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# CORPORATE LEVERAGE AND GROWTH

## The Game-Theoretic Issues

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The equilibrium value of a levered firm facing growth opportunities is shown to involve the valuation of a lottery over (cooperative) games rather than a lottery over specific monetary outcomes. In the absence of assumptions about negotiating risk, the value of the firm's claims is seen to be ambiguous even with zero transactions costs. This ambiguity is compounded if the core of the game is empty. This paper rationalizes specific financial instruments and institutions as means for attenuating negotiation costs and core existence problems. Furthermore, the valuation of these instruments requires determining the certainty-equivalent of a lottery over games.

## 1. Introduction

Is corporate debt policy meaningful in a world in which capital markets are perfect, efficient, and complete? Does the firm's existing capital structure cause it to alter its investment policies in such a frictionless framework? Intuition and years of being subjected to the M–M theorem in a plethora of restatements suggests that the answer to both questions is no. Nevertheless, Myers (1977) argues that, in a perfect capital market, the existence of risky debt in the firm's capital structure may have an adverse incentive effect on the firm's investment behaviour in the presence of growth opportunities. Should the debt mature after these growth opportunities are manifest, bondholders will share with stockholders in any future profitable investments. This externality to shareholding, Myers maintains, curtails the firm's incentive to invest the proper amount in such states of the world. Therefore, in a tax world, the tax advantage of additional debt in the firm's capital structure is offset by this externality yielding, perhaps, an interior solution to the optimal capital structure problem and, thus, a potentially meaningful approach to corporate debt policy.

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This paper has two tasks. The first, and the *least* important of the two, is to demonstrate, utilizing a standard finance model, that the value of the firm and its investment policies are independent of its initial capital structure in a perfect capital market. This is so even in the presence of growth. The reason is simply that it is in shareholders' own interest to internalize the disincentive externality induced by leverage. In a perfect capital market, any number of such internalization procedures are available.[1]

The second task, and the most important, will be to argue that Myers' example raises some fundamental and, heretofore, unexplored issues in finance which will ultimately necessitate the development of new models for the pricing and valuation of risky assets. Such models are necessary because the choice variables in Myers' example are not lotteries over wealth but lotteries over games and the standard expected utility framework, *a fortiori* its derivatives such as the Capital Asset Pricing Model, is unequipped to handle such choice variables.

The implications of this observation for finance theory are complex. On the one hand, we are able to conceptualize a game-theoretic model for which Modigliani and Miller (M–M) (1958, 1963) are still correct even with growth. On the other hand, in the general game-theoretic framework, we show by example that the firm's investment and financing decisions could be interdependent even assuming perfect capital markets.

In what follows, section 2 presents Myers' argument and the counter-argument in the context of a standard two period valuation model. Section 3 analyses the game-theoretic complexities inherent in the valuation of the levered firm. Section 4 provides an example for which financing and investment decisions are interdependent. Section 5 applies these game-theoretic insights to a number of important financial instruments and issues, namely, the call provision, the convertible bond, bankruptcy, and mergers.

## 2. Leverage and externalities

Myers contrasts the investment decision of an all-equity firm with that of an equivalent firm partially financed with debt. Two scenarios are distinguished: one where debt matures before the firm's discretionary investment decisions are implemented, and the other where debt matures after such decisions. Only in the latter case does debt impose a potential externality on the firm's optimal investment strategy, and, therefore, our analysis will be limited to that type of leverage.

[1]One can readily interpret Myers (1977) as agreeing with this statement (see the discussion on p. 149 of his article) but, arguing that, once transaction costs are introduced à la Jensen and Meckling (1976), these internalization procedures become difficult and costly. Our primary concern is to show that, even in the absence of transactions costs, the Myers problem raises fundamental issues in the valuation of the levered firm. By analysing these internalization processes, the valuation problems become apparent.

### 2.1. The all-equity firm

Following Myers, we start with a firm which is entirely equity-financed at $t=0$ and analyze its investment decision at $t=1$, when 'objective' uncertainty governing the firm's investment opportunities is resolved. The firm will invest an amount $I$ in period one as long as $V(S) \geqq I$, where $V(S)$ is the value of the investment at $t=1$ contingent on state $S$. Without loss of generality we divide all states of nature at $t=1$ into two disjoint regions, $S \geqq S_a$ and $S < S_a$, where the former region corresponds to states favourable to the investment $I$ and the latter region corresponds to states unfavourable to the investment.[2] $S_a$ is the breakeven point corresponding to $V(S)=I$. Therefore, given frictionless financial markets, the value of the all-equity firm at $t=0$ is

$$V_A = \int_{S_a}^{\infty} q(S)[V(S)-I]\,\mathrm{d}S, \tag{1}$$

where $q(S)$ is the equilibrium price at $t=0$ of a dollar delivered at $t=1$ if state $S$ occurs.

### 2.2. The levered firm

In this case the firm is partially financed with debt in period zero with a promised payment of $P$ dollars at $t=2$, so that the debt matures after state $S$ is revealed, that is, after the firm makes its investment decision. From the point of view of the firm's stockholders who have discretion over the investment at $t=1$, it will be worth investing only if $V(S) \geqq I+P$. Therefore, the values at $t=0$ of this firm's debt and equity, $V_B$ and $V_E$, are given by

$$\begin{aligned} V_B &= \int_{S_b}^{\infty} q(S)P\,\mathrm{d}S, \quad S \geqq S_b, \\ &= 0, \qquad\qquad\qquad S < S_b, \end{aligned} \tag{2}$$

$$\begin{aligned} V_E &= \int_{S_b}^{\infty} q(S)[V(S)-I-P)\,\mathrm{d}S, \quad S \geqq S_b, \\ &= 0, \qquad\qquad\qquad\qquad S < S_b, \end{aligned} \tag{3}$$

where $S_b$ is the break-even point corresponding to $V(S)=I+P$. Therefore, the total value of the levered firm, $V_L$, is

$$V_L = V_B + V_E = \int_{S_b}^{\infty} q(S)[V(S)-I]\,\mathrm{d}S. \tag{4}$$

[2]As Myers (1977, p. 163) points out, this model is predicated on market-power related imperfections in the real sector since the firm is assumed to be able to appropriate the growth opportunities when $S \geqq S_a$. If the real asset market is competitive, no rents will be generated to any firm even if there is industry wide growth. See Aivazian and Callen (1979).

