Price and Reorder Flexibility for Differentiated Short Lifecycle Products under Competition and Demand Uncertainty

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This paper studies two-period procurement-pricing problems for competing firms selling differentiated short lifecycle products with uncertain demand. Firms place initial orders before demand is known. After demand is realized they compete in prices and, if they have reorder flexibility, place second orders at a higher cost. We show that the expected profit functions need not be unimodal and establish novel equilibrium results, with extensive characterizations of the subgame perfect Nash equilibria in symmetric pure strategies for the games with and without reorder flexibility. We identify two countervailing profit effects of reorder flexibility, gains from downside protection under low demand and losses from intensified price competition under high demand. Reorder flexibility increases expected profits if the downside protection gains dominate, under three interdependent necessary and sufficient conditions that we quantify: demand variability is moderate, reordering sufficiently inexpensive, and competition not too intense. Otherwise, reorder flexibility hurts profitability, which is contrary to prior findings that reorder flexibility can only benefit firms under stochastic demand if prices are exogenous or endogenous under quantity competition. Reorder flexibility is most detrimental when firms can flexibly resupply at a moderate cost, in which case it yields larger initial orders, lower expected profits and higher profit variability.

1. Introduction

Today’s business environment is characterized by significant and growing levels of demand uncertainty, particularly in markets for short lifecycle products such as fashion items or seasonal appliances like air conditioners. As product lifecycles keep getting shorter in many industries, a growing number of companies face the challenging problem of matching supply with demand over a short and unpredictable selling period. However, the accuracy of demand forecasts for short lifecycle products typically increases significantly at the inception of the selling period, a phenomenon that firms can exploit if they are sufficiently responsive after market conditions are revealed. Two important approaches to responsiveness are price flexibility and reorder flexibility.

Price flexibility allows firms to postpone pricing decisions until demand is known, so they can set higher prices to clear inventories in booming markets and charge less if demand is low. Price flexibility also supports downside volume flexibility: under low demand firms may prefer to sell only a fraction of their supply by charging more than the clearance price, and hold back the rest.

Reorder flexibility, a core component of quick response capabilities, is gaining importance as an
operational response in markets for short lifecycle products. The key objective of such initiatives is
to help firms better match supply with demand by dramatically cutting lead times, so that firms
can place and receive additional – typically more expensive – orders right before or at the inception
of the retail season, when better demand information is available. Reorder flexibility offers firms
downside protection in case demand turns out to be low while maintaining their ability to capitalize
on high demand outcomes through *upside volume flexibility*. For example, in the well-known Sport
Obermeyer case (Fisher and Raman 1996), the ability to place a second order closer to the season
results in lower overstocking and higher product availability.

This paper studies price and reorder flexibility under competition in the context of two-period
models of price and inventory control under demand uncertainty. While stochastic dynamic joint
price and inventory control problems have been extensively studied for the monopoly case, the
understanding of competitive models is quite limited. This paper contributes to closing this gap.

We consider a duopoly market in which each firm sells a single differentiated product. Demand
is a linear function of prices with an ex ante unknown intercept. We analyze and compare two
settings, the “no reorder” or $N$ game and the “reorder” or $R$ game, which share the following
structure. Firms simultaneously make first period decisions under demand uncertainty; after this
uncertainty resolves firms simultaneously make second period decisions, sales occur and any leftover
inventory is disposed at a fixed salvage value which is normalized to zero. The $N$ game models
firms that only have price and downside volume flexibility: they can procure inventory only in the
first period, before uncertainty is resolved, and set prices after observing demand. In the $R$ game
the firms have price and reorder flexibility: they place initial orders, before uncertainty is resolved,
and they simultaneously choose their prices and reorder quantities after observing demand. The
initial unit procurement cost is lower than the reorder unit cost, reflecting that early orders and
longer lead times facilitate more cost-effective sourcing, production and distribution of products.

1.1. Overview of Analysis and Results

This paper contributes technical and managerial insights that derive from its distinctive focus on
three problem features: firms engage in *price competition*, sell *differentiated* products, and have
both *downside and upside volume flexibility*. The paper (1) provides equilibrium results for finite-
horizon price and inventory competition under demand uncertainty; (2) specifies conditions for joint
price and reorder flexibility to improve or hurt profits, compared to the case of price flexibility only;
and (3) provides a deeper understanding of how the profit impact of volume flexibility depends on
the mode of competition by contrasting our results with prior findings under quantity competition.
1. Equilibrium analysis and results. We establish the existence of symmetric subgame perfect pure strategy Nash equilibria for the $N$ and $R$ games and explicitly characterize the equilibrium strategies as a function of the demand variability, competition intensity and procurement costs.

(a) $N$ game. We identify two equilibrium strategies that depend on the variation between low and high demand. If the demand variation is below a threshold, firms order as much as in the equivalent riskless problem without uncertainty and price to clear their inventory under either demand realization. If the demand variation exceeds a threshold, firms order more than in the corresponding riskless problem, set prices to clear their inventory if demand is high and to have leftover if demand is low. Both of these strategies are equilibria if the demand variation is moderate. We also find that the coefficient of variation of equilibrium profits increases in procurement costs.

(b) $R$ game. We identify three equilibrium price-reorder strategies that differ in terms of whether the firms price to clear or have leftover inventories under low demand and whether they exercise the reorder option or not under high demand. We characterize how these strategies depend on the demand variation and the order cost ratio between late vs. early procurement. For example, we show that if the demand variation exceeds a threshold, firms cannot match demand with their initial order: they initially order more than they sell under low demand if reordering is relatively expensive, and less than they sell under high demand if reordering is relatively cheap. We also find that the coefficient of variation of equilibrium profits is largest at intermediate order cost ratios.

The equilibrium analyses for $N$ and $R$ games are significantly complicated by the fact that each firm’s first period expected profit function may be bimodal in its own order, which we show to be the direct consequence of price competition and the sequential nature of decisions under uncertainty. This may explain why, to our knowledge, these equilibrium results are novel and virtually the only results for stochastic joint price and inventory competition. A notable exception is the analysis of price competition without reorder flexibility (i.e., the $N$ game) for perfectly homogeneous products (cf. Hviid 1991, Reynolds and Wilson 2000), where a pure strategy symmetric equilibrium need not exist, unlike in our differentiated products model. To our knowledge, our $R$ game equilibrium results are the first ones for a stochastic finite-horizon problem under competition in which firms engage in price competition and can order more than once. Though our two-period structure simplifies the problem, its analysis is quite intricate and it captures key tradeoffs that arise due to competition, demand uncertainty and sequential decisions in a nonstationary environment.

2. Orders and profits under joint price and reorder flexibility, compared to price flexibility only.

(a) Orders. If reordering is sufficiently cheap relative to early procurement, firms with a reorder option order less initially than those without but reorder up to a higher level if demand turns
out high. However, if reordering is moderately expensive, firms with reorder flexibility order more initially than those without and do not reorder. A reorder option therefore yields larger inventories under high demand (unless it is prohibitively expensive), and downside protection for low demand only if the firms order less initially. The higher the demand variation and/or the competition intensity, the larger the range of reorder costs that yield larger initial orders under reorder flexibility.

(b) Expected profits. Compared to price flexibility only, reorder flexibility hurts expected profits whenever it yields larger initial inventories and therefore lower prices and higher procurement costs, which is the case if reordering is moderately expensive. However, reorder flexibility may also hurt expected profits if it leads to smaller initial orders. The profit impact in these cases depends on two countervailing effects: downside protection under low demand improves profits since lower initial orders yield higher prices and lower procurement costs, but intensified competition under high demand hurts profits since the second order yields higher inventories, lower prices and higher procurement costs. The reorder option only yields higher expected profits if the gains from downside protection under low demand dominate the losses of intensified competition under high demand, which holds under three additional conditions that we explicitly specify: (i) the demand variation is neither too small nor too large; (ii) reordering is sufficiently cheap; and (iii) the competition intensity is below a threshold. The latter condition underscores how important it is for firms to sufficiently differentiate their products to reap the benefits of reorder flexibility.

(c) Profit variability. Compared to price flexibility only, under reorder flexibility the coefficient of variation of profits is higher whenever firms order more initially, and lower whenever firms place smaller initial orders. Reorder flexibility therefore benefits firms generally more by reducing profit variability than by increasing expected profits.

We conclude that a reorder option is most detrimental if it is moderately expensive, in which case it yields larger initial orders, lower expected profits and higher profit variability.

3. Profit impact of volume flexibility and the mode of competition that determines prices.

Our results for the N and R games are of interest in their own right in that price competition is a natural model of price formation. They also form the basis for comparison with prior findings on the profit impact of volume flexibility under demand uncertainty (Van Mieghem and Dada 1999; Anupindi and Jiang 2008; Lin and Parlaktürk 2012). These studies assume quantity (Cournot) competition for a perfectly homogeneous product and show that volume flexibility can only increase profits, in contrast to our results under price competition. The assumption of quantity competition is technically appealing for tractability (especially for homogenous products), and it is usually justified with the classic result that single-stage quantity competition yields the same outcome as
two-stage competition where firms first choose supply quantities (capacity, production or inventory) and then prices (Kreps and Scheinkman 1983). However, this equivalence critically hinges on the condition that firms cannot increase their supply while or after demand is formulated (cf. Tirole 1998, p. 217); the presence of volume flexibility may violate this condition. Modeling differentiated products captures a natural market feature, renders stochastic price competition tractable and allows us to study how the mode of competition affects the value of volume flexibility.

(a) Price competition reduces but does not eliminate the value of downside volume flexibility. Anupindi and Jiang (2008) prove that competition under downside volume flexibility yields higher expected profits than competition among inflexible firms (Van Mieghem and Dada 1999 show this numerically). Inflexible firms choose supply quantities before demand is known and sell all supply ex post at the clearance price. Flexible firms also choose capacities ex ante but sales quantities only ex post; the resulting option to hold back supply induces them to build more capacity than inflexible firms and increases their expected (net) revenues more than their capacity cost.

The power of the hold back option rests on the flexible firms’ ability to commit to underutilizing capacity if demand is low. The extent of this hold back commitment in turn depends on the mode of competition in the second stage, following the capacity investments. A model where equilibrium sales quantities and prices are determined in the second stage under quantity competition, as in Anupindi and Jiang (2008)\(^1\), captures settings where firms are not sufficiently flexible to deviate from second-stage production quantities even if they have excess capacity. However, if firms can flexibly supply in the second stage any amount up to their initial capacity choice – due to preproduction or production postponement with highly flexible operations, then no quantity competition equilibrium with excess capacity is sustainable: it leaves each firm with the incentive to lower its price and increase sales. Price competition is more appropriate in such cases as no firm has an incentive to lower its price and increase sales at the corresponding equilibrium, even if it has the capacity to do so; in this sense price competition implies a stronger hold back commitment.

To study how the value of a hold back option depends on the mode of competition, we compare our \(N\) game (which allows hold back under second-period price competition) with two variations, the \(N\) game under second-period clearance and the \(N\) game with a hold back option under second-period quantity competition; these correspond to the models of inflexible and flexible firms, respectively, in Anupindi and Jiang (2008). In contrast to the \(N\) game under price competition, each of these variations has an unique symmetric pure strategy equilibrium. We find that a hold back

\(^1\) They show that this game has the same outcome if the quantity competition subgame is replaced by a two-stage production-price subgame, which proves the stochastic counterpart of the result of Kreps and Scheinkman (1983).
option increases expected profits compared to clearance, also under price competition. However, price competition does yield lower expected profits than quantity competition, consistent with the classic result under deterministic demand. Moreover, unlike in the deterministic case, price competition may yield lower capacity/inventory investments than quantity competition, which holds if demand variability is moderate, and it follows because firms have more control over downside risk under quantity competition – they can hold back a larger share of a given supply.

b) **Upside volume flexibility can hurt profits only under price competition.** Lin and Parlaktürk (2012) focus on a manufacturer’s incentive to offer a reorder option to duopoly retailers that sell a homogeneous product, place initial orders before demand is known and must clear their inventory. The authors assume that quantity competition determines equilibrium reorder quantities, in contrast to our \( R \) game where reorder quantities emerge from price competition. In their analysis competition between “fast” retailers that can reorder after demand is realized yields (weakly) smaller initial orders and larger expected retailer profits, compared to competition between “slow” retailers without a reorder option. These results are in contrast to our findings that the \( R \) game may yield larger initial orders and lower expected profits than the \( N \) game, and moreover, that a reorder option cannot benefit firms if products are sufficiently close substitutes.

In conclusion, the contrast between our results and prior findings show that the mode of competition significantly affects the impact of volume flexibility on equilibrium decisions and profits, particularly in the case of reorder flexibility where price and quantity competition may yield opposite results. Whether quantity or price competition is a more appropriate model of price formation under volume flexibility depends on factors that affect how flexibly firms can increase their supply during the selling season, such as the marginal cost and the delivery time of replenishment orders (cf. Tirole, 1988, p. 224). Our results underscore the importance of understanding these factors in order to better predict the impact of volume flexibility and improve performance as a result.

### 1.2. Literature Review

This paper is at the intersection of two literatures: one studies stochastic price-inventory control problems, the other considers the impact of price and/or operational flexibility on profitability.

Numerous papers in both streams focus on *monopoly* settings. See Chen and Simchi-levi (2012) for a recent survey of integrated price-inventory control models and Petruzzi and Dada (2011) who focus on newsvendor models. Among monopoly studies of flexibility, Van Mieghem and Dada (1999) study the benefits of production and price postponement strategies, with limited analysis of competitive models under quantity competition; Cachon and Swinney (2009) show that quick response capabilities can be significantly more valuable to a retailer in the presence of strategic
consumers than without them; Goyal and Netessine (2011) analyze volume and product flexibility under endogenous pricing for a two-product firm.

The understanding of stochastic price-inventory control under competition is quite limited. At one extreme of the problem space, Bernstein and Federgruen (2005) and Zhao and Atkins (2008) study the single period problem in the classic newsvendor framework where the selling period is so short compared to lead times that firms can order only once, before demand is realized, and also choose prices in advance. At the other extreme Kirman and Sobel (1974) and Bernstein and Federgruen (2004a,b) study periodic-review infinite-horizon oligopoly problems, but under such conditions that they reduce to myopic single period problems where decisions in each period are made before demand is realized. Kirman and Sobel (1974) obtain a partial characterization of a pure strategy Nash equilibrium with a stationary base-stock level. Bernstein and Federgruen (2004a) identify conditions for existence of a pure strategy Nash equilibrium in which each retailer adopts a stationary base-stock policy and a stationary list price. Bernstein and Federgruen (2004b) study games where retailers compete not only on prices but also on service levels.

