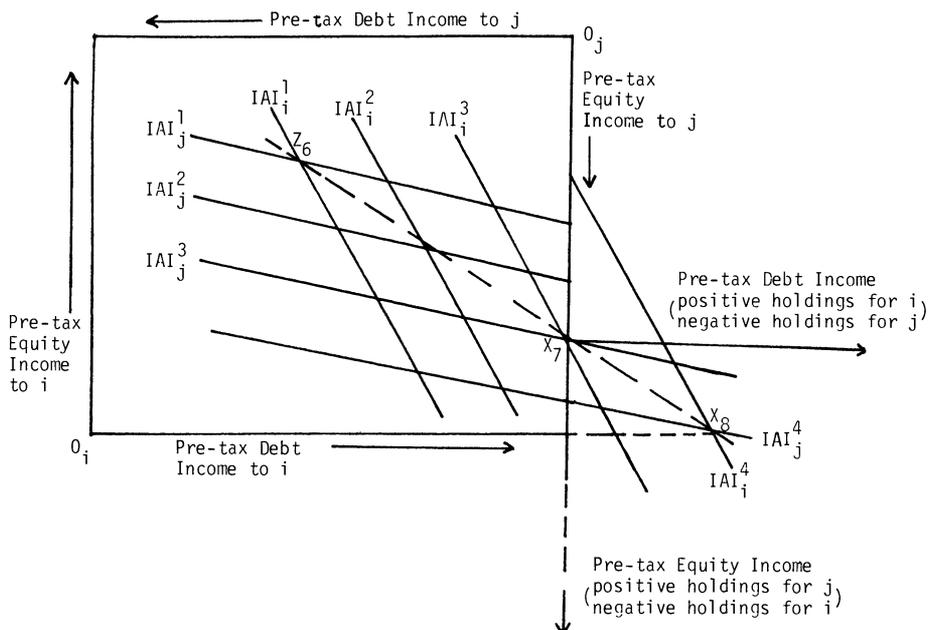


Figure 5. Gains from Exchanging Securities (at P_B/P_E) with Linear Iso-after-Tax Income Curves:

$$IAI_i^1 < IAI_i^2 < \dots < IAI_i^4$$

$$IAI_j^1 < IAI_j^2 < \dots < IAI_j^4$$

We briefly summarize the insights of the geometry so far. Utilizing Figures 1 and 2, we showed that, with equal access and a progressive tax code, corporate capital structure is irrelevant both at the firm level and in aggregate; there are no clientele effects, but there is a determinate level of aggregate personal plus corporate debt. Figure 3 showed how complete short-sale restrictions guarantee the D-M equilibrium under proportional taxes. It showed that, unless short-sale restrictions constrain tax-minimizing trades under a proportional tax code, there will be no equilibrium. Figures 4 and 5 demonstrated how corporate securities make possible tax-minimizing trades in endowments (in effect enlarging the Edgeworth box) and generate clientele effects, when there are limitations on equal access.

IV. Miller's Irrelevance Proposition

We have seen that, for the corporation to be a viable tax intermediary, limitation of equal access is required. On the other hand, this assumption is not sufficient for Miller's proposition since, if corporate securities are required for tax minimization, corporate capital structure is likely to be relevant as well. What makes any corporation's capital structure irrelevant is Miller's assumption that there are perfect corporate substitutes for any firm's securities so that tax surpluses

do not accrue to the firm.⁹ We can see the implications of this assumption by considering the following argument.