Comparing eqs. (4) and (1) it would appear that the value of the levered firm is less than that of the unlevered firm since both equations are the same except for the limits of integration. The presence of debt in the firm's capital structure curtails the stockholders' incentive to invest in certain 'favourable' states of nature $S_k$ (where $S_b > S_k > S_a$), since part of the returns from the investment will accrue to the bondholders. In other words, outstanding debt imposes an externality on the stockholders' optimal investment strategy resulting in a net welfare loss at $t=0$ of

$$\int_{S_a}^{S_b} q(S)[V(S)-I]\,\mathrm{d}S. \tag{5}$$

### 2.3. *Internalizing the externality*

While the level of investment that will maximize shareholder wealth alone (when the firm is levered) is zero in states of nature $S_k$, the level of investment that will maximize the combined wealth of stockholders and bondholders in such states will correpond exactly to that of the all-equity firm, namely $I$. In perfect capital markets, it will be in the interests of both parties to negotiate and internalize the externality generated by debt in the firm's capital structure in period one. In fact, the Coase Theorem guarantees that the internalization will take place in the absence of transactions costs. Thus, it will be in the interest of stockholders to buy up all the debt, or of bondholders to buy up all the stock (or for an outside party to buy up all the firm's securities) and subsequently undertake net-present-value maximization which will generate the optimal level of investment in states $S_k$ in period one.[3]

Consider the case where the stockholders buy up all the debt. The price paid will lie somewhere in the interval $(V(S_k)-I, 0)$, where $V(S_k)-I$ reflects the firm's enhanced investment opportunity due to the elimination of debt from its capital structure. While the price actually paid for the debt is uncertain, the Coase Theorem argues that in the absence of transactions costs some mutually satisfactory price for the debt will be negotiated, enabling stockholders to fully internalize the externality and undertake the optimal level of investment $I$.[4] In the alternative scenarios in which bondholders buy

[3]These arguments, including the relationship between the Coase Theorem and the M–M theorems, were first advanced by Fama (1978, especially pp. 282–283). Nevertheless, Fama did not pursue the game-theoretic implications of his analysis.

[4]It should be pointed out that even in perfect capital markets the problem exists of allocating the gains generated by growth opportunities among the various claimants on the firm. This allocation process is internal to the firm involving negotiations among the claimants. The implication of perfect markets to this allocation process is simply that in such markets negotiation costs are zero so that the Coase Theorem becomes operative. Fama (1978, p. 283) also seems to recognize the ambiguities associated with the allocation of the rents generated by

up the firm's stocks or a third party acquires the firm, the analogous arguments can be made. While the allocation of the rents generated by favourable investment opportunities among the existing claimants on the firm can vary (depending on relative bargaining abilities, number of debtholders vs. stockholders, etc.), the Coase Theorem would guarantee that, regardless of which party ultimately ends up controlling the firm, the firm's optimal investment strategy in period one will be independent of its initial financial structure and will correspond exactly to that of the all-equity firm.

## 3. The game-theoretic complexities

Although the argument in the previous section would appear to be straightforward and non-controversial, nevertheless, we managed to gloss over an important issue which has yet to be addressed by the finance literature. In the case of the all-equity firm the valuation problems are quite simple. In every state of nature, the return to shareholders is a specific monetary value so that shareholders can evaluate their potential investment strategies using the expected utility criterion. In the case of the levered firm, however, we noted in the previous section that shareholder returns are a function of their ability to negotiate with the bondholders in the future. Therefore, in addition to facing risk in the ordinary probabilistic sense, the shareholders also face what may be called 'strategic' risk, that is, the risk inherent in one's ability to bargain and appropriate the attendant returns.[5] In our example, these risks accrue to both the equity and the debt since the bondholders' returns are also a function of their relative negotiating abilities in the future.

To put the Myers problem in its proper perspective, we must recognize the fact that, in some states of nature, bondholders and equity shareholders are involved in a cooperative game. Specifically, if we define $X_E(S)$ and $X_B(S)$ to be the returns to shareholders and bondholders in state $S$, respectively, then the (potential) returns to these claimants are given by

$$\begin{aligned} X_E(S)&=0, \\ X_B(S)&=0, \end{aligned} \quad S<S_a, \tag{6a}$$

$$\begin{aligned} X_E(S)&=V(S)-I, \\ X_B(S)&=P, \end{aligned} \quad S>S_b, \tag{6b}$$

$$\begin{aligned} &X_E(S)+X_B(S)=V(S)-I, \\ &X_E(S)\geqq 0, \qquad X_B(S)\geqq 0, \end{aligned} \quad S_a<S<S_b. \tag{6c}$$

growth opportunities even in a perfect capital market. In his discussion of a firm which does not maximize $V(t)-I(t)$ — his terminology is the same as ours but explicitly time dependent — Fama argues that one option is for outsiders to take over the firm and undertake the optimal investment policy. With respect to the rents, he says: 'The outsiders can even afford to pay a premium for the firm as long as it is no greater than the difference between the maximum value of $V(t)-I(t)$ and the value of $V(t)-I(t)$ under the investment policy chosen by the firm.'

[5]The term strategic risk was first coined by Roth (1977a).

