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Investment, Market Structure, and the Cost of Capital

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THE SEMINAL ARTICLES BY Modigliani and Miller [7 and 8] have led to the misconception that the optimal investment strategy of an all equity firm is to set the marginal rate of return on investment equal to the rate of return required by shareholders. Two recent papers have dispelled this notion by arguing that the firm's optimal investment response is highly sensitive to the nature of the investment process so that the cost of equity capital is not always synonomous with the required rate. Elton and Gruber [3] demonstrate that the marginal rate of return on investment should be less than or greater than the rate of return required by shareholders depending on the evolution of the investment path over time. Gordon and Gould [5] show that, within the context of the constant dividend growth model, the marginal rate of return on investment should be less than the required rate.

In this paper, we will specify the conditions which determine the relationship between the marginal rate of return on investment and the rate of return required by shareholders. It is shown that the nature of the relationship is sensitive not only to the dynamics of the investment process but also to the industry market structure. In what follows, we first provide the intuitive background to motivate our results and then the formal analysis.

I. Investment Externalities and Market Structure

It is not difficult to conceive of the fact that the current investment activities of each firm within an industry determines the future investment opportunities of the industry as a whole. The effect need not be unidirectional. Current investment activities may either increase or decrease future industry investment opportunities. Either of these circumstances implies that externalities may ensue from the firm's current investment activity. The existence of these investment externalities, as well as their extent, depends crucially on how exclusive are property rights over future investment opportunities. The degree of exclusive property rights that a firm has over future industry investment opportunities is, in turn, a function of the firm's market power and, hence, industry structure. Where there are exclusive property rights to these opportunities, as in the case of a monopoly or collusive oligopoly, the externalities will be completely internalized by the firm in arriving at its optimal investment strategy. The externalities are simply co-opted and disappear. On the other hand, as long as property rights are non-

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exclusive, as Coase [2] argues in his pathbreaking article, the externalities remain. Therefore, in the polar case of a competitive industry, future investment opportunities will be treated by each firm in the industry as a common property resource, a free good.[1] Each firm will disregard the impact on future industry investment opportunities engendered by current investment activity. If we extend the common property resource analogy to a non-collusive oligopoly, we should expect that, as the number of firms in the industry is reduced, thereby increasing each firm's potential market power, the firm will internalize a greater proportion of its impact on future industry investment opportunities.[2] Therefore, the rate at which the industry investment opportunities are exploited by each firm is a function of the industry structure as well as the dynamics of the industry investment process.

The Model

We begin the formal analysis by considering the investment strategies of m identical Cournot oligopolists. Each firm is assumed to maximize the (expected) net present value of future cash flows discounted at the rate of return required by shareholders.[3] The oligopolist recognizes that the average rate of return which he earns is a function of the investment behaviour of all market participants both at a given moment in time and over time. Therefore, each firm must estimate the current and future investment behaviour of its $m - 1$ rivals prior to determining its own optimal investment strategy. In his typically myopic fashion the Cournot oligopolist assumes that his investment activity does not generate any reactions in the investment strategies of his rivals and, therefore, has no effect on his estimate of their current and future investment strategy.[4] Specifically, this model

[1] The dynamic welfare implications are almost Schumpterian in nature. To the extent that a competitive industry disregards the effect of its behaviour on future industry investment opportunities society is worse off and the monopolized industry is preferred. From a static resource allocation point of view, the reverse is true.

[2] See Chueng (1970).

[3] We assume throughout our analysis that all firms are competitive with respect to the supply of funds. Thus, the rate of return required by shareholders is constant and independent of market structure. If the use of funds today affects the return that shareholders require in the future, additional time dependent externalities will be operative and the required rate will also be a function of market structure.

[4] Economists continue to display an enduring interest in the stylized Cournot model because (i) there is no comprehensive theory of oligopoly behaviour anyway and (ii) the Cournot model is very useful in contrasting firm behaviour at both ends of the market power spectrum, i.e., monopoly versus perfect competition.

