Bounded Rationality of Dealers in U.S. Treasury Auctions *

David Goldreich†
Rotman School of Management
University of Toronto

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Abstract

This paper provides evidence of bounded rationality by large dealers in U.S. Treasury auction. These dealers use a heuristic of yield-space bidding which, due to price-rounding rules, leads to biases manifested in two ways: they submit dominated bids, i.e. bids that could be improved without raising the price; and they disregard unevenness in the price grid. Consistent with bounded rationality, bidders are less susceptible to bias when the cost of suboptimal bidding is high. These results show that bounded rationality is a factor even for the most sophisticated institutions that are likely to be important for setting asset prices.

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†Mailing address: Rotman School of Management, University of Toronto, 105 St. George St., Toronto, Ontario, M5S 3E6, Canada. Tel: +1-416-946-0833. E-mail:david.goldreich@rotman.utoronto.ca.

1 Introduction

Behavioral biases among individual investors have been documented by a growing literature. In fact, casual empiricism and simple introspection confirm that economic agents do not always behave fully rationally. However, this type of behavior has been less well documented for investors who are likely to be important in the setting of asset prices, i.e., large sophisticated institutions that participate repeatedly in the marketplace.

Primary bond dealers who participate regularly in U.S. Treasury bill auctions are precisely the type of economic agents that should be expected to act according to models of rational behavior. They regularly bid for billions of dollars worth of securities in competitive auctions, which are held several times each week.

Nevertheless, using data from 1983 to 2003, this paper documents that these dealers were subject to bounded rationality when submitting bids. This is evidenced in a number of ways, most strikingly when they often submitted suboptimal bids: in auctions for very-short-term Treasury bills they could have increased the probability of winning the auction without changing the price they would pay for the securities. The observed suboptimal bids can be explained by bounded rationality. Under bounded rationality, economic agents use heuristics to approximate an optimal action (as proposed by Simon (1955) and discussed recently in Kahneman (2003)). Since Treasury auctions are held in yield space and fixed-income securities are usually quoted in yield space, i.e., the bidding decision is "framed" in yield space, I argue that the heuristic used by dealers in Treasury auctions involves choosing a bid in yield space rather than in price space. As explained below, certain bids in yield space can be shown to be suboptimal because of the details of the yield-price conversion in U.S. Treasury auctions. Moreover, as would be expected under bounded rationality, agents rely on the heuristic less when the cost of an error is higher.

Treasury auctions are conducted in yield space, and bidders that submit the lowest yields are awarded the securities. The yield is then converted into a price to be paid for the securities. The key to the main part of this study is that until a rule change in 2004, the rounding rules were such that for very-short-term Treasury bills,two different yields could correspond to the same price. This occured because the grid is finer in yield space than in price space. Since the auction rules specified that the lowest submitted yields win the auction, there could be two (or more) bids that are the same in price space, of which one will win the auction and one will lose. I refer to the bid with the higher yield as being "dominated" by the bid with the lower yield, as it implies the same price, but a lower probability of winning the auction.

Additionally, even for Treasury bills that are not as short, for example, 13-week bills, the rounding rules resulted in uneven grid between bid prices.

This is more than a simple curiosity in the auction rules since it starkly illustrates how bounded rationality manifests itself in even the largest markets and with the largest market participants. The allocation of many billions of dollars of securities are at stake in each auction; a large proportion of possible bids on the bidding grid were dominated (including more than half of the allowable bids for the weekly four-week Treasury bill auction); and most importantly, auction participants submitted dominated bids regularly.

I examine the phenomenon of dominated bids under both the traditional "multiple-price" auction format and the current "single-price" auction format. Under the multiple-price format, dominated bids are never optimal, and any dominated bids should be viewed as evidence of a behavioral bias. Under the single-price format, in certain cases dominated bids can be consistent with rationality, as they could be chosen with the intention of reducing the probability of being rationed at a lower price rather than as an attempt to win at the bid price. However, such bids should be observed only very infrequently.[1] Empirically, under both auction formats I find a large proportion of dominated bids. In auctions that have the potential for dominated bids, 28% of observed bids in multiple-price auctions and 52% of observed bids in single-price auctions are dominated. These results show that when bidding for Treasury bills, dealers do not fully optimize.