Thus, in states of nature in the range $S_a < S < S_b$, bondholders and shareholders face the cooperative game defined by the convex area given above [eqs. (6c)]. This convex area is called the core of the game and delineates the minimum returns the claimants are willing to accept and the maximum returns available from successful negotiations. If the firm does not exercise its option to undertake the investment, bondholders and shareholders earn nothing. Thus, the minimum return acceptable to either group is zero. On the other hand, if the negotiations are successful and the disincentive effect due to leverage is internalized, both bondholders and shareholders will share in the gain $V(S)-I$.

It is quite clear that for the Myers problem the core exists since $V(S_k)-I$ is positive for at least one of the states $S_k$.[6] Therefore, the externality will be internalized and the firm will undertake the optimal investment policy with or without debt in its initial capital structure. The problem we face is one of valuation. Although the total market value of the firm is unambiguous, how is one to determine the *ex ante* equilibrium value of the debt and of the equity? The core does not have merely one solution but there are an infinite number of returns $X_E(S)$ and $X_B(S)$ which satisfy the core conditions. Since the initial values of the firm's equity and debt are a function of future returns which can only be described by the core conditions, how does one go about valuing the firm's claims?

While we would like to provide a definitive solution to the valuation of the levered firm facing growth opportunities, we are constrained to point out that the underlying theoretical developments which are prerequisite for a general valuation for lotteries over games is in its infancy. The best we can do is illustrate the solution for a rather unappealing set of asssumptions. Even so these very same assumptions [e.g. risk neutrality, homogeneous expectations], which are typical of much of the finance and economics literature, yield substantially different valuation formulas when the bargaining complexities are incorporated into the analysis. As we shall see in section 5, this has important implications for the valuation of those financial instruments which are used to eliminate potential bargaining situations. The valuation of such instruments involves the certainty-equivalent of a lottery over games.

Among other assumptions, we assume three axioms developed by Roth (1977a) to extend the expected utility framework from lotteries over specific (monetary) outcomes to lotteries over games. A brief description of the Roth axioms follow.[7] Let $i=1,\ldots,n$ denote the finite number of participants in the

[6]If not, then the only states favourable to the investment lie in the range $S \geqq S_b$ and the Myers problem disappears. On the other hand, if the core exists, the Coase Theorem becomes operative, all positive net present value investments are undertaken, and again the Myers problem disappears.

[7]This description follows Roth and Verrecchia (1979).

game and $v$ the characteristic function of the game. The function $v$ is defined over all potential coalitions in the game and its range is the return to each coalition. In the context of our example, $v(B)=v(E)=0$ and $v(E,B)=V(S)-I$ where $(E,B)$ is the coalition of shareholders and bondholders such that the debt is eliminated from the firm's capital structure. Let $(i,v)$ denote the prospect that participant $i$ finds himself in the game $v$, and let $U(i,v)$ denote the utility of this prospect to participant $i$. The Roth axioms are:

(i) The participant is indifferent among games in which his return from all possible coalitions is zero.

(ii) The participant is risk neutral in the probabilistic (ordinary) sense over games. Thus, if $p$ is the probability of the prospect $(i,v)$ and $(1-p)$ the probability of the prospect $(i,w)$ then the participant is indifferent between a lottery over these games and the prospect $(i,\ pv+(1-p)w)$ for certain.

(iii) The participant is risk neutral in the strategic sense, i.e., he is indifferent between bargaining among $r$ participants or receiving $1/r$ of the return for certain.[8]

Roth has shown these axioms to be necessary and sufficient for the utility function over the game to be the game's Shapley value, i.e.,

$$U(i,w)=\sum_{T\subset N}\frac{(t-1)!\,(n-t)!}{n!}[w(T)-w(T-i)], \tag{7}$$

where $N$ denotes the set of participants in the game and $T$ is a coalition of size $t$.[9] It also follows from the Roth axioms that the utility of a lottery over games is the expected value of the games' Shapely values.[10]

If in addition to the Roth axioms we assume perfect capital markets and homogeneous expectations, the valuation of the levered firm facing future

[8]This is equivalent to saying participants perceive themselves to have equal bargaining power.

[9]In a two participant game, where the participants are denoted $i$ and $j$, their respective utilities are: $U(i,w)=\frac{1}{2}w(i)+\frac{1}{2}[w(i,j)-w(j)]$ and $U(j,w)=\frac{1}{2}w(j)+\frac{1}{2}[w(i,j)-w(i)]$. It is worth noting that the total utility over the game is just equal to the maximum return so that $U(j,w)+U(i,w)=w(i,j)$. In a three participant game, the utilities are more complex because of the greater number of potential coalitions. Thus, $U(i,w)=\frac{1}{3}w(i)+\frac{1}{6}[w(i,j)-w(j)]+\frac{1}{6}[w(i,k)-w(k)]+\frac{1}{3}[w(i,j,k)-w(j,k)]$ and similarly for $U(j,w)$ and $U(k,w)$. Again, it is true that $U(i,w)+U(j,w)+U(k,w)=w(i,j,k)$.

[10]Roth (1978, 1977b) has extended the expected utility framework to cooperative games without side payments and to games which allow for strategic risk aversion. Unfortunately, for his models to be valid, Roth must assume that participants in the game are risk neutral in the ordinary sense.

investment opportunities is immediate.[11] The value of the levered firm's equity is

$$V_E = \int_{S_a}^{S_b} q(S)[(V(S)-I)/2]\,\mathrm{d}S + \int_{S_b}^{\infty} q(S)(V(S)-I-P)\,\mathrm{d}S. \tag{8}$$

Similarly, the value of the firm's debt is

$$V_B = \int_{S_a}^{S_b} q(S)[(V(S)-I)/2]\,\mathrm{d}S + \int_{S_b}^{\infty} q(S)P\,\mathrm{d}S. \tag{9}$$

Thus, the total value of the levered firm at $t=0$ is equal to

$$\begin{aligned} V_L = V_E + V_B &= \int_{S_a}^{S_b} q(S)(V(S)-I)\,\mathrm{d}S + \int_{S_b}^{\infty} q(S)(V(S)-I)\,\mathrm{d}S \\ &= \int_{S_a}^{\infty} q(S)(V(S)-I)\,\mathrm{d}S. \end{aligned} \tag{10}$$

Clearly, the value of the levered firm is equal to the value of the all-equity firm in this game-theoretic setting.

