Optimal Asset and Attention Allocation*

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Abstract

We consider the dynamic optimal strategy of an investor who can simultaneously manage her portfolio and acquire information about the expected returns of the risky asset. We show that the optimal level of attention to news is a hump-shaped function of expected returns and an increasing function of return variance and uncertainty. For unusually high values of variance and uncertainty, however, investment becomes too risky, resulting in a low risky asset position and therefore low attention. The same mechanism generates an inverse relationship between attention and risk aversion. Our empirical analysis lends support to the predictions of the model.

Keywords: Portfolio Choice, Attention to News, Information Acquisition, Learning

JEL Classification. D83, E21, E22, G11

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1 Introduction

Recent studies show that investors’ attention to news is time-varying, counter-cyclical (Da, Engelberg, and Gao, 2011; Lustig and Verdelhan, 2012), and linked to investment strategies (Barber and Odean, 2008). This bears important consequences for financial markets and the economy: fluctuations in investors’ attention can change beliefs in an instant and thus have immediate and real effects. It is therefore essential to understand, theoretically and empirically, what drives investors’ attention.

In studying this question, the theoretical literature mostly focuses on the idea that economic agents have limited ability to process information, often referred to as rational inattention (Sims, 2003). This theory has generated an expansive literature in both macroeconomics and finance. The present paper raises a complementary question: rather than exploring how information should be allocated given an attention capacity constraint, we focus on how much information an investor should acquire. That is, the total flow of information is not bounded by a capacity constraint outside of the model, but is endogenously determined through an optimisation exercise performed by the investor.

We reexamine the standard dynamic portfolio choice problem (Merton, 1969), in which a CRRA investor faces uncertainty about the expected return of a risky asset (Gennotte, 1986). In this context, the investor is simultaneously involved in learning and optimally allocating her portfolio. We depart from this setup by assuming that the investor is able to change her informational environment—and thus her investment opportunity set—by acquiring news relevant to the risky asset (these news could come from consulting, forecasting, financial analysis, and other related activities). This information acquisition decision regulates both the learning and the investment decisions of the investor: by acquiring more accurate information—i.e., by paying more attention to news\(^1\)—the investor is able to better estimate the unobservable expected returns and hence to add value to her portfolio. Since information comes at a cost, the investor faces a dynamic trade-off problem of asset and attention allocation.

To our knowledge, this joint, dynamic information acquisition and asset allocation problem has received little attention in the literature, and not much is known about the dynamic interaction between attention and portfolio choice. This is perhaps surprising, given the amount of resources devoted nowadays to information-related activities. Our paper aims to fill this gap by proposing a theoretical analysis of the joint asset and attention allocation problem, as well as an empirical evaluation providing support to this theoretical analysis.

In our setup, three state variables are found to determine to a great extent the attention

\(^1\)See, e.g., Sims (2003), Peng and Xiong (2006), Kacperczyk, Nieuwerburgh, and Veldkamp (2009), Mondria (2010), and Mondria and Quintana-Domeque (2013) for a similar interpretation of attention.
to news: expected returns, uncertainty, and the variance of stock returns. Our first result is that optimal attention is a hump-shaped function of expected returns, and comes from the following intuition. Because in our setup expected returns are mean-reverting (as commonly observed in financial data), the investor has low incentives to acquire information when expected returns are far below or above their long-term average, because she can easily predict their regression toward the mean. Consider, for instance, an extreme scenario in which expected returns are several standard deviations below or above their long-term mean. The incentive to acquire information in this case is low, for the investor can easily predict the reversion to the mean. On the contrary, when expected returns are close to their long-term average, the investor has strong incentives to acquire information because it is harder now to predict which direction returns will take into the future.

Second, the investor knows at any instant how accurate is her estimate of expected returns or, equivalently, how much uncertainty she faces. We find that high uncertainty implies a strong incentive to acquire accurate information. However, if uncertainty increases further, the investor optimally chooses to invest a smaller share of her wealth into the risky asset and therefore the incentive to acquire information is low. The relation between uncertainty and attention is therefore non-monotonic: attention increases for low to normal levels of uncertainty, but unusually high levels of uncertainty can be associated with low attention to news.

Third, in our setup we assume that the risky asset features stochastic volatility. We find that attention increases with the stock-return variance as long as the latter is not too high. The rationale is similar to the case of uncertainty. An increase in variance implies a deterioration in the investment opportunities and therefore increases the incentive to acquire information. But an increase in variance is also associated with a decrease in the optimal share invested into the risky asset. The latter effect dominates the former for high levels of volatility, generating a non-monotonic relationship.