Since boundedly rational agents trade off the costs and benefits of optimization, when a bidder has a high valuation and the benefit to optimizing his bid is large, he should be less likely to use the yield-space heuristic. Therefore, I distinguish between marginal bids (i.e., those at the market clearing price) and inframarginal bids (i.e., winning bids with a bid price above the market clearing price). If inframarginal bidders anticipate a high probability of winning the auction, they have a greater incentive to choose their bids carefully. I find that among inframarginal bids under the multiple-price format dominated bids are relatively infrequent. The bias is reduced - but not eliminated - when there is both a high probability of winning the auction and the price to be paid depends on the bid. In contrast, in single-price auctions, in which the price paid by inframarginal bidders does not directly depend on their bids, there is little incentive to optimize and we observe many dominated bids.

In addition to the frequency of dominated bids, I provide further evidence that bidders use the heuristic of yield space. In the bidding for three-month Treasury bills there was no possibility of dominated bids, but because of rounding in the price-yield conversion, the equally spaced grid in yield space implies an uneven grid in price space. That is, the bid increment alternates between larger and smaller steps. Presumably, one would expect that the frequency of bids would differ between those that are before a large step and those that are before a small step. Nevertheless, in a sample of 760 auctions, I find that both the marginal bids and the inframarginal bids are almost exactly evenly split between bids before

[1] Fleming, Garbade, and Keane (2005) also identify dominated bids, but only in single-price auctions, and do not recognize that under that auction format such bidding can be consistent with full rationality.

the smaller increment and before the larger increment. While this is not definitive on its own, as the optimal bidding strategy is model dependent, it does add to the evidence that bidders think in yield space and ignore the unevenness in the price grid induced by the rounding rule.

Because this paper considers large dealers, it complements the growing literature documenting behavior inconsistent with rational behavior by individual investors.[2] The behavioral finance literature is supported by a very large experimental literature showing how individual decision making is influenced by psychological effects. In addition, a series of papers going back to Simon (1955) argues that economic agents are only boundedly rational, i.e., they only approximately optimize because of the costs of perfect optimization.[3] The effect of framing on decision making, which in the context of this paper is the fact that the auction is conducted in yield space, goes back to Tversky and Kahneman (1981).

One defence of rational asset pricing models is that individual investors are often small infrequent traders and unimportant in the pricing of securities. More important, the argument goes, is the behavior of large institutions that participate on a large scale on a regular basis which is presumably rational. So the main point of this paper is to demonstrate that even large sophisticated dealers, repeatedly bidding for billions of dollars of securities, are subject to bounded rationality.

The rest of the paper proceeds as follows. Section 2 describes the market and the institutional details, in particular the rounding rules which until 2004 allowed for dominated bids. Section 3 presents the empirical evidence of bounded rationality and the use of the

[2]A very small sample of that literature includes the results that investors do not diversify their portfolios sufficiently (French and Poterba (1991), Huberman (2001), and Grinblatt and Keloharju (2001)); investors trade too frequently (Barber and Odean (2000)); and investors trade based on irrelevant past purchase prices (Odean (1998)). See Barberis and Thaler (2003) for a review of theoretical and empirical behavioral finance. See also Shleifer (2000).

[3]See Camerer (1995) for a survey of the experimental literature on psychology and decision making. See Conlisk (1996) for a review of the literature on bounded rationality.

yield space heuristic. In Section 4, I formally show the conditions under which dominated bids are compatible or incompatible with rationality. Section 5 concludes.

2 The auction and the rounding rules

In this section, I describe the Treasury auction, and explain how rounding rules can lead to two bids of the same price that would have different priorities in the auction, i.e., for a bid to be dominated.

This paper focuses on very-short-term Treasury bills, including “cash management bills” (CMBs) of varying maturities auctioned irregularly to manage the short-term cash needs of the government, and four-week Treasury bills auctioned on a weekly basis since mid 2001. The maturity of CMBs can be from a single day to a year, but in recent years the maturity has typically been just a few weeks. The Treasury also auctions 13-week and 26-week bills on a weekly basis, as well as longer-term coupon-bearing notes, but for reasons that will soon be clear these securities are not subject to the phenomenon of dominated bids. However, longer-term bills will be relevant for the other effects documented in this paper.

The issue size of CMBs varies substantially, but in the auctions covered in this study they averaged close to $20 billion per auction. Some 20 to 30 primary dealers submit the vast majority of bids in the auctions for these securities. Table 1 presents summary statistics of Treasury bill auction characteristics from 1983 through 2003. The auctions are oversubscribed, with the quantity of bids exceeding supply by a factor of 2.3 to 4.3 (depending on the subset considered). For the purposes of this paper, the summary statistics are broken up by auction format and by tick size.