The problems in this paper belong to the intermediate domain between these two extremes in which the selling horizon is finite but firms are sufficiently responsive to exploit information gained over time and make pricing and/or supply decisions more than once. This intermediate case is gaining in importance as businesses are countering shrinking product lifecycles with faster operations and more adaptive pricing, but there is hardly any literature on finite-horizon stochastic price-inventory competition. One notable exception alluded to in Section 1.1 is the analysis of sequential inventory-price competition without reordering in the stochastic Bertrand-Edgeworth model (cf. Hviid 1991, Reynolds and Wilson 2000), which is the homogeneous-product analog of our $N$ game for differentiated products; unlike our $N$ game it need not admit a pure strategy symmetric equilibrium. Recently there is a growing interest in price competition under exogenous capacity levels (cf. Porteus et al. 2010, Liu and Zhang 2012 and the references therein).

In the literature on flexibility under competition, a number of economics papers study the strategic effects of flexibility in the absence of demand uncertainty, cf. Maggi (1996), Boccard and Wauthy (2000), Röller and Tombak (1993). A general insight from these and other studies is that flexibility can be harmful under competition; in other words, not being flexible can have strategic commitment value. However, demand uncertainty is the key reason why matching demand with supply is challenging, and flexibility can have value to hedge against demand risk. One can segment the studies of flexibility under competition with stochastic demand into two streams: one, which includes this paper, focuses on single-product firms with volume flexibility (cf. Vives 1989,
the other focuses on two-product firms with product flexibility (Anand and Girotra 2007, Goyal and Netessine 2007). In contrast to the present paper, none of these studies perform the analysis under price competition. Li and Ha (2008) and Caro and Martínez-de-Albéniz (2010) assume fixed prices. The other papers do consider endogenous pricing but under such assumptions that lead to quantity (Cournot) competition.

In the product flexibility stream, Anand and Girotra (2007) consider two-product firms that choose between early product differentiation before, or delayed differentiation after demand uncertainty is resolved, and show that early differentiation may arise as a dominant strategy for firms. Goyal and Netessine (2007) consider two-product firms that choose whether to invest in flexible or dedicated technology and identify conditions under which flexibility benefits or harms profits. In both papers, unlike in ours, prices are determined by quantity competition and the total supply is determined before demand uncertainty resolves; the essence of product flexibility is that it allows firms to delay product-to-market allocation decisions until demand is known.

The volume flexibility stream has more of a history in economics. Going back to Stigler (1939), these papers often model the degree of flexibility on a continuum, by the slope of the average cost curve around some minimum; cf. Vives (1989) who studies a two-stage homogeneous product oligopoly game in which firms choose their flexibility level, before receiving (private) demand signals, and then choose production quantities in Cournot competition. In contrast, the operations management literature on volume flexibility compares a discrete set of flexibility configurations (typically two) that differ in terms of the timing of supply (capacity/production/inventory) decisions relative to when demand is realized. A key finding in these papers is that competition with volume flexibility increases expected profits compared to competition without, under fixed prices and competition in inventories and/or reactive capacity (Li and Ha 2008, Caro and Martínez-de-Albéniz 2010), and endogenous pricing under quantity competition with downside flexibility through a hold back option (Van Mieghem and Dada 1999, Anupindi and Jiang 2008) or upside flexibility through a reorder option (Lin and Parlaktürk 2012). As discussed in Section 1.1, in the presence of volume flexibility the conditions for the equivalence of price and quantity competition (Kreps and Scheinkman 1983, Anupindi and Jiang 2008) need not hold. Our results show that joint price and reorder flexibility can hurt profits, and compared to quantity competition, price competition may yield (i) lower capacity investments and profit benefits under downside flexibility, and (ii) opposite effects on initial orders and expected profits under upside flexibility.
2. Model, Problem Formulations and Preliminary Analysis

We study symmetric duopoly firms, each selling a single differentiated product with price-sensitive demand. We analyze and compare two games. In the “no reorder” game (N) firms only have price flexibility, and in the “reorder” game (R) firms have price and reorder flexibility, as detailed below. For expository convenience, we also refer to these simply as the N and R games, respectively. In both games, the reorder option is symmetric between firms, i.e., neither has this option in the N game and both have it in the R game. Both games share the following two-period structure. In period one, before observing demand, firms simultaneously choose their initial orders. The outcome of period one is common knowledge. Demand uncertainty is resolved before firms make their second period decisions. The N and R games are identical up to this instant where demand is observed. They differ as follows in the second period decisions. In the N game, each firm simultaneously chooses only the price for its product, but it cannot adjust its inventory. In the R game, each firm simultaneously chooses the price for its product and its reorder quantity. The initial unit procurement cost is typically lower than the reorder unit cost. Demand and sales occur following the second period decisions. Taken literally, this captures a situation where firms gain demand information through factors other than own early-season sales, such as weather, market news and fashion trends. However, the model can also be viewed as a reasonable approximation of settings where sales that materialize between the first order delivery and the second period decisions only make up a small fraction of initial inventory but are still of significant value for demand forecasting. It is quite common that the forecast accuracy for total season demand increases dramatically after observing a few days of early season sales. The model does not specify order delivery lead times; we assume they are short enough so that firms do not lose sales due to delivery delays. If there is leftover inventory, it is disposed at a zero salvage value after the season. We ignore further inventory holding costs that may be incurred during the season, based on the notion that they are insignificant relative to overstocking costs and margins.

We index the firms by $i \in \{1, 2\}$ and denote their variables and functions, such as order quantities, prices, demands, and profits, by corresponding subscripts. We write $-i$ to denote firm $i$’s rival, where $-i \neq i$. We model demand uncertainty within the following linear demand system. Let $p_i$ denote firm $i$’s price and $p = (p_1, p_2)$. Demand for firm $i$’s product is given by

$$d_i(p; \alpha) = \alpha - p_i + \gamma p_{-i} \geq 0, \ i = 1, 2. \quad (1)$$

The intercept $\alpha > 0$ is ex ante uncertain; we call it the market size parameter. We assume that demand is high, i.e., $\alpha = \alpha_H$, or low, i.e., $\alpha = \alpha_L < \alpha_H$, with equal probability. The assumption of equally likely high and low demand scenarios does not change our main qualitative insights.
Uncertainty with respect to \( \alpha \) may be due to a range of factors that equally affect differentiated products in the same product category, e.g., weather in the case of seasonal appliances such as air conditioners, or color in the case of fashion items. The competition intensity parameter \( \gamma \in [0, 1) \) measures the price sensitivity of firm-\( i \) demand to its rival’s price, which reflects the degree of product differentiation and factors such as brand preferences. We assume that \( \gamma \) is ex ante known, based on the notion that brand loyalty and price sensitivity are well understood.

In period one, before knowing whether the market size will be \( \alpha L \) or \( \alpha H \), firm \( i \) chooses its (initial) order size \( x_i \) at unit cost \( c \in [0, C] \), where \( C \) is the unit reorder cost that is available in the presence of reorder flexibility. We normalize \( C = 1 \) in our analysis without loss of generality, but we use the variable \( C \) in our discussion. Let \( \mathbf{x} = (x_1, x_2) \) denote the initial order vector.

We turn to the problem formulations for the \( N \) and \( R \) games. In both cases, our equilibrium analysis focuses on Nash equilibria in pure strategies for the second period subgame and on subgame perfect Nash equilibria in symmetric order strategies for the first period game.

**No reorder game.** Let \( \pi_i^N(p, x_i; \alpha) \) denote firm \( i \)'s second period revenue function in the \( N \) game. It depends on the realized market size \( \alpha \) and its own initial inventory \( x_i \). Given their initial inventory from first period orders \( x \), firms simultaneously choose their prices by solving

\[
\max_{p_i} \pi_i^N(p, x_i; \alpha) = p_i \cdot \min(x_i, d_i(p; \alpha)), \quad i = 1, 2,
\]

where \( \pi_i^N(p, x_i; \alpha) \) is strictly concave in \( p_i \). We assume that excess demand is lost. Since firms observe the market size realization \( \alpha \) prior to choosing their prices, they have no incentive to generate more demand than they can satisfy. However, a firm may find it optimal not to sell all the inventory procured in the first period, if the market turns out to be small. In this case, we say that a firm “prices to have leftover”. As noted above, the salvage value of leftover inventory is zero. Let \( \mathbf{p}^{N*}(\mathbf{x}; \alpha) \) denote the second period equilibrium price vector and \( \pi_i^{N*}(\mathbf{x}; \alpha) = \pi_i^N(\mathbf{p}^{N*}(\mathbf{x}, \alpha), x_i; \alpha) \) the equilibrium firm-\( i \) profit for the \( N \) game, as a function of the initial orders \( \mathbf{x} \) and the market size \( \alpha \). Let \( \Pi_i^N(\mathbf{x}) \) denote firm \( i \)'s expected profit as a function of initial orders \( \mathbf{x} \). In the first period, firms simultaneously choose their orders by solving

\[
\max_{x_i \geq 0} \Pi_i^N(\mathbf{x}) = \frac{1}{2} (\pi_i^{N*}(\mathbf{x}; \alpha_L) + \pi_i^{N*}(\mathbf{x}; \alpha_H)) - cx_i, \quad i = 1, 2.
\]

Let \( \mathbf{x}^{N*} \) denote equilibrium orders, and the scalar \( x^{N*} \) a symmetric equilibrium order quantity.

**Reorder game.** Let \( \pi_i^R(p, x_i; \alpha) \) denote firm \( i \)'s second period profit function in the \( R \) game. In addition to choosing their prices, firms may also order more at a unit cost \( C \geq c \). Since firms observe the market size prior to their second period decisions, they have no incentive to generate more demand than they can satisfy. Therefore, the reorder quantities are determined by the initial inventories and the prices: given \( x_i \) and \( \mathbf{p} \), firm \( i \) orders the amount \( (d_i(\mathbf{p}; \alpha) - x_i)^+ = \)
max \((d_i(p;\alpha) - x_i, 0)\) in the second period. Given initial inventories \(x\), firms simultaneously choose prices and the resulting reorder quantities in the second period by solving

\[
\max_{p_i} \pi_i^R(p, x_i; \alpha) = p_i d_i(p; \alpha) - C(d_i(p; \alpha) - x_i)^+, \quad i = 1, 2,
\]

where \(\pi_i^R(p, x_i; \alpha)\) is unimodal in \(p_i\). Let \(p^{R*}(x; \alpha)\) denote the second period equilibrium price vector and \(\pi_i^{R*}(x; \alpha) = \pi_i^R(p^{R*}(x; \alpha), x_i; \alpha)\) the equilibrium firm-i profit for the \(R\) game. Let \(\Pi_i^R(x)\) denote firm \(i\)'s expected profit for the \(R\) game as a function of the initial order vector. In the first period, firms simultaneously choose their initial orders by solving

\[
\max_{x_i \geq 0} \Pi_i^R(x) = \frac{1}{2} (\pi_i^{R*}(x; \alpha_L) + \pi_i^{R*}(x; \alpha_H)) - cx_i, \quad i = 1, 2.
\]

Let \(x^{R*}\) denote equilibrium initial orders, and the scalar \(x^{R*}\) a symmetric equilibrium initial order.

**Remark.** Each firm’s expected profit may be bimodal in its own initial order, due to the joint effect of uncertainty, price competition and the sequential decisions over a finite horizon, as explained in Section 3. Therefore, our analysis cannot rely on standard equilibrium characterization results for the \(N\) and \(R\) games. Instead, we exploit the structure of the best response problem.

**Second Period Price-Reorder Equilibria.** As a preliminary analysis we characterize the second period subgame equilibria of the \(N\) and \(R\) games, which serve as building blocks for our characterization and comparison of the first period order equilibria in Sections 3-5. Since firms learn the market size \(\alpha\) prior to their second period decisions, these second period subgames are deterministic. Each subgame has an unique equilibrium that depends on the initial order vector \(x\) and the market size \(\alpha\). We discuss the structure of these equilibria. The proof of Lemma 1 provides closed-form expressions for the equilibrium prices and reorder quantities.\(^2\)

**No reorder game.** Given their initial inventory levels \(x\), firms simultaneously choose their prices by solving (2). Let \(p_i^N(x_i, p_{-i})\) denote firm \(i\)'s best response price as a function of its own initial inventory and its rival’s price. Define firm \(i\)'s clearance price \(p_i^c(p_{-i}, x_i)\) as the highest price that generates enough demand for firm \(i\) to sell its entire inventory. Firm \(i\) may choose to charge more than this clearance price and hold back supply. Define firm \(i\)'s revenue management price \(p_i^r(p_{-i})\) as its revenue-maximizing price in the presence of ample inventory. Firm \(i\) prefers to charge this price if it exceeds the clearance price, which results in leftover stock. The best response of firm \(i\) is to charge the larger of these two prices, i.e., \(p_i^N(x_i, p_{-i}) = \max(p_i^c(p_{-i}, x_i), p_i^r(p_{-i}))\). We call these best responses clearance (c) and revenue management (r), respectively.

\(^2\) The second period subgames considered here are also analyzed in Maggi (1996). He assumes deterministic demand. Since his second period equilibrium characterization is less complete and explicit than what we require for our equilibrium analysis under demand uncertainty in Sections 3-5, we provide in Lemma 1 our own result statement and proof.
Both firms follow the same strategies. As shown in Figure 1(a), the firms’ equilibrium strategies partition the initial inventory space \( \{x \geq 0\} \) into four regions, labelled \( N(c,c) \), \( N(r,c) \), \( N(c,r) \) and \( N(r,r) \). Each label identifies the equilibrium strategies for the respective region, the first letter referring to firm 1 and the second to firm 2. For example, region \( N(c,c) \) consists of all initial inventory vectors \( x \) for which the unique equilibrium is for both firms to charge the clearance price. The region boundaries define for each firm, inventory thresholds for its second period equilibrium strategy; see Lemma 1 and its proof for details. For example, for high initial inventory levels \( x \) in region \( N(r,r) \), the unique equilibrium is for both firms to charge the revenue management price, which yields for each firm demand and sales equal to the revenue management quantity \( RM := \alpha/(2 - \gamma) \) and results in leftover inventory; the arrow indicates the shift from an initial inventory vector to the corresponding second period equilibrium sales quantity vector.

