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A note on the economics of exhaustible resources

VAROUJ A. AIVAZIAN and JEFFREY L. CALLEN /
McMaster University

In his seminal paper on the economics of exhaustible resources, Hotelling (1931) shows that the firm's optimal extraction rate, assuming stationary demand and complete exhaustion of the mine, requires that marginal profit increase over time at the firm's cost of capital and that the rate of extraction is always a decreasing function of time. Recently in this JOURNAL Levhari and Liviatan (1977), from now on referred to as L-L, analysed the more realistic case where extraction costs are an increasing function of cumulative output so that the mine need not be completely exhausted. They find that the firm's optimal response is to set the rate of growth of marginal profits less than its cost of capital and that the rate of extraction may be increasing or decreasing over an arbitrary period.

This note shows that the firm's optimal extraction rate is highly sensitive to the nature of the industry structure governing the production of the resource. The case can be made that, for at least one such structure, the firm will set marginal profits equal to zero at every point in time and the rate of extraction will be a globally decreasing function of time. In what follows we provide first the intuitive background to our results and then the formal analysis. We conclude with some public policy observations.

We would like to thank the referees of this JOURNAL for their useful comments. We claim exclusive property rights over any remaining errors.

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PRODUCTION EXTERNALITIES AND MARKET STRUCTURE

When extraction costs are a function of cumulative output, certain property rights arrangements over the resource may result in production externalities. The property rights arrangements governing exhaustible resources which exist in practice are to some extent a function of the resource. In the case of a solid mineral, the land ownership or lease usually provides exclusive property rights. This is primarily the case to which the L-L paper is addressed. On the other hand, in the case of oil and natural gas many producers typically drill the same pool from adjacent leases so that the resource has common property characteristics. Usually the land ownership or lease conveys a property right, called the rule of capture, which gives the producer rights over the oil he extracts. To mitigate the 'beggar thy neighbour' philosophy engendered by the rule of capture, state authorities in the United States and provincial authorities in Canada legislate proration requirements. For example, since 1969 Alberta has employed a market proration scheme which sets ceilings on the amount to be produced from each well depending on market demand.[1] This scheme has not reduced the incentive for overdrilling.[2]

Where the resource has common property characteristics, production externalities could ensue from each firm's current production activity. The existence of these externalities and their extent depend crucially on the exclusive nature of the property rights governing the depletion of the resource. The degree of exclusive rights that a firm has over the resource is in turn a function of the industry structure governing the extraction of the resource. If a monopoly or collusive oligopoly governs the extraction of the resource the externalities will be completely internalized by the firm in arriving at its optimal depletion strategy. The externalities are simply co-opted and disappear. On the other hand as long as property rights are non-exclusive, as Coase (1960) argues in his pathbreaking article, the externalities remain. Therefore, in the polar case of a perfectly competitive structure governing the extraction of the resource, future production opportunities will be treated by each firm as a common property resource, a free good. Each firm will disregard the impact on future industry production opportunities caused by its own current production activities. Extending the common property analogy to a non-collusive oligopoly, we should expect that, as the number of firms extracting the common resource is reduced, each firm internalizes a greater proportion of its impact on future industry production opportunities (see Cheung, 1970). Therefore, the rate at which the exhaustible resource is depleted is a function not only of the industry structure in the output market

1 For an evaluation of the Alberta proration scheme and alternatives to it see Khoury (1969).
2 See Ritchie (1975). The incentive to overdrill could be eliminated by a collusive unitization arrangement which requires producers from a common reservoir to operate as a single unit. Unitization schemes are uncommon both in Canada and the United States.

and the nature of the extraction technology but also of the industry structure governing the extraction of the resource.[3]

THE MODEL

We begin our formal analysis by considering a Cournot oligopoly structure governing the depletion of the resource.[4] We assume that the Cournot oligopolist maximizes the (expected) net present value of future profits discounted at his cost of capital. In typically myopic fashion, the Cournot oligopolist assumes that his depletion strategy does not affect his rivals and takes their behaviour as given. In fact, the extraction costs borne by the oligopolist are a function of all participants both at a given moment in time and over time.[5] This model of oligopoly behaviour can be represented[6] by

$$\text{Maximize} \int_0^T e^{-rt}[R_i(q_1(t), \ldots, q_m(t)) - C(q(t), x(t))q_i(t)]dt, \tag{1}$$

$$\text{subject to } q_i(t) \geq 0, \qquad x(T) \leq a,$$

where $q_i(t)$ is the output of firm i at time t, $i = 1, \ldots, m$; r is the cost of capital; $R_i(q_1(t), \ldots, q_m(t))$ is the total revenue of firm i at time t; $q(t)$ is the total output of firms extracting the resource at time t, $q(t) = \sum_{i=1}^{m} q_i(t)$; $x(t)$ is the cumulative output from time 0 to t of all firms extracting the resource; $C(q(t), x(t))$ is the average cost function of firms extracting the resource at time t; and a is the total availability of the resource.

