The Downside of Reorder Flexibility under Price Competition

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The supply chain literature shows that reorder flexibility increases profits under competition, assuming fixed prices or quantity competition. We show that price competition, arguably a more appropriate price formation model in the presence of reorder flexibility, may yield *opposite* results. We consider a three-stage model of duopoly firms that sell differentiated products with stochastic demand. Firms make reorder-flexibility decisions and then place initial orders, before observing demand. After observing demand, firms set prices and, if they have the option, may reorder at a higher cost. We show that the expected profit functions are not unimodal and provide extensive equilibrium results. These appear to be the first for stochastic finite-horizon price-inventory competition with more than one order opportunity. We show: (i) Unilateral reorder flexibility is *not* an equilibrium. (ii) Reorder flexibility may *increase* initial orders. (iii) Reorder flexibility *hurts profits except* if it reduces initial orders and in addition, demand variability is moderate, reordering is sufficiently inexpensive, and products are sufficiently differentiated. (iv) Firms can avoid the downside of reorder flexibility only in some cases where it hurts profits. In others, firms are trapped in a prisoner's dilemma, whereby *reorder flexibility is the dominant strategy even though it hurts their profits.*

1. Introduction

Fashion apparel retailers face the problem of matching supply with demand over a short and unpredictable selling season. This is particularly challenging if they must make procurement decisions well before the season. However, the accuracy of demand forecasts increases dramatically at the inception of the season, when fashion trends are better understood. Retailers can exploit this improved demand information through *price flexibility* and *reorder flexibility*.

Price flexibility allows retailers to adapt prices to market conditions, contingent on whether demand for an item is high or low. Such flexibility also supports downside volume flexibility, whereby a retailer charges more than the liquidation price, to sell only a fraction of its inventory and hold back the rest. This strategy may boost profits under low demand. Some retailers are unwilling to slash prices even if this means throwing away unsold garments (Dwyer 2010).

However, price flexibility only helps manage demand. To adapt their *supply* and stay on top of fashion trends, reorder flexibility, a core component of *quick response* capabilities, is critical for fashion retailers. Their supply chains have become nimbler than ever. As a result of reduced lead

times, they can order not only well in advance, but also place and receive additional (typically more expensive) orders right before or at the inception of the season, once better demand information is available. This upside volume flexibility also offers *downside protection*, as it allows firms to reduce initial orders and still maintain the ability to satisfy higher demand. For example, in the well-known Sport Obermeyer case (Fisher and Raman 1996), the ability to place a second order reduces overstocking and increases product availability. With their lightning-fast supply chains, fast fashion retailers such as Zara are pushing reorder flexibility to an extreme.

Virtually the entire academic and trade literature focuses on these *benefits* of price and/or reorder flexibility for better matching supply with demand. However, this perspective ignores the following *downside* that we study in this paper: Reorder flexibility may erode profits by fostering more aggressive price competition. This effect is consistent with estimates of management consultants A.T. Kearney, that nowadays apparel retailers sell between 40 and 45 percent of inventory at a promotional price, up from 15 to 20 percent a decade ago (D'Innocenzio 2012). This paper cautions that, given the prevalence of price flexibility in the industry, concerns over intensified price competition are likely to grow as reorder flexibility proliferates. We address two main questions: (1) Why and under what conditions does reorder flexibility hurt or increase profits under price competition? (2) Which reorder flexibility configuration do retailers choose in equilibrium?

We study these questions in the context of a three-stage duopoly model with stochastic demand. Each firm sells a single differentiated product. Demand is a linear function of prices with an ex ante unknown intercept. Both firms simultaneously make reorder-flexibility decisions and then simultaneously place initial orders, before demand uncertainty resolves. In the last stage, after observing demand, firms simultaneously set prices and – if they have the option – reorder more units. The reorder unit cost exceeds the unit cost on initial orders, as shorter lead times increase sourcing and distribution costs. Leftover inventory is disposed with zero salvage value.

1.1. Overview of Main Results

This paper contributes novel managerial insights and technical results that derive from its focus on *price competition* under both *downside and upside volume flexibility*. Managerially, we identify under what conditions reorder flexibility hurts profits, and the resulting equilibrium flexibility choices. Technically, to our knowledge this paper provides the *first* equilibrium results for a stochastic finite-horizon problem under price competition with more than one order opportunity. In contrast, prior flexibility studies assume *quantity competition*. This distinction is important. Managerially, our results are in sharp contrast to those under quantity competition, and price competition may be a more appropriate model of price formation when firms have volume flexibility.

Technically, our analysis overcomes challenges that arise only under price competition.

1. Reorder flexibility can hurt profits under price competition. The ordering and pricing equilibria under the symmetric reorder flexibility configurations yield the following results (see Sections 3-5).

(a) Orders. The key effect of price competition is that it leads to more aggressive ordering. This weakens or even reverses the downside protection of reorder flexibility. Specifically, only if reordering is sufficiently cheap, relative to early procurement, do firms with reorder flexibility order less initially than inflexible firms. Otherwise, however, flexible firms order *more initially* than inflexible firms. Furthermore, even in cases where flexible firms order less initially, if demand turns out high, they reorder so much that they end up with more inventory than inflexible firms.

(b) Expected profits. Reorder flexibility hurts profits whenever it yields larger initial inventories and therefore lower prices and higher procurement costs. However, reorder flexibility may also hurt profits if it leads to smaller initial orders. Two countervailing effects are at work under low versus high demand. Reorder flexibility hurts profits *except* if the gains from downside protection under low demand dominate the losses from intensified competition under high demand, which holds under three conditions: (*i*) products are sufficiently differentiated; (*ii*) the demand variability is neither too small nor too large; and (*iii*) reordering is sufficiently cheap.

2. Reorder flexibility configurations in equilibrium. We show that in the flexibility-selection stage that precedes the procurement-pricing decisions, unilateral reorder flexibility is *not* an equilibrium. Furthermore, bilateral inflexibility is the Pareto-dominant symmetric equilibrium only in some of the cases where reorder flexibility hurts profits. In these cases the firms can avoid the downside of reorder flexibility by committing to inflexibility. However, in other cases they are trapped in a prisoner's dilemma, whereby it is the *dominant strategy for firms to select reorder flexibility even though it hurts their profits* (see Section 6).

These results point to the strategic importance of product differentiation and efficient reorder operations as *complementary* capabilities, not only to reap the benefits of reorder flexibility, but also to avoid its downside.

3. Price competition versus quantity competition. Our results under price competition are in stark contrast to prior findings on the effects of volume flexibility under *quantity (Cournot) competition.*

(a) Upside volume flexibility can hurt profits only under price competition. Lin and Parlaktürk (2012) consider a reorder option for duopoly retailers that sell a homogeneous product under quantity competition. In their analysis competition between "fast" retailers that can reorder after demand is realized yields (weakly) smaller initial orders and larger expected profits, compared to competition between "slow" retailers without a reorder option. These results are the *opposite* of

our findings that the "reorder" game may yield larger initial orders and lower expected profits than the "no reorder" game, and moreover, that a reorder option can only benefit firms if products are sufficiently differentiated (see Sections 5.2-5.3).

(b) Downside volume flexibility may yield lower inventories and profits under price versus quantity competition. Anupindi and Jiang (2008) prove that competition among homogeneous-product firms with downside volume flexibility through a hold back option yields higher expected profits than competition among inflexible firms that sell all supply ex post at the clearance price (Van Mieghem and Dada 1999 show this numerically). In their model of flexible firms, equilibrium sales quantities and prices are determined under quantity competition[1]. We show that compared to quantity competition, under price competition flexible firms not only get lower profits, as expected, but they may also make *lower* inventory investments, because price competition reduces their control of downside risk through the hold back option (see Section 3.2).

The assumption of quantity competition is usually justified with the classic result that single-stage quantity competition yields the same outcome as two-stage competition where firms first choose supply quantities (capacity, production or inventory) and then prices (Kreps and Scheinkman 1983). However, this equivalence critically hinges on the condition that firms *cannot* increase their supply while or after demand is formulated (cf. Tirole 1998, p. 217). By its very nature, volume flexibility may clearly violate this condition, in which case price competition may be a more appropriate model of price formation. This would certainly seem to apply to firms such as Zara that can flexibly increase their inventories above initial levels after observing demand. Whether quantity or price competition is more appropriate under volume flexibility depends on factors that affect how flexibly firms can increase their supply, such as the marginal cost and the delivery time of replenishment orders (cf. Tirole 1998, p. 224). The contrast between the results under price versus quantity competition underscores the importance of understanding these factors in order to better predict and improve performance under volume flexibility.

4. Equilibrium analysis under price competition. We provide extensive results that explicitly characterize the equilibria in terms of the demand and cost characteristics. The analysis is challenging because under price competition, each firm's first-stage expected profit function is generally *not unimodal* in its own order. Quantity competition is much more tractable as it does not face this challenge (see Section 3.1). This may explain why our results appear to be the first for stochastic finite-horizon price competition with more than one order. Our differentiated-products model

[1] They show that quantity competition has the same outcome as a two-stage production-price subgame, which proves the stochastic counterpart of the result of Kreps and Scheinkman (1983).

has the dual appeal of being more plausible and more tractable under price competition than the homogeneous-product model that is prevalent under quantity competition.

1.2. Literature Review

This paper is at the intersection of two literatures. One studies stochastic price-inventory control problems, the other considers the impact of price and/or operational flexibility on profitability.

Numerous papers in both streams focus on *monopoly* settings. See Chen and Simchi-levi (2012) for a recent survey of integrated price-inventory control models and Petruzzi and Dada (2011) who focus on newsvendor models. Among monopoly studies of flexibility, Van Mieghem and Dada (1999) study the benefits of production and price postponement strategies, with limited analysis of quantity competition; Cachon and Swinney (2009) show that quick response can be significantly more valuable to a retailer in the presence of strategic consumers than without them; Goyal and Netessine (2011) analyze volume and product flexibility under endogenous pricing.

The understanding of *stochastic price-inventory control under competition* is limited. At one extreme of the problem space, Bernstein and Federgruen (2005) and Zhao and Atkins (2008) study the *single period* problem in the classic newsvendor framework: The selling period is so short compared to lead times that firms can order only once, before demand is realized, and also choose prices in advance. At the other extreme, Kirman and Sobel (1974) and Bernstein and Federgruen (2004) study periodic-review *infinite-horizon* oligopolies, but under conditions that reduce them to myopic single period problems where decisions in each period are made before demand is realized. Kirman and Sobel (1974) obtain a partial characterization of a pure strategy Nash equilibrium with a stationary base-stock level. Bernstein and Federgruen (2004) identify conditions for existence of a pure strategy Nash equilibrium in which each retailer adopts a stationary base-stock policy and list price. Our paper studies models in the intermediate domain between the single-period and infinite-horizon extremes: The selling horizon is *finite* but firms are sufficiently responsive to exploit information gained over time to *make decisions more than once*. This case is gaining importance as businesses are countering shrinking product lifecycles with faster operations and adaptive pricing. However, this appears to be the *first* paper that considers price competition among firms that can *order more than once over time*. In the stochastic Bertrand-Edgeworth model (cf. Hviid 1991, Reynolds and Wilson 2000) homogeneous-product firms *order only once*, before observing demand, and set prices thereafter. In other studies (cf. Porteus et al. 2010, Liu and Zhang 2013 and references therein) firms only compete in prices whereas capacity levels are *exogenous*.

In the literature on *flexibility under competition*, a number of economics papers study the strategic effects of flexibility in the absence of demand uncertainty, e.g., Maggi (1996), Boccard and

Wauthy (2000), Röller and Tombak (1993). A general insight from these studies is that flexibility can be harmful under competition. However, demand uncertainty is the key challenge to matching supply with demand, and the fundamental reason for supply chain flexibility. Studies of flexibility under competition with stochastic demand can be grouped into two streams. One focuses on product flexibility (Anand and Girotra 2007, Goyal and Netessine 2007), the other, which includes this paper, on volume flexibility (cf. Vives 1989, Van Mieghem and Dada 1999, Anupindi and Jiang 2008, Li and Ha 2008, Caro and Martínez-de-Albéniz 2010, Lin and Parlaktürk 2012). In contrast to this paper, none of these studies consider price competition: They either assume *fixed* prices or consider endogenous pricing under assumptions that lead to *quantity* competition.

In the product flexibility stream, Anand and Girotra (2007) consider two-product firms that choose between early product differentiation before, or delayed differentiation after demand uncertainty is resolved. They show that early differentiation may arise as a dominant strategy. Goyal and Netessine (2007) consider two-product firms that choose whether to invest in flexible or dedicated technology and identify conditions under which flexibility benefits or harms profits. In both papers, unlike in ours, prices are determined by quantity competition and the total supply is determined before demand uncertainty resolves; the essence of product flexibility is that it allows firms to delay product-to-market allocation decisions until demand is known.

The volume flexibility stream has more of a history in economics. Going back to Stigler (1939), these papers often model the degree of flexibility on a continuum, by the slope of the average cost curve around some minimum; cf. Vives (1989) who studies a two-stage homogeneous-product oligopoly in which firms choose their flexibility level before receiving (private) demand signals, and then choose production quantities. In contrast, volume flexibility studies in the operations management literature typically consider two discrete flexibility configurations that differ in terms of the timing of supply decisions (capacity/production/inventory) relative to when demand is realized. The key finding that is common to these papers is that competition with volume flexibility *increases* expected profits compared to competition without such flexibility: This holds for downside flexibility through a hold back option under quantity competition (Van Mieghem and Dada 1999, Anupindi and Jiang 2008), and for upside flexibility through a reorder option or reactive capacity – both under fixed prices (Li and Ha 2008, Caro and Martínez-de-Albéniz 2010) and under quantity competition (Lin and Parlaktürk 2012). As discussed in Section 1.1, the conditions for the equivalence of price and quantity competition (Kreps and Scheinkman 1983, Anupindi and Jiang 2008) may not hold in the presence of volume flexibility. We show that the results under price competition may be the *opposite* of those under quantity competition and fixed prices.

Wu and Zhang (2014) study a sourcing game where homogeneous-product firms first choose between efficient (long lead-time, low cost) and responsive (short lead-time, high cost) sourcing, then place orders, and finally, after demand is realized, sell their inventories at the market-clearing price. Their setup bears some resemblance to ours, but there are important differences. In terms of modeling, we study price competition, and more importantly, we allow *two* orders, early and late, whereas in their model firms can order only *once*, early or late. Our model may be more applicable for fashion retailers such as Sports Obermeyer or Zara that typically order more than once. The results of the two papers are therefore not directly comparable, and they focus on different issues. Indeed, in their model, even without competition, either sourcing option may be preferred depending on the cost-information tradeoff. Wu and Zhang (2014) study the effects of competition and information on this tradeoff. In contrast, in our model a monopoly always prefers reorder flexibility, and we identify under what conditions price competition reverses this preference.

2. Models, Problem Formulations, and Preliminary Analysis

We study duopoly firms with price flexibility, each selling a single differentiated product with price-sensitive demand. The bulk of the paper (Sections 2-5) focuses on the analysis and comparison of two games in which the reorder option is symmetric between firms. In the "no reorder" game, referred to as N game, firms have no reorder flexibility but *only* price flexibility. In the "reorder" game, referred to as R game, firms have price *and* reorder flexibility. In Section 6 we justify this focus on symmetric flexibility configurations: We show that unilateral reorder flexibility is *not* an equilibrium in the flexibility-selection stage that precedes the procurement-pricing decisions.

2.1. Models

The N and R games share the following two-stage structure. In stage one, before observing demand, firms simultaneously choose their initial orders. The outcome of stage one is common knowledge. Demand uncertainty is resolved before firms make their second-stage decisions. The N and R games are identical up to this instant when demand is observed. They differ as follows in the second-stage decisions. In the N game, firms simultaneously choose prices but they cannot reorder. In the R game, firms simultaneously choose prices *and* reorder quantities. The initial unit procurement cost is typically lower than the reorder unit cost. Demand and sales occur following the second-stage decisions. Taken literally, this captures a situation where firms gain demand information through factors other than their own early-season sales, such as weather, market news and fashion trends. However, the model can also be viewed as a reasonable approximation of settings where sales that materialize between the first order delivery and the second-stage decisions only make up a small

fraction of initial inventory but are still of significant value for demand forecasting. It is quite common that the forecast accuracy for total season demand increases dramatically after observing a few days of early season sales. The model does not specify delivery lead times; we assume they are short enough so firms do not lose sales due to delivery delays. Without loss of generality the salvage value of leftover inventory is zero. We ignore further holding costs that may be incurred during the season, as they are insignificant relative to margins and overstocking costs.

We index the firms by $i \in \{1,2\}$ and denote their variables and functions, such as order quantities, prices, demands, and profits, by corresponding subscripts. We write $-i$ to denote firm i's rival, where $-i \neq i$. We model demand uncertainty in a linear demand system, which is widely used in the literature on differentiated products (e.g., McGuire and Staelin 1983, Singh and Vives 1984). The linear form arises as the solution of the optimal consumption problem of a representative consumer with quadratic utility (Vives 2001, Chapter 6.1). Let p_i denote firm i's price and $\mathbf{p} = (p_1, p_2)$. Demand for firm i's product is given by

$$d_i(\mathbf{p};\alpha) = \alpha - p_i + \gamma p_{-i} \geq 0, \; i = 1,2. \tag{1}$$

The intercept $\alpha > 0$ is ex ante uncertain; we call it the *market size parameter*. We assume that demand is high, i.e., $\alpha = \alpha_H$, or low, i.e., $\alpha = \alpha_L < \alpha_H$, with equal probability. The assumption of equally likely high- and low-demand scenarios does not change our main qualitative insights. Uncertainty in α may be due to factors that equally affect differentiated products in the same category, such as color in the case of fashion items. The *product substitution parameter* $\gamma \in [0,1)$ reflects the degree of product differentiation and factors such as brand preferences. We assume that γ is known, based on the notion that brand loyalty and price sensitivity are well understood. Firm-i's demand sensitivity to its rival's price increases in γ. The larger γ the less differentiated the products. For $\gamma \approx 1$ each firm's demand is (approximately) equally sensitive to both prices. The demand system (1) does not explicitly model situations with *perfectly* substitutable products (the higher-price firm gets positive demand even as $\gamma \to 1$), but as discussed in Section 7.1, our main insights continue to hold for a scaled version of (1) that does capture perfect substitution.