**Figure 1 Strategy Regions**

(a) No Reorder Game (\( N \))

(b) Reorder Game (\( R \))

**Reorder game.** Given their initial inventory levels \( x \), firms simultaneously choose their prices and reorder quantities by solving (4). Clearance and revenue management are two possible best responses, as in the \( N \) game. A third possibility is for firm \( i \) to charge the procurement price \( p_i^p(p_i - i) \), defined as its profit-maximizing price in the absence of initial inventory, and to procure the amount required to satisfy its excess demand at that price. The procurement price exceeds the revenue management price since reordering is costly, i.e., \( C > 0 \). This strategy, which we call procurement \( (p) \), is the best response if and only if the procurement price is lower than the clearance price. Otherwise, firm \( i \) has enough inventory to maximize the second period profit, and its best response is to charge the larger of the clearance price and the revenue management price. These equilibrium strategies partition the initial inventory space \( \{x \geq 0\} \) into nine regions as shown in Figure 1(b).
We use the same labeling convention as in the $N$ game. For example, for initial inventory levels in region $R(p,p)$, the unique equilibrium is for both firms to charge the procurement price, which yields for each firm demand equal to the order-up-to level $OU := (\alpha - C (1 - \gamma))/(2 - \gamma)$, and to reorder up to and sell this amount, as indicated by the arrow. For $\alpha \leq C (1 - \gamma)$, the order-up-to level is non-positive, i.e., firms have no incentive to reorder, and the second period $N$ and $R$ subgames are equivalent.

**Lemma 1. (Second Period Subgame Equilibria).** For any initial inventory vector $x$ in the second period, the price subgame of the $N$ game and the price-reorder subgame of the $R$ game each has a unique equilibrium. The equilibrium strategy of each firm depends as follows on $x$.

(N) If firm $i$ cannot reorder, there is a threshold $\tau_i(x_{-i})$ which depends on its rival’s initial inventory: firm $i$ prices to clear its inventory if $x_i \leq \tau_i(x_{-i})$ and prices to have leftover otherwise.

(R) If firm $i$ does have a reorder option then there are two thresholds $\tau_i(x_{-i}) < \tau_i(x_{-i})$ which depend on its rival’s initial inventory: (i) if $x_i < \tau_i(x_{-i})$ then firm $i$ reorders and prices to clear its inventory; (ii) if $\tau_i(x_{-i}) \leq x_i \leq \tau_i(x_{-i})$ then it prices to clear its inventory but does not reorder; (iii) if $x_i > \tau_i(x_{-i})$ then it prices to have leftover and does not reorder.

The second period equilibrium decisions are contingent on the initial inventories and the realized demand. Firms order initial inventories before observing demand. In Section 3 we characterize the $N$ game equilibria, and in Section 4 we present the corresponding results for the $R$ game; to our knowledge, these equilibrium results are novel. In Section 5 we build on these results to discuss the impact of reorder flexibility on equilibrium strategies and profits.

### 3. Price Flexibility without Reorder Flexibility

In the first period, firms simultaneously choose their order quantities by solving (3), i.e.,

$$\max_{x_i \geq 0} \Pi_i^N(x) = \frac{1}{2} \left( \pi_i^{N*}(x; \alpha_L) + \pi_i^{N*}(x; \alpha_H) \right) - cx_i, \ i = 1, 2.$$  

We call $\pi_i^{N*}(x; \alpha_L)$ and $\pi_i^{N*}(x; \alpha_H)$ the conditional second period equilibrium revenue functions: they evaluate the revenue as a function of the second period initial inventories $x$ and the corresponding equilibrium prices, conditional on the market size; see Lemma 1.

**Uncertainty, price competition, and sequential decisions imply bimodal expected profits.** The equilibrium characterization of the $N$ game is significantly complicated by the fact that the conditional second period equilibrium revenue functions $\pi_i^{N*}(x; \alpha_L)$ and $\pi_i^{N*}(x; \alpha_H)$ are not concave in firm $i$’s own order quantity. This fact, combined with the presence of uncertainty, implies that the first period expected revenue and profit functions of each firm may be bimodal in its own order.
The non-concave nature of the conditional second period equilibrium revenue functions is the natural result of price competition, coupled with the sequential nature of decisions over a finite horizon. Consider how \( \pi^*_N(x; \alpha) \) depends on firm \( i \)'s own order \( x_i \), given the market size is \( \alpha \) and its rival orders \( x_{-i} \). By Lemma 1, there is a revenue management threshold \( \pi_i(x_{-i}; \alpha) \) such that firm \( i \) never sells more than \( \pi_i(x_{-i}; \alpha) \). The function \( \pi^*_N(x; \alpha) \) is concave in \( x_i \leq \pi_i(x_{-i}; \alpha) \), constant in \( x_i \geq \pi_i(x_{-i}; \alpha) \), and peaks at an order size \( x_i \) that is smaller than \( \pi_i(x_{-i}; \alpha) \). Figure 2(a) shows a representative example for firm 1, given \( \alpha = 15 \) and \( x_2 = 5 \): firm 1’s second period equilibrium revenue peaks at \( x_1 = 11.0 \), then decreases, before leveling off at the threshold \( \pi_1(x_2; \alpha) = 14.6 \) (indicated by the vertical dotted line). The property that \( \pi^*_N(x; \alpha) \) first decreases before leveling off occurs only under price competition: since the marginal revenue of each firm is higher if it unilaterally drops its price than if it unilaterally increases its inventory, equilibrium prices keep dropping as \( x_1 \) increases from \( x_1 = 11.0 \) to \( x_1 = 14.6 \).

In contrast, if prices are determined under quantity competition, then each firm’s conditional second period revenue for given market size \( \alpha \) is concave in its own order quantity. For given \( \alpha \) and \( x_{-i} \), there also exists a revenue management threshold in this case, but the firm-\( i \) revenue peaks at \( x_i \) equal to this threshold, unlike under price competition. Figure 2(b) shows a representative example: given \( \alpha = 15 \) and \( x_2 = 5 \), the revenue management threshold under quantity competition is \( x_1 = 11 \), and the second period equilibrium revenue of firm 1 peaks at this threshold.

**Figure 2  Conditional Second Period Equilibrium Revenues Given** \( x_2 = 5 \) (\( \alpha = 15, \gamma = 0.7 \))

In summary, the bimodal nature of the first period expected profit functions is due to the joint effect of three factors, uncertainty, price competition and the sequential nature of ordering and pricing decisions over a finite horizon. If any one of these factors is absent, the payoff functions
(in each period) are well-behaved, which seems to be the case virtually throughout the related literature, including in papers where the equilibrium prices are determined by quantity competition (cf., Anupindi and Jiang 2008), and in studies on joint price-inventory competition in stationary infinite-horizon problems (cf. Kirman and Sobel 1974, Bernstein and Federgruen 2004a). A noteworthy exception is the version of the $N$ game with perfectly homogeneous products (cf. Hviid 1991, Reynolds and Wilson 2000). In this extreme case, demand functions are discontinuous in prices, first-period payoffs are not well-behaved and a pure strategy symmetric equilibrium in inventories need not exist if the extent of demand variation exceeds a threshold level. By contrast, our $N$ game with differentiated product admits the following equilibrium result.

**Proposition 1. (First Period Order Equilibria: $N$ Game).** Two thresholds on the high demand market size determine the first period order equilibria in the “no reorder” game:

$$\alpha_H^* := m^*(\gamma)\alpha_L + 2c(1 - \gamma) > \alpha_H^{**} := m^{**}(\gamma)\alpha_L + 2c(1 - \gamma) \text{ for } \gamma > 0,$$

where $m^*(\gamma)$ and $m^{**}(\gamma)$ are explicit functions of the competition intensity $\gamma$ and $m^{**}(\gamma) > m^*(\gamma) > 1$ for $\gamma > 0$. Let $RM(\alpha) := \alpha / (2 - \gamma)$ denote the revenue management quantity for market size $\alpha$. If the high demand market size parameter is:

1. below the smaller threshold, i.e. $\alpha_H \leq \alpha_H^*$, there is a unique symmetric order equilibrium:
   $$x_{N*} = x_{lN*} := (1 + \gamma)(\alpha_H/2 + \alpha_L/2 - c(1 - \gamma)) \leq RM(\alpha_L), \quad (6)$$
   and firms price to clear their inventory in both demand scenarios;

2. larger than the larger threshold, i.e. $\alpha_H \geq \alpha_H^{**}$, there is a unique symmetric order equilibrium:
   $$RM(\alpha_L) < x_{N*} = x_{hN*} := \frac{1 + \gamma}{2 + \gamma}(\alpha_H - 2c(1 - \gamma)) < RM(\alpha_H), \quad (7)$$
   firms price to sell $RM(\alpha_L)$ and have leftover if demand is low, and else clear their inventory;

3. between the two thresholds, i.e. $\alpha_H^* < \alpha_H < \alpha_H^{**}$, there are exactly two symmetric equilibria, one as in 1., the other as in 2.

Given any symmetric initial inventories in the second period subgame, firms sell at most the revenue management quantity $RM(\alpha)$ corresponding to the realized market size $\alpha$; see Figure 2(a).

Since procurement is costly, the equilibrium order size in the $N$ game is therefore smaller than $RM(\alpha_H)$, but it may be smaller or larger than $RM(\alpha_L)$ due to market size uncertainty.

“Small” equilibrium: $x_{lN*} < RM(\alpha_L)$. If the high demand market is not too large, relative to the low demand market ($\alpha_H < \alpha_H^{**}$), the equilibrium order quantity $x_{lN*}$ is such that firms price to clear their inventory under either market size realization. The equilibrium order quantity $x_{lN*}$ in (6) and the expected equilibrium price and profit of each firm are the same as under the corresponding riskless problem, i.e., if the market size were known and equal to the mean $(\alpha_H + \alpha_L)/2$. However,
market size uncertainty leads to variability in equilibrium prices and profits, leaving firms better off under high demand and worse off under low demand, compared to the riskless case.

“Large” equilibrium: \( x^N_h > RM(\alpha_L) \). If the high demand market is relatively large \((\alpha_H > \alpha^*_H)\), the firms order more in equilibrium than in the corresponding riskless problem. The equilibrium order quantity \( x^N_h \) is so large that firms only price to clear their inventory if demand is high. If demand is low, they charge the revenue management price, sell \( RM(\alpha_L) \) and have leftover inventory. The prices and revenues under low demand are therefore independent of how much the firms order in excess of \( RM(\alpha_L) \). The equilibrium quantity \( x^N_h \) in (7) balances the marginal ordering cost with the incremental revenue under high demand.

Equilibrium profit variability. Since the firms balance over- and understocking risks, in equilibrium they order more than they would under known low demand and less than they would under known high demand. Furthermore, as Corollary 1 shows, the spread between the conditional equilibrium profits under low and high demand realizations increases in market size uncertainty.

**Figure 3** No Reorder Game: Equilibrium Profit Variability \((\alpha_L = 0.3, \gamma = 0.7)\)

![Contour Plots of Standard Deviation and Coefficient of Variation](image)

(a) Standard Deviation
(b) Coefficient of Variation

**Corollary 1. (Impact of Market Size Variability).** Fix the low demand market size \( \alpha_L \). The market size coefficient of variation \( cv_\alpha := (\alpha_H - \alpha_L)/(\alpha_L + \alpha_H) \) increases in \( \alpha_H \). For each equilibrium in Proposition 1: (i) The order quantity and the expected profit increase in \( \alpha_H \); (ii) The profit under low demand decreases in \( a_H \); (iii) The profit under high demand increases in \( a_H \).

Figure 3 shows contour plots of the standard deviation (SD) and the coefficient of variation (CV) of equilibrium profits, depending on the market size ratio \( \alpha_H/\alpha_L \) and the order cost \( c \). The low demand market size is fixed \((\alpha_L = 0.3)\), so an increase in the market size ratio increases both the
mean and the CV of the market size. Since firms have no reorder flexibility, the SD of equilibrium profits equals the SD of equilibrium revenues. As Figure 3(a) shows, for given market size ratio, the SD of equilibrium profits and revenues is quite insensitive to the order cost. However, as seen in Figure 3(b), the CV of equilibrium profits increases in the order cost, consistent with the fact that more expensive procurement reduces expected equilibrium profits.

**The impact of price competition on the value of downside volume flexibility.** Earlier studies on volume flexibility with demand uncertainty consider endogenous pricing under assumptions that lead to *quantity* (Cournot) competition. Our equilibrium results under stochastic price competition are novel to our knowledge, and they are of interest in their own right in that price competition is a natural model of price formation. They also form the basis for comparison with prior findings under quantity competition, which allows us to gain a deeper understanding of how the profit impact of volume flexibility depends on the mode of competition. Here we discuss the interplay between the mode of competition and the impact of downside volume flexibility.

In the *N* game firms have downside volume flexibility through the option to hold back supply and charge more than the clearance price under low demand. *Anupindi and Jiang (2008)* prove for perfectly *homogeneous* products that competition under downside volume flexibility through a hold back option yields higher expected profits than competition among inflexible firms. In their paper inflexible firms choose supply quantities in the first period, before demand is known, and sell all supply in the second period at the clearance price, i.e., they have no hold back option. Flexible firms also choose capacities ex ante, but sales quantities and prices are determined in the second stage under quantity competition (they show this game to have the same outcome if the quantity competition subgame is replaced by a two-stage production-price subgame). They find that the hold back option of the flexible firms induces them to build more capacity than inflexible firms and increases their expected (net) revenues more than their capacity cost.

To study how the value of a hold back option depends on the mode of competition, we compare our *N* game (which allows hold back under price competition) with two variations that differ in terms of the second stage, the *N* game with clearance, and the *N* game under quantity competition (with hold back option) which correspond to the models in *Anupindi and Jiang (2008)* of inflexible and flexible firms, respectively (they consider significantly more general demand uncertainty).

The equilibrium analysis of these two *N* game versions is considerably simpler than under price competition, and they each have an unique symmetric pure strategy equilibrium. We omit the technical details and informally summarize our findings from this comparison:

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3 Proofs of these statements are available upon request.
1. **Price competition reduces but does not eliminate the value of downside volume flexibility.** The \( N \) game under quantity competition yields (weakly) higher expected profits than the \( N \) game under price competition, and both yield higher profits than the \( N \) game with clearance.

2. **Price competition yields lower inventory investments than quantity competition under moderate demand variability:** there is an unique threshold \( \alpha^Q_H < \alpha^*_H < \alpha^{**}_H \), such that if \( \alpha_H \in (\alpha^Q_H, \alpha^{**}_H) \) then the pareto-dominant equilibrium order is \( x^N_i \) in the \( N \) game under price competition, and the equilibrium order is \( x^N_i > x^N_i \) in the \( N \) game under quantity competition.

That price competition yields lower expected profits than quantity competition is consistent with the classic result under deterministic demand. However, the second result runs counter to the deterministic case. It follows because firms can better control downside risk under quantity competition – they can hold back a larger share of a given supply, so they invest more upfront, sell more under high demand and less (by exercising their hold back option) under low demand.