## 4. Interdependencies of financing and investment decisions: An example

Placing Myers' problem in its natural game-theoretic valuation framework apparently leads to the same conclusion as before, namely, that the firm's investment decisions are independent of its financial structure with or without growth. But this is merely because we assumed the firm's claimants to be comprised of two homogeneous groups, bondholders and shareholders. Once we allow for heterogeneous claimants or at least three groups of claimants the game-theoretic complications may vitiate the standard M–M results. We will demonstrate this point by an example.

Consider a firm comprised of common shareholders ($E$) and two sets of contractual claimants, bondholders ($B$) and preferred shareholders ($P$).[12] Let the characteristic function $v$ denote the (market) value of each claimant's assets. In the absence of negotiations to internalize the disincentive effect of the contractual claims, the value of each claimant's assets are assumed to be[13]

[11] What follows becomes obvious if one recognizes that, in a two-participant game, the Shapley value splits the gain equally between the participants. Therefore, for all states $S_k$ the Shapley value of the game [eqs. (6c)] is $[(V(S_k)-I)/2]$.

[12] Alternatively, $B$ could represent senior debt and $P$ junior debt.

[13] M stands for millions. The numerics of this example, but not the context, are from Mossin (1968).

$$v(E)=\$24\text{M}, \qquad v(B)=\$8\text{M}, \qquad v(P)=\$3\text{M}.$$

Thus, the value of the firm is $v(F)\equiv v(E)+v(B)+v(P)=\$35\text{M}$.

Without transactions costs, including the cost of negotiating, each of the firm's claimants and, indeed, outside parties as well, will try to attenuate the disincentive effect of the contractual claims on the firm's optimal investment policy. Let $v(E,B,P)$ denote the value of the firm in a (grand) coalition of all claimants such that all contractual claims are eliminated from the firm's capital structure. Similarly, $v(E,B)$ stands for the value of the assets of the coalition comprised of common shareholders and bondholders where the debt is removed from the firm's capital structure. For purposes of our example, we will assume that

$$v(B,P)=\$15\text{M}, \quad v(E,P)=\$31\text{M}, \quad v(E,B)=\$36\text{M},$$
$$v(E,B,P)=\$40\text{M}.$$

Clearly, this example assumes that the value of the assets of a potential coalition is greater than the value of such assets in the absence of a coalition. In the game theory literature, this synergy is incorporated in the function $v$ which is said to be strictly superadditive. Strict superadditivity implies, for example, that

$$v(E,P)>v(E)+v(P), \tag{11}$$

and

$$v(E,B,P)>v(E,P)+v(B). \tag{12}$$

Similar relationships would of course hold for the other potential coalitions. Strict superadditivity is a necessary condition for negotiations to be initiated.

The question arises as to the source of this strict superadditivity. In the case of coalitions involving the firm's common shareholders the answer is obvious. If the common shareholders negotiate successfully with either the bondholders or preferred shareholders, *a fortiori* with both, then the firm will undertake more investment because some or all of the contractual claims will be eliminated from the firm's capital structure. The additional investment will then reflect itself in the market valuation of the coalition's (firm's) assets. However, there is one coalition for which the scenario is incomplete, namely the coalition involving the firm's bondholders and preferred shareholders. It would appear that their coalition confers no real gains and, yet, in our example the gains are \$4M $(V(B,P)-V(P)-V(B))$. One can envision two possibilities which could confer these gains. Bargaining power may be a function of the homogeneity of ownership claims. Therefore, to the extent that bondholders and preferred shareholders form a cohesive group they can

demand a larger minimal portion of the returns from the investment. These bargaining elements are incorporated in the characteristic function. Also, an outside group or firm, which wants to undertake the optimal investment strategy by taking over the firm, may be willing to pay \$15M to the contractual claimants. This sets a reservation price for the value of the contractual claims.

One additional complication is worth noting in the context of the example. Should the common shareholders form a coalition with, say, the preferred shareholders, the bondholders will benefit as well even if the latter are not part of the coalition. This is so because, once the preferred claims are eliminated, the common shareholders will undertake more (but less than the optimal amount of) investment. A similar result would hold for the preferred shareholders, if there is only a bondholder–common shareholder coalition. To account for these complexities, we define $v_{EB}(P)$ to the value of the preferred shareholders' assets under a common shareholder–bondholder coalition and similarly for $v_{EP}(B)$. Thus, $v_{EB}(P) > v(P)$ and $v_{EP}(B) > v(B)$. For our example, we will assume that

$$v_{EB}(P) = \$3.5\text{M}, \qquad v_{EP}(B) = \$8.5\text{M}.$$

Although this example appears somewhat contrived, the issue it raises is fundamental to the theory of finance. To see what the problem is, let us consider some possible coalition bargaining situations. Suppose the common shareholders and bondholders form a coalition excluding the preferred shareholders. Thus, the coalition's assets are worth \$36M and the preferred shareholders' assets are worth \$3.5M. But how is the \$36M to be allocated between the common shareholders and the bondholders? Obviously, the common shareholders must receive at least \$24M and the bondholders at least \$8.5M.