As explained further in the text, the Cournot framework does not assume that the investment levels of all other firm's are exogenously determined in the model. Underlying each Cournot oligopolist's reaction function at any point in time is an *estimate* of the investment behaviour of its rivals. If these estimates are not realized ex-post, the Cournot oligopolist will revise them but again in a myopic fashion. Clearly the game theoretic aspects of oligopoly behaviour are better captured by a more sophisticated oligopoly model or by a differential games approach. Nevertheless, the simpler and, hence, mathematically tractable Cournot framework encompasses the one crucial and fundamental assumption of oligopoly behaviour, namely, that each oligopolist's rate of return is recognized to be a function of all firms in the industry. As long as such dependencies are part of the model, externalities will only be partially internalized and the degree of internalization of future industry investment opportunities will be inversely related to the number of firms in the industry. Intuition

of oligopoly behaviour can be represented by:

Maximize

$$V^j(0) = \int_0^\infty e^{-kt}[E^j(t) - I^j(t)]\,dt \tag{1}$$

subject to:

$$\frac{dE^j(t)}{dt} = r(I(t), K(t), t)I^j(t) \tag{2}$$

$$\frac{dK(t)}{dt} = I(t) \tag{3}$$

where j = the j_{th} firm in the industry, $j = 1, \cdots, m$
$V^j(t)$ = value of firm j at time t
k = rate of return required by shareholders
$E^j(t)$ = earnings of firm j at time t
$I^j(t)$ = investment of firm j at time t
$I(t)$ = total industry investment at time t, $I(t) \equiv \sum_{j=1}^m I^j(t)$
$K(t)$ = total industry accumulated investment (capital stock) at time t, $K(t) = \int_0^t I(\tau)\,d\tau$
$r(I(t), K(t), t)$ = industry average rate of return on investment at time t

The industry average rate of return function which appears in constraint (2) requires some brief comment. This function represents the dynamic evolution of the industry's investment opportunities over time. It is assumed that the industry average rate of return is a function of current and past industry investments and, perhaps, time itself. In addition, the average rate of return is assumed to be inversely related to the rate of current investment for any given level of capital stock so that the average (and marginal) investment opportunities schedule is downward sloping at any point in time. Otherwise, r can take any form whatsoever although three polar cases are representational:[5]

(i) future industry investment opportunities are an increasing function of current (and past) investment decisions, i.e. $\frac{\partial r}{\partial K} > 0$

(ii) future industry investment opportunities are independent of current (and past) investment decisions, i.e. $\frac{\partial r}{\partial K} = 0$

(iii) future industry investment opportunities are a decreasing function of current (and past) investment decisions, i.e. $\frac{\partial r}{\partial K} < 0$

Clearly over any time horizon many combinations of these three cases are possible. In a life cycle theory of industry development, for example, initial

suggests that more sophisticated oligopoly models will affect the quantitative impact of the externality term rather than the qualitative result.

[5] Sufficient conditions for an optimum may require that r take on specific characteristics, i.e., concavity.

investment would result in new investment opportunities, eventually leading to a steady-state and, finally, industrial decline. On the other hand, in the case of IBM, $\frac{\partial r}{\partial K} > 0$ appears to be the rule for any conceivable finite time horizon.

II Market Structure and the Cost of Capital

The optimal solution to the oligopolist's investment problem is solved by standard control theory techniques in the Appendix. It is shown that the oligopolist's optimal response is governed by the equation:[6]

$$MR^j(t) \equiv r + \frac{\partial r}{\partial I} I^j = k - e^{kt} \int_t^\infty e^{-k\tau} \frac{\partial r}{\partial K} I^j d\tau \tag{4}$$

where $MR^j(t)$ is the marginal rate of return on investment at time t for firm j. The last term in equation (4) is the present value at time t of the marginal returns accruing to firm j from future industry investment opportunities engendered by current investment activity. Therefore, this equation can be interpreted to say that the deviation of the current marginal rate of return from the rate of return required by shareholders must equal the (marginal) effect on future industry investment opportunities internalized by firm j.

It is worth noting that equation (4) is a reaction function which describes the Cournot oligopolist's ex-ante behaviour as a function of his estimate of rival current and future investment activity. If these expectations are not realized ex-post (e.g., in the next period) the oligopolist will revise his estimates and solve for a new investment schedule. Again, he assumes myopically that his new investment schedule does not influence the remainder of the industry so that equation (4) is still the appropriate ex-ante decision rule. Equation (4) demonstrates that, whatever the adjustment mechanism, the more of the future industry opportunities which are expected to accrue to the firm $\left(\frac{\partial r}{\partial K} I^j\right)$ the greater the divergence between the current marginal rate of return on investment and the rate of return required by shareholders.