The power of the hold back option rests on the flexible firms’ ability to commit to underutilizing capacity if demand is low. The extent of this hold back commitment in turn depends on the mode of competition in the second stage, following the capacity investments. A model where equilibrium sales quantities and prices are determined in the second stage under quantity competition, as in Anupindi and Jiang (2008), captures settings where firms are not sufficiently flexible to deviate from second-stage production quantities even if they have excess capacity. However, if firms can flexibly supply in the second stage any amount up to their initial capacity choice – due to preproduction or production postponement with highly flexible operations, then no quantity competition equilibrium with excess capacity is sustainable: it leaves each firm with the incentive to lower its price and increase sales. Price competition is more appropriate in such cases as no firm has an incentive to lower its price and increase sales at the corresponding equilibrium, even if it has the capacity to do so; in this sense price competition implies a stronger hold back commitment.

### 4. Price and Reorder Flexibility

In this section we characterize the initial order equilibria for the \( R \) game, i.e., under price and reorder flexibility. Firms simultaneously choose their orders in the first period by solving (5), i.e.,

\[
\max_{x_i \geq 0} \Pi^R_i (x) = \frac{1}{2} (\pi^{R*}_i (x; \alpha_L) + \pi^{R*}_i (x; \alpha_H)) - c x_i, \quad i = 1, 2.
\]

The conditional second period equilibrium profit functions \( \pi^{R*}_i (x; \alpha_L) \) and \( \pi^{R*}_i (x; \alpha_H) \) are specified in Lemma 1. For the same reasons as explained for the \( N \) game, each firm’s first period expected profit function may be bimodal in its own initial order. The equilibrium characterization for the \( R \) game is further complicated by the reorder option.
We henceforth assume that $\alpha_L = C(1 - \gamma)$, which implies that it is not profitable to reorder if demand is low in the second period. This assumption seems reasonable in that firms typically procure enough early on to cover at least what they consider to be their base demand. Relaxing this assumption makes the analysis more cumbersome without generating additional insights.

**Proposition 2. (First Period Order Equilibria: R Game).** Consider the “reorder” game with $\alpha_L = C(1 - \gamma) < \alpha_H$. There exists a symmetric initial order equilibrium $x^{R*}$. Under the Pareto-dominant symmetric equilibrium, the second period price-reorder strategies depend as follows on the demand and the procurement costs.

Call $r_\alpha := \alpha_H/\alpha_L$ the “market size ratio” and $r_c := c/C$ the “order cost ratio”. The revenue management quantity under low demand, and the order-up-to level under high demand, respectively, are

$$ \text{RM}(\alpha_L) = \frac{1 - \gamma}{2 - \gamma} \quad \text{and} \quad \text{OU}(\alpha_H) := \frac{\alpha_H - C(1 - \gamma)}{2 - \gamma}. $$

1. If the market size ratio $r_\alpha$ is below a threshold, in particular, $r_\alpha \leq m^{**}(\gamma)$, then firms price to clear inventory in both scenarios; they reorder up to $\text{OU}(\alpha_H)$ under high demand if and only if the order cost ratio $r_c$ exceeds a threshold, i.e., $r_c > r^c(r_\alpha, \gamma)$.

2. If the market size ratio $r_\alpha$ is in an intermediate range, in particular, $m^{**}(\gamma) < r_\alpha < 2$, the strategies depend on two cost thresholds that satisfy $0 < r^c(r_\alpha, \gamma) < r_c(r_\alpha, \gamma) < 1$:
   (a) Price to clear inventory under low demand if and only if the order cost ratio exceeds the smaller threshold, i.e., $r_c \geq r^c(r_\alpha, \gamma)$; else price to sell $\text{RM}(\alpha_L)$ and have leftover.
   (b) Reorder under high demand up to $\text{OU}(\alpha_H)$ if and only if the order cost ratio exceeds the larger threshold, i.e., $r_c > r_c(r_\alpha, \gamma)$.

3. If the market size ratio $r_\alpha$ exceeds a threshold, in particular, $r_\alpha \geq 2$, then the equilibrium strategies depend on a single cost threshold $r_c(r_\alpha, \gamma) \in (0, 1)$:
   (a) Price to clear inventory under low demand if and only if the order cost ratio exceeds the threshold, i.e., $r_c > r_c(r_\alpha, \gamma)$; else price to sell $\text{RM}(\alpha)$ and have leftover.
   (b) Reorder under high demand up to $\text{OU}(\alpha_H)$ if and only if the order cost ratio exceeds the threshold, i.e., $r_c > r_c(r_\alpha, \gamma)$.

The possible symmetric second-period price-reorder equilibrium strategies differ only in terms of the firms’ pricing under low demand, and whether they reorder under high demand. Table 1 highlights the three possible combinations of low demand pricing and high demand reordering that can occur under a strictly Pareto-dominant equilibrium, as characterized by Proposition 2. The

---

4 In other words, the order-up-to level $\text{OU}$ equals zero under low demand realization.
following reasons rule out other possibilities. For one, firms have no incentive to build inventory in excess of the high demand revenue management quantity $RM(\alpha_H)$, so they never price to have leftover inventory under high demand. Furthermore, firms do not reorder under low demand since doing so is unprofitable for $\alpha_L = C(1 - \gamma)$. Finally, under no strictly Pareto-dominant equilibrium do firms in the second period price to have leftover under low demand but reorder under high demand: these strategies follow only for initial orders in the range $(RM(\alpha_L), OU(\alpha_H))$, but firms weakly prefer ordering initially at most $RM(\alpha_L)$ if $c \geq C/2$, and at least $OU(\alpha_H)$ if $c \leq C/2$.

In particular, the firms’ second period equilibrium revenues are independent of their exact initial orders in the range $[RM(\alpha_L), OU(\alpha_H)]$; if demand is low, they price to sell $RM(\alpha_L)$ and have leftover inventory; if demand is high, they reorder up to and sell $OU(\alpha_H)$. The firms’ preferences over initial order quantities in this range therefore only depend on the relative cost of ordering early, at unit cost $c$, or later at unit cost $C$. Since low and high demand are equally likely, firms weakly prefer ordering initially at most $RM(\alpha_L)$ if $c \geq C/2$, and at least $OU(\alpha_H)$ if $c \leq C/2$.

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Reorder Game: Equilibrium First Period Order and Second Period Price-Reorder Strategies</th>
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<tbody>
<tr>
<td>Order $x^R$</td>
<td>Price ($L$ Demand)</td>
</tr>
<tr>
<td>$RM(\alpha_L) &lt; x^R$, $OU(\alpha_H) \leq x^R$</td>
<td>to have leftover</td>
</tr>
<tr>
<td>$OU(\alpha_H) \leq x^R &lt; RM(\alpha_L)$</td>
<td>to clear</td>
</tr>
<tr>
<td>$x^R \leq RM(\alpha_L)$, $x^R &lt; OU(\alpha_H)$</td>
<td>to clear</td>
</tr>
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</table>

Figure 4 illustrates Proposition 2. It shows, for $\gamma = 0.7$, how the conditions in Proposition 2 partition the parameter space of market size ratios $r_\alpha$ and order cost ratios $r_c$ into three regions, each corresponding to one of the three possible second-period price-reorder strategies highlighted in Table 1. For $\gamma = 0.7$, Part 1 of Proposition 2 applies for market size ratios $r_\alpha \leq 1.32$, Part 2 for $r_\alpha \in (1.32, 2)$ and Part 3 for $r_\alpha \geq 2$.

Figure 4  Proposition 2: Symmetric $R$ Game Equilibrium as a Function of Market Size Ratio and Order Cost Ratio ($\gamma = 0.7$)
Part 1. If the high and low demand markets are of sufficiently similar size (i.e., \( r_\alpha \leq 1.32 \)), firms never end up with excess inventory. Their initial orders are moderate, such that they prefer to clear their inventories even under low demand. The order cost ratio only affects whether the firms make use of reorder flexibility to delay part of their order; this is the case only if reordering is sufficiently cheap, i.e., the order cost ratio \( r_c \) exceeds the threshold \( \tau_c (r_\alpha, \gamma) \).

If the high and low demand market sizes differ more significantly (for \( r_\alpha > 1.32 \)), firms are willing to order more aggressively initially at the risk of overstocking under low demand – provided that initial ordering is cheap enough, i.e., the order cost ratio \( r_c \) is low enough.

Part 2. For moderately different market sizes (i.e., \( 1.32 < r_\alpha < 2 \)), the equilibrium strategies depend on two thresholds on the order cost ratio. If the order cost ratio \( r_c \) is below the lower threshold \( \tau_c (r_\alpha, \gamma) \), the firms initially order enough for high demand, but more than they wish to sell under low demand. If the order cost ratio \( r_c \) exceeds the larger threshold \( \tau_c (r_\alpha, \gamma) \), the firms initially order so little that they reorder under high demand, but they price to clear their inventories under low demand. If the order cost ratio \( r_c \) is in the intermediate range \( [\tau_c (r_\alpha, \gamma), \tau_c (r_\alpha, \gamma)] \), the firms’ initial orders are low enough so they price to clear inventories under low demand, yet large enough so they do not reorder under high demand.\(^5\)

Part 3. For sufficiently different market sizes (i.e., \( r_\alpha \geq 2 \)), firms do not match the demand with their initial order. If early ordering is relatively cheap, firms initially order more than they sell under low demand and do not reorder; otherwise, they initially order less than they need under high demand and do reorder if demand is high.

Equilibrium profit variability. Figure 5 shows contour plots of the standard deviation (SD) and the coefficient of variation (CV) of equilibrium profits, depending on the market size ratio \( r_\alpha \) and the order cost ratio \( r_c \). The low demand market size is fixed (\( \alpha_L = 0.3 \)), so an increase in the market size ratio increases both the mean and the CV of the market size. By Figure 5(a), for given market size ratio, the SD of equilibrium profits is fairly insensitive to procurement costs up to order cost ratios of about one half, at which point it drops and continues to decline in the order cost ratio. The parameter range where the SD of equilibrium profits decreases in the order cost ratio corresponds to cases where firms make use of reorder flexibility under high demand; see Figure 4. By Figure 5(b), for given market size ratio, the CV of equilibrium profits is typically largest at intermediate order cost ratios around one half; for smaller order cost ratios the CV is lower because cheaper initial procurement increases expected profits; for larger order cost ratios the CV is lower because

\(^5\) If \( r_\alpha < 2 \), the low demand revenue management quantity \( RM(\alpha_L) \) exceeds the high demand order-up-to level \( OU(\alpha_H) \).
5. The Impact of Reorder Flexibility on Orders and Profits

In this section we compare the equilibria under price flexibility only (N game) to those under price and reorder flexibility (R game). We call firms inflexible in the N game and flexible in the R game. First, we identify two conditions that determine whether the flexible firms initially order as much as, less than, or more than the inflexible firms in equilibrium. Second, we discuss the impact of reorder flexibility on equilibrium order quantities and on the mean and variability of equilibrium profits, depending on the market size ratio, the competition intensity, and the order cost ratio. Third, we illustrate our results with a numerical example.

**Two conditions determine the impact of reorder flexibility on initial orders.** The availability of a reorder option has two effects on the initial equilibrium orders of flexible firms, in comparison to the equilibrium order quantity $x^{N*}$ of inflexible firms. First, reorder flexibility softens the firms’ output constraints from their initial procurements, which may intensify their second period competition. Second, reorder flexibility allows firms to reduce overstocking risks in matching supply with demand. The first effect may give flexible firms an incentive to sell more than $x^{N*}$ in the second period if demand is high, and the magnitude of the second effect determines in such cases whether they *initially* order more or less than $x^{N*}$.

To make this discussion precise, consider first the N game equilibrium orders. The inflexible
firms hedge their bets between low and high demand, ordering up to the point where their expected marginal second period equilibrium revenue equals the initial unit procurement cost:

$$\frac{1}{2} \left( \frac{\partial \pi_i^N(x^N, \alpha_L)}{\partial x_i} + \frac{\partial \pi_i^H(x^H, \alpha_H)}{\partial x_i} \right) = c. \quad (9)$$

As a result, they order more than optimal for known low demand and less than optimal for known high demand. From (9), we have

$$c - \frac{\partial \pi_i^N(x^N, \alpha_L)}{\partial x_i} = \frac{\partial \pi_i^H(x^H, \alpha_H)}{\partial x_i} - c > 0, \quad (10)$$
i.e., the marginal profit loss if demand is low (the LHS) equals the marginal profit gain if it is high.

The flexible firms initially order more than $x^N$ units, if and only if two conditions hold:

(i) The flexible firms want to sell more than $x^N$ units under high demand. That is, $x^N$ is smaller than the high demand order-up-to level:

$$x^N < OU(\alpha_H). \quad (11)$$

This condition holds if and only if the marginal reorder cost $C$ is lower than a flexible firm’s high demand marginal revenue at $x^N$ if it unilaterally drops its price. This marginal benefit exceeds an inflexible firm’s high demand marginal revenue at $x^N$ if it unilaterally increases its inventory (the term $\frac{\partial \pi_i^N(x^H, \alpha_H)}{\partial x_i}$ in (9)).

(ii) Expected marginal understocking costs at $x^N$ exceed expected marginal overstocking costs. Ordering initially more than $x^N$ units, at a lower cost, realizes procurement cost savings only if condition (i) holds and demand turns out to be high; building more inventory initially results in lower prices and profits if demand turns out to be low. Therefore, flexible firms initially order more than $x^N$ if and only if (i) holds and the expected marginal cost saving from early procurement under high demand, exceeds the expected marginal profit loss at $x^N$ under low demand:  

$$C - c > c - \frac{\partial \pi_i^N(x^N, \alpha_L)}{\partial x_i}. \quad (12)$$

<table>
<thead>
<tr>
<th>Flexible Firms Order Initially</th>
<th>Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Same: $x^R = x^N$</td>
<td>$x^N \geq OU(\alpha_H)$</td>
</tr>
<tr>
<td>More: $x^R &gt; x^N$</td>
<td>$x^N &lt; OU(\alpha_H)$ and $C - c \geq c - \frac{\partial \pi_i^N(x^H, \alpha_H)}{\partial x_i}$</td>
</tr>
<tr>
<td>Less: $x^R &lt; x^N$</td>
<td>$x^N &lt; OU(\alpha_H)$ and $C - c \leq c - \frac{\partial \pi_i^N(x^H, \alpha_H)}{\partial x_i}$</td>
</tr>
</tbody>
</table>

If condition (i) is violated, then the flexible firms initially order the same amount as the inflexible firms, $x^N$, and do not reorder in the second period. If condition (i) holds but condition (ii) is violated, then the flexible firms initially order less than $x^N$, and their order-up-to level under high demand is larger than $x^N$. An important implication is that, whenever the equilibrium orders with

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6 Since low and high demand are equally likely, we omit the probabilities from this expression.
reorder flexibility differ from those without, the flexible firms end up with *more inventory under high demand* than inflexible firms with a single order. As we show below, this over-ordering hurts the flexible firms’ profits under high demand. Table 2 summarizes this discussion.