Let us suppose initially that the common shareholders capture all the gains from the coalition and the bondholders accept their minimum requirements. Defining $X_E$, $X_B$, and $X_P$ to be the value of assets held by common shareholders, bondholders and preferred shareholders, respectively, after all coalition gains have been allocated, we would have in this case that $X_E = \$27.5\text{M}$, $X_B = \$8.5\text{M}$ and $X_P = \$3.5\text{M}$. But this distribution of returns is inherently unstable. In a transaction–costless world, the preferred shareholders will approach the bondholders and offer them, say, an additional \$500 to form their own coalition in which $X_P = \$6\text{M}$ and $X_B = \$9\text{M}$. Since preferred shareholders and bondholders are better off than in the previous case, they can block the original allocation of returns involving only common shareholders and debtholders. Of course the story does not end here, because the common shareholders would come back and make a counteroffer to the bondholders of \$9M to rejoin the original coalition.

Therefore, the bondholders could seemingly play the common shareholders against the preferred shareholders until they capture all the potential benefits from coalition formation with either set of shareholders.

Suppose, in fact, that bondholders and common shareholders form their coalition, as initially hypothesized, but now the bondholders obtain all the benefits. Again, we have an unstable coalition since preferred shareholders will offer the common shareholders an additional \$500 blocking the original coalition. Now the common shareholders are in a position to play the preferred shareholders against the bondholders and try to appropriate the maximum returns.

Even if stable two-participant coalitions could be found, these coalitions are unstable relative to the grand coalition. In our example, every two-member coalition yields total returns no greater than \$39.5M which is less than the benefits from the grand coalition of \$40M. Thus, given a specific distribution of profits in a two-member coalition, one can always find an alternative distribution in the grand coalition which makes each participant better off. Therefore, one might be tempted to argue that the grand coalition must result. But in our example even the grand coalition is unstable. To see why this might be so, consider the grand coalition distribution

$$X_P = \$4.75\text{M}, \qquad X_B = \$9.75\text{M}, \qquad X_E = \$25.50\text{M}.$$

Although each party earns more than it would in the absence of negotiations, there are many two-member coalitions which could, and, presumably, would block this particular distribution. For example, the coalition offering $X_B = \$10\text{M}$ and $X_E = \$26\text{M}$ is preferred by common shareholders and bondholders to the grand coalition solution given above.

Our discussion thus far has tried to highlight a number of the coalition possibilities and why these coalitions are infeasible. Specifically, a coalition is infeasible if any party earns less than it would by independent action. In addition, no coalition can succeed if the resulting distribution of profits can be blocked by an alternative coalition. These two conditions cause all the potential coalitions in our example to be unstable. Ironically, in a world of zero transactions costs, the inherent instability of all coalitions will result in endless recontracting among the claimants.

Formally, coalitional stability requires that the resulting allocation of returns satisfy the following core conditions:

$$X_E \geqq V(E), \quad X_B \geqq V_{EP}(B), \quad X_P \geqq V_{EB}(P), \tag{13}$$

$$X_E + X_B \geqq V(E,B), \quad X_E + X_P \geqq V(E,P), \quad X_B + X_P \geqq V(B,P), \tag{14}$$

$$X_E + X_B + X_P = V(E,B,P). \tag{15}$$

But, in our example, the core is empty. To see this note that if all the two-coalition inequalities in (14) are added together then

$$(X_E+X_B+X_P)\geqq\tfrac{1}{2}[V(E,B)+V(E,P)+V(B,P)], \tag{16}$$

so that substituting (15) into (16) yields

$$V(E,B,P)\geqq\tfrac{1}{2}[V(E,B)+V(E,P)+V(B,P)]. \tag{17}$$

This last condition is violated by our example since the right-hand side of the inequality turns out to be equal to \$41M and the left-hand side to \$40M.

It is worth noting that the existence and size of the core is also a function of the number of participants in the negotiations. Three is the minimum number of participants necessary for the non-existence of the core. As the number of participants in the negotiations increases — in our example, we can assume heterogeneous claimants — the core shrinks and is less likely to exist.[14] Intuitively, the more participants there are, the greater the number of potential coalitions and, thus, constraints.

The implications of our example for finance theory is profound. If the core does not exist, the impact of the firm's initial financing decisions on its investment decisions in a growth situation is unknown. In a world of zero transactions costs, negotiations could be endless. Investment opportunities which are in everyone's interest to undertake, may never be consummated and, in which case, Myers is correct. On the other hand, even if the grand coalition forms, it will only be by happenstance. It is equally probable that one of the two-member coalitions will form and only part of the externality would be internalized. Not only does the non-existence of the core violate the M–M theorems, but it plays havoc with valuation theory in general.[15] What is the *ex ante* value placed by market participants on the firm in our example?[16]

Although the non-existence of the core is the antithesis of financial valuation, it may very well rationalize certain institutional arrangements in financial contracts or for that matter the contracts themselves. For example, convertible debentures and the call feature — the latter is pervasive in corporate bond identures — may simply be useful tools for eliminating

[14]See Johansen (1978), for example.

[15]One could of course assume the Roth axioms as we did in the previous section. The Roth axioms do not depend on the existence of the core. The point is, however, that the Roth axioms are restrictive and, in a more general game-theoretic setting, the value of the levered firm may be indeterminate.

[16]We would like to emphasize that the simplicity of our example belies its potential generality. Examples of negotiations where the core does not exist are easy to concoct and may be quite common in practice. External diseconomies, increasing returns to scale, and non-convex preference functions are all factors which may lead to an empty core. If the core is empty, the M–M theorems may break down.

potential bargaining situations which defy solution. This rationalization is qualitatively different from the agency cost or property rights literature which justify contractual arrangements on the basis of transaction costs. Rationalizing contractual features on the basis of potential bargaining costs which may arise in the absence of such features is the agency approach. When there is no core, it means that, although it is in everyone's interest to resolve the conflict, no solution can be found. If anything, it is the lack of transactions costs which creates the latter problem.