Consider an economy with investor i (in a relatively low debt-tax bracket) and investor j (in a relatively low equity-tax bracket) and two firms, 1 and 2, with identical and constant corporate tax rates, as well as earnings-before-interest-and-taxes X . Suppose that each firm initially supplies one half of the debt and equity claims of society and that investor i divides his or her (constrained) debt claims evenly between the two firms as does investor j with his or her equity claims. The private sector's total initial after-tax cash flows are

$$2[(X - rD)(1 - t_c)(1 - t_{PE}^j) + rD(1 - t_{PB}^i)]. \quad (10)$$

Suppose that firm 1 perturbs its capital structure carrying ΔD dollars less debt.¹⁰ It will be optimal for firm 2 to offset this shift by applying the reverse shift in its capital structure. This is because the contraction in the supply of debt by firm 1 will lower the (pretax) yield on debt and the after-tax cost of debt to firm 2, inducing firm 2 to issue additional debt (to firm 1's previous debt clientele) and to contract its existing supply of equity. With firms 1 and 2 identical in terms of their operating characteristics and fixed (and equal) corporate tax rates, this costless readjustment of security supplies will continue until the old equilibrium yield is reestablished, with the aggregate mix of corporate securities at the same level as before the perturbation. The tax obligations of i and j will remain unchanged.¹¹ The tax-shield reduction to each investor engendered by the activities of any one firm are recouped by the offsetting activities of another firm as the tax-minimizing aggregate corporate capital structure is maintained. Any one firm's debt-equity ratio is therefore irrelevant from the tax-minimizing perspec-

⁹ Note again that the Miller irrelevance argument is not analogous to the "equal-access" irrelevance mechanism proposed by Modigliani and Miller and generalized by Stiglitz [16, 17] and Fama [10]. As we have seen, under equal access, the types of securities that can be issued by firms can be issued by investors on personal account, so that no rents are generated by corporate debt and supply and demand curves for corporate debt are coincident. In Miller's model, tax-induced surpluses are generated by corporate debt. Miller's argument is a tax-world analogue to Fama's so-called "perfect substitutes" (without equal access) irrelevance mechanism. There are always perfect corporate substitutes for any firm's securities, and, if any firm perturbs its capital structure, its actions are fully offset by other firms.

¹⁰ To maintain its total level of financing, firm 1 will, of course, carry out an offsetting increase in its supply of equity.

¹¹ Specifically, in final equilibrium, the after-tax returns to the investors from firm 1's securities are

$$(X - rD + r\Delta D)(1 - t_c)(1 - t_{PE}^j) + r(D - \Delta D)(1 - t_{PB}^i).$$

The after-tax returns from firm 2's securities are

$$(X - rD - r\Delta D)(1 - t_c)(1 - t_{PE}^j) + r(D + \Delta D)(1 - t_{PB}^i).$$

Each investor's total after-tax return will be the same as before. The private sector's after-tax cash flows will also be the same as before the perturbation, namely,

$$2[(X - rD)(1 - t_c)(1 - t_{PE}^j) + rD(1 - t_{PB}^i)].$$

tive of each individual investor.¹² The assumption that the corporate tax rate is constant (or linear) is crucial to this result. With differential (or nonlinear) corporate tax rates, the tax-shield reduction engendered by an arbitrary decrease in debt by one firm cannot be fully offset by the activities of competitors. In equilibrium, tax surpluses from debt will accrue to inframarginal firms (those with tax rates above those for marginal firms and investors) and, hence, capital structure will be relevant for such firms.¹³ One way to generate irrelevance in the nonlinear tax case may be to specify an appropriate reaction function for the government to private-sector tax-minimizing activity. This conjecture remains to be proved.

¹² However, there is a tax-minimizing aggregate corporate debt-equity ratio for the private sector as a whole.

¹³ For further discussion of the effects of differential corporate tax rates, see DeAngelo and Masulis [9], Brealey and Myers [5] (pp. 389–90), Bartholdy, Fisher, and Mintz [4], and Aivazian and Turnbull [1]. The Miller aggregate supply of debt will be a step function in the case of constant (for any firm) but differential (across firms) corporate tax rates. Tax-induced surpluses will then accrue to inframarginal firms, which will, in the absence of offsetting costs to debt, issue only debt. For evidence of differential marginal corporate tax rates in the U.S., see Cordes and Sheffrin [6] and, for Canada, see Bartholdy, Fisher, and Mintz [4].

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