*Initial orders are never larger for a monopoly or under quantity competition.* To see how the mode of competition affects these ordering incentives, consider the conditions (i) – (ii) in the case of a monopoly firm \((\gamma = 0)\), or if second period prices and reorder quantities are determined under quantity competition. In both cases, flexible firms initially order as much as, or less than inflexible firms, because (i) and (ii) are mutually exclusive. Namely, if it is profitable under high demand to order more than the optimal no-reorder quantity \(x^{N*}\), then it is cheaper to delay doing so until the second period. Without competition and under quantity competition, (i) is equivalent to

\[
\frac{\partial \pi^i_{N*} (x^{N*}; \alpha_H)}{\partial x_i} > C,
\]

which, together with (10), implies that condition (ii) cannot hold:

\[
c - \frac{\partial \pi^i_{N*} (x^{N*}; \alpha_L)}{\partial x_i} = \frac{\partial \pi^i_{N*} (x^{N*}; \alpha_H)}{\partial x_i} - c > C - c,
\]

i.e., the marginal profit loss under low demand exceeds the cost savings from early procurement, so firms initially order less than inflexible firms.

This argument sheds light on the finding in *Lin and Parlaktürk (2012)* who consider a reorder option for duopoly retailers that sell a homogeneous product under quantity competition, in contrast to our \(R\) game under price competition. In their analysis competition between “fast” retailers that can reorder after learning demand yields (weakly) smaller initial orders and larger expected retailer profits, compared to competition between “slow” retailers without a reorder option. These results are in stark contrast to our findings: as we discuss next, the \(R\) game may yield larger initial orders and lower expected profits than the \(N\) game, and moreover, a reorder option cannot benefit firms if products are sufficiently close substitutes.

**The impact of reorder flexibility on equilibrium orders and profits.** Proposition 3 summarizes how reorder flexibility affects equilibrium order quantities and expected profits, depending on demand and procurement cost characteristics.

**Proposition 3. (Value of Reorder Flexibility).** Consider the expected profits and the order strategies under the Pareto-dominant symmetric equilibrium of the “no-reorder” game and of the “reorder” game with \(\alpha_L = C(1 - \gamma)\).

1. (Value of reorder flexibility). The firms with reorder flexibility can be more profitable than those without, if and only if:

   (a) the competition intensity is not too high, i.e., \(\gamma < 0.875\), and
(b) the market size ratio is in an intermediate range, i.e., $\underline{r}_\alpha(\gamma) < r_\alpha < \overline{r}_\alpha(\gamma)$, where $\overline{r}_\alpha(\gamma) < \infty$ for $\gamma > 0$.

If (a) – (b) hold, then the firms with the reorder option are more profitable if reordering is relatively inexpensive, i.e., $\underline{\gamma}(\gamma, r_\alpha) < r_c \leq 1$.

2. (Order strategies when reorder flexibility is valuable). Under the conditions of 1., the firms with reorder flexibility order less initially than those without, but under high demand they reorder and end up with more inventory than the inflexible firms, i.e., $x^{R*} < x^{N*} < OU(\alpha_H)$.

3. (Equal equilibrium outcomes). The firms with reorder flexibility order the same amount as those without, and they do not reorder, i.e., $x^{R*} = x^{N*} \geq OU(\alpha_H)$, if and only if:

(a) the market size ratio is below a threshold, i.e., $r_\alpha \leq \overline{r}_\alpha(\gamma)$, where $\overline{r}_\alpha(\gamma) < \infty$ for $\gamma > 0$,

(b) reordering is relatively expensive, i.e., $r_c \leq \overline{r}_c(\gamma, r_\alpha)$.

By Part 2. of Proposition 3, reorder flexibility always hurts profits if the flexible firms order more ex ante than inflexible firms; the additional inventory results in lower prices and higher procurement costs under low and high demand. However, reorder flexibility may also be detrimental to profits in cases where the flexible firms order less initially, compared to inflexible firms; the flexible firms are only better off under the additional conditions of Part 1. of Proposition 3. We first discuss how the orders of flexible and inflexible firms compare, depending on the order cost and market size ratios. We then discuss the conditions under which reorder flexibility improves firm profits. Throughout this discussion, we say “equilibrium” as shorthand for “Pareto-dominant symmetric equilibrium”.

Impact of reorder flexibility on equilibrium orders. Consider the impact of procurement costs on the flexible firms’ incentives to initially deviate from the $N$ game equilibrium order. An increase in the order cost ratio $r_c$ has two effects on the conditions (i) – (ii) discussed above.

1. It reduces the flexibility cost, which creates a stronger incentive for the flexible firms to sell more than $x^{N*}$ units if demand turns out to be high; i.e., condition (i) is likelier to hold.

2. It reduces the early procurement cost savings, and simultaneously increases the expected marginal overstocking costs, which makes it more attractive for the flexible firms to order less initially, and more later only under high demand; i.e., condition (ii) is likelier to be violated.

Figure 6 shows the impact of reorder flexibility on equilibrium order quantities. For competition intensity $\gamma = 0.7$, it partitions the parameter space of market size ratios $r_\alpha$ and order cost ratios $r_c$ into three regions, depending on whether in equilibrium the flexible firms initially order the same as $(x^{R*} = x^{N*})$, more than $(x^{R*} > x^{N*})$, or less than $(x^{R*} < x^{N*})$, the inflexible firms.\(^7\) These regions

\(^7\) A proof of the general partition structure is available from the authors.
correspond to the cases in Table 2. For example, for parameters in the region \(x^R > x^{N*}\), the flexible firms would want to reorder in the second period under high demand (i.e., \(x^{N*} < OU(\alpha_H)\) holds), but they prefer ordering these additional units early, because the procurement cost savings outweigh the overage cost under low demand (i.e., \(C - c \geq c - \partial \pi_i^{N*}(x^{N*};\alpha_L) / \partial x_i\) holds).

If the high demand market size is below a threshold\(^8\), i.e., \(r_\alpha \leq 5.51\) for \(\gamma = 0.7\), the flexible firms order the same as, more than, or less than inflexible firms, depending on whether the order cost ratio is low, intermediate, or high, respectively. A low order cost ratio implies a relatively high flexibility cost, so flexible and inflexible firms order the same amounts; see Part 3. of Proposition 3. For intermediate order cost ratio, the flexibility cost is such that flexible firms want to sell more than \(x^{N*}\) under high demand, but delaying this order is too costly, so they order more upfront. For sufficiently high order cost ratio, the flexibility cost and the early procurement cost savings are negligible, so that flexible firms initially order less than inflexible firms, and more later if needed.

If the high demand market is sufficiently large, i.e., \(r_\alpha > 5.51\) for \(\gamma = 0.7\), then regardless of the order cost ratio, the flexible firms’ initial orders differ from those of inflexible firms. In this case, the flexible firms have an incentive to increase their inventory level beyond \(x^{N*}\) under high demand, even if initial procurement is free \((c = 0)\). In other words, even if flexibility is very costly, a large high demand market fosters such intense price competition that flexible firms look to procure additional units. They procure them in the first period if the initial cost is below a threshold, so that early procurement savings are significant, and in the second period otherwise.

**Figure 6** Equilibrium Orders \((\gamma = 0.7)\)

**Figure 7** Value of Flexibility \((\gamma = 0.7)\)

Value of flexibility: impact of reorder flexibility on equilibrium profits. First consider expected equilibrium profits. By Part 2. of Proposition 3, in cases where firms benefit from reorder flexibility, they order less ex ante and more ex post under high demand, compared to inflexible firms. As noted

\(^8\) The threshold \(\bar{r}_\alpha(\gamma)\) in Part 3(b) of Theorem 3 equals 5.51 for \(\gamma = 0.7\).
above, this condition is only necessary; the flexible firms only gain higher expected profits under the additional conditions in Part 1. of Proposition 3: (i) competition is not too intense, (ii) the high demand market size is in some intermediate range, and (iii) reordering is relatively inexpensive. These conditions are the result of two countervailing profit effects of reorder flexibility:

1. **Downside protection under low demand.** The flexible firms are better off than the inflexible firms under low demand. Keeping their initial inventory low allows them to charge higher prices and eliminate leftover inventory, compared to the inflexible firms. This profit gain increases in the market size ratio $r_\alpha$ and in the order cost ratio $r_c$; the more disparate the market sizes and the more expensive initial procurement, the more valuable the reorder option. If the order cost ratio $r_c$ is sufficiently low, the flexible firms’ gains from downside protection under low demand are too small to offset any losses under high demand, regardless of other factors.

2. **Intensified competition under high demand.** The flexible firms are worse off than the inflexible firms under high demand. Since the reorder option is relatively inexpensive, it drives the flexible firms to over-order up to a larger inventory level and compete more aggressively, compared to the inflexible firms, so that they face larger procurement volumes and unit costs, and lower prices. This profit loss increases in the market size ratio $r_\alpha$ and in the competition intensity $\gamma$; both of these factors foster more intense competition. If the competition intensity parameter is sufficiently high, specifically, if $\gamma \geq 0.875$, the flexible firms’ losses under high demand exceed any gains under low demand, regardless of other factors.

These profit effects also explain why reorder flexibility benefits firms only if the high demand market is of moderate size. If the high and low demand markets are of similar size, reorder flexibility is of limited value as the benefit of downside protection is minimal. If the high demand market is very large, then the lure of high payoffs fosters such intense competition that reorder flexibility is again of limited value. Only in the middle range is the profit gain from downside protection under low demand more significant than the profit loss from intensified competition under high demand.

Figure 7 shows the impact of reorder flexibility on equilibrium expected profits. For competition intensity $\gamma = 0.7$, it partitions the parameter space of market size ratios $r_\alpha$ and order cost ratios $r_c$ into three regions, one where reorder flexibility has no profit effect (Part 3. of Proposition 3), one where it hurts expected profits, and one where it increases expected profits (Part 1. of Proposition 3).\(^9\) Compare with Figure 6 to see that this region is significantly smaller than the set of all $(r_\alpha, r_c)$-pairs where flexible firms order less ex ante than in the N game.

\(^9\) For some market size ratios, the set of order cost ratios for which flexibility has positive value is not an interval: e.g., for $r_\alpha = 2.8$, firms benefit from reorder flexibility if and only if $r_c \in (0.64, 0.67) \cup (0.72, 1)$. This property is likely due to the discrete nature of our demand model.
In addition to the impact of reorder flexibility on the mean equilibrium profits, its impact on equilibrium profit \textit{variability} may also be of importance in evaluating its performance. Figure 8 shows, for competition intensity $\gamma = 0.7$, contour plots of the difference between the coefficient of variation (CV) of equilibrium profits in the \textit{R} game and the CV of equilibrium profits in the \textit{N} game, as a function of the market size ratio $r_\alpha$ and the order cost ratio $r_c$. The parameter region labeled “+”, where reorder flexibility results in a higher CV of equilibrium profits, corresponds to cases where the flexible firms order initially more than inflexible firms (see Figure 6). Similarly, the region labeled “-”, where flexibility lowers the CV of equilibrium profits, corresponds to cases where the flexible firms order initially less than inflexible firms. This region comprises a significant range of cases where reorder flexibility reduces expected profits; see Figure 7.

\textbf{Figure 8} \hspace{1em} \textit{Impact of Reorder Flexibility on Equilibrium Profit CV ($\gamma = 0.7$)}

To conclude, reorder flexibility can benefit equilibrium profits in two ways. First, reorder flexibility reduces profit variability if the flexible firms initially order less than inflexible firms. Second, reorder flexibility also increases expected profits, if in addition, competition is not too intense, the market size ratio is moderate, and reordering is relatively inexpensive.

\textbf{Numerical example.} We illustrate the two profit effects of reorder flexibility, downside protection under low demand and intensified competition under high demand, with a numerical example. Fix the competition intensity $\gamma = 0.7$, the initial order cost $c = 0.8$, and the low demand market size $\alpha_L = C(1 - \gamma) = 0.3$. We discuss the equilibrium order quantities, prices and profits as the market size ratio $r_\alpha$ increases on $[1, 8.5]$, so that $\alpha_H$ varies from 0.3 to 2.55.

Figure 9(a) shows the equilibrium orders, $x^{N*}$ and $x^{R*}$, the revenue management quantity under low demand $RM(\alpha_L)$, and the order-up-to level under high demand $OU(\alpha_H)$, where $RM(\alpha_L) = \alpha_L/(2 - \gamma) = 0.23$ and $OU(\alpha_H) = (\alpha_H - C(1 - \gamma))/(2 - \gamma) = \alpha_H/1.3 - 0.23$ by (8). The flexible firms initially order the same as, or more than, the inflexible firms for $r_\alpha \leq 1.4$, and strictly less for
Beyond this level, the inflexible firms’ order quantity $x^{N^*}$ increases in the size of the high demand market, but the flexible firms’ initial order quantity stays constant at $x^{R^*} = 0.08$, because their initial orders in this range are independent of the high demand market size: they balance the marginal profit loss under low demand with the marginal savings of early procurement.

Figure 9(b) shows the equilibrium prices. Under low demand, the flexible firms price to clear their inventory, and since their initial orders are constant in $\alpha_H$, so are their prices and revenues. The inflexible firms price to clear their inventory only if the high demand market is below a threshold, i.e., $r_\alpha \leq 2.9$, and their prices decrease in this range since their initial order increases in $\alpha_H$. For $r_\alpha > 2.9$, the inflexible firms price to sell the revenue management quantity $RM(\alpha_L)$ at a constant price which is below cost; their low demand revenues are constant while their procurement costs and leftover inventories increase in $\alpha_H$. The flexible firms are better off than the inflexible firms under low demand, as shown in Figure 9(c), since they order less initially and charge higher prices. The flexible firms break even, while the inflexible firms incur losses that increase in the high demand market size, as they order more to prepare for the possibility of high demand.

Figure 9 Numerical Example: $\gamma = 0.7, c = 0.8$

Under high demand, both the flexible and the inflexible firms price to clear inventories, but the flexible firms price more aggressively since they reorder up to $OU(\alpha_H)$ and end up with more inventory than $x^{N^*}$. Since they also incur higher procurement costs, they have lower profits under high demand than the inflexible firms, as shown in Figure 9(c).

The expected profit of the flexible firms is higher for $r_\alpha \in (2.2, 5.0)$, as indicated by the vertical lines in Figure 9(c). For market size ratios in this range, the positive effect of downside protection under low demand dominates the detrimental effect of over-ordering under high demand, and vice versa for $r_\alpha$ outside this range. When $r_\alpha$ is low, the value of reorder flexibility for downside protection is insignificant since even inflexible firms price to clear their inventory under low demand.
The value of downside protection increases in the high demand market size, as inflexible firms are willing to incur the risk of overstocking under low demand. However, the detrimental effect of over-ordering also increases in the high demand market size. The flexible firms over-order and under-price by an increasing amount as $r_\alpha$ increases; see Figures 9(a)-9(b). For sufficiently large $r_\alpha$, this results in lower revenues for the flexible firms, compared to the inflexible firms. Furthermore, the flexible firms buy an increasing share of their total inventory at a high unit cost, since their initial inventory is constant in $r_\alpha$. These effects illustrate why reorder flexibility is only profitable under moderate demand variability.