In stage one, before knowing whether the market size will be α_L or α_H, firm i chooses its (initial) order x_i at unit cost $c \in [0, C]$, where C is the unit reorder cost that is available in the presence of reorder flexibility. We normalize $C \equiv 1$ in our analysis without loss of generality, but use the notation C in our discussion. Let $\mathbf{x} = (x_1, x_2)$ denote the initial order vector.

2.2. Problem Formulations

Both for the N and the R game, our analysis focuses on pure-strategy Nash equilibria in the second stage and on subgame-perfect Nash equilibria in symmetric order strategies in the first stage.

No reorder game. Let $\pi_i^N(\mathbf{p}, x_i; \alpha)$ denote firm i's second-stage revenue function in the N game. It depends on both prices $\mathbf{p}$, firm i's initial inventory x_i, and the realized market size α. Given initial inventories $\mathbf{x}$ and market size α, firms simultaneously choose prices in the second stage:

$$\max_{p_i} \ \pi_i^N(\mathbf{p}, x_i; \alpha) = p_i \cdot \min(x_i, d_i(\mathbf{p};\alpha)), \ i = 1, 2, \tag{2}$$

where $\pi_i^N(\mathbf{p}, x_i; \alpha)$ is strictly concave in p_i. We assume that excess demand is lost. Since firms observe the market size realization α prior to choosing prices, they have no incentive to generate more demand than they can satisfy. However, a firm may find it optimal not to sell all the inventory procured in the first stage, if the market turns out to be small. In this case, we say that the firm prices to *hold back* inventory. As noted above, leftover inventory has zero salvage value. Let $\mathbf{p}^{N*}(\mathbf{x};\alpha)$ denote the second-stage equilibrium price vector and $\pi_i^{N*}(\mathbf{x}; \alpha) = \pi_i^N(\mathbf{p}^{N*}(\mathbf{x}, \alpha), x_i;\alpha)$ firm i's second-stage equilibrium revenue for the N game, as a function of the initial orders $\mathbf{x}$ and the market size α. Let $\Pi_i^N(\mathbf{x})$ denote firm i's expected profit as a function of initial orders $\mathbf{x}$. In the first stage, firms simultaneously choose their orders by solving

$$\max_{x_i \geq 0} \ \Pi_i^N(\mathbf{x}) = \frac{1}{2}\left(\pi_i^{N*}(\mathbf{x}; \alpha_L) + \pi_i^{N*}(\mathbf{x}; \alpha_H)\right) - cx_i, \ i = 1, 2. \tag{3}$$

Let $\mathbf{x}^{N*}$ denote equilibrium orders, and the scalar x^{N*} a symmetric equilibrium order quantity.

Reorder game. Let $\pi_i^R(\mathbf{p}, x_i;\alpha)$ denote firm i's second-stage profit function in the R game. In addition to choosing prices, firms can reorder inventory at a unit cost $C \geq c$. The reorder quantities are determined by the initial inventories and the prices: given x_i and $\mathbf{p}$, firm i orders the amount $(d_i(\mathbf{p};\alpha) - x_i)^+ = \max(d_i(\mathbf{p};\alpha) - x_i, 0)$ in the second stage. Given initial inventories $\mathbf{x}$ and market size α, firms simultaneously choose prices and the resulting reorder quantities in the second stage:

$$\max_{p_i} \ \pi_i^R(\mathbf{p}, x_i;\alpha) = p_i d_i(\mathbf{p};\alpha) - C(d_i(\mathbf{p};\alpha) - x_i)^+, \ i = 1, 2, \tag{4}$$

where $\pi_i^R(\mathbf{p}, x_i; \alpha)$ is concave in p_i. Let $\mathbf{p}^{R*}(\mathbf{x}; \alpha)$ denote the second-stage equilibrium price vector and $\pi_i^{R*}(\mathbf{x}; \alpha) = \pi_i^R(\mathbf{p}^{R*}(\mathbf{x}; \alpha), x_i;\alpha)$ firm i's second-stage equilibrium profit function for the R game. Let $\Pi_i^R(\mathbf{x})$ denote firm i's expected profit for the R game as a function of the initial order vector. In the first stage, firms simultaneously choose their initial orders by solving

$$\max_{x_i \geq 0} \ \Pi_i^R(\mathbf{x}) = \frac{1}{2}\left(\pi_i^{R*}(\mathbf{x}; \alpha_L) + \pi_i^{R*}(\mathbf{x}; \alpha_H)\right) - cx_i, \ i = 1, 2. \tag{5}$$

Let $\mathbf{x}^{R*}$ denote equilibrium initial orders, and the scalar x^{R*} a symmetric equilibrium initial order.

In both games, each firm's expected profit may be bimodal in its own initial order, due to the *joint* effect of price competition, uncertainty, and the sequential decisions over a finite horizon (see Section 3.1). Therefore, our analysis cannot rely on standard equilibrium characterization results. To overcome this challenge we exploit the structure of the best response problem.

Terminology. We refer to the option to hold back inventory also as *downside volume flexibility.* Firms have this flexibility in both games. We refer to the option to reorder inventory also as *upside volume flexibility.* Firms have this flexibility only in the R game. Compared to a hold back option, a reorder option offers firms stronger downside protection against low demand: Firms that hold back a fraction of initial orders still incur the cost on all ordered units. In contrast, the reorder option allows firms to *reduce* initial orders (and costs) and still maintain the ability to satisfy higher demand. Therefore, we use the phrase *downward protection* only in reference to the latter scenario.

2.3. Second-Stage Price-Reorder Equilibria

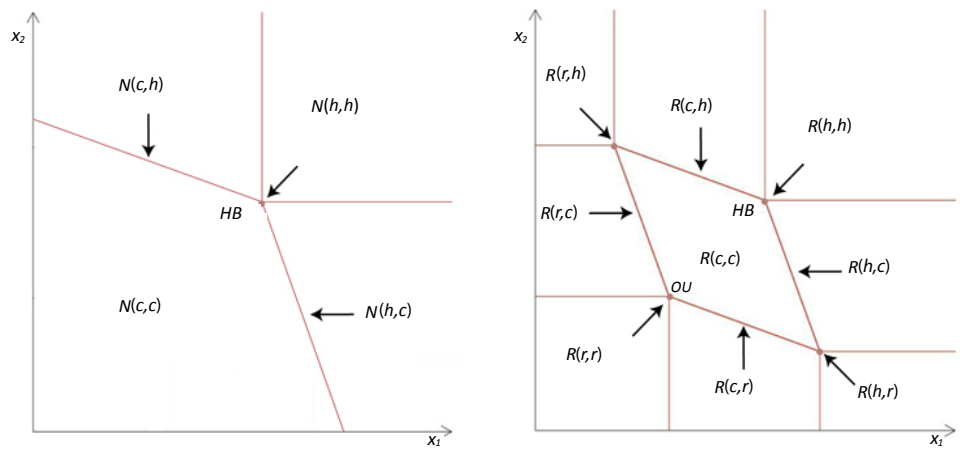
As a preliminary analysis we characterize the second-stage subgame equilibria of the N and R games. These serve as building blocks for our characterization and comparison of the first-stage order equilibria in Sections 3-6. Since firms learn the market size α prior to their second-stage decisions, the second-stage subgames are deterministic. Each subgame has an unique equilibrium that depends as follows on the initial order vector $\mathbf{x}$ and the realized market size α (Lemma 1 below summarizes these results).

No reorder game. Given initial inventories $\mathbf{x}$ and the realized market size α, firms simultaneously choose prices by solving (2). Define firm i's *clearance price* $p_i^c(x_i, p_{-i}; \alpha)$ as the highest price that generates enough demand to sell its inventory x_i, given its rival charges p_{-i} and the market size is α. Firm i may choose to charge more than this clearance price and hold back supply. Define firm i's *hold back price* $p_i^h(p_{-i}; \alpha)$ as its revenue-maximizing price in the absence of inventory constraints. Firm i prefers this price if it exceeds the clearance price, which results in leftover stock. That is, firm i's best response price is the larger of its clearance and hold back prices. We call these best responses *clearance (c)* and *hold back (h)*, respectively.

Both firms follow these strategies. As shown in Figure 1(a) the firms' equilibrium strategies partition the initial inventory space $\{\mathbf{x} \geq \mathbf{0}\}$ into four regions. The labels $N(c,c)$, $N(h,c)$, $N(c,h)$ and $N(h,h)$ identify the equilibrium strategies for each region, the first letter refers to firm 1 and the second to firm 2. For example, for initial inventory vectors $\mathbf{x} \in N(c,c)$ the unique equilibrium is for each firm to charge its clearance price. For high initial inventories $\mathbf{x} \in N(h,h)$ the unique

equilibrium is for each firm to charge the hold back price that generates demand and sales equal to the hold back threshold $HB := \alpha/(2-\gamma)$, so each firm has leftover inventory. The arrows in 1(a) indicate the shift from an initial inventory vector to the corresponding equilibrium sales.

Figure 1 Second Stage Equilibrium Strategies as Functions of Initial Inventories



(a) No Reorder Game (N) (b) Reorder Game (R)

Reorder game. Given initial inventories $\mathbf{x}$ and the realized market size α, firms simultaneously choose their prices and reorder quantities by solving (4). *Clearance* and *hold back* are two possible best responses, as in the N game. A third possibility is for firm i to charge the *reorder price* $p_i^r(p_{-i};\alpha)$, defined as its profit-maximizing price if it has no initial inventory. (The reorder price exceeds the hold back price since reordering is costly.) If its reorder price is lower than its clearance price, firm i's best response is to charge the reorder price and to procure more units to satisfy its excess demand at that price; we call this the *reorder (r)* strategy. Otherwise, firm i's best response is not to reorder and to charge the larger of its clearance and hold back prices.

Both firms follow these strategies. The resulting equilibrium strategies partition the initial inventory space $\{\mathbf{x} \geq \mathbf{0}\}$ into nine regions as shown in Figure 1(b). We use the same labeling convention as in the N game. For example, for initial inventories $\mathbf{x} \in R(r,r)$, the unique equilibrium is for each firm to charge the reorder price that generates demand equal to the order-up-to level $OU := (\alpha - C(1-\gamma))/(2-\gamma)$, and to reorder up to and sell this amount, as indicated by the arrow. For $\alpha \leq C(1-\gamma)$ firms have no incentive to reorder regardless of their initial inventory levels, and the second-stage N and R subgames are equivalent.

Lemma 1 summarizes these results (see proof for closed-form prices and quantities).[2]

[2] These N and R games are also analyzed in Maggi (1996); however, he restricts attention to deterministic demand.

Lemma 1. (Second Stage Subgame Equilibria). *For any initial inventory vector* $\mathbf{x}$ *and market size realization* α*, the price subgame of the* N *game and the price-reorder subgame of the* R *game each has a unique equilibrium. The equilibrium strategy of firm* i *depends as follows on* $\mathbf{x}$*.*

(N) In the N *game there is a hold back threshold* $\overline{x}_i(x_{-i};\alpha)$ *such that: (i) if* $x_i \leq \overline{x}_i(x_{-i};\alpha)$ *then firm* i *prices to clear its inventory; (ii) if* $x_i > \overline{x}_i(x_{-i};\alpha)$ *then it prices to hold back inventory and sells* $\overline{x}_i(x_{-i};\alpha)$*.*

(R) In the R *game there are hold back and order-up-to thresholds* $\underline{x}_i(x_{-i};\alpha) < \overline{x}_i(x_{-i};\alpha)$ *such that: (i) if* $x_i < \underline{x}_i(x_{-i};\alpha)$ *then firm* i *reorders up to* $\underline{x}_i(x_{-i};\alpha)$ *and charges the reorder price to sell this amount; (ii) if* $\underline{x}_i(x_{-i};\alpha) \leq x_i \leq \overline{x}_i(x_{-i};\alpha)$ *then it prices to clear its inventory but does not reorder; (iii) if* $x_i > \overline{x}_i(x_{-i};\alpha)$ *then it prices to hold back inventory and sells* $\overline{x}_i(x_{-i};\alpha)$*.*

In Sections 3 and 4 we characterize the first-stage order equilibria for the N and R games, respectively. These results build on Lemma 1.

3. Price Flexibility without Reorder Flexibility

In this section we first characterize the N game equilibria and then compare these results with those under quantity competition.

3.1. Downside Volume Flexibility under Price Competition: The N Game

In the first stage, firms simultaneously choose their order quantities by solving (3), where the second-stage equilibrium revenue functions $\pi_i^{N*}(\mathbf{x};\alpha_L)$ and $\pi_i^{N*}(\mathbf{x};\alpha_H)$ are given by Lemma 1.

Price competition, uncertainty, and sequential decisions imply bimodal expected profits. The equilibrium characterization of the N game is significantly complicated by the fact that the second-stage equilibrium revenue functions $\pi_i^{N*}(\mathbf{x};\alpha_L)$ and $\pi_i^{N*}(\mathbf{x};\alpha_H)$ are not concave in firm i's own order quantity. This fact, combined with demand uncertainty, implies that the first-stage expected profit functions of each firm may be bimodal in its own order.

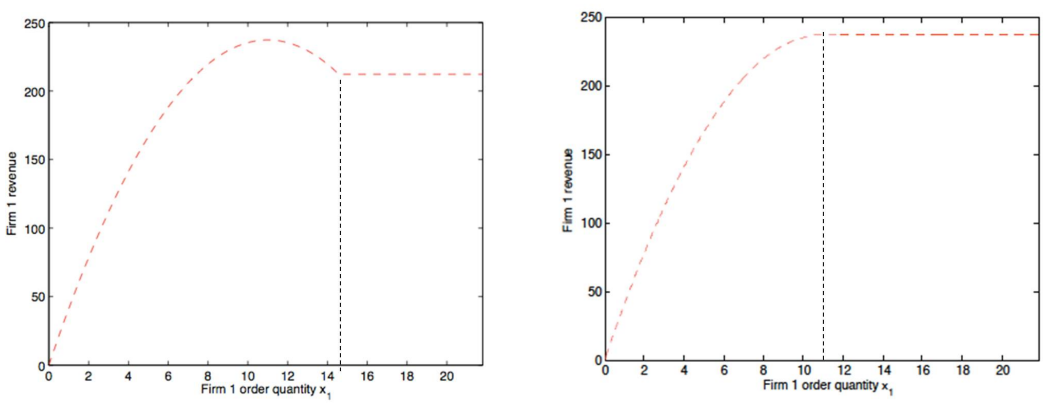
The non-concave nature of the second-stage equilibrium revenue functions is the natural result of *price competition*, coupled with the sequential nature of decisions over a finite horizon. Consider how $\pi_i^{N*}(\mathbf{x};\alpha)$ depends on firm i's order x_i, given the market size is α and its rival orders x_{-i}. By Lemma 1, in the N game equilibrium firm i never sells more than $\overline{x}_i(x_{-i};\alpha)$; it holds back any inventory in excess of this threshold. The function $\pi_i^{N*}(\mathbf{x};\alpha)$ is concave in $x_i \leq \overline{x}_i(x_{-i};\alpha)$, constant in $x_i \geq \overline{x}_i(x_{-i};\alpha)$, and peaks at a *smaller* quantity than the hold back threshold $\overline{x}_i(x_{-i};\alpha)$. Figure 2(a) shows a representative example for firm 1, given $\alpha = 15$ and $x_2 = 5$. Firm 1's second-stage

His second stage equilibrium characterization is less complete and explicit than what we require in Sections 3-6 for our analysis under stochastic demand. Therefore, we provide in Lemma 1 our own results and proof.

equilibrium revenue peaks at $x_1 = 11.0$, then decreases, before leveling off at the hold back threshold $\overline{x}_1(x_2;\alpha) = 14.6$. The property that the hold back threshold exceeds the revenue-maximizing quantity is due to price competition: The marginal revenue of each firm is higher if it unilaterally drops its price than if it unilaterally increases its inventory, so equilibrium prices keep dropping as x_1 increases from the revenue-maximizing quantity $x_1 = 11.0$ to $x_1 = 14.6$. Therefore, firm 1 has an incentive to sell more than 11 units if it has the inventory, even though doing so hurts its revenue.

In contrast, if prices are determined by *quantity competition*, then each firm's second-stage equilibrium revenue function is *concave* in its own order quantity and peaks at the hold back threshold, unlike under price competition. Figure 2(b) shows a representative example. Given $\alpha = 15$ and $x_2 = 5$, firm 1's hold back threshold equals $x_1 = 11$ units under quantity competition, and its second-stage equilibrium revenue function peaks at this threshold.

Figure 2 Effect of Competition Mode on Second Stage Equilibrium Revenue Function ($x_2 = 5,\ \alpha = 15,\ \gamma = 0.7$)



(a) Price Competition (b) Quantity Competition

In summary, the first-stage expected profit functions may be bimodal (and more generally, not unimodal under an arbitrary market size distribution), which is due to the *joint* effect of three factors, price competition, demand uncertainty and the sequential nature of ordering and pricing decisions over a finite horizon. If any one of these factors is absent, the payoff functions (in each stage) are well-behaved. This seems to be the case virtually throughout the literature, including in papers where the equilibrium prices are determined by quantity competition (cf. Anupindi and Jiang 2008), and in studies on joint price-inventory competition in *stationary infinite-horizon* problems (cf. Kirman and Sobel 1974, Bernstein and Federgruen 2004). A noteworthy exception is the version of our N game with perfectly *homogeneous* products (cf. Hviid 1991, Reynolds and Wilson 2000). In this extreme case, demand functions are discontinuous in prices, first-stage payoffs

are not well-behaved, and a pure strategy symmetric equilibrium in inventories need not exist if the extent of demand variation exceeds a threshold level (Reynolds and Wilson 2000). In contrast, our N game with differentiated products admits the following equilibrium result.