The non-monotonic relationships between attention and uncertainty/variance are born out of the interaction between asset and attention allocation. It is precisely because the investor can decide simultaneously how much attention to pay to news and how much wealth to invest into the risky asset that we obtain low levels of attention for extreme values of variance or uncertainty. We believe this prediction has not been formulated before, and could be easily interpreted as the “ostrich effect,” although it is merely optimal behavior.\footnote{See Lo and MacKinlay (1988), Fama and French (1988), and Poterba and Summers (1988) for evidence of mean reversion in stock returns.}

\footnote{Formally, uncertainty is the variance of the expected stock return conditional on investor’s information set. It is commonly called uncertainty because it measures the inaccuracy of the estimated expected return.}

\footnote{The “ostrich effect” is documented by Karlsson, Loewenstein, and Seppi (2009) and Loewenstein, Seppi, Sicherman, and Utkus (2014): investors prefer to pay attention to their portfolios following positive news}
In our model, as in previous theoretical findings, risky investments increase with expected returns (Merton 1971, Kim and Omberg 1996, Campbell and Viceira 1999) and decrease with both variance (Chacko and Viceira 2005, Liu 2007) and uncertainty (Xia, 2001). This theoretical relation between the share of risky investments and variance/uncertainty, coupled with the above relations between attention and variance/uncertainty, predict that attention and risky investments are inversely related (or, equivalently, that attention and risk-free investments are positively related).

Furthermore, we show that attention decreases with risk aversion. We obtain this seemingly counterintuitive result because of the same interaction between asset and attention allocation which generates non-monotonic relationships between attention and risk. High risk aversion implies low share of risky investments, discouraging the investor from gathering information about expected returns.

We further develop an empirical analysis to test the above theoretical predictions. We find that attention is indeed a hump-shaped function of expected returns and an increasing function of uncertainty. The variance of stock returns, however, does not seem to be a significant driver of investors’ attention. In addition, attention is particularly large when risk-free investments are high. These empirical findings support the main theoretical predictions of the model.

This paper complements a large literature which considers portfolio selection problems with stochastic expected returns, stochastic volatility, incomplete information, and uncertainty about predictability. Furthermore, our study builds on the literature which considers costly information acquisition. In this literature, Detemple and Kihlstrom (1987) analyze the problem of an agent who invests in production technologies and in an information market “put their head in the sand” when they expect to see bad news. Andries and Haddad (2014) provide an alternative rationalization of the “ostrich” effect. In their model, agents are disappointment averse and thus riskier environments encourage more inattention. Note also that the “ostrich effect” refers to attention to wealth (Abel, Eberly, and Panageas 2007, Abel, Eberly, and Panageas 2013), whereas here we model investors’ attention to news.

We proxy for expected stock returns with the fitted value of a predictive regression of current dividend yields on future S&P 500 returns, for return variance with a GARCH estimation on realized S&P 500 returns, and for investors’ attention with changes in the S&P 500 trading volume, which has been proposed as a measure of attention of institutional investors by Barber and Odean (2008) and Hou, Peng, and Xiong (2009). We also consider, as a measure of retail investors’ attention, the Google search index proposed by Da et al. (2011). Further, we proxy for uncertainty with the macro-uncertainty index form Jurado, Ludvigson, and Ng (2013), provided on Sydney Ludvigson’s website, and for the allocation in risk-free assets with changes in the Institutional Money Fund index provided by the Federal Reserve Bank of St. Louis.

ket, Veldkamp (2006a,b) show that costly information acquisition helps explain excess co-movement and the simultaneous increases in emerging markets’ media coverage and equity prices, and Huang and Liu (2007) argue that costly information pushes investors to acquire imperfectly accurate signals at a low frequency. None of these studies shed light on the dynamic relations between attention, investments, and key state variables which drive the current economic conditions, namely expected returns, uncertainty, and volatility. Our paper provides both a theoretical and an empirical analysis that help understand these fundamental relations.

The paper proceeds as follows. Section 2 presents and solves the model, Section 3 describes the theoretical implications, Section 4 tests the main predictions of the model, and Section 5 concludes. Derivations and computational details are provided in Appendix A.

2 Optimal Asset and Attention Allocation

This section presents and solves the problem of an investor who optimally chooses her portfolio, consumption, and attention paid to news in order to maximize her expected lifetime utility of consumption. The key feature of the model is that a news signal, whose accuracy can be improved at a cost, provides information on expected returns. Therefore, the investor faces a challenge which consists in optimally trading-off an accurate forecast of future returns at a high cost with an inaccurate forecast but at a low cost.

The investor is allowed to invest in two assets: one stock and one risk free asset. The prices of the stock and the risk-free asset are denoted by $P$ and $M$, respectively. The dynamics of prices and their constituents are

\[
\frac{dP_t}{P_t} = \mu_t dt + \sqrt{V_t} dB_{1t} \\
d\mu_t = \lambda_\mu (\bar{\mu} - \mu_t) dt + \sigma_\mu dB_{2t} \\
dV_t = \lambda_V (\bar{V} - V_t) dt + \sigma_V \sqrt{V_t} dB_{3t} \\
\frac{dM_t}{M_t} = r dt,
\]

where $r$ is the constant risk-free rate, $\mu$ is the expected return on the stock, which mean-reverts (Ornstein and Uhlenbeck, 1930), and $V$ is the stock-return variance, which is stochastic (Heston, 1993). Importantly, the investor observes prices and the variance of stock

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returns, but not the expected return \( \mu \).

In order to get an idea of future market moves, the investor estimates the expected return by observing stock price movements and news releases. The information signal acquired through the processing of news releases has the following dynamics:

\[
ds_t = \mu_t dt + \frac{1}{\sqrt{x_t}} dB_t,
\]

where the term driving the variance of the signal, \( x_t \), is optimally controlled by the investor. The greater the value of \( x_t \) the higher the precision of the signal. We refer thus to the variable \( x_t \) as the optimal attention to news of the investor.

