Interaction of equilibrium selection criteria: Round numbers as focal points in Treasury auctions*

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November 27, 2013

Abstract

In games with multiple equilibria, a number of different equilibrium selection criteria have been proposed. This paper examines how the presence of a cultural focal point interacts with payoff-based selection criteria. The context we consider is a pay-as-bid common value auction with a discrete bidding grid. We show the conditions under which multiple equilibria arise. In the presence of multiple equilibria, the auction can be characterized as a coordination game (specifically a stag hunt game) which has both a Pareto-dominant equilibrium and a risk-dominant equilibrium. Empirically, we consider pay-as-bid Treasury bill auctions. We find that market-clearing bids (submitted as discount rates) more frequently end in a round number of 0 and less frequently end with a 9 than would otherwise be expected. In contrast, the frequency of bids ending with a final digit of 1 is not significantly different from a uniform distribution. We argue that this is evidence of multiple equilibria and a round-number focal point that interacts with Pareto dominance and risk dominance. It suggests that the decreased uncertainty caused by a culturally-based round-number focal point reduces the attractiveness of risk dominance as an equilibrium selection criterion. (JEL: C72, D44)

Keywords: focal point; coordination games; round number; auctions

*We thank Ig Horstmann, Łukasz Pomorski, and participants at the Rotman Finance Brownbag Seminar and the University of Toronto Economic Theory Workshop for helpful comments and suggestions.

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1 Introduction

In games with multiple equilibria, theory does not give a definitive answer as to which equilibrium will occur. When one equilibrium is Pareto dominant (i.e., payoff dominant) and another is risk dominant, it has been debated which of the two equilibria players will coordinate on. It has also been proposed that players may coordinate on a focal equilibrium that is prominent for cultural or other non-payoff related reasons. In this paper, in an auction context, we empirically show that round numbers act as focal points that interact with the payoff dominance and risk dominance selection criteria. We find that a round-number focal point facilitates coordination on the payoff-dominant equilibrium, but does not lead to more frequent coordination on the risk-dominant equilibrium. We argue that the round-number focal point reduces uncertainty about which equilibrium others will play and thus reduces the need to play the risk-dominant equilibrium.

We consider the context of pay-as-bid auctions (i.e., first-price auctions and their multi-unit analogue) for a common-value good with a discrete bidding grid. First, we show (in Section 2) that such an auction environment often becomes a coordination game with multiple equilibria:[1] a high-price equilibrium in which all bids are equal to the value of the auctioned item rounded down to the next allowable bidding point on the grid, and a low-price equilibrium in which all bids are one further increment lower on the grid (and possibly more equilibria at lower points). The coordination game is equivalent to the game known as the stag hunt. The low-price equilibrium is payoff-dominant, but if there is uncertainty about others' bidding strategies, submitting a higher bid has less risk, so the high-price equilibrium can be risk-dominant.

Empirically, we consider auctions for U.S. Treasury bills. Between 1983 and 1998 these auctions used a pay-as-bid format and restricted bids to whole basis points in the discount rate.[2] The final digit of a bid is the number of basis points and, as a benchmark, one should

[1] Despite the voluminous literature on auctions, discrete bidding grids have not been studied extensively, perhaps based on an assumption that the auction outcome will be within one bid increment of the continuous outcome (as shown by Milgrom (2004), p. 273, in the case of open outcry auctions). In the independent private values setting, discrete bidding grids are discussed by Chwe (1989), Rothkopf and Harstad (1994), and Mathews and Sengupta (2008), but multiple equilibria do not arise. In contrast, we show that in pay-as-bid common-value auctions a discrete grid can create additional equilibria — possibly multiple increments away from what would have been the equilibrium bid under a continuous bidding space.