Empirically, however, it is difficult to differentiate between contractual arrangements which arise because of the non-existence of the core from those that arise from transactions costs when there is a core. The non-existence of the core will also manifest itself in transactions costs via the opportunity cost of (negotiation) time. One should not make the mistake, of assuming, however, that once 'time is money' this will force a Pareto-optimal solution. No one knows what actually happens when the core is empty. Do the participants agree to a Pareto-optimal solution or will they merely stop negotiating? It is an empirical question.

## 5. Rationalizing some financial arrangements

Recognizing that financial valuations are problematic once the bargaining complexities are accounted for, leads to new perspectives on the utility of various financial instruments and institutions. This is especially so when the 'real' world phenomena of positive (but usually finite) transactions costs are melded into the analysis.

### *5.1. The call feature and the effective call premium*[17]

Bodie and Taggart (1978) rationalize the pervasiveness of the call feature in corporate bond issues as a means of attenuating the Myers problem. Should future investment opportunities materialize, shareholders may be able to eliminate the disincentive effect of the debt by simply calling it. The crux of the argument, as we have seen, ultimately hinges on the usefulness of the call feature in eliminating negotiation costs and/or in precluding bargaining scenarios which defy solution. If the core exists and there are no negotiating costs, the call provision is simply redundant.

In addition to rationalizing the call provision, the game-theoretic solution is indispensable in understanding the call premium. If they are to be induced to purchase a callable bond, bondholders must be compensated for the (expected) gains they would have received in negotiations with shareholders had new investment opportunities materialized. This compensation, which we term the effective call premium, reflects in equilibrium the certainty-

[17]For a comprehensive discussion of various features in bond indentures including the call provision, see Smith and Warner (1979).

equivalent of a lottery over games. In the simple Myers example we analyzed previously, the equilibrium value of the effective call premium is

$$\int_{S_a}^{S_b} q(S)[(V(S)-I)/2]\,\mathrm{d}S. \tag{18}$$

The conventional call premium is a nominal sum which is equal to the effective call premium provided the bond is selling at par. If the nominal premium is less than the effective premium, the bond must sell at a discount. Similarly, if the nominal premium is greater than the effective premium, the bond must sell for more than its face value. Ultimately, then, the difference between the non-callable and callable bond is that the latter insures shareholders against the contingency and cost of having to bargain with bondholders when investment opportunities arise. The equilibrium price of this insurance, is the effective call premium.

There are still situations, however, when the callable bond may be ineffective against the Myers problem. Implicit in our discussion thus far (and in the Bodie–Taggart framework) is the assumption that the amount the firm pays to call the bond, face value plus accrued interest plus the call premium, is less than the default free value of the bond. A secular upward trend in interest rates, as we have experienced over the last decade, could result in the risk free rate rising above the bond's coupon rate, in which case the bond may never be called. In such circumstances, the decision to call or negotiate depends on whether or not significant investment opportunities have materialized to override this interest-rate effect. While the interest-rate effect lowers the value of the bond, new investment opportunities tend to increase it since bondholders have an indirect claim (through the potential negotiation process) on the rents to be generated within the firm. If stockholders estimate that the negotiated settlement will result in a price which overrides the interest-rate effect so as to exceed the call price, then they will simply call the issue. Otherwise they will negotiate. Furthermore, even if negotiations are the likely alternative, the call feature may still be useful to stockholders. If profitable investment opportunities do materialize, the negotiated value of the bond is limited by an upper bound, namely the call value of the bond.

The above analysis neglects the other known function of the call provision, namely, to allow for refinancing when interest rates fall. In a perfect capital market and in the absence of a call feature, the price that existing bondholders would require to permit refunding will exactly offset the gains from refinancing the debt at the lower interest rate. Any negotiation process would be a zero-sum game unlike the case where refunding is initiated as a result of growth opportunities. In the latter case, imperfections in the real asset market create the potential rents which are to be negotiated. Similarly,

if capital markets are imperfect (due to transactions costs, information costs, etc.) then the refunding process, no matter what its source, may well generate positive returns to the firm and the *ex ante* value of the call privilege will reflect the certainty-equivalent of these gains from refunding.

Looking at the call provision in this fashion has immediate implications for the empirical evaluation of the call provision. First, the benefits of the call provision do not only relate to interest rate changes but also to expectations about future investment opportunities. This is probably why bonds are called even though interest rates are increasing. It also helps to explain why deferred call options are usually immediately callable within the deferral period for reasons other then refunding because of lower interest rates.[18] Profitable investment opportunities may disappear after five or ten years, the usual deferral periods. Second, to determine the *ex ante* value of the call provision, it is not sufficient to measure the differential yield between an immediately callable bond and non-callable (or deferred callable) equivalent instrument. The call premium reflects the present value of the certainty-equivalent of a game involving risky future investment opportunities and/or the gains from refunding because of interest rate changes. In the case where the premium is set 'correctly', equal to the certainty-equivalent, both callable and non-callable bonds must sell at the same yield. It is even conceivable that, if the call premium is greater than the certainty-equivalent of the gamble, the callable bond will sell at a lower yield than the equivalent non-callable instrument.[19] Therefore, the yield differential is meaningless *per se*. It merely represents the residual adjustment of the value of the bond not reflected in the nominal call premium.

[18]For example, Bristol–Myers 25 year, deferred callable sinking fund debentures issued in 1970 to yield 8.625% contains the following restriction: 'Not callable prior to November 1, 1980 through refunding at an interest cost less than 8.625%. Also, callable for sinking fund purposes.' Of the over 140 deferred callable industrial bonds (grades A to AAA) newly issued between 1969 and 1973 only eight have been called. Of these, two were called for sinking fund purposes. The remainder were called when interest rates were increasing. It is possible that those bonds called within two years of maturity reflected expectations of even higher interest rates at maturity. Rather than refund at the higher rate at maturity, the bonds were called and refunded at a lower rate. This would not explain, however, why some of the bonds were called eight years before maturity. Except for the sinking fund debentures, none of the others were called within the deferral period.