6. Concluding Remarks

We study price and reorder flexibility under competition in two-period models of price-inventory control under demand uncertainty. This paper contributes technical and managerial insights that derive from its distinctive focus on three problem features: firms engage in price competition, sell differentiated products, and have both downside and upside volume flexibility. We (1) provide novel equilibrium results for finite-horizon price and inventory competition under demand uncertainty; (2) specify conditions for joint price and reorder flexibility to improve or hurt profits, compared to the case of price flexibility only; and (3) provide a deeper understanding of how the profit impact of volume flexibility depends on the mode of competition by contrasting our results with prior findings under quantity competition. We refer to Section 1 for a summary of these results.

Our results have several implications on the value of joint marketing and operations flexibility – contingent pricing and inventory replenishment flexibility. First, they demonstrate how the impact of joint reorder and price flexibility on decisions and profitability depends on intricate interaction effects between operational and demand characteristics. Firms need to be aware of these effects in order to improve performance. For example, firms cannot benefit from reorder flexibility unless they differentiate their products sufficiently from their competitors. Second, the detrimental effect of reorder flexibility through intensified price competition depends on factors that determine how flexibly firms can increase their supply. Limitations on upside volume flexibility, such as convex reordering costs or hard capacity constraints, can help mitigate its detrimental effect, but they also reduce the ability of firms to capitalize on demand surges. It is therefore of strategic importance for firms to determine levels of reactive capacity which appropriately balance competitive considerations with the upside and downside risks due to stochastic demand. Third, while the specific pricing and ordering prescriptions from our pre-season replenishment model do not directly transfer to settings with in-season replenishment, we believe that our main qualitative insights regarding the factors that determine the profit effects of reorder flexibility do extend to such settings.
We conclude by outlining some important avenues for future research that builds on our analysis and results. First, we focus on the procurement-pricing game for given symmetric flexibility configurations; in the tradition of the flexibility literature, it is of interest to also study the setting where only one firm has a reorder option, and to analyze the three-stage game where reorder flexibility selection precedes the resulting procurement-pricing game. Second, in our analysis firms make pricing and reordering decisions after obtaining a perfect demand signal. If firms receive a noisy demand signal, they face residual demand uncertainty and need to balance over- and understocking risks in their second period decisions. Third, we assume that firms sell substitutable products; it would be interesting to see how their decisions and profitability change under complementary products. Finally, studying the sensitivity of our results to different demand uncertainty models may yield additional insights (cf. Anupindi and Jiang 2008).

References


Appendix. Proofs.

Proof of Lemma 1. (R) We start with the R game. First, we characterize firm $i$'s best response for a given competitor’s price $p_{-i}$ and its own initial inventory level $x_i$. Second, we use this result to solve for the equilibrium prices for each strategy pair. Lastly, we use the equilibrium prices to identify the valid $(x_i, x_{-i})$ regions, i.e., the partition in Figure 2(b), corresponding to the strategy pairs. The results of the no-reorder game can be considered as a special case of the reorder game with $C$ is infinity.

First, from problem (4), we see that the marginal profit with respect to $p_i$ is

$$\frac{\partial \pi_i^R}{\partial p_i} = d_i(p; \alpha) + p_i \frac{\partial d_i}{\partial p_i} - C \cdot 1_{(d_i(p; \alpha) > x_i)} \frac{\partial d_i}{\partial p_i} = \alpha - 2p_i + \gamma p_{-i} + C \cdot 1_{(p_i < p_i^*(p_{-i}, x_i))},$$

where $p_i^*(p_{-i}, x_i) = \alpha + \gamma p_{-i} - x_i$ is the clearance price, and hence $\pi_i^R(p, x_i; \alpha)$ is concave in $p_i$. Setting the marginal profit to be zero yields the revenue management price $p_i^c(p_{-i}) = (\alpha + \gamma p_{-i})/2$ or the procurement price $p_i^c(p_{-i}) = (\alpha + \gamma p_{-i} + C)/2$.

Second, we solve for all possible equilibrium prices. Note that the best response price $p_i^{R*}(p_{-i}, x_i)$ can be expressed in a general form as $p_i^{R*}(p_{-i}, x_i) = m_i(k_i + \gamma p_{-i})$. The choices of $m_i$ and $k_i$ depend on which of the three potential best-response prices is the optimal one. For example, $m_i = 1$ and $k_i = \alpha - x_i$ if $p_i^{R*}(p_{-i}, x_i) = p_i^c(p_{-i}, x_i)$. Thus, the equilibrium prices can be solved from the following system of linear equations:

$$\begin{cases}
    p_i^{R*}(x_i) = m_i(k_i + \gamma p_i^R(x_i)) \\
    p_i^{R*}(x_{-i}) = m_{-i}(k_{-i} + \gamma p_i^c(x_i)) \iff \\
    p_i^{R*}(x_{-i}) = m_{-i}(k_{-i} + m_{-i}k_{-i} \gamma)/(1 - m_i m_{-i} \gamma^2).
\end{cases} \tag{13}$$

Table 3 lists the choices of $m_i, m_{-i}, k_i, k_{-i}$ for each strategy pair. Substituting the appropriate coefficients back into system (13), we can obtain all possible equilibrium prices.

<table>
<thead>
<tr>
<th>Table 3</th>
<th>Coefficients of Best-Response Prices</th>
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<tr>
<td>$R(p, p)$</td>
<td>$R(p, c)$</td>
</tr>
<tr>
<td>$m_i$</td>
<td>$1/2$</td>
</tr>
<tr>
<td>$m_{-i}$</td>
<td>$1/2$</td>
</tr>
<tr>
<td>$k_i$</td>
<td>$\alpha + C$</td>
</tr>
<tr>
<td>$k_{-i}$</td>
<td>$\alpha + C$</td>
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Lastly, we identify the region boundaries in the initial inventory space for each specific strategy pair to arise as an equilibrium. Note that a specific price pair is indeed in equilibrium if and only if the initial inventory levels are in a certain region as illustrated in Figure 2(b). For any fixed competitor’s initial inventory level $x_{-i}$, a horizontal line with intercept $x_{-i}$ intersects with the region boundaries at two points, which serve as the desired thresholds claimed in the stipulation. Using the prices we have from system (13), we can solve for the valid region boundaries. To illustrate, we work with equilibrium strategy pair $(p, p)$ as an example. For this particular strategy pair to be in equilibrium, we require $x_i < (\alpha + \gamma p_i^{R*}(x_{-i}) - C)/2$ and $x_{-i} < (\alpha + \gamma p_i^{R*}(x_i) - C)/2$. 


Since \( p_{i}^{R}(x_{i}) \) and \( p_{-i}^{R}(x_{-i}) \) are available from Table 3, we have that \((p,p)\) arises as an equilibrium strategy pair if and only if \( x_{i} < OU = (\alpha - C + \gamma C)/(2 - \gamma) \). In a similar way, we can solve the boundaries of \( R(r,p) \), \( R(r,r) \), and \( R(p,r) \) as illustrated in Figure 2(b). This gives us other three vertices of the diamond area \( R(c,c) \) as illustrated in Figure 2(b): starting from the bottom-right vertex, their coordinates, in the counterclockwise direction, are \( \left( \frac{\alpha(2+\gamma)+C}{4-\gamma}, \frac{\alpha(2+\gamma)+(\gamma^2-2)C}{4-\gamma} \right), \left( \frac{\alpha}{2-\gamma}, \frac{\alpha}{2-\gamma} \right) \), and \( \left( \frac{\alpha(2+\gamma)+(\gamma^2-2)C}{4-\gamma}, \frac{\alpha(2+\gamma)+\gamma C}{4-\gamma} \right) \). One can also easily verify that lines connecting every adjacent vertices of the diamond \( R(c,c) \) give other boundaries as well. For records, these are \( g(x_{i},x_{-i},1) = 0 \), \( g(x_{i},x_{-i},0) = 0 \), \( g(x_{-i},x_{i},1) = 0 \), and \( g(x_{-i},x_{i},0) = 0 \), where \( g(x,y,k) = (2 - \gamma^2)x + \gamma y - (1 + \gamma)(\alpha - k(1 - \gamma)C) \). Moreover, \( g(x_{i},x_{-i},1) \) intersects with \( g(x_{-i},x_{i},1) \) at \( OU \) and \( g(x_{i},x_{-i},0) \) intersects with \( g(x_{-i},x_{i},0) \) at \( RM \).

(N) In the \( N \) game, the potential best-response prices are left with either the revenue management price \( p_{i}^{R}(p_{-i}) = (\alpha + \gamma p_{-i})/2 \) or the clearance price \( p_{i}^{F}(p_{-i},x_{i}) \). The equilibrium region boundaries can be similarly determined as in the \( R \) game. Equivalently, one can view the region partition generated in the \( N \) game as setting \( C \) very large in the \( R \) game (i.e., the reorder option is too expensive) when the point \((OU,OU)\) goes below the origin and into the third quadrant. □

**Proof of Proposition 1.** The general idea of the equilibrium proof is straightforward: we identify best-response functions and the equilibrium is where the best response functions intersect with each other. To execute this idea, we partition the initial inventory space \( \{x \geq 0\} \) into nine regions dependent on ex-post equilibrium strategies in the second period. We use the ex-post equilibrium strategies to label the region. For instance, the region \( N(c_{L}H_{y}c_{L}H) \) consists of all initial inventory vectors \( x \) for which the unique price equilibrium in both the low and high demand scenario is for both firms to charge the clearance price. We follow the same notation convention for other regions, where the first and second components refer to ex-post equilibrium strategies in the second period for firm \( i \) and \(-i\) respectively, and the first and the second letter of each component represent the firm’s equilibrium strategy in low and high demand scenarios, respectively. It is easy to see that an equilibrium cannot be in region \( N(r,r) \) in a deterministic market size, hence when we have two-point distributed demand uncertainty, an equilibrium cannot be in region \( N(rr,rr) \). Therefore a symmetric equilibrium can only be in region \( N(cc,cc) \) or \( N(rc,rc) \). We solve for an equilibrium candidate by concatenating the two first order conditions (FOCs) of each firm’s profit maximization problem with the profit function corresponding to \( N(cc,cc) \) or \( N(rc,rc) \). Then we identify conditions under which the candidate is indeed in the intended region and it is indeed an equilibrium.

First, we consider region \( N(cc,cc) \). The expected profit function of firm \( i \) with \( x \in N(cc,cc) \) is \( \Pi_{i}^{N}(x) = \left( \frac{\pi_{1}}{1+\gamma} - \frac{1}{1+\gamma}x_{i} - \frac{\gamma}{1+\gamma}x_{-i} \right) x_{i} - c_{x_{i}} \), where \( \pi = (\alpha_{L} + \alpha_{H})/2 \). For any fixed \( x_{-i} \in \left[ 0, \frac{\alpha_{H}}{2-\gamma} \right] \), we solve for a line from the FOC of firm \( i \)'s profit maximization problem: \( x_{i} = \frac{1}{2} ( (1+\gamma)(\pi - c(1-\gamma)) - \gamma x_{-i} ) \). Since the game is symmetric, these two lines, \( i = 1,2 \), solved from FOCs intersect on the diagonal suggesting a symmetric
equilibrium candidate \( x_i^{N*} = \left( \frac{1 + \sqrt{1 - c(1 - \gamma)}}{\gamma + 2 - c(1 - \gamma)} \right) \geq 0 \), where the inequality is due to \( \alpha - c(1 - \gamma) \geq 0 \). To ensure that the equilibrium candidate is indeed in the region \( N(cc, cc) \), we require \( \alpha_H \geq \frac{\gamma^2 + 2 + 2}{(1 + \gamma)(2 - \gamma)} \alpha_L + 2 =: B_1 \alpha_L + 2 \), where \( \alpha_L = \frac{\alpha}{c(1 - \gamma)} \) and \( \alpha_H = \frac{\alpha}{c(1 - \gamma)} \). Next, we need to characterize the conditions under which \( x_i = x_i^{N*} \) is a best-response in maximizing \( \Pi_i^N(x) \) among \( x_i \geq 0 \) while fixing \( x_{-i} = x_i^{N*} \). It is clear that \( x_i = x_i^{N*} \) is the expected profit maximizer for any \( (x_i, x_{-i} = x_i^{N*}) \in N(cc, cc) \). Moreover, \( \Pi_i^N(x) \) with \( x \in N(rr, cc) \) is decreasing in \( x_i \) for any fixed \( x_{-i} \). Thus, only the local maximizer of \( \Pi_i^N(x) \) with \( x = (x_i, x_{-i} = x_i^{N*}) \in N(rc, cc) \), which is quadratic in \( x_i \), could be a possible best response. It can be shown that if this local maximizer in \( N(rc, cc) \) is on the the boundary, i.e., \( \alpha_H < \frac{8 + \gamma \sqrt{\frac{3}{\gamma}}}{5 + 4 \sqrt{\frac{3}{\gamma}}} \alpha_L + 2 := m^*(\gamma) \alpha_L + 2 \), it must reside on the boundary between \( N(rc, cc) \) and \( N(cc, cc) \). By the continuity of profit over the boundary, such \( x \) that \( x_i \in N(rc, cc) \cap N(cc, cc) \) and \( x_{-i} = x_i^{N*} \) is dominated by \( x_i = x_i^{N*} \) that is the local maximizer over \( N(cc, cc) \). If the local maximizer of \( \Pi_i^N(x) \) with \( x = (x_i, x_{-i} = x_i^{N*}) \in N(rc, cc) \) is an interior point of \( N(rc, cc) \), i.e., \( \alpha_H \geq m^*(\gamma) \alpha_L + 2 \), then \( x_i = x_{-i} = x_i^{N*} \) yields equal or more profit than the local maximizer in \( N(rc, cc) \) if and only if \( K_1 \alpha_L + 2 \leq \alpha_H \leq K_2 \alpha_L + 2 \), where \( K_1 := \frac{\sqrt{2} - 4 + (2\sqrt{2} - 4)\sqrt{2}}{(\sqrt{2} - 1)3^3 + 2(2\sqrt{2} - 4)\sqrt{2}} \) and \( K_2 := \frac{\sqrt{2} - 4 + (2\sqrt{2} - 4)\sqrt{2}}{(\sqrt{2} - 1)3^3 + 2(2\sqrt{2} - 4)\sqrt{2}} \). We verify that \( K_1 \leq m^*(\gamma) \leq K_2 \leq B_1 \) for any given \( \gamma \in [0, 1] \), then we have \( x_i = x_i^{N*} \) is indeed a best response in profit maximization with \( x_{-i} = x_i^{N*} \) if and only if \( \alpha_H \leq K_2 \alpha_L + 2 \). Thus \( x_i = x_{-i} = x_i^{N*} \) is a symmetric equilibrium if and only if \( \alpha_H \leq K_2 \alpha_L + 2 \).