[2] Discount rates, the convention for quoting and bidding for Treasury bills, are similar to yields, and are linearly and inversely related to price. See Section 3.1 for details. A basis point is one hundredth of a percentage point.

expect all digits between 0 and 9 to appear equally often. When examining the announced market-clearing winning bids, we observe (in Section 3) that bidders more frequently submit bids with discount rates ending in a 0, and less frequently submit bids with discount rates ending in a 9. In contrast, the frequency of bids ending in a 1 is not significantly different from the uniform distribution benchmark. We argue that the existence of multiple equilibria gives scope for bidders to choose among the equilibria using a "round-number" focal point. The tendency to submit bids ending with 0 rather than 9 is consistent with a round-number focal point that helps coordination on the payoff-dominant (i.e., low-price/high-discount-rate) equilibrium. However, when available equilibria have final digits of 0 and 1, the high-price equilibrium with the final digit 0 is potentially risk dominant, but the round-number focal point does not cause players to coordinate on this equilibrium more often.[3]

The concept of focal equilibria in coordination games goes back to Schelling (1960). In the presence of multiple equilibria, players will have the tendency to coordinate and choose an equilibrium at a focal point — a salient feature or label on a strategy.[4] Schelling famously gave the example of Grand Central Station as a meeting point in New York City. Focality may be the result of a game's payoffs, or may simply be an arbitrary or culturally-based salient point. When players agree on the relative attractiveness of the equilibria, i.e., they are Pareto ranked, one can argue that the Pareto-dominant equilibrium which has higher payoffs for all players is a natural focal point. Schelling (1960) and Harsanyi and Selten (1988) suggest that players will coordinate on the Pareto-dominant equilibrium. However, Carlsson and van Damme (1993) and Harsanyi (1995) show that a Pareto-inferior equilibrium may be desired if it is risk-dominant, and thus becomes focal. Risk dominance can arise when there is uncertainty over which equilibrium will be played by the opponent. When evaluating strategies, players evaluate which strategy is less risky when the opponent's strategy is unknown. The equilibrium that results is known as the risk-dominant equilibrium and may become focal.

In our empirical setting, a round number for the discount rate of a Treasury bill is more salient than other numbers and acts as a cultural focal point that interacts with the risk-

[3]The finance literature documents a tendency toward trading on round numbers on stock exchanges (e.g., Harris (1991), interpreted as reducing bargaining costs), and IPO auctions (Kandel, Sarig and Wohl (2001), interpreted as a behavioral bias). The only paper that we are aware of that interprets round numbers as a coordination device is Kandel and Marx (1997) in the context of market makers' avoidance of odd eighths on Nasdaq.

[4]For a formal presentation, see Sugden (1995).

dominance and payoff-dominance selection criteria. Our evidence shows that it increases the tendency to coordinate on the payoff-dominant equilibrium. We argue that a round-number focal point reduces uncertainty over the opponent's strategy, and thus makes risk dominance less important.

2 Model: Coordination game in auctions

In order to investigate equilibrium selection and the role of focal points, we first present an auction model to demonstrate how multiple Pareto-ranked equilibria arise. In Section 3 we will consider the setting of U.S. Treasury bill auctions to demonstrate the empirical interaction of equilibrium selection criteria.

Consider an auction in which q items are being sold to $n > q$ bidders. Each item has the same common value, V. Each bidder, $i = 1, \ldots, n$, observes the common value, demands one unit, and submits a bid b_i. The goods are sold to the q highest bidders with each winning bidder paying a price equal to his own bid.

A key feature of the model is that the bids are restricted to a discrete grid. Without loss of generality, we set the bidding increment to one, so that bids must be chosen from the set of integers. Each bidder submits a bid so as to maximize his expected payoff.

Each winning bidder has a payoff of $V - b_i$, where b_i is his own bid. If there is a tie at the margin, then the units at the margin are randomly assigned to the tied bidders. Specifically, suppose m bidders tie for the final $\hat{q} < m$ units with a bid of b_m (and $q - \hat{q}$ bidders each bid a price higher than b_m). Each bidder i who bids higher than b_m has a payoff of $V - b_i$, while the m tied bidders each have an expected payoff of $\frac{\hat{q}}{m}(V - b_m)$.

Define $\underline{V} = int(V)$ as the integer portion of V and $\alpha \in [0, 1)$ as the fractional portion of V, so that $V = \underline{V} + \alpha$. In other words, $\underline{V}$ is the highest point on the bidding grid that is less than or equal to the true value, V.

First, we show that there always exists an equilibrium in which all bidders bid $\underline{V}$.