Proposition 1. (First Stage Order Equilibria: N Game). *Two thresholds on the market size ratio* $r_\alpha := \frac{\alpha_H}{\alpha_L}$ *determine the first stage order equilibria in the "no reorder" game:*

$$r_\alpha^{**} := m^{**}(\gamma) + 2c(1-\gamma)/\alpha_L > r_\alpha^{*} := m^{*}(\gamma) + 2c(1-\gamma)/\alpha_L \text{ for } \gamma > 0,$$

where $m^*(\gamma)$ *and* $m^{**}(\gamma)$ *are explicit functions of* γ *and* $m^{**}(\gamma) > m^*(\gamma) > 1$ *for* $\gamma > 0$. *Let* $HB(\alpha) := \alpha/(2-\gamma)$ *denote the hold back threshold for market size* α. *If the market size ratio is:*

(i) below the smaller threshold, i.e., $r_\alpha \le r_\alpha^*$, *there is a unique symmetric order equilibrium:*

$$x^{N*} = x_l^{N*} := \frac{(1+\gamma)(\alpha_H/2 + \alpha_L/2 - c(1-\gamma))}{2+\gamma} \le HB(\alpha_L), \tag{6}$$

and firms price to clear their inventory in both demand scenarios;

(ii) larger than the larger threshold, i.e., $r_\alpha \ge r_\alpha^{**}$, *there is a unique symmetric order equilibrium:*

$$HB(\alpha_L) < x^{N*} = x_h^{N*} := \frac{1+\gamma}{2+\gamma}(\alpha_H - 2c(1-\gamma)) < HB(\alpha_H), \tag{7}$$

firms price to sell $HB(\alpha_L)$ *and hold back inventory if demand is low, and they price to clear their inventory if demand is high;*

(iii) between the two thresholds, i.e., $r_\alpha^* < r_\alpha < r_\alpha^{**}$, *there are exactly two symmetric equilibria, one as in (i), the other as in (ii). Moreover,* x_l^{N*} *Pareto-dominates* x_h^{N*}.

Given any symmetric initial inventories in the second stage, firms sell at most the hold back threshold $HB(\alpha)$ corresponding to the realized market size α. The equilibrium order is therefore smaller than $HB(\alpha_H)$, but it may be smaller or larger than $HB(\alpha_L)$ due to market size uncertainty.

"Small" equilibrium: $x_l^{N*} < HB(\alpha_L)$. If the high-demand market is not too large, relative to the low-demand market ($r_\alpha < r_\alpha^{**}$), the Pareto-dominant equilibrium order is such that firms price to clear their inventory under either market size realization. The equilibrium order x_l^{N*} in (6) and the expected equilibrium price and profit of each firm are the same as under the corresponding *riskless* problem, i.e., if the market size were known and equal to the mean $(\alpha_H + \alpha_L)/2$. However, market size uncertainty leads to variability in equilibrium profits, leaving firms better off under high demand and worse off under low demand, compared to the riskless case.

"Large" equilibrium: $x_h^{N*} > HB(\alpha_L)$. If the high-demand market is relatively large ($r_\alpha > r_\alpha^{**}$), the firms order more in equilibrium than in the corresponding riskless problem. Their order x_h^{N*}

is so large that they only price to clear their inventory if demand is high. If demand is low, they charge the hold back price, sell $HB(\alpha_L)$ and have leftover inventory, so that their revenues are independent of how much the firms order in excess of $HB(\alpha_L)$. The equilibrium quantity x_h^{N*} in (7) balances the marginal ordering cost with the incremental revenue under high demand.

3.2. Downside Volume Flexibility under Price versus Quantity Competition

Earlier studies of volume flexibility with stochastic demand consider endogenous pricing under *quantity* competition. This assumption is usually justified with the classic result that single-stage quantity competition yields the same outcome as two-stage competition where firms first choose supply quantities (capacity, production or inventory) and then prices (Kreps and Scheinkman 1983). However, this equivalence critically hinges on the condition that firms *cannot* increase their supply while or after demand is formulated (cf. Tirole 1998, p. 217). By its very nature, volume flexibility may clearly violate this condition, in which case price competition may be a more appropriate model of price formation. Our novel price competition results are relevant in this light. They also allow us to compare the effects of volume flexibility under price versus quantity competition. Here we focus on downside volume flexibility, in Sections 5.2-5.3 on reorder flexibility.

In the N game firms have downside volume flexibility through a hold back option. Anupindi and Jiang (2008) prove for perfectly *homogeneous* products that competition under downside volume flexibility through a hold back option yields higher expected profits and capacity investments than competition among inflexible firms. In their model inflexible firms choose supply quantities in the first stage, before demand is known, and sell all supply in the second stage at the clearance price, i.e., they have no hold back option. Flexible firms also choose capacities ex ante, but sales quantities and prices are determined in the second stage under *quantity competition*. (They show the quantity competition subgame to be equivalent to a two-stage production-pricing subgame).

To study how the effects of a hold back option depend on the mode of competition, we compare the results of our N game (with hold back under price competition) to those of two variations, the N *game with clearance*, and the N *game under quantity competition* (with hold back). These variations correspond to the models in Anupindi and Jiang (2008) of inflexible and flexible firms, respectively (they consider significantly more general demand uncertainty). The analysis of these N game versions is much simpler than under price competition, and they each have a unique symmetric pure strategy equilibrium. We omit the details[3] and summarize our results informally:

[3] Proofs of these statements are available upon request.

1. *Price competition reduces but does not eliminate the value of downside volume flexibility. The N game under quantity competition yields (weakly) higher expected profits than the N game under price competition, and both yield higher profits than the N game with clearance.*
2. *Price competition yields lower inventory investments than quantity competition under moderate demand variability: There is an unique threshold $r_\alpha^Q (< r_\alpha^* < r_\alpha^{**})$, such that if $r_\alpha \in (r_\alpha^Q, r_\alpha^{**})$ then the Pareto-dominant equilibrium order is x_l^{N*} in the N game under price competition, and the equilibrium order is $x_h^{N*} > x_l^{N*}$ in the N game under quantity competition.*

That price competition may yield *lower* inventories than quantity competition, runs counter to the deterministic case. It follows because under quantity competition, firms have better control of downside risk – they can hold back more inventory. Hence they invest more upfront, sell more under high demand and less (by exercising their hold back option) under low demand.

The power of the hold back option rests on the flexible firms' ability to commit to underutilizing capacity if demand is low. The extent of this hold back commitment in turn depends on the mode of competition in the second stage, following the capacity investments. A model where equilibrium sales quantities and prices are determined in the second stage under quantity competition, as in Anupindi and Jiang (2008), captures settings where firms are not sufficiently flexible to deviate from second-stage production quantities even if they have excess capacity. However, if firms can flexibly supply in the second stage any amount up to their initial capacity choice – due to preproduction or production postponement with highly flexible operations, then no quantity competition equilibrium with excess capacity is sustainable: Each firm has the incentive to lower its price and increase sales. Price competition is more appropriate in such cases as no firm has an incentive to lower its price and increase sales at the corresponding equilibrium, even if it has the capacity to do so. In this sense the hold back commitment is more credible under the price competition equilibrium.

4. Price and Reorder Flexibility

In this section we characterize the initial order equilibria for the R game, i.e., under price and reorder flexibility. Firms simultaneously choose their initial orders by solving (5). The second-stage equilibrium profit functions $\pi_i^{R*}(\mathbf{x}; \alpha_L)$ and $\pi_i^{R*}(\mathbf{x}; \alpha_H)$ are specified by Lemma 1. As in the N game, due to price competition, each firm's first-stage expected profit function may be bimodal in its initial order. The R game analysis is further complicated by the reorder option.

We henceforth assume that $\alpha_L = C(1-\gamma)$, which implies that it is not profitable to reorder if demand is low (i.e., $OU(\alpha_L) = 0$). This assumption seems reasonable in that firms typically procure enough early on to cover at least what they consider to be their base demand. Relaxing

this assumption makes the analysis more cumbersome without generating additional insights, as confirmed through extensive numerical experiments with different values of α_L.

Proposition 2. (First Stage Order Equilibria: R Game). *Consider the "reorder" game with $\alpha_L = C(1-\gamma) < \alpha_H$. There exists a symmetric initial order equilibrium x^{R*}. Under the strictly Pareto-dominant symmetric equilibrium, in the second stage the firms do not reorder under low demand and they price to clear inventory under high demand; their price-reorder strategies depend as follows on the market size ratio $r_\alpha := \alpha_H/\alpha_L$ and the order cost ratio $r_c := c/C$:*

Market Size Ratio	*Pricing Strategy under Low Demand*	*Reorder Strategy under High Demand*
(i) $r_\alpha \le m^{**}(\gamma)$	*clear inventory*	*reorder to* $OU(\alpha_H)$ *iff* $r_c > \overline{r}_c(r_\alpha,\gamma)$
(ii) $m^{**}(\gamma) < r_\alpha < 2$	*clear inventory if* $r_c \ge \underline{r}_c(r_\alpha,\gamma)$; *sell* $HB(\alpha_L)$ *with leftover otherwise*	
(iii) $r_\alpha \ge 2$	*clear inventory if* $r_c \ge r_c(r_\alpha,\gamma)$; *sell* $HB(\alpha_L)$ *with leftover otherwise*	*reorder to* $OU(\alpha_H)$ *iff* $r_c > r_c(r_\alpha,\gamma)$

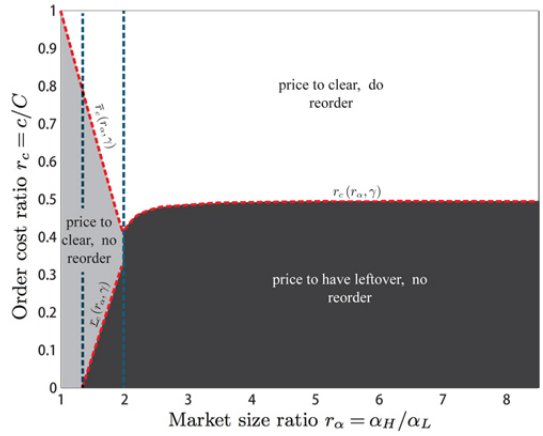
The thresholds $m^{**}(\gamma)$, $\underline{r}_c(r_\alpha,\gamma)$, $\overline{r}_c(r_\alpha,\gamma)$, and $r_c(r_\alpha,\gamma)$ are explicit functions and $\underline{r}_c(r_\alpha,\gamma) < \overline{r}_c(r_\alpha,\gamma)$. The hold back threshold under low demand, and the order-up-to level under high demand are, respectively,

$$HB(\alpha_L) = C\frac{1-\gamma}{2-\gamma} \text{ and } OU(\alpha_H) := \frac{\alpha_H - C(1-\gamma)}{2-\gamma}.$$

Proposition 2 identifies three combinations of low-demand pricing and high-demand reordering that can occur under a strictly Pareto-dominant equilibrium. The firms do not reorder under low demand since doing so is unprofitable for $\alpha_L = C(1-\gamma)$. They price to clear inventory under high demand because they have no incentive to order more than the high-demand hold back threshold $HB(\alpha_H)$. Note that, under no *strictly* Pareto-dominant equilibrium do firms in the second stage price to have leftover under low demand but reorder under high demand. These strategies hold only for initial orders in the range $(HB(\alpha_L), OU(\alpha_H))$. However, firms weakly prefer ordering initially at most $HB(\alpha_L)$ or at least $OU(\alpha_H)$, because their second-stage equilibrium revenues are *independent* of their initial orders in the range $[HB(\alpha_L), OU(\alpha_H)]$: If demand is low, they price to sell $HB(\alpha_L)$ and have leftover inventory; if demand is high, they reorder up to and sell $OU(\alpha_H)$. Therefore, the firms' preferences over initial order quantities in this range only depend on the relative cost of ordering early, at unit cost c, or later at unit cost C. The firms weakly prefer ordering initially at most $HB(\alpha_L)$ if $c \ge C/2$, and at least $OU(\alpha_H)$ if $c \le C/2$.

Figure 3 illustrates, for $\gamma = 0.7$, how the conditions in Proposition 2 partition the parameter space of market size ratios r_α and order cost ratios r_c into three regions, each corresponding to one of the three possible second-stage price-reorder strategies. For $\gamma = 0.7$, Part (i) of Proposition 2 applies for market size ratios $r_\alpha \le 1.32$, Part (ii) for $r_\alpha \in (1.32, 2)$, and Part (iii) for $r_\alpha \ge 2$.

Figure 3 **Proposition 2: Symmetric R Game Equilibrium as a Function of Market Size Ratio and Order Cost Ratio ($\gamma = 0.7$)**



Part (i). If the high- and low-demand markets are of sufficiently similar size ($r_\alpha \leq 1.32$), firms never end up with excess inventory. Their initial orders are moderate, such that they prefer to clear their inventories even under low demand. The order cost ratio only affects whether the firms make use of reorder flexibility to delay part of their order; this is the case only if reordering is sufficiently cheap, i.e., the order cost ratio r_c exceeds the threshold $\overline{r}_c(r_\alpha, \gamma)$.

If the high- and low-demand market sizes differ more significantly ($r_\alpha > 1.32$), firms are willing to order more aggressively initially, at the risk of overstocking under low demand – provided that initial ordering is cheap enough, i.e., the order cost ratio r_c is low enough.

Part (ii). For moderately different market sizes ($1.32 < r_\alpha < 2$), the equilibrium strategies depend on two thresholds on the order cost ratio. If the order cost ratio r_c is below the lower threshold $\underline{r}_c(r_\alpha, \gamma)$, the firms initially order enough for high demand, but more than they wish to sell under low demand. If the order cost ratio r_c exceeds the larger threshold $\overline{r}_c(r_\alpha, \gamma)$, the firms initially order so little that they reorder under high demand, but they price to clear their inventories under low demand. If the order cost ratio r_c is in the intermediate range $[\underline{r}_c(r_\alpha, \gamma), \overline{r}_c(r_\alpha, \gamma)]$, the firms' initial orders are low enough so they price to clear inventories under low demand, yet large enough so they do not reorder under high demand.

Part (iii). For sufficiently different market sizes ($r_\alpha \geq 2$), firms do not match the demand with their initial order. If early ordering is relatively cheap, that is, $r_c \leq r_c(r_\alpha, \gamma)$, then firms initially order more than they sell under low demand and do not reorder; otherwise, they initially order less than they need under high demand and do reorder if demand is high.

5. The Impact of Reorder Flexibility on Orders and Profits

In this section we compare the equilibria with and without reorder flexibility. We call firms inflexible in the N game and flexible in the R game. In Section 5.1 we identify two conditions that determine

the impact of reorder flexibility on initial orders. In Section 5.2 we use these conditions to explain the known results that under *quantity competition*, reorder flexibility reduces initial orders and improves expected profits. These result are in stark contrast to ours: In Section 5.3 we show that under *price competition*, reorder flexibility may increase initial orders and lower profits, and moreover, a reorder option cannot benefit firms if products are sufficiently close substitutes.

5.1. Two Conditions Determine the Impact of Reorder Flexibility on Orders

The availability of a reorder option has two effects on the initial equilibrium orders of flexible firms, in comparison to the equilibrium order quantity x^{N*} of inflexible firms. First, reorder flexibility softens the firms' output constraints from their initial procurements, which may intensify their second-stage competition. Second, reorder flexibility allows firms to reduce overstocking risks in matching supply with demand. The first effect may give flexible firms an incentive to sell more than x^{N*} in the second stage *if* demand is high, and the magnitude of the second effect determines in such cases whether they *initially* order more or less than x^{N*}.

To make this discussion precise, consider first the N game equilibrium orders. The inflexible firms hedge their bets between low and high demand, ordering up to the point where their expected marginal second-stage equilibrium revenue equals the initial unit procurement cost:

$$\frac{1}{2}\left(\frac{\partial \pi_i^{N*}\left(\mathbf{x}^{N*};\alpha_L\right)}{\partial x_i}+\frac{\partial \pi_i^{N*}\left(\mathbf{x}^{N*};\alpha_H\right)}{\partial x_i}\right)=c. \tag{8}$$

As a result, they order more than optimal for known low demand and less than optimal for known high demand. From (8), we have

$$c-\frac{\partial \pi_i^{N*}\left(\mathbf{x}^{N*};\alpha_L\right)}{\partial x_i}=\frac{\partial \pi_i^{N*}\left(\mathbf{x}^{N*};\alpha_H\right)}{\partial x_i}-c>0, \tag{9}$$

i.e., the marginal profit loss if demand is low (the LHS) equals the marginal profit gain if it is high.

The flexible firms initially order more than x^{N*} units, if and only if two conditions hold:

(A) The flexible firms want to sell more than x^{N*} units under high demand. That is, x^{N*} is smaller than the high-demand order-up-to level:

$$x^{N*}<OU\left(\alpha_H\right). \tag{10}$$

This condition holds if and only if the marginal reorder cost C is lower than a flexible firm's high-demand marginal revenue at x^{N*} from unilaterally dropping its price. This marginal revenue exceeds an inflexible firm's high-demand marginal revenue at x^{N*} from unilaterally increasing its inventory (the term $\partial \pi_i^{N*}\left(\mathbf{x}^{N*};\alpha_H\right)/\partial x_i$ in (8)). In the degenerate case without demand uncertainty, i.e.,

$\alpha_L = \alpha_H$, whenever condition *(A)* holds, the flexible firms initially order more than x^{N*} units and have lower profits than inflexible firms. However, under stochastic demand, firms must also evaluate under- and overstocking costs.

(B) Expected marginal understocking costs at x^{N*} exceed expected marginal overstocking costs. Ordering initially more than x^{N*} units, at a lower cost, realizes procurement cost savings only if condition *(A)* holds *and* demand turns out to be high; otherwise, if demand turns out to be low, more initial inventory yields lower prices and profits. Therefore, flexible firms initially order more than x^{N*} if, and only if, *(A)* holds and the expected marginal cost saving from early procurement under high demand exceeds the expected marginal profit loss at x^{N*} under low demand:[4]

$$C - c > c - \frac{\partial \pi_i^{N*}(\mathbf{x}^{N*}; \alpha_L)}{\partial x_i}. \tag{11}$$

If condition *(A)* is violated, then the flexible firms initially order the same amount as the inflexible firms, x^{N*}, and do not reorder in the second stage. If condition *(A)* holds but *(B)* is violated, then the flexible firms initially order (weakly) *less than* x^{N*} and their order-up-to level under high demand strictly exceeds x^{N*}. An important implication is that, whenever the equilibrium orders with reorder flexibility differ from those without, the flexible firms end up with *more inventory under high demand* than inflexible firms with a single order. As shown in Section 5.3, this over-ordering hurts the flexible firms' profits under high demand. Table 1 summarizes this discussion.

Table 1 Comparison of Equilibrium Order Strategies in N and R Games.