The average cost function $C(q(t), x(t))$ requires some brief comment. Average cost each period is a function of both the rate of output and the level of cumulative output. By assumption, all Cournot firms face the same industry cost function, so that two qualitatively different externalities ensue from each firm's current extraction activity. Each firm's extraction affects the instantaneous extraction costs of its rivals while simultaneously increasing future industry extraction costs.

3 The structure governing the depletion of the resource need not necessarily bear any relationship to the over-all output market structure. To see this, consider an oil reservoir tapped by a small number of producers relative to the size of the reservoir who are nevertheless competitive in the output market.

4 Our analysis ignores the possibility of storage, so that the rate of extraction of the resource is equivalent to the sales rate. Where storage is feasible, these two rates might diverge. Intuition suggests that the divergence will also be a function of the market structure governing extraction of the resource.

5 Clearly the game-theoretic aspects of oligopoly behaviour are better captured by a more sophisticated oligopoly model or by a differential games approach. Nevertheless, our simpler and hence mathematically tractable Cournot framework encompasses the one crucial and fundamental assumption of oligopoly behaviour, namely, that each oligopolist's extraction costs are recognized to be a function of all firms in the industry.

6 This model is patterned on the L-L framework.

THE SOLUTION

The Euler necessary condition to solve this model[7] is

$$\frac{d}{dt}[e^{-rt}(R_{ii}(q_1, \ldots, q_m) - C(q, x) - C_q(q, x)q_i)] = -e^{-rt}C_x(q, x)q_i, \quad (2)$$

where $R_{ii}(q_1, \ldots, q_m)$, the marginal revenue of firm i at time t, equals $\partial R_i(q_1, \ldots, q_m)/\partial q_i$; $C_x(q, x) = \partial C(q, x)/\partial x > 0$; and $C_q(q, x) = \partial C(q, x)/\partial q$. Integrating both sides of (2) gives

$$\int_t^T \frac{d}{ds}[e^{-rs}(R_{ii}(q_1, \ldots, q_m) - C(q, x) - C_q(q, x)q_i)]ds = -\int_t^T e^{-rs}C_x(q, x)q_i ds. \quad (3)$$

Completing the integration of the left-hand side of (3) and solving for $R_{ii}(q_1, \ldots, q_m)$ yields

$$\begin{aligned} R_{ii}(q_1, \ldots, q_m) = {} & C(q, x) + C_q(q, x)q_i \\ & + e^{-r(T-t)}[R_{ii}(q_1(T), \ldots, q_m(T)) - C(q(T), x(T)) \\ & - C_q(q(T), x(T))q_i(T)] + \int_t^T e^{-r(s-t)}C_x(q, x)q_i ds. \quad (4) \end{aligned}$$

Since the mine need not be completely exhausted, the transversality conditions restrict marginal revenue to equal marginal cost at the terminal date T. Therefore, equation (4) reduces to

$$R_{ii}(q_1, \ldots, q_m) = C(q, x) + C_q(q, x)q_i + \int_t^T e^{-r(s-t)}C_x(q, x)q_i ds \quad (5)$$

or

$$\mathrm{MR}_t^i = \mathrm{MC}_t^i + \int_t^T e^{-r(s-t)}C_x(q, x)q_i ds, \quad (5')$$

where MR_t^i and MC_t^i denote the marginal revenue and the marginal cost of firm i at time t. Equation (5′) can be interpreted to say that marginal profits each period must equal the discounted value of the increase in future extraction costs internalized by firm i.

The optimal behaviour of the monopolist and the competitive firm is determined by a common technique. Sum equation (5) over all the m firms extracting the resource to obtain:

$$\sum_{i=1}^{m} R_{ii}(q_1, \ldots, q_m) = mC(q, x) + C_q(q, x)q + \int_t^T e^{-r(s-t)}C_x(q, x)q ds. \quad (6)$$

7 From this point onwards, the time variable is normally deleted for notational simplicity.

Define $\epsilon = 1/C_q(q, x) \cdot C(q, x)/q$ and divide (6) by m to yield

$$\left(\sum_{i=1}^{m} R_{ii}(q_1, \ldots, q_m)\right) \Big/ m = C(q, x)\left(1 + \frac{1}{m\epsilon}\right) + \left(\int_t^T e^{-r(s-t)} C_x(q, x) q\, ds\right) \Big/ m. \quad (7)$$

The monopolist's behaviour is obtained by setting $m = 1$, whereas the competitive firm's response is seen by setting $m = \infty$.[8] In the competitive case equation (7) becomes

$$R^A = C(q, x), \quad (8)$$

where R^A is price (at time t) if the output market is perfectly competitive, or the average of the marginal revenues of the firms extracting the resource if the output market is non-competitive.[9]

It follows from the above that in contrast to the oligopolist the monopolist fully internalizes his impact on future extraction opportunities while the competitive firm disregards it completely. Furthermore, the greater the degree of monopoly power the firm has over the resource (i.e. the smaller is m) the greater the divergence between marginal revenue and marginal cost at each point in time.