Lemma 1 (high-price equilibrium) *For any q, $n > q$, and V, it is an equilibrium for all bidders to bid $\underline{V}$.*

Proof. Suppose all other bidders, $j \neq i$, bid $b_j = \underline{V}$. For bidder i, bidding $b_i = \underline{V}$ gives a payoff of $\frac{q}{n}\alpha \geq 0$. There is no profitable deviation: If he bids less, $b_i < \underline{V}$, he loses the auction and gets a payoff of zero. If he bids more, $b_i = \underline{V} + 1$ or higher, he wins the item, but overpays, resulting in negative payoff, since $V - (\underline{V} + 1) = \alpha - 1 < 0$. ■

While $b_i = \underline{V}$ for all i is always an equilibrium, there may also be other equilibria. Specifically, $b_i = \underline{V} - 1$ for all i is often an equilibrium as well. To see this, first consider the following numerical example:

Example 1 *Suppose that a single item is being sold to two potential buyers in a sealed-bid, first-price auction. The item has a value of $91.60 which is common to both bidders and is known to both bidders. In the absence of a discrete bidding grid, the equilibrium auction price would be $91.60, resulting in a zero profit for the bidders.*

When the bids are restricted to whole dollar amounts, $91 is the equilibrium from Lemma 1. If both bidders bid $91, they will each have an expected payoff of 0.5 · \$0.60 = \$0.30. *Neither bidder would deviate, since a bid of $90 (or less) would result in losing the auction and a zero payoff, while a bid of $92 (or more) would result in a negative payoff.*

However, both bidders bidding $90 is also an equilibrium outcome. If both bidders bid $90, they will each have an expected payoff of 0.5 · \$1.60 = \$0.80. *A unilateral deviation upward to $91 only yields $0.60 to the deviating bidder. (A downward deviation results in a zero payoff.) Thus, $90 is sustained as an equilibrium. We refer to $91 as the high-price equilibrium and $90 as the low-price equilibrium. (Note that in this example, $89 cannot be sustained as an equilibrium.)*

	$\underline{V}$	$\underline{V}$-1
$\underline{V}$	$\frac{1}{2}\alpha$, $\frac{1}{2}\alpha$	α , 0
$\underline{V}$-1	0, α	$\frac{1}{2}(\alpha+1)$, $\frac{1}{2}(\alpha+1)$

Figure 1: Payoffs to bidders in an auction with two bidders and one good.

In fact, those two equilibria exist in every auction with one item and two bidders. The payoff matrix of such an auction is presented in Figure 1.[5] As we can see from the payoff matrix, two equilibria exist: one where both bidders bid $\underline{V}$ and one where they both bid $\underline{V} - 1$. Notice that the low-price equilibrium, $\underline{V} - 1$, is payoff dominant because it gives both

[5] This payoff matrix with two equilibria is an example of what is known in the game theory literature as the *stag hunt* game.

players a higher payoff. The $\underline{V}$ equilibrium, however, is often risk dominant because if there is uncertainty over which equilibrium will be played by the opponent, then playing $\underline{V}$ is less risky.

The low-price (payoff-dominant) equilibrium arises since, by construction, $\alpha < 1$, so it is always true that $\alpha < \frac{1}{2}(\alpha + 1)$. However, $\underline{V} - 2$ is not an equilibrium with one item and two bidders since $\alpha + 1 > \frac{1}{2}(\alpha + 2)$, so each bidder would prefer to deviate to $\underline{V} - 1$. More generally, the number of equilibria depends on the number of bidders and the number of units being sold. Proposition 1 formalizes the relation between the set of pure-strategies equilibria and the parameters of the environment.

Proposition 1 *When n bidders bid for $q < n$ items with a common value $V = \underline{V} + \alpha$, the pure-strategy equilibria are for all bidders to bid $\underline{V} - \varphi$ for integers $\varphi \in [0, \bar{\varphi}]$, where*

$$\bar{\varphi} = int\left(\frac{n}{n-q} - \alpha\right).$$

Proof. Consider when all bidders bid $\underline{V} - \varphi$, for some positive integer φ. (Since it is never rational to bid $\underline{V} + 1$ or higher, we only consider $\varphi \geq 0$.) All bidders get expected payoff of $\frac{q}{n}(\alpha + \varphi)$. Deviating downward is never profitable since it leads to a zero profit. Deviating upward and bidding $\underline{V} - \varphi + 1$ gives a profit of $\alpha + \varphi - 1$. This deviation is not profitable when

$$\frac{q}{n}(\alpha + \varphi) \geq \alpha + \varphi - 1 \iff \varphi \leq \frac{n}{n-q} - \alpha \tag{1}$$

and is profitable otherwise. Therefore, all bidders bidding $\underline{V} - \varphi$ is an equilibrium for any φ satisfying (1), and is not an equilibrium otherwise. Let $\bar{\varphi}$ be the the largest integer satisfying (1), i.e., $\bar{\varphi} = int\left(\frac{n}{n-q} - \alpha\right)$. Then for all integers $\varphi \in [0, \bar{\varphi}]$, and only those, $\underline{V} - \varphi$ is an equlibirium.