Second, we consider region \( N(rc, rc) \). For any fixed \( x_{-i} \geq \frac{2lc}{2 + \gamma} \), firm \( i \) solves the FOC of the expected profit function in \( N(rc, rc) \), which corresponds to a best-response line \( x_i = \frac{1}{2} (\alpha_H (1 + \gamma) - \gamma x_{-i}) - c(1 - \gamma^2) \). This line intersects on the diagonal suggesting a symmetric equilibrium candidate \( x_i = x_{-i} = x_i^{N*} = \alpha_H \frac{1 + \gamma}{2 + \gamma} - 2C \frac{1 - \gamma^2}{2 + \gamma} \), which is equal to or larger than \( RM(\alpha_L) \) if and only if \( \alpha_H \geq B_2 \alpha_L + 2 \), where \( B_2 := \frac{2 + \gamma}{2} \frac{1 + \gamma}{1 + \gamma} \). Next, we identify the conditions under which \( x_i = x_i^{N*} \) is a best response of firm \( i \)'s profit maximization with \( x_{-i} = x_i^{N*} \). Similar to the case in \( N(cc, cc) \), there may exist two local maximizers in \( N(cc, cc) \) and \( N(rc, rc) \) respectively. Following similar procedures, we prove that: (i) \( x_i = x_i^{N*} \) yields equal or more profit than the local maximizer in \( N(cc, cc) \) if and only if \( \alpha_H \geq T_1 \alpha_L + 2 \), where \( T_1 := \frac{2 + \gamma}{2} \frac{1 + \gamma}{1 + \gamma} \left( 2 + \gamma + \frac{2 + \gamma}{2 + \gamma} \sqrt{\frac{1 - \gamma^2}{2 + \gamma}} \right) \) and (ii) \( x_i = x_i^{N*} \) yields equal or more profit than the local maximizer in \( N(rc, cc) \) if and only if \( \alpha_H \geq T_2 \alpha_L + 2 \), where \( T_2 := (2 + \gamma)(1 + \gamma)(2 - \gamma)^{3/2} \frac{1 + \gamma}{(1 + \gamma)(1 - \gamma^2 + 4 + \gamma)} \left( 1 + \gamma \right) \). Moreover, we verify that \( B_2 \leq \max(T_1, T_2) \) for any given \( \gamma \in [0, 1] \) (the order of \( T_1 \) and \( T_2 \) depends on \( \gamma \)). Hence, it is implied that \( x_i = x_i^{N*} \) is a best response of \( \Pi_i^N(x) \) among \( x_i \geq 0 \) with \( x_{-i} = x_i^{N*} \) if and only if \( \alpha_H \geq \max(T_1, T_2) \alpha_L + 2 \). Thus, \( x_i = x_{-i} = x_i^{N*} \) is a symmetric equilibrium if and only if \( \alpha_H \leq \max(T_1, T_2) \alpha_L + 2 \).

Let \( m^*(\gamma) = K_2 \) and \( m^*(\gamma) = \max(T_1, T_2) \). We verify that \( m^*(\gamma) > m^*(\gamma) > 0 \) for any given \( \gamma \in (0, 1) \). Then we have the desired results on the symmetric equilibria. Lastly, we eliminate the existence of asymmetric equilibria. We first consider the case that \( x_i = x_i^{N*} \) is not a best response to \( x_{-i} = x_i^{N*} \). Then, the local maximizer of \( \Pi_i^N(x) \) in \( N(rc, cc) \) must be the best response, which requires \( \alpha_H > K_2 \alpha_L + 2 \). To have an
asymmetric equilibrium in region $N(rc, cc)$, it also requires that the best response mapping of firm $-i$ is in $N(rc, cc)$ for $\hat{\alpha}_H > K_2 \hat{\alpha}_L + 2$. By symmetry, this is equivalent to that the best response mapping for firm $i$ is in $N(cc, rc)$ if $\hat{\alpha}_H > K_2 \hat{\alpha}_L + 2$, which contradicts with the fact that $x_i = x_{-i} = x_{h}^{N*} \in N(rc, rc)$ is a symmetric equilibrium if $\hat{\alpha}_H > K_2 \hat{\alpha}_L + 2$. Similar arguments apply when $\hat{\alpha}_H \leq \max \{K_4, K_5\}$ and we can eliminate all the possibilities of asymmetric equilibria. □

Proof of Corollary 1. (i). That the order quantity increases in $\alpha_H$ is immediate from (6)-(7) in Proposition 1. To show that the expected profit increases in $\alpha_H$, let $p_i(x; \alpha)$ denote firm $i$’s inverse demand function corresponding to the demand functions $d_i(p; \alpha)$ in (1). For any symmetric quantities $x_1 = x_2 = x$, the corresponding prices equal $p_i(x; \alpha) = \frac{4 - c}{\gamma}$. Let $\bar{\alpha} = (\alpha_H + \alpha_L)/2$ denote the expected market size.

Under the equilibrium quantity $x_i^{N*}$, the expected profit satisfies

$$x_i^{N*} p_i(x^{N*}; \alpha_H) + p_i(x^{N*}; \alpha_H) - cx^{N*} = x_i^{N*} \left( \frac{\bar{\alpha} - x_i^{N*}}{1 - \gamma} - c \right) = x_i^{N*} \frac{\bar{\alpha} - c (1 - \gamma)}{(2 + \gamma)(1 - \gamma)}.$$

Both the quantity and the margin are increasing in $\alpha_H$.

Under the equilibrium quantity $x_h^{N*}$, the revenue under low demand is independent of $\alpha_H$: firms price to sell the revenue management quantity $RM(\alpha_L)$ which does not depend on $\alpha_H$. The expected profit therefore depends on how $\alpha_H$ affects half the high demand revenue, minus the procurement cost; this measure equals

$$\frac{1}{2} x_h^{N*} \frac{\alpha_H - x_h^{N*}}{1 - \gamma} - cx_h^{N*} = \frac{1}{2} x_h^{N*} \frac{\alpha_H - 2c (1 - \gamma)}{(2 + \gamma)(1 - \gamma)},$$

which increases in $\alpha_H$.

(ii). The equilibrium profit under low demand realization decreases in $\alpha_H$. This is immediate for the equilibrium quantity $x_i^{N*}$: in this case the second period equilibrium revenue under low demand is constant in $\alpha_H$ as noted above, whereas the order quantity and the procurement cost increase in $\alpha_H$. For the equilibrium quantity $x_h^{N*}$, the following simple argument implies that the equilibrium profit under low demand realization decreases in $\alpha_H$. Suppose that the market size is known to be $\alpha_L$ and a monopoly manages the system. Then the monopoly quantities, which are symmetric for both products, maximize industry profits under known market size $\alpha_L$. Furthermore, beyond the monopoly levels, industry profits decrease in the order quantity. The argument is complete by noting that $x_i^{N*}$ increases in $\alpha_H$ and is larger than the $N$ game equilibrium quantity for known market size $\alpha_L$, which in turn is larger than the monopoly quantity.

(iii). That the profit under high demand increases in $\alpha_H$ is immediate from parts (ii)-(iii). □

Proof of Proposition 2. We call a point on the diagonal a symmetric equilibrium candidate if it is a local maximizer of problem $\max_{x_i \geq 0} \Pi_i^R(x)$. We solve for the symmetric equilibrium candidate and then identify conditions under which it is indeed an equilibrium.

First, we consider $r_a \geq 2$, i.e., $\alpha_H \geq 2 \alpha_L$. Since $\alpha_L = C(1 - \gamma)$, then $0 = OU(\alpha_L) < RM(\alpha_L) \leq OU(\alpha_H) < RM(\alpha_H)$. Depending on the relative position of a symmetric point with respect to the RM and OU points
of the low and high demand scenario which determines ex-post equilibrium strategies in the second period, by Table 3, the marginal value of initial inventory level is

\[
\frac{\partial \Pi^R_i(x)}{\partial x_i} \mid_{x_i=x_{-i}} = \begin{cases} 
\frac{1}{2} \left( \frac{\alpha}{1-\gamma} - x_i + \frac{2+\gamma}{1-\gamma} \right) + \frac{C}{2} - c & \text{if } x_i = x_{-i} \in [0, \alpha_L], \\
\frac{C}{2} - c & \text{if } x_i = x_{-i} \in (\alpha_L, \alpha_H), \\
\frac{1}{2} \left( \frac{\alpha_H}{1-\gamma} - x_i + \frac{2+\gamma}{1-\gamma} \right) - c & \text{if } x_i = x_{-i} \in (\alpha_H, \alpha_L).
\end{cases}
\]

Hence we have the following symmetric equilibrium candidates.

1. For \( x_i = x_{-i} \in [0, \alpha_L] \), setting the derivative to zero yields \( x^R_i := \frac{2(1-\gamma)}{2+\gamma} (C - r_c) \). Note that \( x^R_i \in [0, \alpha_L] \) is equivalent to \( r_c \in [r_{c_2}, 1] \), where \( r_{c_2} := \frac{2^2 - \gamma}{2(1+\gamma)(1-\gamma)} \).

2. For \( x_i = x_{-i} \in (\alpha_L, \alpha_H) \), setting the derivative to zero yields \( x^R_i := \frac{1-\gamma}{2+\gamma} (r_\alpha - 2r_c) \). Note that \( x^R_i > \alpha_H \) is equivalent to \( r_c < r_{c_1} := -\frac{2^{2+\gamma}}{2(1+\gamma)(1-\gamma)} r_\alpha + \frac{2+\gamma}{2(1+\gamma)(1-\gamma)} \). Recall that the noreorder equilibrium is always smaller than \( \alpha_H \) and the profit functions are the same for points along the diagonal between \( \alpha_H \) and \( \alpha_L \) for both \( R \) and \( N \) games; thus it is implied that \( x^R_i < \alpha_L \).

3. For \( x_i = x_{-i} \in (\alpha_L, \alpha_H) \), the derivative of firm \( i \)'s expected profit is constant. (i) If \( c \neq C/2 \), then there cannot exist a symmetric equilibrium in \( (\alpha_L, \alpha_H) \) because an equilibrium has to be a local maximizer. (ii) If \( c = C/2 \), then any point in \( (\alpha_L, \alpha_H) \) is an equilibrium candidate. However, if \( c = C/2 \), the point \( x_i = x_{-i} = \alpha_L \) cannot be an equilibrium because \( r_{c_2} < 1/2 = r_c \) which implies that \( x^R_i \) is a local maximum but \( \alpha_L \) is not.

4. For the point \( x_i = x_{-i} = \alpha_H \), it is a symmetric equilibrium candidate if and only if \( r_c \geq r_{c_1} \) and \( c \leq C/2 \). For \( x_i = \alpha_H \), \( \Pi^R(x) \) is not differentiable at \( x_i = \alpha_H \) and it is a local maximizer if its left derivative is \( C/2 - c \geq 0 \) and its right derivative is \( \frac{1}{2} \left( \frac{\alpha}{1-\gamma} - x_i + \frac{2+\gamma}{1-\gamma} \right) - c \), which is nonpositive if and only if \( r_c \geq r_{c_1} \).

Second, we consider \( 1 < r_c < 2 \), i.e., \( \alpha_L \leq \alpha_H < 2\alpha_L \). Then \( 0 = \alpha_L < \alpha_H < \alpha_L < \alpha_H \). Following the same procedure, we have four equilibrium candidates as summarized in the following table.

<table>
<thead>
<tr>
<th>Equilibrium Candidate (1 ( \leq r_c \leq 2 ))</th>
<th>Cost Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x^R_h := \frac{1-\gamma}{2+\gamma} C(r_c - 2r_c) &gt; \alpha_L )</td>
<td>( 0 \leq r_c \leq r_{c_4} := \frac{1}{2} r_\alpha - \frac{1}{2} m^*(\gamma) )</td>
</tr>
<tr>
<td>( x^R_p := \frac{1-\gamma}{2(2+\gamma)} C(r_c - 2r_c + 1) \in (\alpha_H, \alpha_L) )</td>
<td>( 0 \leq r_c \leq r_{c_5} := -\frac{\gamma^2 + 4\gamma + 2}{2(2+\gamma)(1+\gamma)} r_\alpha + \frac{\gamma^2 + 4\gamma + 6}{2(2+\gamma)(1+\gamma)} )</td>
</tr>
<tr>
<td>( x^R_o := \frac{1-\gamma}{2-\gamma} C(r_c - 1) = \alpha_H )</td>
<td>( r_{c_5} &lt; r_c &lt; r_{c_6} := -\frac{2^{2+\gamma}}{2(2-\gamma)(1+\gamma)} r_\alpha + \frac{2^{2+\gamma} + 6}{2(2-\gamma)(1+\gamma)} )</td>
</tr>
<tr>
<td>( x^R_l := \frac{2(1-\gamma)}{2+\gamma} C(1 - r_c) \leq \alpha_H )</td>
<td>( r_{c_6} \leq r_c \leq 1 )</td>
</tr>
</tbody>
</table>

In summary, the parameter space \( \{(r_\alpha, r_c) \mid r_\alpha > 1, 0 < r_c < 1\} \) is partitioned and summarized as follows.

We verify that in each subset of this partition the equilibrium candidate is indeed an equilibrium from first principles by means of a computer program (submitted as a supplementary document). Note that in view of the rather involved analytic equilibrium verification for the \( N \) game (see the Proof of Proposition...
3, e.g., the comparison of thresholds \( K_1 \) and \( K_2 \), for the considerably more complicated \( R \) game, such analytic verification would be extremely cumbersome because it literally amounts to dozens of comparisons of thresholds as a function of \( \gamma \). Numeric verification in this case is rigorous in the sense that the profit functions are continuous in the parameters and within each subset of the parameter space partition, if a symmetric equilibrium candidate is not an equilibrium, such a case will be detected by the computer program because it has enough granularity.

1. Consider \( r_\alpha \leq 2 \). For any \( r_c \in (\max\{0, r_{c_4}\}, r_{c_5}) \), there exists a unique Pareto-dominant symmetric equilibrium that is in the form of \( x_p^{R*} \) and for any \( r_c \in [r_{c_5}, r_{c_6}] \), there exists a unique Pareto-dominant symmetric equilibrium that is \( OU(\alpha_H) \). In both cases, firms price to clear inventory in both low and high scenarios. For any \( r_c \in (r_{c_5}, 1) \), there exists a unique Pareto-dominant symmetric equilibrium that is in the form of \( x_l^{R*} \), where firms reorder up to \( OU(\alpha_H) \) under high demand and price to clear inventory under low demand. Hence, let \( r_c(r_\alpha, \gamma) = r_{c_6} \), which is the desired threshold.