Flexible Firms Order Initially		Conditions
Same:	$x^{R*} = x^{N*}$	$x^{N*} \geq OU(\alpha_H)$
More:	$x^{R*} > x^{N*}$	$x^{N*} < OU(\alpha_H)$ and $C - c > c - \frac{\partial \pi_i^{N*}(\mathbf{x}^{N*};\alpha_L)}{\partial x_i}$
Less:	$x^{R*} \leq x^{N*}$	$x^{N*} < OU(\alpha_H)$ and $C - c \leq c - \frac{\partial \pi_i^{N*}(\mathbf{x}^{N*};\alpha_L)}{\partial x_i}$

5.2. Quantity Competition: Reorder Flexibility Reduces Initial Orders, Improves Profits

Before we discuss the effects of reorder flexibility in our model with price competition, consider the case where second-stage prices and reorder quantities are determined under *quantity competition*. This mode of competition yields the same results as for the monopoly case ($\gamma = 0$): Flexible firms initially order (weakly) less than inflexible firms. This follows because conditions *(A)* and *(B)* are mutually exclusive. Namely, if it is profitable under high demand to order more than the optimal

[4] Since low and high demand are equally likely, we omit the probabilities from this expression.

no-reorder quantity x^{N*}, then it is cheaper to delay doing so until the second stage. Mathematically, under quantity competition (and for a monopoly) condition *(A)* is equivalent to

$$\frac{\partial \pi_i^{N*}(\mathbf{x}^{N*};\alpha_H)}{\partial x_i} > C,$$

which, together with (9), implies that condition *(B)* cannot hold:

$$c - \frac{\partial \pi_i^{N*}(\mathbf{x}^{N*};\alpha_L)}{\partial x_i} = \frac{\partial \pi_i^{N*}(\mathbf{x}^{N*};\alpha_H)}{\partial x_i} - c > C - c.$$

That is, the marginal profit loss under low demand exceeds the cost savings from early procurement.

This argument sheds light on the finding in Lin and Parlaktürk (2012) who consider a reorder option for duopoly retailers that sell a homogeneous product under *quantity competition*, in contrast to our R game under price competition. In their model competition between "fast" retailers that can reorder after learning demand yields (weakly) smaller initial orders and larger expected retailer profits, compared to competition between "slow" retailers without a reorder option.

5.3. Price Competition: Reorder Flexibility May Increase Initial Orders, Hurt Profits

The results under quantity competition are in stark contrast to ours under price competition. As we discuss in this section, the R game may yield larger initial orders and lower expected profits than the N game, and moreover, a reorder option cannot benefit firms if products are sufficiently close substitutes. The following proposition summarizes these results.

Proposition 3. (Effects of Reorder Flexibility). *Assume $\alpha_L = C(1-\gamma)$. Consider the expected profits and the order strategies under the Pareto-dominant symmetric equilibrium of the "no-reorder" game and of the "reorder" game.*

1. *(Value of reorder flexibility). The firms with reorder flexibility are not more profitable than those without, except if the following three conditions hold:*
   (a) *the products are sufficiently differentiated, i.e., $\gamma < 0.875$;*
   (b) *the market size variability is moderate, i.e., $\underline{r}_\alpha(\gamma) < r_\alpha < \overline{r}_\alpha(\gamma)$, where $\overline{r}_\alpha(\gamma) < \infty$ for $\gamma > 0$;*
   (c) *reordering is relatively inexpensive, i.e., $\underline{r}_c(r_\alpha, \gamma) < r_c \leq 1$.*
2. *(Order strategies when reorder flexibility is valuable). Whenever the firms with reorder flexibility are more profitable than inflexible firms, they order less initially, but under high demand they reorder and sell more inventory, than the inflexible firms, i.e., $x^{R*} < x^{N*} < OU(\alpha_H)$.*
3. *(Equal equilibrium outcomes). The firms with reorder flexibility order the same amount as those without, and they do not reorder, i.e., $x^{R*} = x^{N*} \geq OU(\alpha_H)$, if and only if:*
   (a) *the market size ratio is below a threshold, i.e., $r_\alpha \leq \overline{\overline{r}}_\alpha(\gamma)$, where $\overline{\overline{r}}_\alpha(\gamma) < \infty$ for $\gamma > 0$;*

(b) *reordering is relatively expensive, i.e.,* $r_c \leq \overline{r}_c(r_\alpha, \gamma)$.

By part 2. of Proposition 3, reorder flexibility always hurts profits if the flexible firms order more ex ante than inflexible firms; the additional inventory increases procurement costs and lowers prices. However, reorder flexibility may also hurt profits in cases where the flexible firms order less initially, compared to inflexible firms; the flexible firms are only better off under the additional conditions 1.(a)-(c) of Proposition 3. Next we compare the orders of flexible and inflexible firms. Then we discuss the conditions under which reorder flexibility improves firm profits. In this discussion, we say "equilibrium" as shorthand for "Pareto-dominant symmetric equilibrium".

Impact of reorder flexibility on equilibrium orders. Consider the impact of procurement costs on the flexible firms' initial order incentives, relative to the N game equilibrium. An increase in the order cost ratio r_c has two effects on the conditions *(A)* and *(B)* of Section 5.1.

1. It reduces the flexibility cost, which creates a stronger incentive for the flexible firms to sell more than x^{N*} units under high demand. That is, *(A)* is likelier to hold.
2. It reduces the early procurement cost savings under high demand and increases the expected marginal overstocking costs under low demand, making it more attractive for the flexible firms to order less initially and reorder only under high demand. That is, *(B)* is likelier to be violated.

Figure 4 shows the impact of reorder flexibility on equilibrium orders for $\gamma = 0.7$. It partitions the parameter space of market size ratios r_α and order cost ratios r_c into three regions, depending on whether the flexible firms initially order the same as ($x^{R*} = x^{N*}$), more than ($x^{R*} > x^{N*}$), or less than ($x^{R*} < x^{N*}$), the inflexible firms. These regions correspond to the cases in Table 1.

Figure 4 **Equilibrium Orders** ($\gamma = 0.7$)

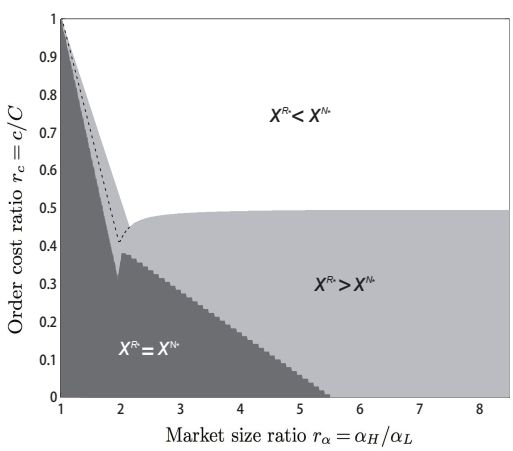
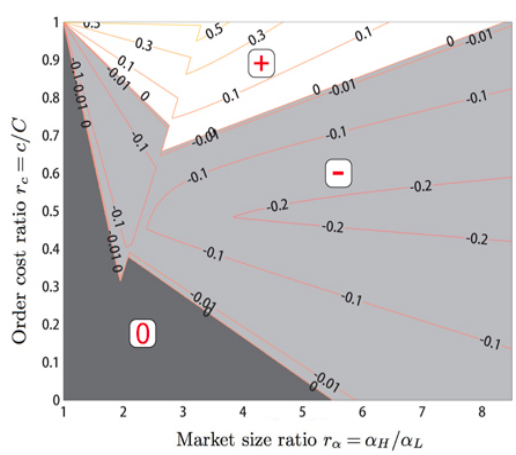


Figure 5 **Value of Flexibility** ($\gamma = 0.7$)



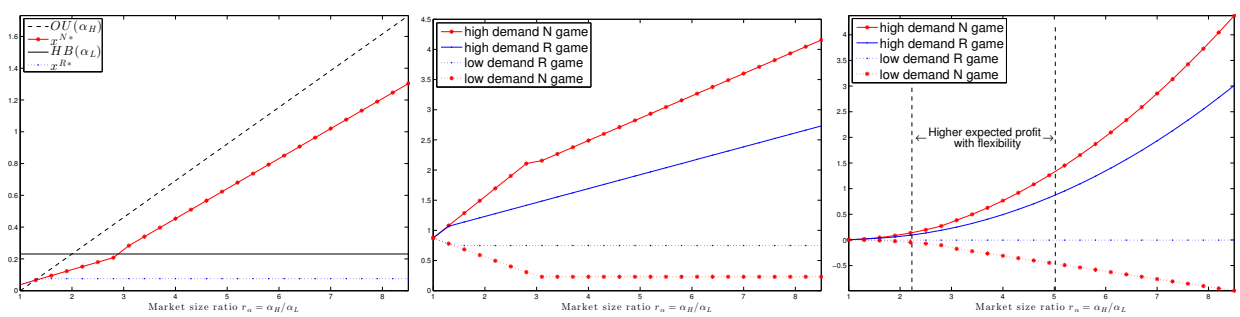
If the high-demand market size is below a threshold ($r_\alpha \leq 5.51$ where $\overline{\overline{r}}_\alpha(\gamma) = 5.51$ for $\gamma = 0.7$ is the threshold in part 3(a) of Proposition 3), the flexible firms order the same as, more than,

or less than inflexible firms, depending on whether the order cost ratio is low, intermediate, or high, respectively. A low order cost ratio implies a relatively high flexibility cost, so flexible and inflexible firms order the same amounts; see part 3(b) of Proposition 3. For an intermediate order cost ratio, the flexibility cost is such that flexible firms want to sell more than x^{N*} under high demand, but delaying this order is too costly, so they order more upfront. For a sufficiently high order cost ratio, the flexibility cost and the early procurement cost savings are negligible, so that flexible firms initially order less than inflexible firms, and more later if needed.

If the high-demand market is sufficiently large ($r_\alpha > 5.51$ for $\gamma = 0.7$), the flexible firms' initial orders differ from those of inflexible firms, regardless of the order cost ratio. In this case the high-demand market fosters such intense price competition that the flexible firms want to sell more than x^{N*} units under high demand, even if initial procurement is free ($c = 0$). If the initial cost is below a threshold, they procure the extra units in the first stage. Otherwise, they order less than x^{N*} initially and reorder up to a higher inventory level under high demand.

Figure 6(a) shows, for order cost ratio $r_c = 0.8$, how the equilibrium orders x^{N*} and x^{R*} depend on the market size ratio r_α ($\gamma = 0.7$ as in Figure 4). The flexible firms initially order the same as or more than the inflexible firms for $r_\alpha \leq 1.4$, and strictly less for $r_\alpha > 1.4$. Above this level, only the inflexible firms' order x^{N*} increases in the high-demand market size. The flexible firms' initial order x^{R*} stays constant as it balances the marginal profit loss from overstocking under low demand with the marginal cost saving from early procurement under high demand.

Figure 6 **Numerical Example** ($\gamma = 0.7, r_c = 0.8$)



(a) Quantities (b) Prices (c) Conditional Profits

Value of flexibility: impact of reorder flexibility on equilibrium profits. By part 2. of Proposition 3, in cases where firms benefit from reorder flexibility, they order less ex ante and more ex post under high demand, compared to inflexible firms. This condition is only necessary, as noted above. The flexible firms only gain higher expected profits under the additional conditions in

part 1 of Proposition 3: (a) products are sufficiently differentiated, (b) the market size variability is moderate, and (c) reordering is relatively inexpensive. Figure 5 illustrates these conditions for $\gamma = 0.7$. It partitions the parameter space of market size ratios r_α and order cost ratios r_c into three regions, one where reorder flexibility has no profit effect (part 3. of Proposition 3), one where it hurts expected profits, and one where it increases expected profits (part 1. of Proposition 3). The region of positive reorder benefit is significantly smaller than the set of all (r_α, r_c)-pairs where flexible firms order less ex ante than in the N game (shown in Figure 4). Conditions 1.(a)-(c) of Proposition 3 result from two countervailing profit effects of reorder flexibility:

1. *Downside protection under low demand.* The flexible firms are better off than the inflexible firms under low demand. Keeping their initial inventory low allows them to charge higher prices (see Figure 6(b)) and eliminate leftover inventory, compared to the inflexible firms (see Figure 6(a)). The value of this downside protection increases in the market size ratio r_α (see Figure 6(c)) and in the order cost ratio r_c: The more disparate the market sizes and the more expensive initial procurement, the more valuable the reorder option. If the order cost ratio r_c is sufficiently low, the flexible firms' gains from downside protection under low demand are too small to offset any losses under high demand, regardless of other factors.
2. *Intensified competition under high demand.* The flexible firms are worse off than the inflexible firms under high demand. If reordering is relatively inexpensive, the flexible firms over-order to a larger inventory level ($OU(\alpha_H) > x^{N*}$ as shown in Figure 6(a)) and compete more aggressively in price, compared to the inflexible firms (see Figure 6(b)), so they have larger procurement volumes and unit costs, and lower prices. This profit loss increases in the market size ratio r_α (Figure 6(c)) and in the product substitution parameter γ: both effects foster more intense competition. If the products are insufficiently differentiated ($\gamma \geq 0.875$), the flexible firms' losses under high demand exceed any gains under low demand, regardless of other factors.

These two profit effects explain why reorder flexibility benefits firms only if the market size variability is moderate (condition 1.(b) of Proposition 3). If the high- and low-demand markets are of similar size, the value of reorder flexibility for downside protection is insignificant because even inflexible firms price to clear their inventory under low demand. The value of downside protection increases in the high-demand market size, as inflexible firms are willing to incur the risk of overstocking under low demand. However, the detrimental effect of over-ordering also increases in the high-demand market size. Therefore, only for intermediate high-demand market size (in Figure 6(c) for $r_\alpha \in (2.2, 5.0)$) does the profit gain from downside protection under low demand exceed the profit loss from intensified competition under high demand.

Summary. Reorder flexibility under *price competition* may lead to smaller or larger initial orders. Flexible firms generate higher profits only if they order less initially, and in addition, products are sufficiently differentiated, the market size variability is moderate, and reordering is relatively inexpensive. Otherwise, reorder flexibility *hurts* profits. As discussed in Section 3.2, under volume flexibility the conditions for the equivalence of price and quantity competition may be violated, and price competition may be a more appropriate model of price formation. Our results are in stark contrast to those under *quantity competition*: In that case reorder flexibility consistently yields lower initial orders and higher profits, as in the monopoly case. These contrasting results underscore the importance of understanding how flexibly firms can increase their supply, in order to better predict and improve performance under reorder flexibility.

6. Flexibility Selection: Unilateral Flexibility is Not An Equilibrium

We have so far restricted attention to symmetric flexibility configurations. In this section we justify this focus: We show that in the flexibility-selection stage that precedes the procurement-pricing decisions, unilateral reorder flexibility is not an equilibrium. In Section 6.1 we characterize the initial order equilibria under unilateral reorder flexibility. In Section 6.2 we characterize the flexibility-selection equilibria and explain why unilateral reorder flexibility is not an equilibrium. In Section 6.3 we highlight the profit implications of the symmetric flexibility equilibria.

6.1. Initial Order Equilibria Under Unilateral Reorder Flexibility

Consider the two-stage procurement-pricing decisions under *unilateral* reorder flexibility, referred to as the U game. Both firms place initial orders before, but only one firm has the option to reorder after observing the market size; as before both firms have price flexibility. We call the firm who can reorder *flexible* and the one who cannot *inflexible*. Let $\mathbf{x}^{U*} = (x_I^{U*}, x_F^{U*})$ denote equilibrium initial orders in the U game, where x_I^{U*} and x_F^{U*} are the orders of the inflexible and flexible firm, respectively. The following result establishes that there exists at least one initial order equilibrium.

Proposition 4. (First Stage Order Equilibria: U Game). *Consider the U game with $\alpha_L = C(1-\gamma) < \alpha_H$. There exists at least one initial order equilibrium $\mathbf{x}^{U*}$. There are two thresholds $\underline{r}_c(r_\alpha, \gamma) < \tilde{r}_c(r_\alpha, \gamma)$ on the order cost ratio such that:*

(i) if $r_c \leq \underline{r}_c(r_\alpha, \gamma)$, there is a unique equilibrium and it is symmetric. Moreover, $\mathbf{x}^{U*} = \mathbf{x}^{R*} = \mathbf{x}^{N*}$;

(ii) if $\underline{r}_c(r_\alpha, \gamma) < r_c < \tilde{r}_c(r_\alpha, \gamma)$, there is a continuum of equilibria. Moreover, $x_F^{U*} > x^{N*}$;

(iii) if $\tilde{r}_c(r_\alpha, \gamma) \leq r_c \leq 1$, there is a unique equilibrium and it is asymmetric. Moreover, $x_F^{U*} < x^{R*} < x^{N*}$.

Figure 7 illustrates Proposition 4 for $\gamma = 0.7$ and shows under what conditions the flexible firm in the U game initially orders as much as ($x_F^{U*} = x^{N*}$), more than ($x_F^{U*} > x^{N*}$), or less than ($x_F^{U*} < x^{N*}$) the firms in the N game. As seen by comparison with Figure 4, unilateral and bilateral reorder flexibility have similar effects on the flexible firm's initial order.

Figure 7 Equilibrium Orders of U Game ($\gamma = 0.7$)

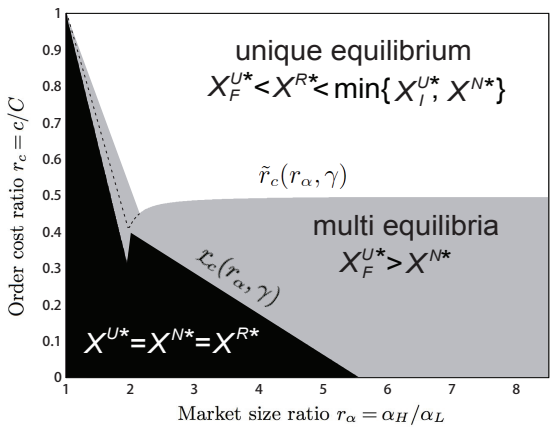


Figure 8 Payoff Matrix of Flexibility Strategies

		Firm 2	
		N	R
Firm 1	N	Π^{N*}, Π^{N*}	Π_I^{U*}, Π_F^{U*}
	R	Π_F^{U*}, Π_I^{U*}	Π^{R*}, Π^{R*}

If the order cost and market size ratios are sufficiently low (part (i) of Prop. 4: $r_c \leq \underline{r}_c(r_\alpha, \gamma)$ and $r_\alpha \leq 5.51$ for $\gamma = 0.7$, the black region in Figure 7), the firm with unilateral reorder flexibility has no incentive to sell more than x^{N*} under high demand, so that the U game equilibrium is the same as in the N and R games. Otherwise, the flexible firm has an incentive to sell more than x^{N*} under high demand. If the order cost ratio is in some intermediate range (part (ii) of Prop. 4: $\underline{r}_c(r_\alpha, \gamma) < r_c < \tilde{r}_c(r_\alpha, \gamma)$, the grey region in Figure 7), deferring the procurement is too costly, so the flexible firm orders all inventory upfront ($x_F^{U*} > x^{N*}$). However, for sufficiently high order cost ratio (part (iii) of Prop. 4: $r_c \geq \tilde{r}_c(r_\alpha, \gamma)$, the white region in Figure 7), reordering is so cheap that the flexible firm initially orders less than symmetrically inflexible firms ($x_F^{U*} < x^{N*}$) and reorders if demand is high. In this case the flexible firm initially orders even less, whereas the inflexible firm orders more, than under symmetric reorder flexibility, that is, $x_F^{U*} < x^{R*} < x^{N*}$ and $x_I^{U*} > x^{R*}$. This follows because the only way for the inflexible firm to prepare for potentially high demand is to place a relatively large initial order. In response, the flexible firm reduces its initial order to further mitigate a potential loss under low demand, knowing that it can use the relatively cheap reorder option under high demand.

