THE "UNANIMITY" LITERATURE AND THE SECURITY MARKET LINE CRITERION: THE ADDITIVE RISK CASE

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Introduction

The burgeoning literature on shareholder unanimity (e.g. Stiglitz (1970), Long (1972), Ekern and Wilson (1974), Leland (1974), Nielsen (1977), Krouse (1978), Grossman and Stiglitz (1977), and Baron (1979)) jeopardizes much of the received doctrine on capital budgeting rules.[1] Most, if not all, textbooks, for example, advocate value maximization as the premier objective for financial managers (e.g. Weston and Brigham (1978), and Van Horne (1977)). The immediate result is that these textbooks also advocate utilizing the security market line criterion for resolving capital budgeting decisions under uncertainty. The "unanimity" literature, on the other hand, has demonstrated that the value maximization objective will be acceptable to all shareholders only under very restrictive conditions. In fact, it is now known that only in specific cases will there be any objective function at all which is unanimously supported by all shareholders. One of the most important ramifications of this literature is to severely circumscribe the potential usefulness of the security market line criterion for resolving capital budgeting decisions under uncertainty.

This note will demonstrate that, at least when the return on industry investment has an additive risk structure, even a firm with some monopoly power in the real asset market should use the security market line criterion to evaluate potential investment projects. Not only is the security market line the optimal and unanimously acceptable criterion for project evaluation in the additive risk case, but this criterion is independent of the industry structure in which the firm finds itself. The remainder of the paper is devoted to proving these contentions.

The Model

The authors' formal analysis utilizes the familiar one period Sharpe-Lintner-Mossin Capital Asset Pricing Model (CAPM) where individual investors are price-takers in frictionless markets for financial claims. This model predicts that the equilibrium value of firm j is given by:[2]

$$V_j = Rf^{-1}[E(D_j) - \lambda \sum_{k=1}^{n+1} \sigma_{jk}] \quad \ldots \quad (1)$$

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where

D_j = the stochastic total end-of-period cashflow to firm j,

$j = 1, ..., n + 1$

Rf = one plus the risk-free borrowing-lending rate

$$\sigma_{jk} = \begin{cases} \text{var}(D_j), & j = k \\ \text{cov}(D_j, D_k), & j \neq k \end{cases}$$

λ = market price of risk assumed constant[3]

Market structure is introduced into the formal analysis by assuming that n identical Cournot firms belong to the same industry. The subscript n+1 denotes all other firms in the economy. Each firm j is assumed to have existing assets which yield, in the absence of further investment, the cash-flows D_j. A new (and risky) industry investment opportunity arises in the current period whose return is in general not independent of past investment behaviour. Uncertainty of the firm's rate of return on this new investment is introduced via an industry average rate of return function $r(I,\theta)$ where r is one plus the rate of return on investment, I is total industry investment and θ is a random variable. In the additive risk case $r(I,\theta) = r(I) + \theta$ whereas in the multiplicative risk case $r(I,\theta) = \theta r(I)$.

A Sufficient Condition for Unanimity

There are two known conditions which are sufficient to yield a unanimously supported objective function in incomplete markets. One of these is the competitivity assumption where each firm is assumed to act as if a change in its output does not affect the value of contingent claims on the output mix of all firms in the economy for all states of nature.[4] However, the competitivity assumption is difficult to rationalize unless firms are competitive[5] and the authors wish to consider the more general case which allows the firm some monopoly power in the real asset market. Therefore, the alternative assumption will be made that all individuals are in long-run steady state equilibrium with respect to their portfolio holdings. This so called *ex post* unanimity condition ensures that shareholders will be unanimous in their decision to a proposed change in output, although the unanimous response is not generally consistent with value maximization.

The *ex post* unanimity assumption will now be used to derive the firm's optimal investment policy. But, rather than begin from first principles, one can borrow from the extant literature. Specifically, Nielsen (1976, p.593) has shown –see his equation (3) – that investor p prefers an increase in firm L's investment (I_L) from its equilibrium value if[6]

$$\sum_{k=1}^{n} \sum_{\theta} \Pi_p(\theta)\, U^1_{p\theta} \alpha_{pk} \frac{\partial X_k(\theta)}{\partial I_L} > 0 \quad \ldots \quad (2)$$

where

θ = state of nature

$U^1_{p\theta}$ = investor p's state dependent marginal utility function

$\Pi_p(\theta)$ = investor p's subjective probability assessment for state θ

$X_k(\theta)$ = state dependent output produced by firm k

α_{pk} = fraction of the k'th firm owned by investor p (in equilibrium)

Assuming the CAPM assumptions are operative, it can be shown that the equilibrium value of firm k is[7]

$$V_k = \sum_{\theta} \Pi_p(\theta)\; U^1_{p\theta}\, X_k(\theta) \quad \ldots \quad (3)$$

so that investor p prefers I_L provided

$$\sum_{k=1}^{n} \alpha_{pk} \frac{\partial V_k}{\partial I_L} > 0 \quad \ldots \quad (4)$$