2. Consider \( m^*(\gamma) < r_\alpha < 2 \), where \( m^*(\gamma) \) is the intercept of line \( r_c = r_{c_4} \) with \( r_\alpha \)-axis. For any \( r_c \in (0, r_{c_4}) \), there exists a unique Pareto-dominant symmetric equilibrium that is in the form of \( x_h^{R*} \), where firms price to sell \( RM(\alpha_L) \) and have leftover under low demand and price to clear inventory under high demand. Hence, let \( r_c(r_\alpha, \gamma) = r_{c_4} \), which is the desired threshold.

3. Consider \( r_\alpha \geq 2 \). There exists \( r_c(r_\alpha, \gamma) \in (0, 1) \) such that for any \( r_c \in [r_{c_2}, r_c(r_\alpha, \gamma)) \), \( OU(\alpha_H) \) is the unique Pareto-dominant symmetric equilibrium and for any \( r_c \in (0, r_{c_1}) \), \( x_h^{R*} \) is the unique Pareto-dominant symmetric equilibrium. In both cases, firms price to sell \( RM(\alpha_L) \) and have leftover under low demand and price to clear inventory under high demand. For any \( r_c \in (r_\alpha(r_\alpha, \gamma), 1) \), there exists a unique Pareto-dominant symmetric equilibrium that is in the form of \( x_l^{R*} \), where firms reorder up to \( OU(\alpha_H) \) under high demand and price to clear inventory under low demand. □

**Proof of Proposition 3.** First, we focus on the case where \( r_\alpha \geq 2 \) and then extend the results to the case where \( 1 \leq r_\alpha < 2 \).

<table>
<thead>
<tr>
<th>Table 4</th>
<th>Equilibrium Pairs</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r_\alpha \geq 2 )</td>
<td>( r_c \in [0, r_{c_1}) )</td>
</tr>
<tr>
<td>( r_c \geq \frac{r_\alpha - m^*(\gamma)}{2} )</td>
<td>( x_h^{R*}, x_h^{R*} )</td>
</tr>
<tr>
<td>( r_c &lt; \frac{r_\alpha}{2} )</td>
<td>( x_h^{R*}, x_h^{R*} )</td>
</tr>
<tr>
<td>( 1 \leq r_\alpha &lt; 2 )</td>
<td>( r_c \in [0, r_{c_4}) )</td>
</tr>
<tr>
<td>( r_c \leq \frac{r_\alpha}{2} )</td>
<td>( x_h^{R*}, x_h^{R*} )</td>
</tr>
<tr>
<td>( r_c \in (r_{c_5}, 1) )</td>
<td>( x_h^{R*}, x_h^{R*} )</td>
</tr>
</tbody>
</table>

Table 4 shows that there are 6 possible no-reorder/reorder equilibrium combinations for any given \( r_c \) and \( r_\alpha \) such that \( r_\alpha \geq 2 \). We need to quantify the expected profit differences for all equilibrium combinations. By Proposition 1, we know that there is at least one symmetric equilibrium in the \( N \) game for a given parameter pair \((r_\alpha, r_c)\). If \( \alpha_L = C(1 - \gamma) \) and \( \alpha_H \geq 2\alpha_L \), we assume that both firms adopt the Pareto-dominant
\( x_i^{N*} \) over \( x_h^{N*} \) when two symmetric equilibria exist. Propositions 1 and 2 give us the initial equilibrium ordering quantities. Referring to Table 3, we can also calculate the corresponding equilibrium prices. Thus, the expected equilibrium profit can be obtained. Table 5 lists the conditional revenue and cost in low and high scenarios for all equilibria.

**Table 5  Revenue and Cost at Equilibrium for \( \alpha_H \geq 2\alpha_L \)**

<table>
<thead>
<tr>
<th>Equilibrium</th>
<th>Conditional on the Low Scenario</th>
<th>Conditional on the High Scenario</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_i^{R*} )</td>
<td>( \frac{\alpha L}{1 - \gamma} - \frac{1}{2} x_i^{R*} )</td>
<td>( c x_i^{R*} )</td>
</tr>
<tr>
<td>( x_i^{O*} )</td>
<td>( \frac{\alpha H + C}{2 - \gamma} )</td>
<td>( c x_i^{R*} )</td>
</tr>
<tr>
<td>( x_h^{N*} )</td>
<td>( \frac{\alpha H}{1 - \gamma} - \frac{1}{2} x_h^{N*} )</td>
<td>( c x_h^{N*} )</td>
</tr>
<tr>
<td>( x_h^{O*} )</td>
<td>( \frac{\alpha L}{1 - \gamma} - \frac{1}{2} x_h^{O*} )</td>
<td>( c x_h^{O*} )</td>
</tr>
</tbody>
</table>

We compare the expected profits of each equilibrium combination as follows.

(i) \( \{ (r_o, r_c) \mid r_c < r_o / 2 - m^{**}(\gamma) / 2 \) and \( r_{o2} \leq r_c \leq 1 \}. In this case, firms choose \( x_h^{N*} \) in the \( N \) game and \( x_i^{R*} \) in the \( R \) game. By Table 5, we have \( x_i^{R*} \) generates more expected profit than \( x_i^{N*} \) if and only if \( (r_o, r_c) \in F_1 := \{ (r_o, r_c) \mid r_o > 2 \text{ and } \frac{\alpha L}{(1 + 2(\gamma) - \gamma)} r_o + \frac{1}{2} < r_c < r_o / 2 - m^{**}(\gamma) / 2 \leq 1 \} \). The set of \( F_1 \) is nonempty if and only if \( 2 + m^{**}(\gamma) \geq \frac{\gamma(-1)(2 + \gamma)}{\gamma} \), i.e., \( \gamma \leq 0.875 \). Note that for a nonempty \( F_1 \), we have \( r_c > r_{c3} \), which implies that \( x_i^{R*} \) is the unique symmetric equilibrium candidate for the \( R \) game.

(ii) \( \{ (r_o, r_c) \mid r_c \geq r_o / 2 - m^{**}(\gamma) / 2 \) and \( r_{o2} \leq r_c \leq 1 \}. In this case, firms choose \( x_i^{N*} \) in the \( N \) game and \( x_i^{R*} \) in the \( R \) game. By Table 5, we have \( x_i^{N*} \) generates more profit than \( x_i^{R*} \) if and only if \( (r_o, r_c) \in F_2 := \{ (r_o, r_c) \mid 2 \leq r_o \leq 2 r_c + m^{**}(\gamma) \) and \( 1 - \frac{1}{2} (r_o - 1) \left( 1 - 2 \frac{\gamma (\gamma + 1)}{(\gamma + 1)(2 - \gamma)} \right) < r_c \leq 1 \} \). The set of \( F_2 \) is nonempty if and only if \( \gamma \leq \frac{1}{3} ((17 + 12 \sqrt{2})^{1/3} + (17 + 12 \sqrt{2})^{-1/3} - 1) \approx 0.849 \). Note that for a nonempty \( F_2 \), we have \( r_c > r_{c3} \), which implies that \( x_i^{R*} \) is the unique symmetric equilibrium for the \( R \) game.

(iii) \( \{ (r_o, r_c) \mid r_c < r_o / 2 - m^{**}(\gamma) / 2 \) and \( r_{c1} \leq r_c \leq r_{c3} \}. In this case, although \( x_i^{R*} \) may also be a reorder symmetric equilibrium, the analysis is similar to case (i). Thus here we only compare the equilibrium combination \( x_h^{N*} \) and \( x_o^{R*} \). Algebraically, \( x_o^{R*} \) generates more profit than \( x_h^{N*} \) if and only if \( (r_o, r_c) \in \{ (r_o, r_c) \mid r_o \geq 2 \text{ and } \frac{2 - 2 \gamma}{\gamma} r_o + \frac{2 + \gamma}{2 - \gamma} \leq r_o \leq \frac{2 (\gamma + 1) (\gamma - 2) r_c + 2 + \gamma}{\gamma} \} \), which is exclusive of the validity set \( \{ (r_o, r_c) \mid r_c < r_o / 2 - m^{**}(\gamma) / 2 \) and \( r_{c1} \leq r_c \leq r_{c3} \} \). Thus, we conclude that the expected profit of \( x_o^{R*} \) is no more than that of \( x_h^{N*} \) in this case.

(iv) \( \{ (r_o, r_c) \mid r_c \geq r_o / 2 - m^{**}(\gamma) / 2 \) and \( r_{c1} \leq r_c \leq r_{c3} \}. In this case, although \( x_i^{R*} \) may also be a reorder symmetric equilibrium, the analysis is similar to case (ii). We here only compare the equilibrium combination \( x_i^{N*} \) and \( x_h^{R*} \). By Table 5, we calculate the profit difference \( \Pi_i^N (x_i = x_{-i} = x_i^{N*}) - \Pi_i^R (x_i = x_{-i} = x_i^{R*}) \)

\[
= \left(\frac{1 - \gamma^2}{4(2 + \gamma)^2} - \frac{1}{2} \left(\frac{1 - \gamma^2}{2 - \gamma}\right)^2 \right) r_o r_c + \frac{1 - \gamma^2}{(2 + \gamma)^2} r_c^2
\]
Now we prove that this lower bound is nonnegative. Substituting \( r_c = r_\alpha/2 - m^{**}(\gamma)/2 \) into \( \Pi^N(x_i = x_{-i} = x^{N*}) - \Pi^R(x_i = x_{-i} = x^{R*}) \) and noticing \( 0 \leq \gamma \leq 1 \), we have the profit difference
\[
= \left( -\frac{1}{2} \left( \frac{1 - \gamma}{2 - \gamma} \right)^2 + \frac{1 - \gamma}{2 - \gamma} \right) r_\alpha + \left( \frac{1 - \gamma}{2 - \gamma} \right)^2 r_\alpha + \frac{1 - \gamma^2}{4(2 + \gamma)^2} - \frac{1}{2} \frac{1 - \gamma}{2 - \gamma} r_\alpha.
\]

(v) \( \{ (r_\alpha, r_c) \mid r_c < r_\alpha/2 - m^{**}(\gamma)/2 \) and \( 0 \leq r_c < r_\alpha \} \). In this case, we compare the equilibrium combination \( x^{N*} \) and \( x^{R*} \). By Propositions 3 and 4, \( x^{R*} = x^{N*} \).

(vi) \( \{ (r_\alpha, r_c) \mid r_c \geq r_\alpha/2 - m^{**}(\gamma)/2 \) and \( 0 \leq r_c < r_\alpha \} \). In this case, we compare the equilibrium combination \( x^{N*} \) and \( x^{R*} \). Note that the profit of \( x^{R*} \) has an identical expression as that of \( x^{N*} \) and we can verify that if \( \alpha_\alpha = C(1 - \gamma) \) and \( \alpha_{H} \geq 3\alpha_\alpha \), then \( \Pi^N(x_i = x_{-i} = x^{N*}) \geq \Pi^N(x_i = x_{-i} = x^{N*}) \) if and only if \( (r_\alpha, r_c) \in \{(r_\alpha, r_c) \mid \frac{1}{2} \left( r_\alpha - 1 - \frac{2\gamma^2}{2\gamma - 1} \right) \leq r_c \leq \frac{1}{2} \left( r_\alpha - 1 + \frac{2\gamma^2}{2\gamma - 1} \right) \} \), which contains the set of \( \{ (r_\alpha, r_c) \mid r_c \geq r_\alpha/2 - m^{**}(\gamma)/2 \) and \( 0 \leq r_c < r_\alpha \} \). Thus, we conclude that \( x^{N*} \) generates more profit than \( x^{R*} \).

From (i) to (vi), we see that reorder flexibility benefits firms only in (i) and (ii), where firms play only \( x^{R*} \) in the R game but have different no-reorder equilibrium quantity \( x^{N*} \) and \( x^{R*} \) dependent on \( (r_\alpha, r_c) \). Moreover, \( x^{R*} < x^{N*} \) if and only if \( (r_\alpha, r_c) \in \{(r_\alpha, r_c) \mid r_c \geq -r_\alpha/2 + 3/2 \} \cap F_1 \cup F_2 \). Thus, we conclude that \( x^{R*} < x^{N*} \). The parameter subset where reorder flexibility benefits firms is \( F_1 \cup F_2 \). Such a set is nonempty if \( \gamma \leq 0.849 \).

Second, let us consider the case where \( 1 \leq r_\alpha < 2 \). (i) Whenever \( x^{R*} \) and \( x^{R*} \) are equilibria, the equilibrium outcomes of the R game are respectively equivalent to the outcome of \( x^{N*} \) and \( x^{N*} \) in the N game. Thus, the reorder flexibility has no value when the equilibrium is either \( x^{N*} \) or \( x^{R*} \). (ii) We explore the value of reorder flexibility when \( x^{R*} \) is an equilibrium. The algebraic expression \( \Pi^N(x_i = x_{-i} = x^{N*}) - \Pi^R(x_i = x_{-i} = x^{R*}) > 0 \) if and only if \( (r_\alpha, r_c) \in \{(r_\alpha, r_c) \mid r_\alpha \geq 1 - \frac{3\gamma + 1}{3\gamma - 1} (r_\alpha - 1) < r_c < 1 - \frac{3\gamma - 1}{3\gamma - 1} (r_\alpha - 1) \} \). However, in such a region, \( x^{R*} \) is not an equilibrium. Thus, the reorder flexibility has no positive value when \( x^{R*} \) is an equilibrium. (iii) We consider \( x^{R*} \). From case (ii) of \( r_\alpha \geq 2 \), we have that the reorder flexibility has a positive value for \( (r_\alpha, r_c) \in F_3 \) and \( r_c = \max\{r_c, r_{c_0}\} \). In summary, the value of flexibility has a positive value only when \( x^{R*} \) is an equilibrium and \( (r_\alpha, r_c) \in F_1 \cup F_2 \cup F_3 \). Let \( \bar{r}_\alpha(\gamma) : = 1 + \left( \frac{3\gamma}{3\gamma - 1} - 1 \right) m^{**}(\gamma) + 1 \) and \( \bar{r}_\alpha(\gamma) \) is a solution of \( \{ (r_\alpha, r_c) \mid 1 \leq r_\alpha < 2 \} \) and \( \bar{r}_\alpha(\gamma) \) is a solution of \( \{ (r_\alpha, r_c) \mid 0 \leq r_\alpha < 2 \} \). Moreover, the R game has the same equilibrium as the N game if and only if \( r_\alpha \leq \bar{r}_\alpha(\gamma) = \frac{3\gamma}{3\gamma - 1} \) and \( r_c \leq \bar{r}_\alpha(\gamma) = \max\{r_{c_4}, r_{c_0}\} \). □