6.2. Reorder Flexibility Selection: Unilateral Flexibility is Not An Equilibrium

Consider the flexibility-selection stage that precedes the procurement-pricing decisions discussed so far. Each firm chooses whether to have reorder flexibility (R) or not (N). Let (R, R) and (N, N)

denote the symmetric flexibility strategies, (N, R) and (R, N) the asymmetric strategies, where the first letter refers to firm 1. Our analysis assumes a zero fixed cost for reorder flexibility. This assumption isolates the strategic effects of flexibility selection without biasing the comparison against reorder flexibility by imposing an upfront investment in addition to the higher unit cost. In Section 6.3 we explain why our main results are robust if this assumption is relaxed.

We show that unilateral reorder flexibility is *not* an equilibrium. From the payoff matrix in Figure 8, it is easy to see that the asymmetric flexibility strategies (R, N) or (N, R) can be an equilibrium if and only if (1) unilateral flexibility is at least as profitable as bilateral inflexibility (i.e., $\Pi_F^{U*} \geq \Pi^{N*}$), and (2) unilateral inflexibility is at least as profitable as bilateral flexibility (i.e., $\Pi_I^{U*} \geq \Pi^{R*}$). We find that these conditions cannot hold. By part (i) of Proposition 4, all flexibility configurations yield the same equilibrium if $r_c \leq \mathfrak{L}_c(r_\alpha, \gamma)$. Proposition 5 establishes for a large set of cases (part (iii) of Prop. 4: $\tilde{r}_c(r_\alpha, \gamma) \leq r_c \leq 1$) that conditions (1) and (2) are mutually exclusive. Numerical results confirm for the remaining cases (part (ii) of Prop. 4: $\mathfrak{L}_c(r_\alpha, \gamma) < r_c < \tilde{r}_c(r_\alpha, \gamma)$) that condition (1) is violated, that is, bilateral inflexibility is strictly more profitable than unilateral flexibility (i.e., $\Pi^{N*} > \Pi_F^{U*}$).

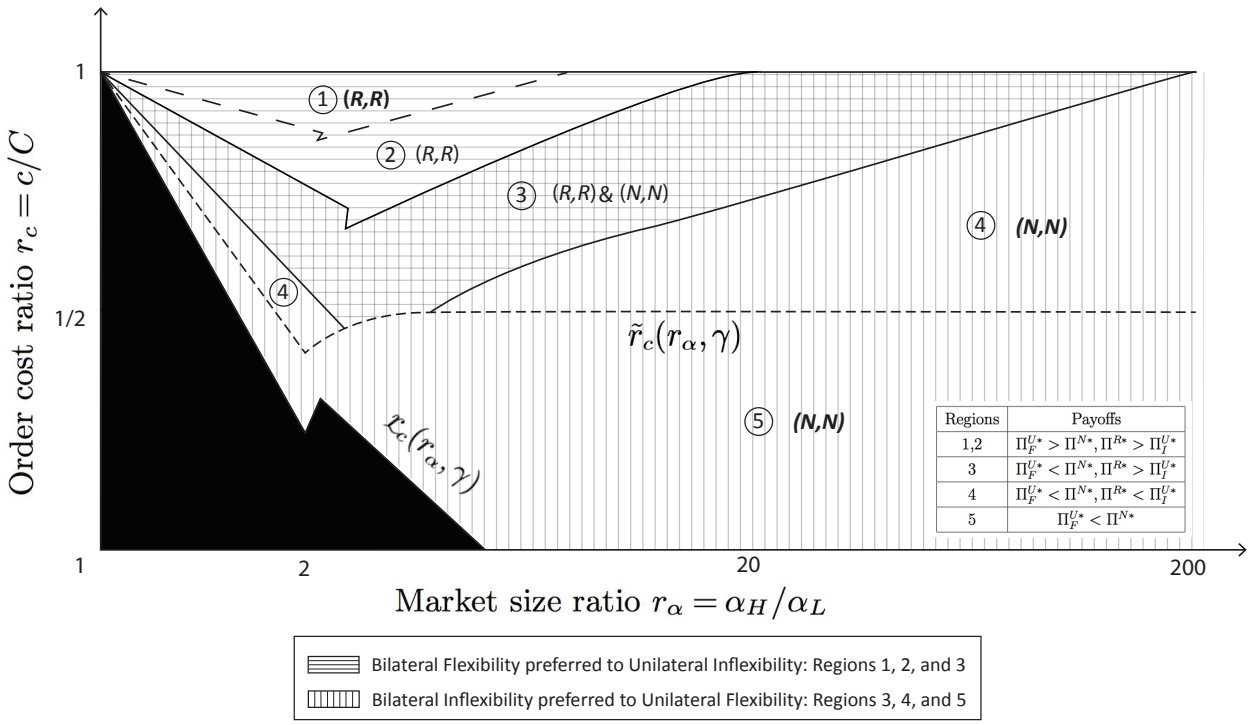
Proposition 5. *If $\tilde{r}_c(r_\alpha, \gamma) \leq r_c \leq 1$, then asymmetric flexibility strategies cannot be an equilibrium in the reorder flexibility selection game.*

Figure 9 illustrates the results for $\gamma = 0.7$ (Proposition 5 applies to the regions 1-4, the numerical analysis to region 5).

(i) Unilateral reorder flexibility is strictly more profitable than bilateral inflexibility (i.e., $\Pi_F^{U*} > \Pi^{N*}$), if and only if the order cost ratio r_c is sufficiently high and the market size ratio r_α is in an intermediate range. However, in this case the firms also prefer bilateral flexibility over unilateral inflexibility (i.e., $\Pi^{R*} > \Pi_I^{U*}$). That is, selecting reorder flexibility is the dominant strategy. The parameter conditions for this case correspond to regions 1 and 2 in Figure 9. These preference conditions for unilateral reorder flexibility over bilateral inflexibility parallel those for bilateral reorder flexibility to increase profits over bilateral inflexibility (part 1 of Proposition 3), and the underlying intuition discussed in Section 5.3 applies here as well: First, reorder flexibility can be of value only if reordering is so cheap (i.e., the order cost ratio is so high) that the flexible firm reduces its initial order and reorders only under high demand. Second, only for an intermediate high-demand market size does reorder flexibility yield a sufficiently high profit gain from downside protection under low demand to offset the loss from intensified competition under high demand.

(ii) Otherwise, the firms strictly prefer bilateral inflexibility to unilateral reorder flexibility (i.e., $\Pi^{N*} > \Pi_F^{U*}$), see regions 3-5 in Figure 9. The intuition for these cases parallels that for bilateral

Figure 9 **Preferences over Flexibility Configurations ($\gamma = 0.7$; Figure is Re-Scaled).**



reorder flexibility to reduce profits relative to bilateral inflexibility (refer to the discussion of Figures 4-5 in Section 5.3). In regions 3 and 4, the firm with unilateral flexibility initially orders less than in the N game (i.e., $x_F^{U*} < x^{N*}$ by part (iii) of Prop. 4) and therefore enjoys profit gains from downside protection under low demand, but these are overwhelmed by losses from intensified competition under high demand. In region 5, however, the firm with unilateral flexibility enjoys no downside protection, because it initially orders *more* than in the N game (i.e., $x_F^{U*} > x^{N*}$ by part (ii) of Prop. 4). As a result, it faces higher procurement costs and lower prices than in the N game.

6.3. Symmetric Flexibility Equilibria: Profit Implications

Profits are as follows under the symmetric flexibility equilibria (refer to Figure 9).

In regions 1 and 2, selecting reorder flexibility is the *dominant* strategy, so that bilateral reorder flexibility (R, R) is the unique equilibrium. However, by Proposition 3, the resulting expected profits Pareto-dominate those under bilateral inflexibility (i.e., $\Pi^{R*} > \Pi^{N*}$) only in region 1, whereas the *opposite* holds in region 2 (and also in regions 3-5). Therefore, parameters in region 2 give rise to the *worst-case scenario*: Bilateral reorder flexibility is the unique equilibrium but both firms are better off under bilateral inflexibility (i.e., $\Pi^{N*} > \Pi^{R*}$). In this case, which parallels the traditional "prisoner's dilemma", reordering is so cheap that firms cannot credibly commit to inflexibility

under price competition, yet still so expensive that the losses under high demand exceed the gains under low demand. As a result, each firm individually prefers reorder flexibility (i.e., $\Pi_F^{U*} > \Pi^{N*}$ and $\Pi^{R*} > \Pi_I^{U*}$) although the outcome hurts their profits.

In regions 3-5, bilateral inflexibility is the Pareto-dominant equilibrium. In region 3, bilateral flexibility is also an equilibrium, but profits are higher without reorder option (Proposition 3).

To summarize, reorder flexibility benefits firms only under fairly restrictive conditions (region 1). Absent these conditions, firms can commit to inflexibility and avoid the downside of reorder flexibility only in some cases (regions 3-5); in other cases price competition compels them to have reorder flexibility even though it hurts their profits (region 2).

Effect of fixed cost for reorder flexibility. Our analysis assumes a zero fixed cost for reorder flexibility. The presence of a positive fixed cost (or a flat-fee as part of the procurement tariff) would not alter our key findings about the downside of reorder flexibility: The main effect of a fixed cost is that reorder flexibility becomes *less*, not more, attractive. Therefore, the set of parameters for which firms prefer unilateral reorder flexibility to bilateral inflexibility (i.e., $\Pi_F^{U*} > \Pi^{N*}$) is smaller if there is a fixed cost. That is, regions 1 and 2 shrink, whereas regions 3-5 expand. An asymmetric flexibility equilibrium may potentially arise in a subset of these reduced regions 1 and 2. However, the worst-case scenario, whereby choosing reorder flexibility is the dominant strategy even though firms are more profitable with bilateral inflexibility, will still arise for certain parameters. Furthermore, in the expanded regions 3-5, bilateral inflexibility would Pareto-dominate bilateral flexibility (i.e., $\Pi^{N*} > \Pi^{R*}$) and outperform unilateral flexibility (i.e., $\Pi^{N*} > \Pi_F^{U*}$) even more strongly.

7. Discussion and Concluding Remarks

7.1. Discussion

Demand function. Our analysis assumes linear demand for tractability but our main results do not rely on this assumption: With nonlinear demand, reorder flexibility may also yield larger orders under price competition, which is the key driver of profit losses. Furthermore, as noted in Section 2 the demand system (1) does not explicitly model perfectly substitutable products. However, (1) is a rescaling of the alternative form $d_i(\mathbf{p};\alpha) = [(1-\gamma)\alpha - p_i + \gamma p_{-i}]/(1-\gamma^2)$ which does capture perfectly substitutable products and also yields the same managerial insights. First, all comparisons of reorder configurations for a *fixed* γ in our model remain intact when (1) is scaled back to the alternative form. Second, comparative statics on γ obtained in our model remain qualitatively the same as under the alternative system, because there is a one-to-one increasing correspondence between γ in our model and the cross-elasticity $\frac{\gamma}{1-\gamma^2}$ in the alternative demand system.

Binary market size distribution. Relaxing the assumption that the market size has a binary distribution does not change our main results that under price competition, reorder flexibility may increase initial orders and hurt profits. (Lin and Parlaktürk 2012 also assume a two-point market size distribution in their study of reorder flexibility under quantity competition.) Let the market size α follow a general distribution with probability density function $f(\cdot)$ over $[\underline{\alpha}, \overline{\alpha}]$. Following Section 5.1, it is easy to see from (8) that the N game equilibrium order should satisfy: $\int_{\underline{\alpha}}^{\overline{\alpha}} \frac{\partial \pi_i^{N*}(\mathbf{x}^{N*};\alpha)}{\partial x_i} f(\alpha) d\alpha = c$. For any market size α such that $OU(\alpha) > x^{N*}$, the flexible firms want to sell more than x^{N*}. They will over-order ex ante or ex post for these market size realizations, depending on the costs and outcome probabilities (generalizing the conditions in Table 1), which intensifies price competition and hurts their profits. For a sufficiently dispersed market size distribution $f(\cdot)$, the incidence of such larger market sizes and the resulting losses would offset the gains from downside protection under lower demand.

Information about initial orders. The assumption that firms know their competitor's past supply decisions (i.e., initial orders in our model) is standard, both in the flexibility literature and in dynamic models of inventory competition (e.g., Van Mieghem and Dada 1999, Netessine et al. 2006, Anupindi and Jiang 2008, Olsen and Parker 2008, Caro and Martínez-de-Albéniz 2010, Lin and Parlaktürk 2012). In practice it is not uncommon for third-party watchdogs who scout industry news to disseminate information about firms' orders and inventory positions.

7.2. Conclusion

This paper provides the first analysis of reorder flexibility under price competition and identifies its downside: higher initial orders and lower profits. These results are contrary to prior findings under fixed prices or quantity competition. Unilateral reorder flexibility is not an equilibrium. Furthermore, firms can commit to bilateral inflexibility and avoid the downside of reorder flexibility only in some of the cases where it hurts profits. In other cases firms are trapped in a prisoner's dilemma, whereby reorder flexibility is the dominant strategy even though it hurts their profits.

Our results have several implications for marketing and operations. To reap the benefits and avoid the downside of reorder flexibility, firms need to better understand its effects on competition and profitability. First, firms must differentiate their products sufficiently from their competitors. In this sense, reorder flexibility and product innovation must be viewed and managed as complementary capabilities. Second, the detrimental effect of reorder flexibility through intensified price competition depends on factors that determine how flexibly firms can increase their supply. Limitations on upside volume flexibility, such as convex reordering costs or hard capacity constraints, can help mitigate its detrimental effect, but they also reduce the ability of firms to capitalize on demand

surges. It is therefore of strategic importance for firms to determine levels of reactive capacity which appropriately balance competitive considerations with the upside and downside risks due to stochastic demand. Third, although the specific pricing and ordering prescriptions from our pre-season replenishment model do not directly transfer to settings with in-season replenishment, our main findings should continue to hold, because in such settings reorder flexibility under price competition may also yield larger orders.

We conclude by outlining some important research avenues. (1) In our analysis firms base their second-stage decisions on a perfect demand signal. If firms receive a noisy demand signal, they need to balance over- and under-stocking risks in their second-stage decisions. (2) It would be similarly interesting to relax the standard assumption of perfect initial-order information and consider how noisy information on competitor inventories affects the results. (3) By endogenizing product differentiation decisions one could study the interplay of strategic product positioning, pricing, and reorder flexibility selection. (4) We assume that firms are symmetric in terms of their demand and costs. Accounting for asymmetric firms is a challenging opportunity for future research. (5) Studying the sensitivity of our results to different demand uncertainty models may yield additional insights (cf. Anupindi and Jiang 2008). (6) We model substitutable products. It would be interesting to compare our results with those for complementary products.

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Online Appendix to
"The Downside of Reorder Flexibility under Price Competition"

For ease of exposition, we sometimes use $\rho_n^x(a_n,\ldots,a_0)$ to represent an n-degree polynomial in x with coefficients a_i for the ith-exponent term. For example, $3\gamma^2+1$ is denoted as $\rho_2^\gamma(3,0,1)$.

Proof of Lemma 1. (R) We start with the R game. First, we characterize firm i's best response given its competitor's price p_{-i} and its own initial inventory x_i. Second, we use this result to solve for the equilibrium prices for each strategy pair. Lastly, we use the equilibrium prices to identify the valid (x_i,x_{-i}) regions, i.e., the partition in Figure 2(b), corresponding to the strategy pairs. The results of the N game can be considered as a special case of the reorder game where $C=\infty$.

First, from problem (4), we see that the marginal profit with respect to p_i is

$$\frac{\partial \pi_i^R}{\partial p_i} = d_i(\mathbf{p};\alpha) + p_i\frac{\partial d_i}{\partial p_i} - C\cdot \mathbf{1}_{\{d_i(\mathbf{p};\alpha)>x_i\}}\frac{\partial d_i}{\partial p_i} = \alpha - 2p_i + \gamma p_{-i} + C\cdot \mathbf{1}_{\{p_i<p_i^c(p_{-i},x_i)\}},$$

where $p_i^c(p_{-i},x_i)=\alpha+\gamma p_{-i}-x_i$ is the clearance price, and hence $\pi_i^R(\mathbf{p},x_i;\alpha)$ is concave in p_i. Equating the marginal profit to zero yields the hold back price $p_i^h(p_{-i})=(\alpha+\gamma p_{-i})/2$ or the reorder price $p_i^r(p_{-i})=(\alpha+\gamma p_{-i}+C)/2$.

Second, we solve for all possible equilibrium prices. Note that the best response price $p_i^{R*}(x_i,p_{-i})$ can be expressed in a general form as $p_i^{R*}(x_i,p_{-i})=m_i(k_i+\gamma p_{-i})$. The choices of m_i and k_i depend on which of the three potential best-response prices is the optimal one. For example, $m_i=1$ and $k_i=\alpha-x_i$ if $p_i^{R*}(x_i,p_{-i})=p_i^c(x_i,p_{-i})$. Thus, the equilibrium prices can be solved from the following system of linear equations:

$$\begin{cases} p_i^{R*}(x_i) &= m_i(k_i+\gamma p_{-i}^{R*}(x_{-i})) \\ p_{-i}^{R*}(x_{-i}) &= m_{-i}(k_{-i}+\gamma p_i^*(x_i)) \end{cases} \Longleftrightarrow \begin{cases} p_i^{R*}(x_i) &= m_i(k_i+m_{-i}k_{-i}\gamma)/(1-m_im_{-i}\gamma^2) \\ p_{-i}^{R*}(x_{-i}) &= m_{-i}(k_{-i}+m_ik_i\gamma)/(1-m_im_{-i}\gamma^2) \end{cases}. \tag{12}$$

Table 2 lists the choices of m_i, m_{-i}, k_i, and k_{-i} for each strategy pair. Substituting the appropriate coefficients back into system (12), we can obtain all possible equilibrium prices.