The assumption that rival expectations are mutually consistent across all firms in the industry transforms equation (4) from a reaction function to an equilibrium relationship.[7] Thus, we can sum equation (4) over all firms in the industry, m in number, to obtain:

$$rm + \frac{\partial r}{\partial I} I = km - e^{kt} \int_t^\infty e^{-k\tau} \frac{\partial r}{\partial K} I d\tau \tag{5}$$

Define $\epsilon(t)$ to be the elasticity of the industry average rate of return function

[6] From this point onwards, the time variable will normally be deleted for notational simplicity.

[7] Whether or not there is an equilibrium will depend on the specific adjustment process assumed for the revision of the estimates of rival investment activity. See, for example, Fisher (1961).

with respect to I at time t and divide equation (5) by m to yield for the marginal rate of return of the individual firm:

$$r\left(1+\frac{1}{m\epsilon}\right)=k-\frac{e^{kt}\int_t^{\infty} e^{-k\tau}\frac{\partial r}{\partial K}Id\tau}{m} \tag{6}$$

In equilibrium, the oligopolist's optimal response is an explicit function of the parameter m, the number of firms in the industry. This will permit us to contrast the optimal response of the Cournot oligopolist with that of the monopolist and competitive firm. In particular, equation (6) shows that the greater the oligopolist's market power (i.e. the smaller is m) over expected future industry opportunities the greater the divergence between the current marginal rate of return on investment and the rate of return required by shareholders. The monopolist's behaviour is obtained by setting $m = 1$ in equation (6). Therefore, in contrast to the oligopolist, the monopolist fully internalizes his impact on future investment opportunities.

The competitive firm's behaviour is obtained by letting m become arbitrarily large in equation (6). As the number of firms in the industry increases, the elasticity (me) of each firm's average rate of return function on the left-hand side of equation (6) increases by the multiplier m. In the limit, as m approaches infinity, the firm's average rate of return function becomes infinitely elastic—marginal rate of return on investment equals the average rate of return—in reflection of perfect competition. Also, as m approaches infinity, the second term on the right-hand side of equation (6) become progressively smaller so that the firm internalizes less of its impact on future investment opportunities. In the limit, under perfect competition, the firm completely disregards its impact on future investment opportunities.[8]

Formally, we have proved the following theorems:

THEOREM 1. *The monopolist (oligopolist) will set the current marginal rate of return on investment*

greater than / *equal to* / *less than* } *the rate of return required by shareholders if future industry investment opportunities*

[8] Intuitively, as the number of firms in the Cournot setting becomes infinitely large, a natural extension to the competitive industry obtains. In fact, the rather stylized Cournot assumption that oligopolists disregard mutual interdependencies is an accurate description of the behaviour of competitive firms. Since each competitive firm alone is powerless to change industry parameters, mutual interdependencies, even if recognized, are immaterial to the firm's investment behaviour. The competitive firm's powerlessness is manifest in its inability to earn a rate of return above the industry average should superior investment opportunities become available. As each firm tries to appropriate the potential benefits, similar efforts by a multitude of rivals will bid up the price of the investment and lower the yield so that each firm earns the industry average rate of return. Similarly, the competitive firm disregards its impact on future industry investment opportunities since the potential benefits cannot be appropriated by the individual firm.

are a

$$\left.\begin{array}{l}\textit{decreasing}\\ \textit{constant}\\ \textit{increasing}\end{array}\right\}\ \textit{function of current investment for all } t.^{9}$$

THEOREM 2. *The greater the degree of potential monopoly power which the firm has over future industry investment opportunities, the greater the divergence between the current marginal rate of return on investment and the rate of return required by shareholders.*

THEOREM 3. *The competitive firm always sets the average (marginal) rate of return equal to the rate of return required by shareholders independent of future industry investment opportunities.*

III Conclusion

We have shown that the implicit assumption underlying the traditional analysis of the cost of capital is that the industry is perfectly competitive or in long-run steady-state equilibrium. Otherwise, the cost of capital is sensitive to market structure, since alternative structures have different implications for the rate of exploitation of industry investment opportunities over time. Further research is imperative, however, if a definitive statement relating market structure and investment strategy is to be made. For example, market structure needs to be treated as a dynamic variable which can be affected by alternative industry investment rates. Also, the game-theoretic complexities of oligopoly behaviour should be endogenized in the modelling process. Such a theory presupposes a far more sophisticated model of dynamic oligopoly behaviour than we have presented.