The processes \( B_1, B_2, B_3, \) and \( B_4 \) are Brownian motions with correlation matrix \( \Lambda \) defined as follows:

\[
\Lambda = \begin{pmatrix}
1 & \rho_{P\mu} & \rho_{PV} & 0 \\
\rho_{P\mu} & 1 & \rho_{\mu V} & 0 \\
\rho_{PV} & \rho_{\mu V} & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}.
\]

Let us denote by \( \hat{\mu} \) the estimated expected return, henceforth called the filter. The filtered expected return \( \hat{\mu} \) and the posterior variance \( \gamma \) are such that

\[
\mu_t \sim N(\hat{\mu}_t, \gamma_t),
\]

where \( N(m, v) \) denotes the Gaussian distribution with mean \( m \) and variance \( v \). For the rest of this paper, we call the posterior variance \( \gamma \) uncertainty. It measures the accuracy of the filter: low uncertainty implies that the filter \( \hat{\mu} \) is likely to be close to the true expected return \( \mu \) whereas high uncertainty makes the filter likely to be inaccurate.

Using standard results from filtering theory (Lipster and Shiryaev, 2001), the vector of state variables observed by the investor is:

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8The expected returns are hard to estimate precisely from historical returns (Merton, 1980).

9The structure of the signal is adapted from Detemple (1986), Veronesi (2000), Scheinkman and Xiong (2003), and Bansal and Shaliastovich (2011) among others. In this case, the drift of the signal is correlated with the expected stock return. An alternative is to have signals correlated with unexpected innovations of stock returns, as in Dumas, Kurshev, and Uppal (2009) or Andrei and Hasler (2014). Both approaches give similar results. See Buraschi and Whelan (2012) for a discussion of the two types of signals.
Proposition 1. (Learning) The dynamics of the observed state variables satisfy

\[
\frac{dP_t}{P_t} = \hat{\mu}_t dt + \left( \sqrt{V_t} \ 0 \right) d\hat{B}_t \tag{1}
\]

\[
dV_t = \lambda_V (\bar{V} - V_t) dt + \sigma_V \sqrt{V_t} \left( \rho_{PV} \sqrt{1 - \rho_{PV}^2} \right) d\hat{B}_t \tag{2}
\]

\[
d\hat{\mu}_t = \lambda_{\mu} (\bar{\mu} - \hat{\mu}_t) dt + \left( \frac{\gamma_t}{\sqrt{V_t}} + \rho_{PV} \sigma_{\mu} \right) \sqrt{V_t} \left( \rho_{PV} \mu - \rho_{PV} \right) \left( \rho_{PV} \rho_{PV} - \rho_{PV} \right) \left( \rho_{PV} \rho_{PV} - \rho_{PV} \right) \sigma_{\mu}^2 \tag{3}
\]

\[
d\gamma_t = -\gamma_t^2 \left( x_t + \frac{1}{V_t (1 - \rho_{PV}^2)} \right) - 2\gamma_t \left( \lambda_{\mu} + \frac{\left( \rho_{PV} \mu - \rho_{PV} \rho_{PV} \right) \sigma_{\mu}}{\sqrt{V_t} (1 - \rho_{PV}^2)} \right) + \frac{1}{1 - \rho_{PV}^2}, \tag{4}
\]

where \( \hat{B} = \left( \hat{B}_1 \ \hat{B}_2 \ \hat{B}_3 \right)^\top \) is a 3-dimensional vector of independent and observable Brownian motions. The Brownians \( \hat{B}_1 \) and \( \hat{B}_2 \) are defined by Equations (1) and (2), and \( \hat{B}_3 \) is such that \( ds_t = \hat{\mu}_t dt + \sigma_{\mu} d\hat{B}_3 t \).


The investor estimates expected returns by using three sources of information: returns, stock-return variance, and the news signal. As Equation (3) shows, when performing her learning exercise, the investor assigns stochastic weights to these three sources of information. Each weight depends on the relative quality of each information flow and therefore corresponds to the signal-to-noise ratio of each source of information. The relative importance of these weights is driven by three variables, \( x, \gamma, \) and \( V \). The investor directly controls \( x \), indirectly controls \( \gamma \) (see Equation 4), and has no control over \( V \).

A straightforward way to understand the evolution of uncertainty is to fix all the correlations form matrix \( \Lambda \) to zero. Equation (4) then becomes:

\[
d\gamma_t = -\gamma_t^2 \left( x_t + \frac{1}{V_t} \right) - 2\lambda_{\mu} \gamma_t + \sigma_{\mu}^2
\]

Uncertainty is locally deterministic and decreases rapidly when the attention \( x \) is high. However, the decrease in uncertainty is slower when expected returns are persistent (low \( \lambda_{\mu} \)) or when the volatility of realized returns \( V_t \) is high. When the volatility of expected returns \( \sigma_{\mu} \) is sufficiently high, the last component of uncertainty outweighs the first two negative components and therefore uncertainty increases. It is worth noting that, because the stock-return variance and investor’s attention are time-varying, uncertainty never converges to a constant steady-state. This feature holds in the studies of Xia (2001) and Andrei and Hasler (2014) and contrasts with the constant steady-state uncertainty obtained by Brennan and Xia (2001), Scheinkman and Xiong (2003), Dumas et al. (2009), and Xiong and Yan (2010).
2.1 Optimization Problem