It can easily be shown that bidders choosing different bids from each other is not an equilibrium, except in the special case of $\underline{V} = V$, i.e., $\alpha = 0$. (In that case, it is an equilibrium for at least q bidders to bid $\underline{V}$ and the remaining bidders to bid lower, with each bidder earning zero profit.) ■

From Proposition 1 we see that the are $\bar{\varphi} + 1$ equilibria, ranging from $\underline{V}$ to $\underline{V} - \bar{\varphi}$. The number of equilibria depends on the number of units being sold, q, the number of bidders, n, and the fractional portion of the value, α.

When there are exactly twice as many bidders as there are units for sale, i.e., $\frac{n}{q} = 2$ (and positive α), there are exactly two equilibria: all bidders bidding $\underline{V}$ and all bidders bidding $\underline{V}-1$. To have a third equilibrium (or more), there must be fewer than twice as many bidders as there are units for sale, i.e., $\frac{n}{q} < 2$. If there are more than twice as many bidders as units for sale, i.e., $\frac{n}{q} > 2$, then there is either one equilibrium ($\underline{V}$) or two equilibria, depending on the fractional value α. The second, low-price equilibrium ($\underline{V} - 1$) exists as long as $\alpha \leq \frac{q}{n-q}$. Thus, regardless of how large $\frac{n}{q}$ is, for sufficiently small α two equilibria exist.

Focal equilibrium

Now that we have established the possibility of multiple equilibria, the question arises of which equilibrium will be seen in practice. The game theory literature proposes various refinements to choose among equilibria, one of which is the concept of focal equilibrium.

Suppose two equilibria exist, $\underline{V}$ and $\underline{V} - 1$ (as in Figure 1). Then $\underline{V} - 1$ (the low-price) equilibrium could be focal when it exists because it is payoff dominant for bidders. Alternatively, $\underline{V}$ (the high-price) equilibrium could be focal when it is risk dominant. Harsanyi and Selten (1988) argue that players will coordinate on the payoff-dominant equilibrium. However, Carlson and van Damme (1993) and Harsanyi (1995) suggest that the risk-dominant equilibrium will be chosen.

We propose that a third possible focal equilibrium is round numbers. Indeed, there is a literature showing that price negotiations often settle on round numbers (e.g., Harris (1991) and Ikenberry and Weston (2007), regarding stock prices). If one of the multiple equilibria is a round number, then the salience of the round number could encourage bidders to choose it as an equilibrium. Thus, all else equal, round numbers will be observed more often than other numbers. In the earlier example, the high-price equilibrium is \$91 and the low-price equilibrium is \$90. If bidders focus on round numbers, then we would be more likely to observe the low-value equilibrium of \$90 than in a similar auction with equilibria that do not include round numbers. It is also possible that if the two equilibria are \$89 and \$90, then the focus on round numbers could lead to the high-price equilibrium. However, as the data in Section 3 show, the effect of a round number on the decision to coordinate on a high-price or low-price equilibrium is not symmetric.

3 Empirical setting and results

3.1 Institutional setting

The regular auction for United States Treasury bills is an archetypical example of a common-value auction. Every week the U.S. Treasury auctions off many billions of dollars of short-term Treasury bills to large dealers. Shortly after buying Treasury bills in the auction, dealers sell them to customers or trade them on the very liquid inter-dealer market. Thus, the Treasury auction is best modeled as common value. Moreover, a liquid secondary market for the same securities (either existing securities or on a forward when-issued basis) is active at the time of the auction, allowing bidders to observe the secondary market value.[6]

From 1983 until 1998, Treasury auctions were held using a “pay-as-bid” format, the multi-unit analogue of first-price auctions.[7] Bids in Treasury bills are submitted as discount rates rather than as prices. The price per \$100 of face value corresponding to the discount rate is given by the linear transformation $price = 100 \times [1 - rate \cdot ndays/360]$, where $ndays$ is the number of days until the bill matures. Over the time period we consider, bids were restricted to multiples of whole basis points, i.e., 3.23% and 3.24% were acceptable bids, but 3.231% was not.[8]

Since discount rates and prices are inversely related, we clarify that in this paper “higher bid” refers to a lower rate — because it corresponds to a higher price — and “lower bid” refers to a higher rate (lower price).