Table 2 Coefficients of Best-Response Prices

	$R(r,r)$	$R(r,c)$	$R(r,h)$	$R(c,r)$	$R(c,c)$	$R(c,h)$	$R(h,r)$	$R(h,c)$	$R(h,h)$
m_i	$1/2$	$1/2$	$1/2$	1	1	1	$1/2$	$1/2$	$1/2$
m_{-i}	$1/2$	1	$1/2$	$1/2$	1	$1/2$	$1/2$	1	$1/2$
k_i	$\alpha+C$	$\alpha+C$	$\alpha+C$	$\alpha-x_i$	$\alpha-x_i$	$\alpha-x_i$	α	α	α
k_{-i}	$\alpha+C$	$\alpha-x_{-i}$	α	$\alpha+C$	$\alpha-x_{-i}$	α	$\alpha+C$	$\alpha-x_{-i}$	α

Lastly, we identify region boundaries in the initial inventory space for each specific strategy pair to arise as an equilibrium. Note that a specific price pair is indeed an equilibrium if and only if the initial inventory levels are in a certain region as illustrated in Figure 2(b). For any fixed competitor's initial inventory level x_{-i}, a horizontal line with intercept x_{-i} intersects with the region boundaries at two points, which serve as the desired thresholds claimed in the stipulation. Using the prices from system (12), we can solve for the valid region boundaries. To illustrate, we work with equilibrium strategy pair (r,r) as an example. For this particular strategy pair to be an equilibrium, we require $x_i<(\alpha+\gamma p_{-i}^{R*}(x_{-i})-C)/2$ and $x_{-i}<(\alpha+\gamma p_i^{R*}(x_i)-C)/2$. Since $p_i^{R*}(x_i)$ and $p_{-i}^{R*}(x_{-i})$ are available from Table 2, we have that (r,r) arises as an equilibrium strategy pair if and only if $x_i<OU=(\alpha-C+\gamma C)/(2-\gamma)$. In a similar way, we can determine the boundaries of $R(h,r)$, $R(h,h)$, and $R(r,h)$ as in Figure 2(b). This gives us the other three vertices of $R(c,c)$ as in Figure 2(b): starting from the bottom-right and going counterclockwise, their coordinates are $\left(\frac{\alpha(2+\gamma)+\gamma C}{4-\gamma^2},\frac{\alpha(2+\gamma)+(\gamma^2-2)C}{4-\gamma^2}\right)$, $\left(\frac{\alpha}{2-\gamma},\frac{\alpha}{2-\gamma}\right)$, and $\left(\frac{\alpha(2+\gamma)+(\gamma^2-2)C}{4-\gamma^2},\frac{\alpha(2+\gamma)+\gamma C}{4-\gamma^2}\right)$. One can also easily

verify that the lines connecting every pair of adjacent vertices of the diamond $R(c,c)$ give the other boundaries.

(N) In the N game, the potential best-response prices are left with either the hold back price $p_i^h(p_{-i}) = (\alpha + \gamma p_{-i})/2$ or the clearance price $p_i^c(p_{-i}, x_i)$. The equilibrium region boundaries can be similarly determined as in the R game. Equivalently, one can view the region partition generated in the N game as setting C very large in the R game (i.e., the reorder option is too expensive) when the point (OU, OU) goes below the origin and into the third quadrant. □

Proof of Proposition 1. The general idea of the equilibrium proof is straightforward: we identify the best-response functions, and the equilibrium is where the best response functions intersect. To execute this idea, we partition the initial inventory space $\{\mathbf{x} \geq 0\}$ into nine regions dependent on the second-stage equilibrium strategies. We use the second-stage equilibrium strategies to label the regions. For instance, the region $N(\underbrace{\overbrace{c}^{L}\overbrace{c}^{H}}_{i}, \underbrace{\overbrace{c}^{L}\overbrace{c}^{H}}_{-i})$ consists of all initial inventory vectors $\mathbf{x}$ for which the unique price equilibrium in both the low and high demand scenarios is for both firms to charge the clearance price. We follow the same notation convention for the other regions: the first and second components refer to the second-stage equilibrium strategies for firm i and $-i$, respectively, and the first and second letters of each component represent the firm's equilibrium strategy under low and high demand, respectively. It is easy to see that, because procurement is costly, it cannot be an equilibrium for firms to hold back inventory under high demand. Therefore a symmetric equilibrium can only be in region $N(cc,cc)$ or $N(hc,hc)$. We solve for an equilibrium candidate by concatenating the two first order conditions (FOCs) of each firm's profit maximization problem for the profit functions corresponding to $N(cc,cc)$ or $N(hc,hc)$. Then we identify conditions under which the candidate is indeed in the intended region and is indeed an equilibrium.

First, we consider region $N(cc,cc)$. The expected profit function of firm i with $\mathbf{x} \in N(cc,cc)$ is $\Pi_i^N(\mathbf{x}) = \left(\frac{\overline{\alpha}}{1-\gamma} - \frac{1}{1-\gamma^2}x_i - \frac{\gamma}{1-\gamma^2}x_{-i}\right)x_i - cx_i$, where $\overline{\alpha} = (\alpha_L + \alpha_H)/2$. For any fixed $x_{-i} \in \left[0, \frac{\alpha_L}{2-\gamma}\right]$, we solve for a line from the FOC of firm i's profit maximization problem: $x_i = \frac{1}{2}\left((1+\gamma)(\overline{\alpha} - c(1-\gamma)) - \gamma x_{-i}\right)$. Since the game is symmetric, these two lines, for $i = 1,2$, intersect on the diagonal, which yields the symmetric equilibrium candidate $x_l^{N*} = \frac{(1+\gamma)(\overline{\alpha}-c(1-\gamma))}{\gamma+2} \geq 0$, where the inequality is due to $\overline{\alpha} - c(1-\gamma) \geq 0$. To ensure that the equilibrium candidate is indeed in the region $N(cc,cc)$, we require $\hat{\alpha}_H \leq \frac{\gamma^2+\gamma+2}{(1+\gamma)(2-\gamma)}\hat{\alpha}_L + 2 := B_1\hat{\alpha}_L + 2$, where $\hat{\alpha}_L = \frac{\alpha_L}{c(1-\gamma)}$ and $\hat{\alpha}_H = \frac{\alpha_H}{c(1-\gamma)}$. Next, we need to characterize the conditions under which $x_i = x_l^{N*}$ is a best-response in maximizing $\Pi_i^N(\mathbf{x})$ among $x_i \geq 0$ while fixing $x_{-i} = x_l^{N*}$. It is clear that $x_i = x_l^{N*}$ is the expected profit maximizer for any $(x_i, x_{-i} = x_l^{N*}) \in N(cc,cc)$. Moreover, $\Pi_i^N(\mathbf{x})$ with $\mathbf{x} \in N(hh,cc)$ is decreasing in x_i for any fixed x_{-i}. Thus, only the local maximizer of $\Pi_i^N(\mathbf{x})$ with $\mathbf{x} = (x_i, x_{-i} = x_l^{N*}) \in N(hc,cc)$, which is quadratic in x_i, could be a possible best response. It can be shown that if this local maximizer in $N(hc,cc)$ is on the boundary, i.e., $\hat{\alpha}_H < \frac{8+4\gamma-\gamma^3}{8+4\gamma-4\gamma^3-\gamma^3}\hat{\alpha}_L + 2 := m^{**}(\gamma)\hat{\alpha}_L + 2$, it must reside on the boundary between $N(hc,cc)$ and $N(cc,cc)$. By the continuity of the profit at the boundary, such $\mathbf{x}$ with $x_i \in N(hc,cc) \cap N(cc,cc)$ and $x_{-i} = x_l^{N*}$ is dominated by $x_i = x_{-i} = x_l^{N*}$ that is the local maximizer over $N(cc,cc)$. If the local maximizer of $\Pi_i^N(\mathbf{x})$ with $\mathbf{x} = (x_i, x_{-i} = x_l^{N*}) \in N(hc,cc)$ is an interior point of $N(hc,cc)$, i.e., $\hat{\alpha}_H \geq= m^{**}(\gamma)\hat{\alpha}_L + 2$, then $x_i = x_{-i} = x_l^{N*}$ yields equal or more profit than the local maximizer in $N(hc,cc)$ if and only if $K_1\hat{\alpha}_L + 2 \leq \hat{\alpha}_H \leq K_2\hat{\alpha}_L + 2$, where $K_1 := \frac{(\sqrt{2}+1)\gamma^3+(2\sqrt{2}+4)\gamma^2-2\sqrt{2}\gamma-4\sqrt{2}}{(\sqrt{2}+1)\gamma^3+2\sqrt{2}\gamma^2-2\sqrt{2}\gamma-4\sqrt{2}}$ and $K_2 := \frac{(\sqrt{2}-1)\gamma^3+(2\sqrt{2}-4)\gamma^2-2\sqrt{2}\gamma-4\sqrt{2}}{(\sqrt{2}-1)\gamma^3+2\sqrt{2}\gamma^2-2\sqrt{2}\gamma-4\sqrt{2}}$. We verify that $K_1 \leq m^{**}(\gamma) \leq K_2 \leq B_1$ for any given $\gamma \in [0,1]$, so that $x_i = x_l^{N*}$ is indeed a best response in profit maximization with $x_{-i} = x_l^{N*}$ if and only if $\hat{\alpha}_H \leq K_2\hat{\alpha}_L + 2$. Thus $x_i = x_{-i} = x_l^{N*}$ is a symmetric equilibrium if and only if $\hat{\alpha}_H \leq K_2\hat{\alpha}_L + 2$.

Second, we consider region $N(hc,hc)$. For any fixed $x_{-i} \geq \frac{\alpha_L}{2-\gamma}$, firm i solves the FOC of the expected profit function in $N(hc,hc)$, which yields the line $x_i = \frac{1}{2}\left(\alpha_H(1+\gamma) - \gamma x_{-i}\right) - c(1-\gamma^2)$. This line intersects on the diagonal suggesting the symmetric equilibrium candidate $x_i = x_{-i} = x_h^{N*} = \alpha_H \frac{1+\gamma}{2+\gamma} - 2c\frac{1-\gamma^2}{2+\gamma}$, which is equal to or larger than $HB(\alpha_L)$ if and only if $\hat{\alpha}_H \geq B_2\hat{\alpha}_L + 2$, where $B_2 := \frac{2+\gamma}{(2-\gamma)(1+\gamma)}$. Next, we identify the conditions under which $x_i = x_h^{N*}$ is a best response of firm i's profit maximization with $x_{-i} = x_h^{N*}$. Similar to the case in $N(cc,cc)$, there may exist two local maximizers in $N(cc,cc)$ and $N(cc,hc)$ respectively. Following similar procedures, we prove that: (i) $x_i = x_h^{N*}$ yields equal or more profit than the local maximizer in $N(cc,cc)$ if and only if $\hat{\alpha}_H \geq T_1\hat{\alpha}_L + 2$, where $T_1 := \frac{2+\gamma}{4(1+\gamma)}\left(2+\gamma+\frac{\gamma^2}{2-\gamma}\sqrt{\frac{4-3\gamma^2}{2-\gamma^2}}\right)$, and (ii) $x_i = x_h^{N*}$ yields equal or more profit than the local maximizer in $N(hc,cc)$ if and only if $\hat{\alpha}_H \geq T_2\hat{\alpha}_L + 2$, where $T_2 := \frac{(2+\gamma)(1+\gamma)(2-\gamma)+4\gamma(2+\gamma)\sqrt{1+\gamma}}{(1+\gamma)(-\gamma^2+4\gamma+4)(2-\gamma)}$. Moreover, we verify that $B_2 \leq \max(T_1,T_2)$ for any given $\gamma \in [0,1]$ (the order of T_1 and T_2 depends on γ). Hence, it follows that $x_i = x_h^{N*}$ is a best response of $\Pi_i^N(\mathbf{x})$ among $x_i \geq 0$ with $x_{-i} = x_h^{N*}$ if and only if $\hat{\alpha}_H \geq \max(T_1,T_2)\hat{\alpha}_L + 2$. Thus, $x_i = x_{-i} = x_h^{N*}$ is a symmetric equilibrium if and only if $\hat{\alpha}_H \geq \max(T_1,T_2)\hat{\alpha}_L + 2$.

Let $m^{**}(\gamma) = K_2$ and $m^*(\gamma) = \max(T_1,T_2)$. We verify that $m^{**}(\gamma) > m^*(\gamma) > 0$ for any given $\gamma \in (0,1)$. Then we have the desired results on the symmetric equilibria.

Lastly, we eliminate the existence of asymmetric equilibria. We first consider the case that $x_i = x_l^{N*}$ is not a best response to $x_{-i} = x_l^{N*}$. Then, the local maximizer of $\Pi_i^N(\mathbf{x})$ in $N(hc,cc)$ must be the best response, which requires $\hat{\alpha}_H > K_2\hat{\alpha}_L + 2$. To have an asymmetric equilibrium in region $N(hc,cc)$, it is also required that the best response of firm $-i$ is in $N(hc,cc)$ for $\hat{\alpha}_H > K_2\hat{\alpha}_L + 2$. By symmetry, this is equivalent to requiring that the best response of firm i is in $N(cc,hc)$ if $\hat{\alpha}_H > K_2\hat{\alpha}_L + 2$, which contradicts the fact that $x_i = x_{-i} = x_h^{N*} \in N(hc,hc)$ is a symmetric equilibrium if $\hat{\alpha}_H > K_2\hat{\alpha}_L + 2$. Similar arguments apply when $\hat{\alpha}_H \leq K_2\hat{\alpha}_L + 2$ and we can eliminate all the possibilities of asymmetric equilibria. □

Proof of Proposition 2. We call a point on the diagonal of initial order quantities a *symmetric* equilibrium candidate if it is a local maximizer of problem $\max_{x_i \geq 0} \Pi_i^R(\mathbf{x})$. We solve for all symmetric equilibrium candidates and then identify for each candidate conditions under which it is indeed an equilibrium.

Table 3 Notation in Proof of Proposition 2

$w_0 = (\gamma+2)^2(\gamma^2-2)^2,\ w_0' = (\gamma-2)^{-1}(\gamma+2)^{-1}/2$	$r_{c_2} = w_3 r_\alpha - w_1(\sqrt{w_2 r_\alpha^2 + w_4 r_\alpha + w_5} - w_6)$
$w_1 = (4\gamma^4 - 10\gamma^2 + 8)^{-1} w_0',\ w_2 = (\gamma^2-2)^2 w_0$	$r_{c_5} = -(\rho_1^\gamma(1,1,2) r_\alpha + \rho_2^\gamma(1,-3,-6)) w_0'$
$w_3 = w_0 w_1, w_4 = \rho_6^\gamma(2,-20,-2^4,72,2^6,-2^6,-2^6) w_0$	$r_{c_6} = -(\rho_1^\gamma(1,2) r_\alpha + \rho_2^\gamma(2,-3,-6)) w_0'$
$w_5 = -\rho_6^\gamma(7,-24,-32,80,80,-64,-64) w_0$	$r_{c_7} = (2\rho_4^\gamma(1,-1,-4,2,4))^{-1}\big((-2\gamma^2+\gamma^4) r_\alpha +$
$w_6 = \rho_6^\gamma(5,-14,-26,46,60,-40,-48)$	$\rho_3^\gamma(-1,-2,2,4) - 2\sqrt{\rho_2^\gamma(1,-1,-2)(2-r_\alpha)}$
$r_{c_1} = (-\gamma^2 r_\alpha + 2 + \gamma) w_0',\ r_{c_4} = (r_\alpha - m^{**}(\gamma))/2$	$\cdot\sqrt{\rho_3^\gamma(1,1,-1,-2) r_\alpha + \rho_4^\gamma(1,-2,-4,2,4)}\ \big)$

First, we consider $r_\alpha \geq 2$, i.e., $\alpha_H \geq 2\alpha_L$. Since $\alpha_L = C(1-\gamma)$, then $0 = OU(\alpha_L) < HB(\alpha_L) \leq OU(\alpha_H) < HB(\alpha_H)$. Depending on the relative position of a symmetric point with respect to the HB and OU points of the low and high demand scenarios which determine the second-stage equilibrium strategies, by Table 2, the marginal value of initial inventory is

$$\left.\frac{\partial \Pi_i^R(\mathbf{x})}{\partial x_i}\right|_{x_i = x_{-i}} = \begin{cases} \frac{1}{2}\left(\frac{\alpha_L}{1-\gamma} - x_i\frac{2+\gamma}{1-\gamma^2}\right) + \frac{C}{2} - c & \text{if } x_i = x_{-i} \in [0, HB(\alpha_L)], \\ \frac{C}{2} - c & \text{if } x_i = x_{-i} \in (HB(\alpha_L), OU(\alpha_H)), \\ \frac{1}{2}\left(\frac{\alpha_H}{1-\gamma} - x_i\frac{2+\gamma}{1-\gamma^2}\right) - c & \text{if } x_i = x_{-i} \in (OU(\alpha_H), HB(\alpha_H)]. \end{cases}$$

Hence we have the following symmetric equilibrium candidates.

1. For $x_i = x_{-i} \in [0, HB(\alpha_L)]$, setting the derivative to zero yields $x_l^{R*} := \frac{2(1-\gamma^2)}{2+\gamma} C(1-r_c)$. Note that $x_l^{R*} \in [0, HB(\alpha_L)]$ is equivalent to $r_c \in [r'_{c_2}, 1]$, where $r'_{c_2} := \frac{2\gamma^2-\gamma-2}{2(1+\gamma)(\gamma-2)}$. As in the proof of Proposition 1, we need to further identify when x_l^{R*} is indeed a best response while fixing her rival's inventory at $\frac{2(1-\gamma^2)}{2+\gamma}(C-r_c)$. After comparing with all possible local optima and noting that $r'_{c_2} \le r_{c_2}$, we conclude that x_l^{R*} is an equilibrium if and only if $r_{c_2} \le r_c \le 1$ when $r_\alpha \ge 2$. Under this equilibrium, firms clear inventory in low demand and reorder in high demand.