Since each investor holds the same proportions of every firm in the economy in the CAPM equilibrium ($\alpha_{pk} = \alpha_p$ all k) (Mossin, 1973, pp.69–70), (4) becomes

$$\alpha_p \sum_{k=1}^{n} \frac{\partial V_k}{\partial I_L} > 0 \quad \ldots \quad (5)$$

Thus, all shareholders prefer the change in I_L if p does. Finally, substituting equation (1) into (5), it is readily seen that the equilbrium unanimously preferred investment must satisfy the optimality condition[8]

$$\sum_{k=1}^{n} \left[\frac{\partial ED_k(I_k)}{\partial I_L} - \lambda \sum_{j=1}^{n+1} \frac{\partial Cov(I_j, I_k)}{\partial I_L} \right] = 0 \quad \ldots \quad (6)$$

It follows immediately from equation (6), that each firm's optimal investment criterion is independent of its market structure. This can be seen by noting that equation (6) sums over all firms in the industry netting out firm specific behaviour.[9] One can now proceed to evaluate equation (6), first for the additive risk case which has been much neglected by the literature, and then for the multiplicative risk case.

The Optimal Investment Criterion

(i) The Additive Risk Case

If all the firms in the industry invest a total of I dollars, and the industry's rate of return function is additive, then firm k's expected end of period cashflow from the investment (i.e. net of cashflows from existing assets) is

$$ED_k(I_k) = E\left((r(I) + \theta - Rf)\, I^k\right) = (r(I) + E(\theta) - Rf)\, I^k \quad \ldots\ldots \quad (7)$$

Thus,

$$\frac{\partial ED_k(I_k)}{\partial I_L} = \begin{cases} r(I) + r'(I)\, I^k + E(\theta) - Rf & L = k \\ r'(I)\, I^k & L \neq k \end{cases} \quad \ldots\ldots \quad (8)$$

where $r'(I) = \dfrac{dr(I)}{dI}$

Summing equation (8) over all firms in the industry yields

$$\sum_{k=1}^{n} \frac{\partial ED_k(I_k)}{\partial I_L} = r(I) + r'(I)\, I + E(\theta) - Rf$$

$$= r(I)\left(1 + \frac{1}{\epsilon}\right) + E(\theta) - Rf \quad \ldots\ldots \quad (9)$$

where ϵ is the elasticity of the industry average rate of return function.[10]

The covariance between firm j and k's terminal cashflows, after both firms have undertaken their investments, is

$$Cov(I_j, I_k) \equiv Cov\{(D_j + r(I)\, I^j + \theta I^j),\ (D_k + r(I)\, I^k + \theta I^k)\}$$

$$= \sigma_{jk} + I^k \sigma_{j\theta} + I^j \sigma_{\theta k} + I^j I^k \sigma_\theta^2 \quad \ldots\ldots \quad (10)$$

where

$$\sigma_\theta^2 = var\ (\theta)$$

$$\sigma_{\theta k} = cov\ (\theta, D_k) = \sigma_{k\theta}$$

Therefore

$$\frac{\partial Cov(I_j, I_k)}{\partial I_L} = \begin{cases} \sigma_{j\theta} + I^j \sigma_\theta^2 & L = k \\ 0 & L \neq k \end{cases} \quad \ldots\ldots \quad (11)$$

for j = 1, ..., n + 1.

Summing equation (11) twice, once over all firms in the economy and then over

all firms in the industry, gives

$$\sum_{k=1}^{n} \sum_{j=1}^{n+1} \frac{\partial Cov\ (I_j, I_k)}{\partial\ I_L} = \sum_{j=1}^{n+1} \sigma_{j\theta} + I\sigma_\theta^2 \quad \ldots\ldots\ldots\ldots \quad (12)$$

Substituting equations (9) and (12) into equation (6) yields the optimality condition:

$$r(I)\ (1 + \tfrac{1}{\epsilon}) + E(\theta) = Rf + \lambda\ [\sum_{j=1}^{n+1} \sigma_{\theta j} + I\sigma_\theta^2] \quad \ldots\ldots\ldots\ldots \quad (13)$$

This condition lends itself to the following familiar interpretation: the firm should invest until the expected marginal return on its investment is equal to the expected marginal opportunity cost of the investment adjusted for risk. In this additive risk case, the opportunity cost of the investment is nothing more than the security market line. To see this note that the right-hand side of equation (13) can be rewritten as

$$Rf + \lambda Cov\ [r(I) + \theta,\ \sum_{j=1}^{n+1} (D_j + (r(I) + \theta)\ I^j)] \quad \ldots\ldots\ldots\ldots \quad (14)$$

where the covariance term relates the average return on the investment and the return on the market portfolio.[11]