[19]In equilibrium, and abstracting from tax effects, bondholders should be indifferent between receiving their return from the yield or from the call premium. Shareholders should also be indifferent to the form of payment except that, from a risk sharing perspective, they may wish to guarantee themselves a certain price for the bonds. Since the certainty-equivalent of the gamble is undoubtedly difficult to measure, the shareholders may avail themselves the services of an underwriter to guarantee the price. Of course, the underwriter's fee will in turn reflect the risks involved. It may very well be that even underwriters find it difficult to measure the risks except for relatively large and well established firms for which some average certainty-equivalent measurement is available. This may answer why premia tend to be set according to some heuristic, for example, one year's interest for call premia. This may also explain why underwriters are unwilling to insure smaller firms and so distribute their bonds on a 'best efforts' arrangement. For a more comprehensive analysis of underwriting contracts, see Mandelker and Raviv (1977).

### 5.2. The convertible bond

This instrument is most commonly regarded as deferred equity financing which permits new firms to issue equity at a price above current share value. Accompanying this textbook description of the convertible is the usual presumption that management (current shareholders) is more aware of future investment opportunities than are other participants in the equity market. Thus, management is loath to sell equity at current prices and thereby dilute earnings per share.

Most of us have undoubtedly felt uncomfortable with the above rationalization of the convertible bond. Again, it bears repeating, in a perfect capital market *and* where the existence of the core is not problematic, the conversion feature is redundant. The conversion feature is useful because it eliminates bargaining costs and/or core existence problems just like the callable bond. Therefore, it is apparent that the convertible bond is not simply deferred equity financing. Only if future investment opportunities materialize is it in the interest of bondholders to convert. Unlike the pure callable bond, however, once conversion is completed the Myers externality, it would seem, does not disappear. On the contrary, these new shareholders will join with existing shareholders in exploiting the gains from new investment opportunities, apparently creating a disincentive problem for existing shareholders. However, *ex ante* when the convertible is issued, the bondholders compensate existing shareholders for the right to participate in new investment opportunities if and when they materialize. This compensation is in the form of the conversion premium. In fact, in equilibrium, the conversion premium represents the present value of the certainty-equivalent of a lottery over games which would arise in the absence of the conversion privilege. Therefore, unlike the pure callable bond in which the bondholders are paid not to participate in future investments, convertible bondholders compensate existing shareholders for the right to participate.[20] In both cases, negotiations are avoided.[21]

Again, the convertible need not sell at a yield less than the yield on a similar non-convertible bond. It will do so only if the conversion premium is

[20]The conversion premium is analogous to charging a fixed fee to join a club.

[21]The texbook description of the convertible typically argues that bondholders have little incentive to convert since the price of the convertible will in any case reflect positive investment opportunities. Therefore, convertible debentures have a call feature or a step-up provision to force conversion. This argument denies the Myers problem by implicitly assuming that shareholders will undertake the optimal investment even without conversion. But if the firm's investment policy is contingent on conversion it is in bondholders' interest to convert since they have already obtained the rights to (some of) the gains from the optimal investment policy. Why then should shareholders have to force conversion? Perhaps, there is a free-rider problem here in the sense that if investments are lumpy and most of the bondholders convert, then the optimal investment will be undertaken irrespective of the remaining bondholders. Consequently, each bondholder presumes that the others will convert, the investment will be undertaken, and he can continue to benefit from both the coupon payments and the appreciation of the convertible.

too small relative to the certainty-equivalent compensation which must be paid to existing shareholders. In the latter case, the yield on the convertible will fall to compensate for the difference.

Since both the pure callable bond and the convertible can eliminate the Myers problem, why do we see both instruments in use?[22] First, as we noted earlier, the call feature may be ineffective, especially when there is an upward secular trend in interest rates. Second, the perception that small growing firms are more likely to issue the convertible is germane to the issue. The convertible debenture is more effective for economizing on transactions costs, a consideration which is of paramount importance to small firms. If the pure bond is called, shareholders will have to issue new bonds (or equity) to refinance. In the case of the convertible, on the other hand, the financing remains after the conversion, only the form changes from debt to equity. Since small firms face higher average placement costs (per dollar of financing) than do large firms, the former are more likely to issue convertibles. Third, differential attitudes towards risk among managers (stockholders) of different firms may provide yet another rationale for preferring one instrument over the other. More risk averse firms would have a preference for risk sharing via the convertible rather than calling the debt and undertaking all of the gamble. However, risk sharing arguments are also predicated on transaction costs. In the absence of transactions costs, risk sharing could take place in many formal and informal markets and, thus, it would be difficult to justify any particular risk sharing arrangement such as the convertible.

### 5.3. *Bankruptcy as a reorganization decision*

On the whole, the finance literature has modelled the bankruptcy decision as a liquidation process.[23] It is assumed that the firm's creditors find it in their power and in their interest to initiate bankruptcy proceedings and,

[22]Our referee has suggested that sinking funds and coupon payments also help to attenuate the Myers problem. In Myers' scenario, all of the debt payments are due at maturity, after the investment is to be undertaken. In the absence of negotiations or some other incentive, shareholders will simply forego the option to invest if the opportunity is not sufficiently profitable for them although it is profitable for bondholders. However, if some of the debt must be repaid prior to maturity (specifically at $t=1$ when the investment decision is to made by the firm), either in the form of coupon or sinking fund payments, shareholders will default on the payment if the project is not sufficiently profitable to them (i.e., to cover the coupon and the market value of the principal at $t=1$), and bondholders will take over and undertake the investment. The need for negotiations is eliminated. Of course, as we discuss below, the default process may in turn involve (costly) negotiations so that the renegotiation ambiguities may not be eliminated. Also, this solution to the investment incentive problem applies only in the case where the firm is constituted for that one project (as in the Myers example), or that it can be costlessly broken down into its component projects. In general, the firm will have existing assets or other projects which may mitigate against using the default mechanism for solving the Myers problem.