2. For $x_i = x_{-i} \in (OU(\alpha_H), HB(\alpha_H)]$, setting the derivative to zero yields $x_h^{R*} := \frac{1-\gamma^2}{2+\gamma} C(r_\alpha - 2r_c)$. Note that $x_h^{R*} > OU(\alpha_H)$ is equivalent to $r_c < r_{c_1}$. Recall that the no-reorder equilibrium is always smaller than $HB(\alpha_H)$ and the profit functions are the same for points along the diagonal between $OU(\alpha_H)$ and $HB(\alpha_H)$ for both R and N games; thus it is implied that $x_h^{R*} < HB(\alpha_H)$. Again, we need to characterize when x_h^{R*} is indeed a best response. We can show that x_h^{R*} is an equilibrium if and only if $r_c < r_{c_1}$ when $r_\alpha \ge 2$. Under this equilibrium, firms have leftovers in low demand and do not reorder in high demand.

3. For $x_i = x_{-i} \in (HB(\alpha_L), OU(\alpha_H))$, the derivative of firm i's expected profit is constant. (i) If $c \ne C/2$, the derivative is nonzero, so no point in $(HB(\alpha_L), OU(\alpha_H))$ can be a symmetric equilibrium because an equilibrium has to be a local maximizer. (ii) If $c = C/2$, then the derivative is zero, so every point in $(HB(\alpha_L), OU(\alpha_H))$ is an equilibrium candidate. However, no such point can be a Pareto-dominant equilibrium. This follows because $c = C/2$ implies $r_{c_2} < 1/2 = r_c$ which implies by point 1. above that x_l^{R*} is a symmetric equilibrium. Noting that $x_l^{R*} < HB(\alpha_L)$, it follows that the expected profit for $x_i = x_{-i} = x_l^{R*}$ strictly exceeds that for $x_i = x_{-i} \in [HB(\alpha_L), OU(\alpha_H)]$.

4. Let $x_o^{R*} := OU(\alpha_H)$. The point $x_i = x_{-i} = x_o^{R*}$ is a symmetric equilibrium candidate if and only if $c \le C/2$ and $r_c \ge r_{c_1}$. This holds because for $x_{-i} = OU(\alpha_H)$, $\Pi_i^R(\mathbf{x})$ is not differentiable at $x_i = OU(\alpha_H)$ which is a local maximizer if the left derivative $C/2 - c \ge 0$ and the right derivative $\frac{1}{2}\left(\frac{\alpha_H}{1-\gamma} - x_i \frac{2+\gamma}{1-\gamma^2}\right) c$ is nonpositive, which holds if and only if $r_c \ge r_{c_1}$. Furthermore, we can show that x_o^{R*} is a symmetric equilibrium if $r_c \in [r_{c_1}, r_{c_2}]$ when $r_\alpha \ge 2$. Under this equilibrium, firms have leftovers in low demand and reorder in high demand. Finally, x_o^{R*} is not a Pareto-dominant equilibrium if $r_c \ge r_{c_2}$: In this case x_l^{R*} is a symmetric equilibrium by point 1. above, and it is straightforward to verify that the resulting expected profit exceeds that for $x_i = x_{-i} = x_o^{R*}$.

Table 4 Sufficient and Necessary Conditions for Equilibria ($1 \le r_\alpha < 2$)

Equilibrium Candidate ($1 \le r_\alpha < 2$)	Conditions	Strategy in Low Demand	Strategy in High Demand
$x_h^{R*} := \frac{1-\gamma^2}{2+\gamma} C(r_\alpha - 2r_c) > HB(\alpha_L)$	$0 \le r_c < r_{c_4}$ or $\max\{r_{c_4}, r_{c_5}\} \le r_c \le r_{c_7}$	leftover	no reorder
$x_p^{R*} := \frac{1-\gamma^2}{2(2+\gamma)} C(r_\alpha - 2r_c + 1) \in (OU(\alpha_H), HB(\alpha_L))$	$\max\{0, r_{c_4}\} \le r_c \le r_{c_5}$ and $0 \le r_\alpha \le \hat{r}_\alpha$	clear	no reorder
$x_o^{R*} := \frac{1-\gamma}{2-\gamma} C(r_\alpha - 1) = OU(\alpha_H)$	$\max\{r_{c_5}, r_{c_7}\} < r_c < r_{c_6}$	clear	no reorder
$x_l^{R*} := \frac{2(1-\gamma^2)}{2+\gamma} C(1-r_c) \le OU(\alpha_H)$	$r_{c_6} \le r_c \le 1$	clear	reorder

Second, we consider $1 < r_\alpha < 2$, i.e., $\alpha_L \le \alpha_H < 2\alpha_L$. Then $0 = OU(\alpha_L) < OU(\alpha_H) < HB(\alpha_L) < HB(\alpha_H)$. Following a similar procedure, we show that there are four equilibrium candidates for $r_\alpha \in (1, 2)$. Table 4 summarizes the sufficient and necessary equilibrium conditions for each candidate, where $\hat{r}_\alpha$ is the r_α-axis coordinate of the intersection of the functions r_{c_4} and r_{c_5}.

The above arguments prove the existence of a symmetric equilibrium. The equilibrium strategies for the cases (i)-(iii) in the statement of Proposition 2 are obtained by summarizing points 1-4 for $r_\alpha \ge 2$ and Table 4 for $1 < r_\alpha < 2$, letting $r_c(r_\alpha, \gamma) = r_{c_2} \mathbf{1}_{\{r_\alpha \ge 2\}}$, $\overline{r}_c(r_\alpha, \gamma) = r_{c_6} \mathbf{1}_{\{1 \le r_\alpha < 2\}}$, and $\underline{r}_c(r_\alpha, \gamma) = \left(r_{c_4} \mathbf{1}_{\{r_{c_5} \ge r_{c_4}\}} + r_{c_7} \mathbf{1}_{\{r_{c_5} \le r_{c_7}\}}\right) \mathbf{1}_{\{1 \le r_\alpha < 2\}}$. (Note that $r_{c_4} \le 0$ for $r_\alpha \le m^{**}(\gamma)$, in which case the conditions on r_c in the first two rows in Table 4 simplify.) $\square$

Proof of Proposition 3. We first prove the results for $r_\alpha \geq 2$ and then extend to $1 \leq r_\alpha < 2$.

Table 5 shows that there are 6 possible no-reorder/reorder equilibrium combinations for any given r_c and r_α such that $r_\alpha \geq 2$. We need to quantify the expected profit differences for all equilibrium combinations. By Proposition 1, we know that there is at least one symmetric equilibrium in the N game for a given parameter pair (r_α, r_c). If $\alpha_L = C(1-\gamma)$ and $\alpha_H \geq 2\alpha_L$, we assume that both firms adopt the Pareto-dominant x_l^{N*} over x_h^{N*} when two symmetric equilibria exist. Propositions 1 and 2 give us the initial equilibrium ordering quantities. Referring to Table 2, we can also calculate the corresponding equilibrium prices and obtain the expected equilibrium profits. Table 6 lists the conditional revenue and cost in low and high scenarios for all equilibria. Note that conditions 1.(a)-(c) hold whenever the firms with reorder flexibility are more profitable than inflexible firms. We show that 1.(a)-(c) imply $x^{R*} < x^{N*} < OU(\alpha_H)$.

Table 5 Equilibrium Pairs

	$r_\alpha \geq 2$			$1 \leq r_\alpha < 2$			
	$r_c \in [0, r_{c_1})$	$[r_{c_1}, r_{c_3}]$	$[r_{c_2}, 1]$	$r_c \in [0, r_{c_4}]$	$[0, r_{c_5}]$	(r_{c_5}, r_{c_6})	$[r_{c_6}, 1]$
$r_c \geq \frac{r_\alpha}{2} - \frac{m^{**}(\gamma)}{2}$	(x_l^{N*}, x_h^{R*})	(x_l^{N*}, x_o^{R*})	(x_l^{N*}, x_l^{R*})	(x_l^{N*}, x_h^{R*})	(x_l^{N*}, x_p^{R*})	(x_l^{N*}, x_o^{R*})	(x_l^{N*}, x_l^{R*})
$r_c < \frac{r_\alpha}{2} - \frac{m^{**}(\gamma)}{2}$	(x_h^{N*}, x_h^{R*})	(x_h^{N*}, x_o^{R*})	(x_h^{N*}, x_l^{R*})	(x_h^{N*}, x_h^{R*})	(x_h^{N*}, x_p^{R*})	(x_h^{N*}, x_o^{R*})	N/A

We compare the expected profits of each equilibrium combination as follows.

(i) $\{(r_\alpha, r_c) \mid r_c < r_\alpha/2 - m^{**}(\gamma)/2 \text{ and } r_{c_2} \leq r_c \leq 1\}$. In this case, firms choose x_h^{N*} in the N game and x_l^{R*} in the R game. By Table 6, we have x_l^{R*} generates more expected profit than x_l^{N*} if and only if $(r_\alpha, r_c) \in F_1 := \left\{(r_\alpha, r_c) \mid r_\alpha > 2 \text{ and } \frac{\gamma^3}{2(1+\gamma)(2-\gamma)^2} r_\alpha + \frac{1}{2} < r_c < r_\alpha/2 - m^{**}(\gamma)/2 \leq 1\right\}$. The set F_1 is nonempty if and only if $2 + m^{**}(\gamma) \geq \frac{(\gamma+1)(2-\gamma)^2}{\gamma^3}$, i.e, $\gamma \leq 0.875$. Note that for nonempty F_1, we have $r_c > r_{c_3}$, which implies that x_l^{R*} is the unique symmetric R game equilibrium candidate.

(ii) $\{(r_\alpha, r_c) \mid r_c \geq r_\alpha/2 - m^{**}(\gamma)/2 \text{ and } r_{c_2} \leq r_c \leq 1\}$. In this case, firms choose x_l^{N*} in the N game and x_l^{R*} in the R game. By Table 6, we have x_l^{N*} generates more profit than x_l^{R*} if and only if $(r_\alpha, r_c) \in F_2 := \{(r_\alpha, r_c) \mid 2 \leq r_\alpha \leq 2r_c + m^{**}(\gamma) \text{ and } 1 - \frac{1}{2}(r_\alpha - 1)(1 - 2\frac{\gamma\sqrt{\gamma(\gamma+1)}}{(\gamma+1)(2-\gamma)}) < r_c \leq 1\}$. The set F_2 is nonempty if and only if $\gamma \leq \frac{1}{3}\left((17+12\sqrt{2})^{1/3} + (17+12\sqrt{2})^{-1/3} - 1\right) \approx 0.849$. Note that for nonempty F_2, we have $r_c > r_{c_3}$, which implies that x_l^{R*} is the unique symmetric R game equilibrium.

Table 6 Revenue and Cost at Equilibrium for $\alpha_H \geq 2\alpha_L$

	Equilibrium	Conditional on the Low Scenario		Conditional on the High Scenario	
		Revenue	Cost	Revenue	Cost
R game	x_l^{R*}	$\left(\frac{\alpha_L}{1-\gamma} - \frac{1}{1-\gamma}x_l^{R*}\right)x_l^{R*}$	cx_l^{R*}	$\frac{\alpha_H + C}{2-\gamma}OU(\alpha_H)$	$cx_l^{R*} + C(OU(\alpha_H) - x_l^{R*})$
	x_o^{R*}	$\frac{\alpha_L}{2-\gamma}HB(\alpha_L)$	cx_o^{R*}	$\frac{\alpha_H + C}{2-\gamma}OU(\alpha_H)$	cx_o^{R*}
	x_h^{R*}	$\frac{\alpha_L}{2-\gamma}HB(\alpha_L)$	cx_h^{R*}	$\left(\frac{\alpha_H}{1-\gamma} - \frac{1}{1-\gamma}x_h^{R*}\right)x_h^{R*}$	cx_h^{R*}
N game	x_l^{N*}	$\left(\frac{\alpha_L}{1-\gamma} - \frac{1}{1-\gamma}x_l^{N*}\right)x_l^{N*}$	cx_l^{N*}	$\left(\frac{\alpha_H}{1-\gamma} - \frac{1}{1-\gamma}x_l^{N*}\right)x_l^{N*}$	cx_l^{N*}
	x_h^{N*}	$\frac{\alpha_L}{2-\gamma}HB(\alpha_L)$	cx_h^{N*}	$\left(\frac{\alpha_H}{1-\gamma} - \frac{1}{1-\gamma}x_h^{N*}\right)x_h^{N*}$	cx_h^{N*}

(iii) $\{(r_\alpha, r_c) \mid r_c < r_\alpha/2 - m^{**}(\gamma)/2 \text{ and } r_{c_1} \leq r_c \leq r_{c_3}\}$. In this case, although x_l^{R*} may also be a reorder symmetric equilibrium, the analysis is similar to case (i). Thus here we only compare the equilibrium combination x_h^{N*} and x_o^{R*}. Algebraically, x_o^{R*} generates more profit than x_h^{N*} if and only if $(r_\alpha, r_c) \in \left\{(r_\alpha, r_c) \mid r_\alpha \geq 2 \text{ and } \frac{\gamma-2}{\gamma}r_c + \frac{2+\gamma}{2\gamma} \leq r_\alpha \leq \frac{2(\gamma+1)(\gamma-2)}{\gamma^2}r_c + \frac{2+\gamma}{\gamma^2}\right\}$, which is exclusive of the validity set $\{(r_\alpha, r_c) \mid r_c < r_\alpha/2 - m^{**}(\gamma)/2 \text{ and } r_{c_1} \leq r_c \leq r_{c_3}\}$. Thus, we conclude that the expected profit of x_o^{R*} is no more than that of x_h^{N*} in this case.

(iv) $\{(r_\alpha, r_c) \mid r_c \geq r_\alpha/2 - m^{**}(\gamma)/2 \text{ and } r_{c_1} \leq r_c \leq r_{c_3}\}$. In this case, although x_l^{R*} may also be a reorder symmetric equilibrium, the analysis is similar to case (ii). We here only compare the

equilibrium combination x_l^{N*} and x_o^{R*}. By Table 6, we calculate the profit difference $\Pi_i^N(x_i = x_{-i} = x_l^{N*}) - \Pi_i^R(x_i = x_{-i} = x_o^{R*})$

$$= \left(\frac{1-\gamma^2}{4(2+\gamma)^2} - \frac{1}{2}\left(\frac{1-\gamma}{2-\gamma}\right)^2\right) r_\alpha^2 - \left(\frac{1-\gamma^2}{(2+\gamma)^2} - \frac{1-\gamma}{2-\gamma}\right) r_\alpha r_c + \frac{1-\gamma^2}{(2+\gamma)^2} r_c^2$$
$$+ \left(\frac{1-\gamma^2}{2(2+\gamma)^2} + \left(\frac{1-\gamma}{2-\gamma}\right)^2 - \frac{1-\gamma}{2(2-\gamma)}\right) r_\alpha + \left(-\frac{1-\gamma^2}{(2+\gamma)^2} - \frac{1-\gamma}{2-\gamma}\right) r_c + \frac{1-\gamma^2}{4(2+\gamma)^2} - \frac{1}{2}\frac{1-\gamma}{(2+\gamma)^2}.$$

Now we prove that this lower bound is nonnegative. Substituting $r_c = r_\alpha/2 - m^{**}(\gamma)/2$ into $\Pi_i^N(x_i = x_{-i} = x_l^{N*}) - \Pi_i^R(x_i = x_{-i} = x_o^{R*})$ and noticing $0 \le \gamma \le 1$, we can show the profit difference is nonnegative.

(v) $\{(r_\alpha, r_c) \mid r_c < r_\alpha/2 - m^{**}(\gamma)/2 \text{ and } 0 \le r_c < r_{c_1}\}$. In this case, we compare the equilibrium combination x_h^{N*} and x_h^{R*}. By Propositions 3 and 4, $x_h^{R*} = x_h^{N*}$.

(vi) $\{(r_\alpha, r_c) \mid r_c \ge r_\alpha/2 - m^{**}(\gamma)/2 \text{ and } 0 \le r_c < r_{c_1}\}$. In this case, we compare the equilibrium combination x_l^{N*} and x_h^{R*}. Note that the profit of x_h^{R*} has an identical expression as that of x_h^{N*} and we can verify that if $\alpha_L = C(1-\gamma)$ and $\alpha_H \ge 2\alpha_L$, then $\Pi_i^N(x_i = x_{-i} = x_l^{N*}) \ge \Pi_i^N(x_i = x_{-i} = x_h^{N*})$ if and only if $(r_\alpha, r_c) \in \left\{(r_\alpha, r_c) \mid \frac{1}{2}\left(r_\alpha - 1 - \frac{2\gamma}{2-\gamma}\sqrt{\frac{\gamma}{1+\gamma}}\right) \le r_c \le \frac{1}{2}\left(r_\alpha - 1 + \frac{2\gamma}{2-\gamma}\sqrt{\frac{\gamma}{1+\gamma}}\right)\right\}$, which contains the set of $\{(r_\alpha, r_c) \mid r_c \ge r_\alpha/2 - m^{**}(\gamma)/2 \text{ and } 0 \le r_c < r_{c_1}\}$. Thus in this case, x_l^{N*} generates more profit than x_h^{R*}.

From (i) to (vi), we see that reorder flexibility benefits firms only in (i) and (ii), where firms play only x_l^{R*} in the R game but have different no-reorder equilibrium quantity x_l^{N*} and x_h^{N*} dependent on (r_α, r_c). Moreover, $x_l^{R*} \le x_l^{N*}$ if and only if $(r_\alpha, r_c) \in \{(r_\alpha, r_c) \mid r_c \ge -r_\alpha/2 + 3/2\} \supset F_1 \cup F_2$. Thus, we conclude that $x_l^{R*} < x_l^{N*} < OU(\alpha_H) < x_h^{N*}$. The parameter subset where reorder flexibility benefits firms is $F_1 \cup F_2$. This set is nonempty if $\gamma \le 0.849$.