Although the authors' derivation of the security market line criterion is an apparently straightforward exercise, it is worth remembering that this criterion was not generated from the usual value maximizing approach. Rather, this result depends primarily on the assumption of an additive risk structure. By contrast, where the risk of the investment is multiplicative, an alternative, but, unfortunately, rather complex investment criterion becomes operative.[12]

(ii) The Multiplicative Risk

Employing the same techniques as in the additive risk case, one finds that

$$\sum_{k=1}^{n} \frac{\partial ED_k}{\partial I_L} = E(\theta)\ r(I)\ (1 + \tfrac{1}{\epsilon}) - Rf \quad \ldots\ldots\ldots\ldots \quad (15)$$

and

$$\sum_{k=1}^{n} \sum_{j=1}^{n+1} \frac{\partial\ Cov\ (I_j, I_k)}{\partial I_L} = r'\ (I)I \sum_{k=1}^{n} \sigma_{\theta k} + (r(I) + r'\ (I)I) \sum_{j=1}^{n+1} \sigma_{\theta j}$$

$$+ (2r(I)r'\ (I)I^2 + r(I)^2 I)\ \sigma_\theta^2 \quad \ldots\ldots \quad (16)$$

Substituting equations (15) and (16) into the optimality condition (Equation (6)) gives

$$E(\theta)\ r\ (I)\ (1 + \tfrac{1}{\epsilon}) = Rf + \lambda[\frac{r(I)}{\epsilon} \sum_{k=1}^{n} \sigma_{\theta k} + r(I)\ (1+\tfrac{1}{\epsilon}) \sum_{j=1}^{n+1} \sigma_{\theta j}$$

$$+ (2r(I)r'(I)\,I^2 + r(I)^2 I)\,\sigma_\theta^2] \quad \ldots\ldots\ldots\ldots \quad (17)$$

This time the optimal investment criterion is not only a function of the covariance between the project return and the market portfolio, but also of the elasticity of the industry return function and the own variance of the project. Unfortunately, it is impossible to specify *a priori* whether the opportunity cost of capital in the multiplicative risk case (the right hand-side of equation (17)) is greater or less than the opportunity cost of capital specified by the security market line criterion. One can readily concoct examples where either result obtains depending on the elasticity of the industry return function.

Conclusions

The authors' conclusions are straightforward. In general, it is difficult to specify whether or not risk is additive, multiplicative or some other structure. Nevertheless, it has been demonstrated that, in the simple and potentially useful additive risk case, the security market line criterion is still optimal and unanimously supported by all investors even though the firm has market power in the real asset market. Moreover, this criterion is independent of industry structure.

NOTES

1 The Baron piece provides a comprehensive review of a very confusing literature.

2 One should not confuse the perfect (competitive) capital markets assumptions underlying the CAPM with competition in the real asset market. A firm can readily possess market power in the real asset market and yet its securities trade in frictionless and competitive capital markets. In short, the CAPM can be used to value firms which posses monopoly power in the real asset market.

3 This assumption, which is maintained throughout the paper, is consistent with constant absolute risk aversion on the part of investors. This assumption simplifies the mathematics but is not strictly necessary as Ekern and Wilson, and Baron have shown.

4 Technically, the competitivity assumption states that if any firm increases its output by some percentage in each and every state of nature, the value of the firm increases by the same percentage. See Baron, for example.

5 In fact, it is difficult to rationalize unless the firm is in a competitive market structure in the output market and its production function exhibits constant returns to scale.

6 Because of the *ex post* unanimity assumption, the last term in Nielsen's equation (3) is zero and is therefore absent from the authors' equation (2). Equation (2) also makes the usual Cournot-Nash assumption that all n firms act as if their investment activity has no impact on their rival's optimal investment decisions.

7 See Ekern and Wilson's equations (5) and (10). The Ekern and Wilson unanimity criterion differs from those of Krouse, Nielsen, and Baron since Ekern and Wilson assumed that firm j's investment has no impact on all other firms in the economy. Therefore, the Ekern and Wilson unanimity rule (their equation (11)) is the same as the authors' equation (6) without the initial summation sign and L = k.

8 In what follows, expected cashflows and the covariances are written explicitly as functions of the kth firm's investment I_k. Note that

$$\sum_{k=1}^{n} I_k = I.$$

9 The distinction between the firm and the industry becomes blurred since the unanimously supported objective takes into account the portfolio holdings of each investor. But, in the Mean-Variance CAPM framework, as noted, each shareholder owns the same fraction of all firms in the economy. Therefore, all firms in the industry are led, as if they were colluding, to take their investors' total interest into account.

10 That is $\epsilon = \dfrac{r'(I)}{I/r(I)}$

11 The market portfolio now includes the new investments undertaken by the industry.

12 The complexity of the multiplicative risk case is relatively well-known. See Baron, for example. What the literature seems to have neglected is the potentially important additive risk case. The authors briefly derive the multiplicative risk case to contrast the two results.

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