After each auction, the Treasury releases summary statistics, including the market-clearing discount rate. Individual bids are never publicly revealed, but the market-clearing rate is the bid submitted by the marginal bidder. It is the lowest discount rate (highest price) of all winning bids.

3.2 Empirical results

Our data is comprised of the market-clearing bids in 1826 auctions held between 1983 and 1998. These are all of the Treasury bill auctions held under the pay-as-bid format with a

[6]For more details on Treasury auction procedures see Sundaresan (2009). See also Fleming (2007).

[7]The Treasury refers to the pay-as-bid format as “multiple price”. The academic literature often refers to such auctions as “discriminatory”. Starting in late 1998, Treasury bill auctions have been held under the “single-price” or “uniform-price” format, analogous to a second-price auction for a single unit.

[8]Since late 1997 (1998 for Cash Management Bills) the Treasury has changed the grid to multiples of a half basis point.

Table 1: Final digit of market-clearing discount rate.
This table reports the distribution of the final digit of observed market-clearing bids. The data includes all Treasury bill auctions conducted under the pay-as-bid format with a bid increment of one basis point. The data includes 13-week, 26-week and 52-week Treasury bill auctions from 1983 until 1997 and cash management bills from 1983 until 1998.

	Final digit in bid discount rate	Frequency	$H_0: p = 10\%$ p-value
Least frequent	9	140	0.1%
.	6	154	2.6%
.	1	165	17.0%
.	3	175	55.3%
.	7	180	83.9%
.	2	188	67.4%
.	8	188	67.4%
.	5	207	5.7%
.	4	208	4.8%
Most frequent	0	221	0.3%
Join test of discrete uniform distribution		$\chi^2_9 = 31.7$ p-value=0.0%	
# of auctions=1826			

single basis point grid. On average, \$9.9 billion of Treasury bills were sold in each auction, with total bids averaging \$34.4 billion. The median difference between the highest winning rate (i.e., the marginal bid) and the lowest winning rate is 2 basis points across the auctions.

We focus on the distribution of the final digit (single basis points) of the marginal winning bids. Over the 15-year time period of this study, the market-clearing discount rate varied between 2.67% and 11.30%. A priori, we have no reason to believe that any particular digit is more likely than others to occur as the the final digit (i.e., the second decimal place). Thus, it is most natural to start with a null hypothesis that the final digit is uniformly distributed between 0 and 9.

Table 1 displays the observed distribution of the final digits of the marginal winning bids over all 1826 auctions. The hypothesis of a discrete uniform distribution of bids is strongly rejected. Most noticeably, a final digit of 0 is observed far more frequently, while the digit 9 is observed less frequently, than would be expected under a uniform distribution. The frequencies of 0 and 9 are individually statistically significantly different from one tenth at the 1% level. The frequencies of the other possible digits are not statistically different from one tenth.

3.3 Interpretation of results

For the purpose of interpreting the empirical results, we define $g(\underline{V})$ as the final digit of the discount rate corresponding to $\underline{V}$. Because discount rates and prices are inversely related, lowering the bid price by a unit increment on the grid corresponds to a final digit of $g(\underline{V})+1$. Under this notation, lowering a bid by an increment increases the final digit by one; lowering a bid from a final digit of 9 results in a final digit of 0.

Starting from the premise that the final digit of the true value, $g(\underline{V})$, is uniformly distributed, the fact that the bid distribution is not uniform suggests the existence of multiple equilibria. If there were only unique equilibria, there would be no scope for bidders to favor one digit over the others, and observed bids would be uniformly distributed with each final digit observed one tenth of the time.

Empirically, the quantity of bids is almost always at least twice the quantity of securities being auctioned (and is usually more than three times the offering quantity).[9] Thus, theory suggests that there are at most two equilibria. In our context, those two equilibria would have final digits of $g(\underline{V})$ and $g(\underline{V}) + 1$. The low-price equilibrium $g(\underline{V}) + 1$ may be focal because it is payoff dominant, while the high-price equilibrium $g(\underline{V})$ may be focal if it is risk dominant. Importantly, whether an equilibrium is payoff dominant or risk dominant is unrelated to its final digit. Thus, if the choice between high-price and low-price equilibria is unaffected by the final digit of $\underline{V}$, then the distribution of the final digits in the auction outcome would remain uniform.