The investor chooses consumption $c$, risky investment share $q$, and the signal volatility $\sigma_s$ to maximizes her expected lifetime utility of consumption. The investor’s problem writes

$$J(W_t, \hat{\mu}_t, V, \gamma) = \max_{c \in \mathbb{R}, q \in \mathbb{R}, \sigma_s \in \mathbb{R}^+} \mathbb{E}_t \left( \int_t^\infty e^{-\Delta(u-t)} \frac{c_{u-\alpha}}{1-\alpha} du \right)$$

s.t. $dW_t = [(r - K(x_t))W_t + q_tW_t(\hat{\mu}_t - r) - c_t] dt + q_tW_t \left( \sqrt{V_t} \ 0 \right) d\hat{B}_t,$

(5)

where $\Delta$ is the subjective discount factor, $\alpha$ is the coefficient of relative risk aversion, $W$ the wealth of the investor, and the vector of state variables $(\hat{\mu}, V, \gamma)$ is defined in Proposition 1. We specify a quadratic information cost function $K(x)$:

$$K(x) \equiv kx^2.$$ 

(6)

The HJB equation associated to problem (5) is given in Appendix A.1. For tractability purposes, we follow Bansal and Shaliastovich (2011) and assume that the total information cost is linear in wealth (see Equation 5). This assumption conserves the homogeneity of the value function and simplifies considerably the solution. In addition, the per-unit of wealth cost function $K(x)$ is assumed to be quadratic\(^{10}\) (increasing and convex) in attention, which implies that perfect information ($x = \infty$) can never be achieved. That is, the investor is never able to observe the true level of the expected return. If the investor chooses to pay no attention to news ($x = 0$ and $K(x) = 0$) and to learn using the information provided by the price and the variance of stock returns only, then her entire wealth is invested in the financial market. If, however, the investor decides to pay attention to news ($x > 0$ and $K(x) > 0$), then a positive fraction of her wealth flows to the information market.

To summarize, the attention choice variable $x$ enters in (i) the drift of wealth $W$, (ii) the diffusion of expected returns $\hat{\mu}$, and (iii) the drift of uncertainty $\gamma$. Therefore, by optimally choosing the attention level $x$ the agent changes three of the state variables, as opposed to the classical investment problem (Merton, 1971) in which only the wealth is changing by choosing the optimal risky share. The investor’s optimal consumption, risky investment share, and attention are characterized in Proposition 2 below.

Proposition 2. The optimal consumption $c^*$, risky investment share $q^*$, and attention $x^*$

\(^{10}\)Although in different settings, quadratic cost functions are considered by Garleanu and Pedersen (2013) and Collin-Dufresne, Daniel, Moallemi, and Saglam (2014) among others.
satisfy

\[ c^* = F^{-1/\alpha} W \]

\[ q^* = \frac{\hat{\mu} - r}{\alpha V} + \left( \gamma + \sqrt{\nu} \rho \sigma \mu \right) \frac{F \hat{\mu}}{\alpha VF} + \frac{\rho \nu \sigma V F_V}{\alpha F} \]

\[ x^* = \begin{cases} \bar{x}^* = \frac{\gamma^2(2F \gamma - F \hat{\mu})}{4k(\alpha - 1)F^2}, & \text{if } \bar{x}^* \in \mathbb{R}_{>0} \\ 0 & \text{otherwise,} \end{cases} \quad (7) \]

where the function \( F(\hat{\mu}, V, \gamma) \) is such that the value function satisfies

\[ J(W, \hat{\mu}, V, \gamma) = \frac{W^{1-\alpha}}{1-\alpha} F(\hat{\mu}, V, \gamma). \]

Note that \( F_y \) stands for the partial derivative of the function \( F \) with respect to the variable \( y \).

**Proof.** See Appendix A.1.

The optimal consumption rule (Equation 2) is standard, as in Merton (1971). The optimal risky share consists in a myopic term (Merton, 1971) and two hedging terms which depend on the sensitivities of the value function to changes in the filter and the stock-return variance (Liu, 2007). Finally, the optimal attention is given in Equation (7). Since the investor is constrained to choose her optimal attention on the non-negative real line, the optimal attention \( x^* \) is nil as long as the term \( \bar{x}^* \notin \mathbb{R}_{>0} \).

According to Equation (7), uncertainty has a direct effect on attention (ignoring indirect effects arising through the function \( F \) and its derivatives). Intuitively, high uncertainty implies an inaccurate filter and therefore a willingness to pay more attention to news. Equation (7) also shows that higher risk aversion decreases the optimal level of attention. High risk aversion implies a low risky investment share (see Equation 2) and thus the incentive to gather information about the expected stock return is weaker.\(^{11}\)

An approximate solution of the HJB equation associated to problem (5) is obtained by conjecturing an exponential-quadratic form for the function \( F \) and by solving the associated linearized system of equations numerically.\(^{12}\) All derivations and computational details are provided in Appendix A.1. We proceed now to discuss the dynamics of optimal attention in detail.

\(^{11}\)Note however that a zero investment share does not necessarily imply that attention should also be zero, since the investor knows that she might still invest in the risky asset in the future.