Second, let us consider the case where $1 \le r_\alpha < 2$. (i) Whenever x_h^{R*} and x_p^{R*} are equilibria, the equilibrium outcomes of the R game are respectively equivalent to the outcome of x_h^{N*} and x_l^{N*} in the N game. Thus, reorder flexibility has no value when the equilibrium is either x_h^{R*} or x_p^{R*}. (ii) We explore the value of reorder flexibility when x_o^{R*} is an equilibrium. The algebraic expression $\Pi_i^N(x_i = x_{-i} = x_l^{N*}) - \Pi_i^R(x_i = x_{-i} = x_o^{R*}) > 0$ if and only if $(r_\alpha, r_c) \in \left\{(r_\alpha, r_c) \mid r_\alpha \ge 1 \text{ and } 1 - \frac{2+3\gamma}{2(2-\gamma)}(r_\alpha - 1) < r_c < 1 - \frac{\gamma^2+\gamma+2}{2(2-\gamma)(1+\gamma)}(r_\alpha - 1)\right\}$. However, in this region x_o^{R*} is not an equilibrium. Thus, reorder flexibility has no positive value when x_o^{R*} is an equilibrium. (iii) Consider x_l^{R*}. From case (ii) of $r_\alpha \ge 2$, we have that reorder flexibility has a positive value for $(r_\alpha, r_c) \in F_3 := \left\{(r_\alpha, r_c) \mid 1 \le r_\alpha < 2 \text{ and } 1 - \frac{1}{2}(r_\alpha - 1)\left(1 - 2\frac{\gamma\sqrt{\gamma(\gamma+1)}}{(\gamma+1)(2-\gamma)}\right) < r_c \le 1\right\}$.

In sum, the reorder flexibility has a positive value only when x_l^{R*} is an equilibrium and $(r_\alpha, r_c) \in F_1 \cup F_2 \cup F_3$. Let $\underline{r}_\alpha(\gamma) := 1 + \left((\frac{\gamma^3}{4-3\gamma^2} - 1)m^{**}(\gamma) + (\frac{\gamma^3}{4-3\gamma^2} + 1)\right)\mathbf{1}_{\{0.849 \le \gamma < 0.875\}}$, $\overline{r}_\alpha(\gamma) := (1+\gamma)(2-\gamma)^2/\gamma^3$ and

$$\underline{r}_c(\gamma, r_\alpha) := \begin{cases} 1 + \left(\frac{\gamma}{2-\gamma}\sqrt{\frac{\gamma}{1+\gamma}} - \frac{1}{2}\right)(r_\alpha - 1) & \text{if } \underline{r}_\alpha(\gamma) \le r_\alpha < \tilde{r}_\alpha(\gamma), \\ \frac{\gamma^3}{2(1+\gamma)(2-\gamma)^2} r_\alpha + \frac{1}{2} & \text{if } \tilde{r}_\alpha(\gamma) \le r_\alpha < \overline{r}_\alpha(\gamma), \end{cases}$$

where $\tilde{r}_\alpha(\gamma) = 1 + (m^{**}(\gamma) + 1)/\left(2 - \frac{\gamma}{2-\gamma}\sqrt{\frac{\gamma}{1+\gamma}}\right)$. Moreover, the R game has the same equilibrium as the N game if and only if $r_\alpha \le \overline{\overline{r}}_\alpha(\gamma) = \frac{2+\gamma}{\gamma^2}$ and $r_c \le \overline{r}_c(\gamma, r_\alpha) = \max\{r_{c_4}, r_{c_5}\}\mathbf{1}_{\{1 \le r_\alpha < 2\}} + r_{c_1}\mathbf{1}_{\{r_\alpha \ge 2\}}$. □

Proof of Proposition 4. We first solve the 2nd-stage pricing-ordering game given the 1st-stage orders, and then determine the equilibrium orders in the 1st stage.

Define $\hat{x}_I = \frac{(2+\gamma)\alpha + \gamma C}{4-\gamma^2}$ and $\hat{x}_F = \frac{(2+\gamma)\alpha - (2-\gamma^2)C}{4-\gamma^2}$. Following the proof of Lemma 1, we can show that for any initial inventory vector $\mathbf{x} = (x_I, x_F)$ where x_I and x_F represent inflexible and flexible firms' inventory, the price subgame of the U game has a unique equilibrium as follows:

Table 7 Notation in Proof of Proposition 4

$y_1 = 1/(2\rho_9^\gamma(1,-10,26,4,-100,88,104,-144,0,64))$	$-904,880,1568,-768,-1408,256,512)$
$y_2 = \rho_{11}^\gamma(-2,3,21,-20,-101,43,252,0,-304,$ $-96,128,64),\ y_3' = (\gamma^2-\gamma-2)^2$	$y_{10} = -2^{\frac{3}{2}}\gamma^2(\gamma^2-2)(\gamma^4-14\gamma^2+16)\rho_3^\gamma(2,-1,-4,4)$
$y_3 = \rho_7^\gamma(5,12,-34,-74,68,136,-32,-64)y_3'$	$y_{11} = \rho_{13}^\gamma(-1,-48,280,-464,-1416,3152,3144,$ $-7056,4064,7744,3072,-4352,-1024,1024)$
$y_4 = \rho_{11}^\gamma(2,16,-56,-126,337,441,-816,$ $-848,832,816,-256,-256)$	$r_{c_8} = (8(1-\gamma)^{\frac{3}{2}}(y_2r_\alpha^2+y_3r_\alpha+y_4)^{\frac{1}{2}}+y_5r_\alpha+y_6)y_1$
$y_5 = \rho_9^\gamma(1,-5,-4,28,32,-100,-64,144,32,-64)$	$r_{c_9} = ((3\gamma^4-7\gamma^2+4)^{\frac{1}{2}}(y_8r_\alpha+y_{10})+y_9r_\alpha+y_{11})/y_7$
$y_6 = \rho_8^\gamma(-9,54,-6^2,-236,284,312,-432,-2^5,192)$	$r_{c_{10}} = (-2\gamma^2-\gamma+4)(2(2-\gamma^2))^{-1},$
$y_7 = 4\rho_{14}^\gamma(0.5,0,-33,96,136,-656,-388,1808,$ $1048,-2528,-1792,,1792,1536,-512,-512)$	$r_{c_{11}} = (-\gamma(\gamma+2)r_\alpha+\rho_3^\gamma(1,-13,-2,16))\times$ $(\rho_3^\gamma(2,-12,-4,16))^{-1}$
$y_8 = 2\sqrt{2}\gamma^3(\gamma^2+\gamma-2)(\gamma^2-2)(\gamma^4-14\gamma^2+16)$	$r_{c_{12}} = (1/\rho_4^\gamma(1,1,-4,-2,4))\left((2\gamma^4-8\gamma^2+8)r_\alpha+\right.$
$y_9 = (\gamma-1)(\gamma+2)\rho_{12}^\gamma(1,0,-40,96,280,-464,$	$\left.\rho_4^\gamma(2,1,-6,-2,4)+\gamma^2\sqrt{2(3\gamma^4-7\gamma^2+4)}\ \right)$

- for the inflexible firm, it prices to clear its inventory if $x_I \le \overline{x}_I(x_F)$ and prices to have leftover otherwise;
- for the flexible firm, (i) if $x_F < \underline{x}_F(x_I)$ then it reorders and prices to clear its inventory; (ii) if $\underline{x}_F(x_I) \le x_F \le \overline{x}_F(x_I)$ then it prices to clear its inventory but does not reorder; (iii) if $x_F > \overline{x}_F(x_I)$ then it prices to have leftover and does not reorder;

where $\underline{x}_F(x_I) = \begin{cases} -\frac{\gamma}{2-\gamma^2}x_F + \frac{(1+\gamma)(\alpha-C+\gamma C)}{2-\gamma^2} & \text{if } x_I < \hat{x}_I \\ \hat{x}_F, & \text{if } x_I \ge \hat{x}_I \end{cases}$, $\overline{x}_F(x_I) = \begin{cases} -\frac{\gamma}{2-\gamma^2}x_F + \frac{(1+\gamma)\alpha}{2-\gamma^2} & \text{if } x_I < HB(\alpha) \\ HB(\alpha) & \text{if } x_I \ge HB(\alpha) \end{cases}$, and

$$\overline{x}_I(x_F) = \begin{cases} \hat{x}_I & \text{if } x_F \le \hat{x}_F \\ -\frac{\gamma}{2-\gamma^2}x_F + \frac{(1+\gamma)\alpha}{2-\gamma^2} & \text{if } \hat{x}_F < x_F < HB(\alpha). \\ HB(\alpha) & \text{if } x_F \ge HB(\alpha) \end{cases}$$

Solving the 1st stage ordering game proceeds similarly as in the proofs of Propositions 1 and 2. We first identify all possible equilibrium candidates and then characterize the conditions which ensure that a candidate is the best response. As reorder flexibility is asymmetric in the U game, we need to establish these conditions both for the flexible and for the inflexible firm. As the idea of the proof is essentially the same as Propositions 1 and 2, we omit the details of the rather complicated algebra and directly present the results. Let us define $\check{r}_\alpha$ as the r_α-axis coordinate of the intersection of r_{c_4} and r_{c_7}. Then the following holds for the three cases:

(i) If $r_c \le \mathcal{L}_c(r_\alpha,\gamma) := r_{c_5}\mathbf{1}_{\{1\le r_\alpha<\check{r}_\alpha\}} + r_{c_7}\mathbf{1}_{\{\check{r}_\alpha\le r_\alpha<2\}} + r_{c_1}\mathbf{1}_{\{r_\alpha\ge 2\}}$, then the U game has the same initial order equilibrium as the R and N games. In particular,

$$\mathbf{x}^{U*} = \begin{cases} \mathbf{x}_h^{R*} = \mathbf{x}_h^{N*}, \text{ if } (r_\alpha,r_c) \in \left\{(r_\alpha,r_c)|0\le r_c < r_{c_1}\mathbf{1}_{\{r_\alpha\ge 2\}} + r_{c_4}\mathbf{1}_{\{1\le r_\alpha<2\}}\right\} \\ \qquad\qquad \cup\left\{(r_\alpha,r_c)|\check{r}_\alpha \le r_\alpha < 2 \text{ and } \max\{r_{c_4},r_{c_5}\} \le r_c < r_{c_7}\right\}; \\ \mathbf{x}_p^{R*} = \mathbf{x}_l^{N*}, \text{ if } (r_\alpha,r_c) \in \left\{(r_\alpha,r_c)|1\le r_\alpha < \hat{r}_\alpha \text{ and } \max\{0,r_{c_4}\} \le r_c \le r_{r_5}\right\}. \end{cases}$$

(ii) If $\mathcal{L}_c(r_\alpha,\gamma) < r_c < \tilde{r}_c(r_\alpha,\gamma)$, there is a continuum of equilibria on the line $y = -\frac{\gamma}{2-\gamma^2}x + \frac{(1-\gamma^2)(r_\alpha-1)C}{2-\gamma^2}$ in the one-sided ϵ-neighbourhood $(x_l-\epsilon, x_l]$ of x where $x_l = \frac{C}{4}\left(-2(2-\gamma^2)r_c + (\gamma+2)(1-\gamma)+\gamma\right)$.

(iii) If $\tilde{r}_c(r_\alpha,\gamma) := r_{c_6}\mathbf{1}_{\{1\le r_\alpha<2\}} + \max\{r_{c_8},1/2\}\mathbf{1}_{\{r_\alpha\ge 2\}} \le r_c \le 1$, then the U game has a unique

asymmetric initial order equilibrium. At equilibrium, the flexible firm reorders if the market is realized as high. In particular[5],

$$\mathbf{x}^{U*} = \begin{cases} ((x_I^{U*})_{cn}, (x_F^{U*})_{cn}), \text{ if } (r_\alpha, r_c) \in \{(r_\alpha, r_c) | \max\{r_{c_9}, \max\{r_{c_8}, 1/2\}\}\mathbf{1}_{\{r_\alpha \geq 2\}} + r_{c_6}\mathbf{1}_{\{1 \leq r_\alpha < 2\}} \leq r_c < r_{c_{12}}\}; \\ ((x_I^{U*})_{ln}, (x_F^{U*})_{ln}), \text{ if } (r_\alpha, r_c) \in \{(r_\alpha, r_c) | 1/2 \leq r_c < \max\{r_{c_9}, r_{c_{10}}\}\}; \\ ((x_I^{U*})_{cz}, (x_F^{U*})_{cz}), \text{ if } (r_\alpha, r_c) \in \{(r_\alpha, r_c) | \max\{r_{c_{11}}, r_{c_{12}}\} \leq r_c \leq 1\}; \\ ((x_I^{U*})_{lz}, (x_F^{U*})_{lz}), \text{ otherwise;} \end{cases}$$

$$\begin{aligned} \text{where } (x_I^{A*})_{cn} &= \tfrac{2(1-\gamma^2)C}{\gamma^4-14\gamma^2+16}\left(\gamma^3-\gamma^2-\gamma+2+(-\gamma^3+2\gamma^2+2\gamma-4)r_c+(\gamma+2)(1-\gamma)r_\alpha\right); \\ (x_F^{U*})_{cn} &= \tfrac{(1-\gamma^2)C}{\gamma^4-14\gamma^2+16}\left(\gamma^3-13\gamma^2-2\gamma+16-(2\gamma^3-12\gamma^2-4\gamma+16)r_c-(\gamma^2+2\gamma)(1-\gamma)r_\alpha\right); \\ (x_I^{U*})_{cz} &= \tfrac{(1-\gamma^2)C}{2(4-3\gamma^2)}\left((1+\gamma)(2-\gamma)-2(2-\gamma^2)r_c+(\gamma+2)(1-\gamma)r_\alpha\right); \quad (x_F^{U*})_{cz} = (x_F^{U*})_{lz} = 0; \\ (x_F^{U*})_{lz} &= \tfrac{C}{4}\left(-2\gamma^2-\gamma+4+2(\gamma^2-2)r_c\right); \ (x_I^{U*})_{ln} = (x_I^{U*})_{lz} = \tfrac{C}{4}\left(\gamma+2(\gamma^2-2)r_c+(\gamma+2)(1-\gamma)r_\alpha\right). \end{aligned}$$

The proofs of $x_F^{U*} > x^{N*}$ for case (ii) and $x_F^{U*} < x^{R*} < x^{N*}$ for case (iii) involve lengthy but straightforward algebra and are therefore omitted. □

Table 8 Notation in Proof of Proposition 5

$z_1 = \rho_{11}^\gamma(1, 4, -44, -192, 168, 1280, 344, -3008, -2000, 2944, 2560, -1024, -1024)$	$z_{11} = 4(\gamma-1)(\gamma-2)^2(2-\gamma^2)^2(\gamma+2)^3$
$z_2 = \rho_{12}^\gamma(-3, -6, 58, 188, -116, -1104, -528, 2592, 2128, -2688, -2560, 1024, 1024)$	$z_{12} = 2\rho_9^\gamma(1, 9, 4, -66, -44, 192, 96, -2^8, -2^6, 2^7)$
$z_3 = \rho_7^\gamma(-1, -2, 16, 32, -44, -88, 32, 64)$	$z_{13} = 8(1-\gamma)^2(4-\gamma^2)^2(\gamma^2-2)^3\rho_8^\gamma(1, 4, 4, -64, -16, 256, 192, -256, -256)$
$z_4 = \rho_7^\gamma(4, -8, -9, 64, 6, -120, 0, 64)$	$z_{14} = 8(1-\gamma)^2(4-\gamma^2)^2(\gamma^2-2)^2\rho_{10}^\gamma(2, 5, 8, -88, -64, 400, 256, -768, -640, 512, 512)$
$z_5 = \left((1-\gamma)(\gamma-2)(\gamma^4-\gamma^3+16\gamma+16)\gamma^3\right)^{-1}$	$z_{15} = \rho_{20}^\gamma(8, -8, -70, -148, 310, 3568, -3180, -26448, 26608, 102144, -116096, -239104, 289280, 364544, -442368, -368640, 425984, 229376, -245760, -65536, 65536)$
$z_6 = (1-\gamma)(2-\gamma)(2-\gamma^2)\rho_5^\gamma(1, 2, 8, -8, -48, -32)$	
$z_7 = (1-\gamma)(2-\gamma)(3\gamma^2-4)(\gamma^2-2\gamma-4)(\gamma+2)^2$	
$z_8 = 4(1-\gamma^2)(4-\gamma^2)^2(3\gamma^2-4)^2(2-\gamma^2)^2$	
$z_9 = (\gamma+2)^2(2-\gamma^2)^2\rho_{10}^\gamma(1, -4, -3, 48, -34, -132, 124, 144, -144, -64, 64)$	$\chi_{cn} = 1 + 2z_1(r_c-1)/(z_2 - \gamma z_3\sqrt{2\gamma z_4})$
	$\chi_{lz} = z_5\left(z_6 r_c + z_7 - 4\sqrt{(r_\alpha^2 - r_\alpha)z_8 + z_9}\right)$
$z_{10} = \left(2\gamma^4(\gamma^2-2)(\gamma+2)^2(1-\gamma)^2\right)^{-1}$	$\chi_{ln} = z_{10}\left(z_{11}r_c + z_{12} - \sqrt{z_{13}r_c^2 - z_{14}r_c + z_{15}}\right)$

Proof of Proposition 5. For a given (r_α, r_c) in this region, the N, R, and U games each have a unique equilibrium. The proof proceeds by showing at least one of the inequalities $\Pi_F^{U*} < \Pi^{N*}$ and $\Pi_I^{U*} < \Pi^{R*}$ holds. Table 9 summarizes under what condition which firm would deviate from the asymmetric flexibility strategies (N, R). □

Table 9 Deviating Firm in Asymmetric Reorder Flexibility Endowment (N, R)

Equil.	Condition	Deviating Firm	Equil.	Condition	Deviating Firm
$\mathbf{x}_{cz}^{U*}$	any (r_α, r_c)	inflexible $(\Pi_I^{U*} < \Pi^{R*})$	$\mathbf{x}_{lz}^{U*}$	$r_\alpha < \chi_{lz}(r_c, \lambda)$	inflexible $(\Pi_I^{U*} < \Pi^{R*})$
				$r_\alpha > \chi_{lz}(r_c, \lambda)$	flexible $(\Pi_F^{U*} < \Pi^{N*})$
$\mathbf{x}_{cn}^{U*}$	$r_\alpha < \chi_{cn}(r_c, \lambda)$	inflexible $(\Pi_I^{U*} < \Pi^{R*})$	$\mathbf{x}_{ln}^{U*}$	$r_\alpha < \chi_{ln}(r_c, \lambda)$	inflexible $(\Pi_I^{U*} < \Pi^{R*})$
	$r_\alpha > \chi_{cn}(r_c, \lambda)$	flexible $(\Pi_F^{U*} < \Pi^{N*})$		$r_\alpha > \chi_{ln}(r_c, \lambda)$	flexible $(\Pi_F^{U*} < \Pi^{N*})$

[5] The first subscript c or l represents the inflexible firm's strategy, c for *clear-out* and l for *left-over*. The second subscript represents the flexible firm's strategy, n for *non-zero* and z for *zero*.