THE OPTIMAL PATH OF MARGINAL PROFITS

The optimal path of marginal profits is a function of the market structure over the resource. To see this, differentiate equation (7) with respect to time to yield

$$\frac{dM\pi_t^i}{dt} = \left(r \int_t^T e^{-r(s-t)} C_x(q, x) q\, ds\right) \Big/ m - C_x(q, x) q/m, \quad (9)$$

where $M\pi_t^i$ is marginal profits for firm i at time t. Since the sign of the right-hand side of equation (9) is ambiguous, the oligopolist's (monopolist's) marginal profits may decrease or increase over an arbitrary period. However, by the terminal date marginal profits must decrease to zero. For the competitive firm, marginal profits equal zero at each point in time.

As L-L show, the rate of growth of marginal profits is less than r for the

8 Intuitively, as the number of firms in the Cournot setting becomes very large, a natural extension to competitive industry obtains. In fact the rather stylized Cournot assumption that oligopolists disregard mutual interdependencies is an accurate description of the behaviour of competitive firms. Since each competitive firm alone is powerless to change industry parameters, mutual interdependencies, while recognized, are immaterial to the firm's extraction behaviour.

9 It is worth noting that as long as industry average cost, for every level of cumulative output, is an eventually increasing function of q, the mine need not be exhausted instantaneously under perfectly competitive extraction.

monopolist. In fact, the rate of growth of marginal profits will be the same for both the monopolist and Cournot oligopolist. This rate of growth is

$$\frac{dM\pi_t^i/dt}{M\pi_t^i} = r - \frac{C_x(q, x)q}{\int_t^T e^{-r(s-t)}C_x(q, x)qds}. \tag{10}$$

OUTPUT EFFECTS

The effect of market structure on the optimal output path is difficult to specify. L-L demonstrate that the optimal output path of the monopolist (and therefore the oligopolist) is ambiguous, i.e. the rate of extraction may increase or decrease over time. We will show, however, that if the market structure governing the extraction of the resource is competitive, output decreases globally over time. To see this, totally differentiate equation (5) with respect to time to yield, after minor manipulation.

$$\left(\frac{\partial M\pi_t^i}{\partial q_i}\right)\frac{dq_i}{dt} = qC_x\left(1 + \frac{1}{mu}\right) + \left(r\int_t^T e^{-r(s-t)}C_x(q, x)qds\right)\Big/ m - \frac{C_x(q, x)q}{m}, \tag{11}$$

where $u = 1/C_{qx}\cdot C_x/q \gtrless 0$ and $\partial M\pi_t^i/\partial q_i < 0$ by the Legendre condition. In the limit, under perfect competition,[10]

$$dq_i/dt = qC_x/\partial M\pi_t^i/\partial q_i < 0, \tag{12}$$

so that $dq/dt < 0$.

The rate at which the mine is exhausted is also a function of the market structure governing the extraction of the resource. If the output market is competitive, the transversality conditions require that AC$(q(T), x(T))$ = MC$(q(T), x(T))$ = price, so that cumulative output at the terminal date is independent of market structure over the resource. In view of equation (7), the greater the monopoly power the smaller the rate of output q for every level of cumulative output x. Therefore, since all market structures have the same cumulative output, the less market power that firms have over the resource the faster the rate of exhaustion. Where the output market is non-competitive, the effect of industry structure over the resource on the life of the mine becomes difficult to specify without considering the specific market demand conditions.

10 The Legendre condition will hold in the competitive case if the industry average cost function is an eventually increasing function of q.

WELFARE IMPLICATIONS

In their paper, L-L claim that a competitive industry will effect the same optimal depletion decisions as a social welfare maximizing central planner. Critical to their analysis, however, is the assumption that each firm in the competitive industry has exclusive property rights over the resource. But to repeat, property rights arrangements over natural resources such as oil and natural gas do not convey exclusive rights. Therefore, the competitive industry is bound to make suboptimal dynamic production decisions. Hence, from society's point of view, the choice between a monopoly or competitive industry in the provision of an exhaustible resource is not unambiguous and the ramifications for regulation are complex.

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The role of money in the Canadian economy: fixed vs flexible exchange rates

CHRISTOPHER A. SARLO / *McMaster University*

INTRODUCTION

The purpose of this note is to comment briefly on recent attempts to determine the direction of causality between money and income for the Canadian economy. Dy Reyes (1974), Barth and Bennett (1974), and Sharpe and Miller (1975) have all applied a test developed by Sims (1972) to Canadian data. The results of these tests are given in Table 1.

On the basis of these results, Dy Reyes concluded that there exists 'no significant causal influence between money and income in either direction'

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