\(^{12}\)Our exponential-quadratic conjecture is motivated by solutions derived by Wachter (2002) and Liu (2007). Alternatively, solving the HJB equation using the Chebyshev collocation method described in Judd (1998) yields a function with the same properties as those of the exponential-quadratic form. In other words, the exponential-quadratic form considered here is likely to accurately approximate the true solution.
The “steady-state” uncertainty, $\gamma_{ss}\{x=0, V=\bar{V}\}$, is defined in Equation (16) in the Appendix.

3 Theoretical Implications

This section presents and discusses the dependence of optimal attention and risky investment share on expected stock returns, stock-return variance, and uncertainty.

If not otherwise specified, parameters and values of the state variables used for the rest of this paper are provided in Table 1. Variance parameters are adapted from Christoffersen, Jacobs, and Mimouni (2010), expected return parameters from Xia (2001), and utility parameters are standard. The correlation between return and variance is negative (Christoffersen et al., 2010) and reflects the leverage effect (Black 1976, Christie 1982). The correlation between returns and expected returns is negative (Fama and French, 1992). Because both expected returns and volatilities are counter-cyclical, we fix the correlation between them to be positive (Campbell and Shiller 1988a,b, Schwert 1989, Mele 2007). Although the cost parameter $k$ looks particularly small, we will show below that it implies a non-negligible information cost together with a reasonably accurate news signal.

3.1 Attention, Investments, and Economic Conditions

Panel (a) of Figure 1 shows that the optimal attention level $x^*$ is a hump-shaped function of the filtered expected return $\hat{\mu}$. When the filter is either sufficiently large or sufficiently low, the investor knows that the probability of the expected return reverting back to its
mean is relatively high. Indeed, in these states of the world the drift of the expected return is large in absolute terms (see Proposition 1, Equation 3); this gives no incentive to pay a cost and be attentive to news, i.e., constraint (7) binds. In contrast, when the filter is close to its long-term mean, its main driver is now the noise term and thus the investor has an incentive to pay attention to news and gather accurate information. It is worth noting that, even though the cost parameter $k$ is small, the investor places up to 15 bps of her wealth per unit of time into the information market, which is non-negligible—in this particular case, the parameters from Table 1 imply that the investor acquires a news signal with a volatility of about 5%.

Turning now to the risky investment share $q^*$, Panel (b) of Figure 1 shows that $q^*$ is an increasing function of the filtered expected return $\hat{\mu}$. This is because the filter measures the quality of risky investment opportunities. Since the relation between attention and the filter is hump-shaped and that between the risky share and the filter is monotone increasing, attention is an endogenous hump-shaped function of the risky share.

Panel (a) of Figure 2 depicts a non-monotone relation between attention and stock-return variance. Two opposite forces are at work in this case. First, the investor pays more attention to news as variance increases, because a large stock-return variance implies that information provided by the observation of returns is inaccurate. A second force, however, creates a decreasing relation between attention and variance, because, as Panel (b) of Figure 2 shows, a high stock-return variance implies a low risky investment share (Chacko and Viceira 2005, Liu 2007) and therefore a weaker incentive to gather information on expected stock returns. These competing forces imply that attention increases with the variance of stock returns as long as the latter is not too large.
Figure 2: Attention and Risky Investment Share against Stock-Return Variance

The effect of variance on attention is, however, less strong that the effect coming from changes in expected returns. Comparing Figures 1 and 2 shows that changes in expected returns impact attention significantly more than changes in variance do.

A similar mechanism implies a non-monotonic relation between attention and uncertainty, which we depict in Panel (a) of Figure 3. First, attention tends to increase with uncertainty because higher uncertainty means a more inaccurate filter. However, attention tends also to decrease with uncertainty because high uncertainty implies low risky investments (Panel (b) of Figure 3) and therefore the investor is less willing to acquire accurate information regarding expected stock returns.\footnote{Intuitively, the risky investment share decreases monotonically with uncertainty (Panel (b) of Figure 3) because uncertainty is, similar to the variance of stock return, a measure of the risk faced by the investor.} Panel (a) of Figure 3 shows that the former effect dominates the latter as long as uncertainty is not too large.

Figures 1, 2, and 3 show that changes in attention are mainly driven by changes in expected returns and uncertainty, while changes in the risky investment share are mainly driven by changes in expected returns and stock-return variance. Optimal attention is a hump-shaped function of expected stock returns and an increasing function stock-return variance and uncertainty (except for large levels of variance and uncertainty, which have the tendency to decrease the optimal attention paid by the investor). Overall, Figures 2 and 3 suggest that, holding expected returns fixed, high attention is associated with low risky investment share.
3.2 Impact of Risk Aversion and Cost of Ignoring Information

Proposition 2 shows that attention to news is a decreasing function of investor’s risk aversion. This relationship is confirmed in Figure 4: the left hand side panels show that higher risk aversions lead in all cases to lower attention allocation. Although this effect seems counterintuitive at first, it is easily rationalized by the fact that high risk aversion implies low risky investments, as the right hand side panels of Figure 4 show. Therefore, the incentive to gather accurate information about expected stock returns is larger for a low risk averse investor than for a high risk averse investor.