Since the final digit of 0 is more frequently observed, we argue that when a final digit of 0 is among the possible equilibria, it becomes a focal point and bidders may have a tendency to coordinate on it. For example, if the potential equilibria are 3.29% and 3.30%, an outcome of 3.30% would be more likely, whereas no such effect would be observed if the potential equilibria are 3.28% and 3.29%. This is consistent with the empirical result showing a shift away from 9 as a final digit toward 0. Indeed, the percentage over-representation of 0 is very close to the percentage under-representation of 9. At the same time, we do not see a

[9] In our sample, the aggregate bid quantity was at least twice the offering amount in 99.5% of auctions. It was at least three times the offering amount in 69.5% of auctions.

tendency away from 1 toward 0. Indeed, the observed frequency of the final digit of 1 is not statistically significantly different from one tenth.[10]

The combination of an increased frequency on 0, a decreased frequency on 9, but a frequency on 1 that does not significantly differ from one tenth, has an interesting interpretation when considering the tension between payoff dominance and risk dominance. The result suggests that the focality of 0 helps participants coordinate on the payoff-dominant low-price equilibrium, but does not facilitate coordination on the high-price equilibrium.

The reason for this asymmetry may be that when a round number is one of the equilibria, risk dominance becomes less important. Agents would prefer the low-price payoff-dominant equilibrium in the absence of risk, but may coordinate on the high-price risk-dominant equilibrium because of uncertainty about which equilibrium others might play. When the culturally distinctive final digit of 0 is available as a focal point, it can alleviate the uncertainty about which equilibrium others will play, reducing the rationale for coordinating based on risk dominance. Thus, when the payoff-dominant equilibrium ends in a 0, the agents more easily coordinate on their preferred high-payoff equilibrium. But when the risk-dominant equilibrium ends in a 0, the round-number focal point does not affect players' behavior.

Thus, the results show that people's preference for round numbers — the focality of 0 as a final digit — interacts with other equilibrium selection criteria. In the context of Treasury auctions, we find that the focal point of 0 helps coordination on the low-price payoff-dominant equilibrium, but we do not find evidence that a round number makes coordination on the high-price equilibrium more frequent.

4 Conclusion

In this paper we investigate the interaction between equilibrium selection criteria. In the context of a pay-as-bid (or first-price) auction with a discrete bidding grid, we show that there will often be multiple equilibria. Empirically, we infer the existence of such multiple equilibria in the context of Treasury bill auctions. We find that a final digit of 0 is observed more frequently than would otherwise be expected and a final digit of 9 is less frequently observed. This is consistent with the existence of multiple equilibria and bidders coordinating

[10]When the digits 0 and 9 are excluded, the joint test of discrete uniform distribution over the remaining eight digits does not reject uniformity. Moreover, the fact that the empirical frequency of final digits of 8 and 2 are each in line with a uniform distribution further suggests that there are at most two equilibria.

more often on 0 than on 9, when both are available. At the same time the frequency of 1 as the final digit is not statistically different from the expected one tenth.

We explain this phenomenon as an interaction between equilibrium selection criteria when players in a coordination game choose from among multiple equilibria. On the one hand, it has been hypothesized that they will choose the (low-price) payoff-dominant equilibrium as a focal equilibrium. On the other hand, it is possible that players will focus on the risk-dominant equilibrium. Moreover, players may coordinate on a focal equilibrium that is more salient for arbitrary or cultural reasons, for example, the preference for round numbers ending in 0.

We propose that the focality of round numbers affects agents' decision whether to coordinate on the payoff-dominant equilibrium or the risk-dominant equilibrium. When a round number is one of the equilibria, players focus on it, and thus there is less uncertainty over others' bidding strategy. With less uncertainty, risk-dominance becomes less important as an equilibrium selection criterion, and players can more easily coordinate on the payoff-dominant equilibrium. In short, the evidence suggests that the preference for round numbers facilitates coordination on the payoff-dominant low-price equilibrium but not the risk-dominant high-price equilibrium.

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