Appendix

The Cournot oligopoly model is solved as a control theory problem.[10] Form the Hamiltonian:

$$H(t) = e^{-kt}[E^j(t) - I^j(t) + \lambda_E(t)r(I(t), K(t), t)I^j(t) + \lambda_K(t)I(t)] \quad \text{(A1)}$$

where $\lambda_E(t)$ and $\lambda_K(t)$ are the adjoint variables. The Maximum Principle requires

[9] Otherwise the sign of the externality term

$$\int_t^{\infty} e^{-k(\tau-t)} \frac{\partial r}{\partial K} I^j \, d\tau$$

is ambiguous. In the constant dividend growth model, for example, the marginal rate of return is set less than the rate of return required by shareholders. This result is obtained because the constant dividend growth model makes the restrictive assumption that future investment opportunities are growing at every point in time. In fact, this result is correct only if the firm has some potential market power.

[10] See, for example, Hadley and Kemp (1971).

that the optimal investment path satisfy the condition[11]

$$\frac{\partial H}{\partial I^j} = 0 \text{ for all } t \tag{A2}$$

while the adjoint variables must satisfy the differential equations

$$-\frac{\partial H}{\partial E^j} = \frac{d}{dt}(e^{-kt}\lambda_E) \tag{A3}$$

$$-\frac{\partial H}{\partial K} = \frac{d}{dt}(e^{-kt}\lambda_K) \tag{A4}$$

Also, the transversality conditions

$$\lim e^{-kt}\lambda_K = 0 \text{ as } t \to \infty \tag{A5}$$

$$\lim e^{-kt}\lambda_E = 0 \text{ as } t \to \infty \tag{A6}$$

are assumed to hold so that the necessary conditions (A2, A3, A4) are also sufficient.

Applying the Maximum Principle to our model yields

$$r + \frac{\partial r}{\partial I} I^j = \frac{1}{\lambda_E}(1 - \lambda_K) \tag{A7}$$

$$\frac{d\lambda_E}{dt} = k\lambda_E - 1 \tag{A8}$$

$$\frac{d\lambda_K}{dt} = k\lambda_K - \lambda_E \frac{\partial r}{\partial K} I^j \tag{A9}$$

Using the transversality conditions gives the following solutions to the differential equations (A8) and (A9):

$$\lambda_E = \frac{1}{k} \tag{A10}$$

$$\lambda_K = \frac{e^{kt}}{k} \int_t^\infty e^{-k\tau} \frac{\partial r}{\partial K} I^j \, d\tau \tag{A11}$$

Substituting these latter solutions into equation (A7) yields the optimality condition:

$$r + \frac{\partial r}{\partial I} I^j = k - e^{kt} \int_t^\infty e^{-k\tau} \frac{\partial r}{\partial K} I^j \, d\tau. \tag{A12}$$

[11] In addition to the differential equations (2) and (3). Note that equation (A2) assumes that the optimal control I^j takes an interior solution if I^j is constrained to be non-negative.

REFERENCES

1. S. Cheung, "The Structure of a Contract and the Theory of a Non-Exclusive Resource," Journal of Law and Economics, April, 1970.
2. R. Coase, "The Problem of Social Cost," *Journal of Law and Economics,* October, 1960.
3. E. J. Elton and M. J. Gruber, "Valuation and Asset Selection Under Alternative Investment Opportunities," *Journal of Finance,* May, 1976.
4. F. M. Fisher, "The Stability of Cournot Oligopoly Solution: The Effects of Speeds of Adjustment and Increasing Marginal Costs," *Review of Economic Studies,* February, 1961.
5. M. J. Gordon and L. I. Gould, "The Cost of Equity Capital: A Reconsideration and Extension," *Journal of Finance,* forthcoming.
6. G. Hadley and M. C. Kemp, *Variational Methods in Economics,* North-Holland, 1971.
7. F. Modigliani and M. M. Miller, "The Cost of Capital, Corporation Finance, and the Theory of Investment," *American Economic Review,* June, 1958.
8. and , "Taxes and the Cost of Capital: A Correction, *American Economic Review,* June, 1963.