We aim to quantify the benefits associated with the acquisition of information. In order to do so, we compute the certainty equivalent measure proposed by Das and Uppal (2004). This measure is defined as the additional fraction of wealth required by an investor who ignores the information signal (or equivalently, an investor who faces an information cost function with parameter $k \to \infty$) to reach the same expected utility of an investor who optimally pays attention to news. We calculate several certainty equivalent measures in Table 2, for different values of risk aversion $\alpha$ and information cost $k$. The overall message is that the cost of ignoring the information signal is non-negligible, reaching as much as 8% of wealth, especially when both the risk aversion and the information cost are small. The lower the risk aversion and the information cost, the larger the incentive to gather information (as shown in Figure 4), and thus the higher the opportunity cost of information acquisition. It is worth noting that irrespective of the value of the cost parameter $k$, a highly risk averse investor (e.g., $\alpha \approx 20$) enjoys only a marginal benefit from information acquisition, simply because most of her wealth is invested in the risk-free asset.
Figure 4: Attention and Risky Investment Share against Risk Aversion
Table 2: The Cost of Ignoring the News Signal

The cost of ignoring news represents the additional fraction of wealth required by an investor who ignores the news signal to reach the expected utility of an investor who optimally pays attention to news.

<table>
<thead>
<tr>
<th>Risk Aversion $\alpha$</th>
<th>Cost Parameter $k$</th>
<th>Cost of Ignoring News</th>
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<tr>
<td>3</td>
<td>$10^{-8}$</td>
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</tr>
<tr>
<td>7</td>
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4 Empirical Analysis

This section provides evidence supporting the main predictions of the model. First, investors’ attention is a hump-shaped function of expected returns and an increasing function of both stock-return variance and uncertainty. Second, attention increases with the share of risk-free investments and therefore decreases with the share risky investments. The following testable hypotheses formulate these two predictions:

**Hypothesis 1.** At the optimum, investors’ attention is a hump-shaped function of expected returns and an increasing function of both stock return variance and uncertainty.

**Hypothesis 2.** At the optimum, investors’ attention attention increases with the share of risk-free investments (alternatively, attention decreases with the share of risky investments).

To test these two hypotheses, we run regressions of the following form

$$ Attention_t = a + b_1 ER_t + b_2 ER_t^2 + b_3 \Delta \text{Variance}_t + b_4 \Delta \text{Uncertainty}_t + b_5 RFI_t $$  \hspace{1cm} (8)  

where $ER_t$ is the expected return and $RFI_t$ is our proxy for the risk-free investments. We expect the signs of the regression coefficients to be $b_1 > 0$, $b_2 < 0$, $b_3 > 0$, $b_4 > 0$ (according to Hypothesis 1), and $b_5 > 0$ (according to Hypothesis 2). Since both the variance of stock returns and uncertainty are highly autocorrelated by construction, we consider changes in these variables. We also control for the autocorrelation in attention by including the 1-month lagged attention in the set of predictors.

Our empirical analysis is based on the following dataset. The expected stock return represents the fitted value of a predictive regression that infers the future S&P 500 return...
using the current dividend yield and S&P 500 return. The variance of stock returns is obtained by fitting a GARCH(1,1) model (Bollerslev, 1986) to the demeaned S&P 500 returns. We use the 1-month ahead macro-uncertainty index constructed by Jurado et al. (2013) as a proxy for uncertainty.

We consider two measures of investors’ attention. Our first measure is motivated by Barber and Odean (2008) and Hou et al. (2009) and represents the growth rate of the S&P 500 trading volume. As this proxy is more likely to measure the attention of institutional investors, we also consider a measure of retail investors’ attention, namely the Google attention index built first by Da et al. (2011).

Finally, we use the growth rate of the “Institutional Money Fund” index to proxy risk-free investments. The “Institutional Money Fund” index is obtained from the Federal Reserve Bank of St. Louis. Our monthly dataset spans the period January 1974 - January 2012 when we use the first proxy of attention and January 2004 - January 2012 when we use the second proxy of attention.

In order to test whether attention is a hump-shaped function of expected returns and an increasing function of both stock-return variance and uncertainty, we perform Ordinary Least Squares (OLS) regression analyses as in Equation (8). The model-implied hump-shaped relation between attention and expected returns is captured by including the expected return and its square in the set of predictors.

The regression results in column (1) of Table 3 confirm a clear hump-shaped relation between attention and expected returns. The quadratic coefficient is negative, significant, and substantially larger than the linear coefficient in absolute terms. In addition, the top of the hump shape is close to the mean of expected returns (which is approximately 1% over the studied sample), as the theoretical model predicts. Moreover, the same regression shows that uncertainty is an important driver of attention, bearing a significant positive coefficient, whereas stock-return variance is not; although the coefficient of the variance is positive, it is not statistically different from zero. Coming back to Figures 1, 2, and 3, the model also predicts a strong relation between attention and expected returns/uncertainty but a weaker relation between attention and variance, consistent thus with the empirical findings in column (1) of Table 3.