[23]See, for example, Kim (1978), Scott (1976, 1977) and Van Horne (1976).

ultimately, the firm's assets are divided among the creditors according to some well-defined set of 'me-first' rules. However, as Warner (1977) has emphasized, in most formal bankruptcy proceedings, the courts appoint a trustee, who, together with the representatives of the different classes of claimants, try to reorganize the firm's capital structure by scaling down its fixed contractual obligations. The reorganization plan is submitted to all the creditors and it must be ratified by two-thirds of each class of creditors. If the plan is not ratified, the trustee in conjunction with the creditors tries to come up with a more acceptable alternative. In other words, the formal bankruptcy process recognizes that the value of the ongoing concern may be greater than the liquidation value of the firm's assets so that it is in the interest of the creditors to reorganize rather than liquidate. It is the trustee's job to recognize when the reorganization is preferable and to initiate negotiations among the creditors concerning the redistribution of their claims. Therefore, the bankruptcy decision involves potential negotiations and, hence, the valuation of a lottery over games.

Looking at the bankruptcy decision as a lottery over games has two immediate implications for the finance literature. First, the valuation of a levered firm or any firm with fixed contractual obligations requires the valuation of a lottery over games whether or not the firm has potential growth opportunities. Second, current normative models of the bankruptcy decision, such as Van Horne's (1976) dynamic programming solution to the optimal initiation of bankruptcy proceedings, are much too simplistic. His model does not allow for the reorganization option which is at the heart of the bankruptcy decision (at least as it concerns large corporations) and its inherent gaming complexities.[24]

### 5.4. *Bankruptcy laws*

One of the outstanding questions in the theory of financial distress is why is it that the reorganization process takes place in a formal milieu? If it is in everyone's interest to reorganize the firm's capital structure because the value of the ongoing firm is greater than its liquidation value, why involve the courts? Surely the costs of the external reorganization (court costs, lawyers fees, and so on) are greater than the costs of an informal process.

A first approach to answering this question might be to argue that formal reorganizations are preceded by an informal process. The theory of the core tells us, however, that even though it is in everyone's interest to negotiate a (Pareto-optimal) solution, this does not mean that a solution will be forthcoming. With more than two participants in the negotiating process, the core may very well be empty. If the core is empty, the bargaining process

[24]These same criticisms can be levelled against the positive models of the bankruptcy process developed by Kim and Scott.

may be extremely prolonged and costly or negotiations may simply break down so that a formal process becomes mandatory.

But what is it about the formal process which yields a solution? The answer we believe lies in the two-third's rule mentioned above. In an informal reorganization, all claimants must agree or be compensated. One can readily envisage small bondholders or shareholders threatening to abort the reorganization unless they are fully compensated.[25] These threats are far less effective in a formal reorganization since the Bankruptcy Act requires that only two-thirds agreement (by face value of the claims) need be obtained for the reorganization. Therefore, the external reorganization may be less costly because non-cooperative gaming strategies are prescribed by the formal bankruptcy apparatus.

The formal bankruptcy stratagem may also be optimal for certain creditors if they perceive coalition-formation behaviour on the part of other claimants which are inimical to their own interests. The preferential transfer statute, for example, prevents shareholders from transferring part or all of the firm's assets to one particular group of creditors at the expense of the others.

### 5.5. *Mergers*

How should the acquired and acquiring firm distribute their gains from the merger? This issue was first addressed by Mossin (1968). Mossin's solution was to say that if the firms involved are willing to accept a constitution of three seemingly innocuous axioms developed by Shapley (1953), then the only acceptable division of the merger benefits is its Shapley value. Perhaps, because Mossin's problem was placed in a purely normative framework or perhaps because the Shapley axioms are somewhat arbitrary (i.e., unrelated to risk considerations and/or the general expected utility criterion), the Mossin approach has been overlooked by the merger literature. But if one is willing to assume, at least as a first approximation, that merger participants are risk neutral in the ordinary and strategic risk senses, then each firm would be indifferent between negotiating over the merger benefits or receiving its Shapley value. Therefore, Mossin's suggestion represents a potentially positive, as well as normative, solution to the distribution of merger benefits. Such a solution is much more likely where negotiations are costly and a solution can be found, which is both consistent with firms' risk preferences (assuming they are) and economizes on negotiation costs.

If the Mossin solution is of a positive nature then one should expect the empirical merger literature to either corroborate or reject the theory. There

[25]A referee has suggested to us that small bondholders had inordinate power in the negotiations to reorganize shaky REITs because all classes of creditors had to agree. The returns to these small bondholders came at the expense of the larger creditors, primarily the banks.

are, it would appear, two pieces of information, one which bears directly and the other indirectly, on this issue. Halpern (1973) has in fact measured the benefits from merger activity and the distribution of these benefits in absolute dollar terms between the acquired and acquiring firms. He finds that, on average, the benefits from the distribution are divided evenly between the firms, independent of firm size. Assuming that on average merger negotiations involve only two firms, then the Shapley value will split the benefits evenly between the two firms. Somewhat more indirectly, the pervasive finding that the acquired firm captures a greater proportion of the merger benefits, where the proportion is a function of each firm's pre-merger size, is also consistent with a Shapley solution. This result is to be expected if the dollar benefits are split evenly between the firms and the acquired firm is typically much smaller than the acquiring firm. Of course, this result is also consistent with competition in the demand for acquisitions so that the acquired firm appropriates most of the rental. The point is that this result is also consistent with a Shapley solution.

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