We turn now to the relationship between attention and the risky investment share. As discussed in Section 3, the model predicts that, controlling for the hump-shaped relation between attention and expected returns, attention tends to increase with the risky investment share. We test this prediction by performing OLS regressions of the type (8). As mentioned earlier, we use the growth rate of the “Institutional Money Fund” index provided by the

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14The empirical findings of Chordia and Swaminathan (2000), Lo and Wang (2000), and Gervais, Kaniel, and Mingelgrin (2001) suggest that trading volume is a reasonable proxy of investors’ attention.
Federal Reserve Bank of St. Louis to proxy the risk-free investment share. Column (2) of Table 3 controls for the impact of the expected return and the auto-correlation of attention, while Column (3) of Table 3 also controls for the impact of the stock-return variance and uncertainty.

Both regressions show that attention and risk-free investments are strongly and positively related. Indeed, the risk-free investment (RFI) coefficients are positive and significant at the 99% confidence level, confirming the prediction that institutional investors are likely to pay high attention when risk-free investments are large, and therefore when risky investments are low. This prediction is also in line with Vlastakis and Markellos (2012) and Andrei and Hasler (2014), who show that investors’ attention is most likely counter-cyclical.

We also construct an alternative measure of investors attention. To do so, we follow Da et al. (2011) and use the Google search volumes, a publicly available measure which is more likely to capture retail investors’ attention. The search volume associated to each

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**Table 3: Attention (proxied as the growth rate of the S&P 500 Trading Volume) against Expected Stock Returns, Stock-Return Variance, Uncertainty, and Risk-Free Investments**

Data is at monthly frequency from January 1974 to January 2012. Standard errors are adjusted using the procedure of Newey and West (1987). $t$-statistics are reported in brackets and statistical significance at the 10%, 5%, and 1% levels is labeled with */**/***, respectively. The last two rows present the adjusted $R^2$ and the number of observations.

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>0.0171**</td>
<td>0.0114*</td>
<td>0.0121*</td>
</tr>
<tr>
<td>$RFI_t$</td>
<td>(2.5583)</td>
<td>(1.8730)</td>
<td>(1.7871)</td>
</tr>
<tr>
<td>$ER_t$</td>
<td>0.2203***</td>
<td>0.1924***</td>
<td>0.1924***</td>
</tr>
<tr>
<td>$ER^2_t$</td>
<td>(3.0719)</td>
<td>(2.8616)</td>
<td></td>
</tr>
<tr>
<td>$\Delta Variance_t$</td>
<td>1.9622***</td>
<td>1.2769**</td>
<td>1.8966***</td>
</tr>
<tr>
<td>$\Delta Uncertainty_t$</td>
<td>(2.9828)</td>
<td>(2.1226)</td>
<td>(2.9205)</td>
</tr>
<tr>
<td>Attention$_{t-1}$</td>
<td>-95.0355**</td>
<td>-68.9174**</td>
<td>-88.1649**</td>
</tr>
<tr>
<td>$\Delta Variance_t$</td>
<td>(-2.4182)</td>
<td>(-2.5221)</td>
<td>(-2.2303)</td>
</tr>
<tr>
<td>$\Delta Uncertainty_t$</td>
<td>0.9307**</td>
<td>0.8936*</td>
<td></td>
</tr>
<tr>
<td>Attention$_{t-1}$</td>
<td>(2.0050)</td>
<td>(1.8434)</td>
<td></td>
</tr>
<tr>
<td>Adj. $R^2$</td>
<td>0.0812</td>
<td>0.0771</td>
<td>0.0879</td>
</tr>
<tr>
<td>N</td>
<td>455</td>
<td>456</td>
<td>455</td>
</tr>
</tbody>
</table>
query is recorded at weekly frequency from the first week of January 2004 to the last week of January 2012. We demean these time-series by their respective sample average for scaling purposes and sum them to obtain an attention index at weekly frequency. Then, we compute averages over each month to obtain a proxy for investors’ attention at monthly frequency. The measures of expected return, variance, uncertainty, and risk-free investments are the same as in the previous section.

Results are provided in Table 4. Although statistical significance is weaker when considering the Google-based measure of attention than when considering the trading-based measure of attention, attention is again a hump-shaped function of expected returns with a significant negative quadratic coefficient, an increasing function of uncertainty and an increasing function of stock-return variance, although the variance coefficient is once again not significant. Column (1) of Table 4 therefore shows that the relation between attention, expected returns, variance, and uncertainty is robust to changes in attention measures.

Columns (2) and (3) of Table 4 further confirm that our empirical results depend only weakly on the attention proxy considered. Indeed, attention is, as in the previous section, an increasing function of risk-free investments. Consistent with the prediction of the model, investors tend to pay high attention when risky investments are low.

5 Conclusion

In this paper we analyze the theoretical link between attention to news, asset allocation, and economic conditions. When acquiring information is costly, investors’ attention to news is a hump-shaped function of expected returns and an increasing function of both stock-return variance and uncertainty. Therefore, attention to news and risky investments tend to be inversely related. In addition, we show that attention decreases with risk aversion.

Our empirical analysis confirms that attention is a hump-shaped function of expected returns and an increasing function of uncertainty. The predicted positive relation between attention and stock-return variance is insignificant, but nonetheless positive. Finally, investors pay high attention to news when investing substantially in risk-free assets, lending support to the prediction of the model.

16Google Search Volume data are recorded until the end of January 2012 only, because the uncertainty index provided by Jurado et al. (2013) ends at that particular time.
Table 4: Attention (proxied from Google search data) against Expected Stock Returns, Stock-Return Variance, Uncertainty, and Risk-Free Investments

Data is at monthly frequency from January 2004 to January 2012. Standard errors are adjusted using the procedure of Newey and West (1987)’s. t-statistics are reported in brackets and statistical significance at the 10%, 5%, and 1% levels is labeled with */**/*** respectively. The last two rows present the adjusted $R^2$ and the number of observations.

A Appendix

A.1 HJB Equation

The HJB equation associated to problem (5) is written

$$
\max_{c \in \mathbb{R}^+, q \in \mathbb{R}, \sigma \in \mathbb{R}^+} \left\{ -\Delta J + \frac{c^{1-\alpha}}{1 - \alpha} + \mathcal{L}^{W, \hat{\mu}, V, \gamma} J \right\} = 0,
$$

(9)

where $\mathcal{L}^{W, \hat{\mu}, V, \gamma}$ is the infinitesimal generator of the vector $(W, \hat{\mu}, V, \gamma)$. Equations (4) and (5) show that the signal volatility drives the drifts of uncertainty and wealth, and the third diffusion component of the filter. Since the third diffusion component is zero for all state variables but the filter, the signal volatility enters Equation (9) through the cost function $\bar{K}(\sigma_s)$ and terms of the order of $\frac{1}{\sigma_s^2}$ only.

The change of variable and the cost function defined in Equation (6) imply the following HJB equation

$$
\max_{c \in \mathbb{R}^+, q \in \mathbb{R}, x \in \mathbb{R}^+} \left\{ -\Delta J + \frac{c^{1-\alpha}}{1 - \alpha} + \mathcal{L}^{W, \hat{\mu}, V, \gamma} J \right\} = 0.
$$

(10)
Solving the first order conditions yields

\[ c^* = J_w^{-1/\alpha} \]

\[ q^* = \frac{(r - \mu)J_w - V \rho_{PV} \sigma_V J_{W,V} - \left( \gamma + \sqrt{V \rho_{P\mu} \sigma_\mu} \right) J_{W,\hat{\mu}}}{V W J_{W,W}} \]  \hspace{1cm} (12)

\[ x^* = \begin{cases} \bar{x}^* = \frac{\gamma^2 (-2 J_\gamma + J_{\bar{\mu},\bar{\gamma}})}{4 J_W J_\gamma}, & \text{if } \bar{x}^* \in \mathbb{R}_{>0} \\ 0 & \text{otherwise} \end{cases} \]  \hspace{1cm} (13)

where \( J_y \) stands for the partial derivative of the function \( J \) with respect to the variable \( y \).

The candidate solution to Equation (10) is written

\[ J(W, \hat{\mu}, V, \gamma) = \frac{W^{1-\alpha}}{1-\alpha} F(\hat{\mu}, V, \gamma). \]  \hspace{1cm} (14)

Substituting Equation (14) in Equations (11), (12), and (13) yields the expressions provided in Proposition 2.

Substituting the optimality conditions (11), (12), and (13) and the candidate solution (14) in the HJB equation (10) yields a 3-dimensional non-linear PDE.

Motivated by the exponential-quadratic forms presented by Wachter (2002) and Liu (2007), the PDE is solved by approximating the candidate solution as follows

\[ J(W, \hat{\mu}, V, \gamma) \approx \frac{W^{1-\alpha}}{1-\alpha} e^{A_0 + A_1 \hat{\mu} + A_2 \hat{\mu}^2 + A_3 V + A_4 V^2 + A_5 \gamma + A_6 \gamma^2 + A_7 \hat{\mu} V + A_8 \hat{\mu}^2 + A_9 \gamma}. \]  \hspace{1cm} (15)

The solution (15) is substituted in the PDE. Then, the resulting expression is approximated by a second order Taylor expansion around \( \hat{\mu} = \bar{\mu}, V = \bar{V} \), and \( \gamma = \frac{1}{2} \gamma_{ss}|_{\{x=0,V=\bar{V}\}} \), where \( \gamma_{ss}|_{\{x=0,V=\bar{V}\}} \) stands for the “steady-state” uncertainty conditional on \( x = 0 \) and \( V = \bar{V} \). The “steady-state” uncertainty is obtained by setting \( x = 0, V = \bar{V}, \) and \( \frac{d\mu}{dt} = 0 \) in Equation (4). The solution to the aforementioned equation is

\[ \gamma_{ss}|_{\{x=0,V=\bar{V}\}} = \bar{V} \lambda_\mu (\rho_{PV}^2 - 1) + \sqrt{\bar{V}} (-\rho_{P\mu} + \rho_{PV} \rho_{\mu V}) \sigma_\mu \]

\[ + \sqrt{\bar{V}} (\rho_{PV}^2 - 1) \left( \hat{\bar{V}} \lambda_\mu^2 (\rho_{PV}^2 - 1) - 2 \sqrt{\bar{V}} \lambda_\mu (\rho_{P\mu} - \rho_{PV} \rho_{\mu V}) \sigma_\mu + (\rho_{PV}^2 - 1) \sigma_\mu^2 \right). \]  \hspace{1cm} (16)

The resulting Taylor expansion yields a system of 10 equations and 10 unknowns (the \( A_i \)s) that is solved numerically.
References