The auction proceeds as follows: A number of days prior to the auction the quantity and maturity of the securities to be issued are announced. Immediately before the auction deadline, each bidder submits (possibly multiple) yield-quantity pairs. The yields submitted by bidders are constrained to fall on a discrete grid. Currently, the bidding increment for all Treasury bills is a half basis point (0.005%), e.g., 3.240%, 3.245% etc. In the past, bids were submitted as multiples of a whole basis point.

The auctioneer determines the stop-out yield as the lowest yield at which the quantity demanded equals or exceeds the supply of securities. Bids at lower yields are awarded their demand in full. Bids exactly at the stop-out yield are awarded a fraction of their demand on a pro-rata basis to clear the market.

Under the traditional "multiple-price" format, used until November 1998, each winning bidder pays a price corresponding to his submitted yield. Under the current "single-price" auction format, all winning bidders pay a price corresponding to the market-clearing stop-out yield.[4] This paper considers auctions under both formats.

Given the yield in the auction, before rounding, the price per \$100 of face value is

$$P = 100(1 - yield \times ndays/360),$$

where *ndays* is the maturity of the bill measured in days. The yield used in this calculation is known as the "banker's discount rate".[5] Importantly for the purpose of this paper, until a rule change in 2004, the price was then rounded to the nearest \$0.001. (It is now rounded much more finely to six decimal places to the right of the decimal for each \$100 in face value.)

The rounding of P to the nearest \$0.001 is crucial to the present puzzle, since it generates a second grid: first, the bids are constrained to a grid of one basis point (or, more recently, a half basis point) in yield space (the "bidding grid"), second, the corresponding dollar price is rounded to fall on a grid with a tick size of \$0.001 (the "pricing grid").

[4]In the auction literature, these formats are usually referred to as "uniform-price" and "discriminatory" mechanisms. In this paper, I use the Treasury's terminology.

[5]See Federal Register (1999) for the official rules of the auction including the yield-price conversion.

Consider a hypothetical Treasury bill with 36 days until maturity and a tick size of one basis point. For such a security, each basis point increment in the yield corresponds to a reduction of exactly $0.001 in price. Hence, there is a one-to-one correspondence between the bidding grid and the pricing grid. However, for bills of less than 36-days maturity, each basis point change in yield corresponds to a change of less than $0.001 in price space. This implies that two different bids in yield space could be rounded to the same price, even though a pair of such bids would have different priorities in the determination of who wins the auction. The bid with the higher yield is dominated by the bid with the lower yield, since by lowering the yield a bidder could increase his probability of winning the auction without changing the price.

Under the half basis point tick size, the same reasoning implies that during the sample period dominated bids exist for auctions of securities with maturities of less than 72 days. For four-week Treasury bills (which have a half basis point tick size), for example, the pricing grid is $72/28 = 2.57$ times as wide as the bidding grid. As a result, some 61% $(= (2.57 - 1)/2.57)$ of all possible bids are dominated. See Table 2 for a numerical example of dominated bids.

The interpretation of dominated bids depends on the auction format. Under the multiple-price format, a rational bidder would never submit a dominated bid. Indeed, by submitting a bid at the next lower yield increment (corresponding to the same price), he could increase the likelihood of being awarded the securities precisely in those states when the auction outcome implies a high value to the securities. Under the single-price format dominated bids are not necessarily suboptimal. It can be rational if the bidder wants to ensure that his allocation will not be reduced by rationing at the next lower price, but values the bills at less than the price of his bid. In Section 4, I formalize these arguments.

The rounding rule has a second implication. Indeed, for longer term bills there is no possibility of dominated bids. Nevertheless, the rounding rule results in a bidding grid that is not uniformly spaced in price space. (Indeed, dominated bids are merely an extreme example of unevenly spaced grid in price space.) For example, in a 13-week Treasury bill auction with a one basis point bidding increment in yield space, the increments in price space would alternate between $0.002 and $0.003.

3 Empirical results

In this section, I show that the observed bidding patterns in Treasury bill auctions are consistent with boundedly rational bidders using a heuristic of choosing bids in yield space. Such bidders are more likely to abandon the heuristic and optimize when the cost of deviating from optimality is high. The main evidence is the frequency of dominated bids. As corroborating evidence, I discuss the bidding patterns in auctions with unevenly spaced grids in price space.

The data used in this paper are drawn from the summary statistics revealed by the Treasury after each auction. The statistics released by the Treasury depend on the auction format. For multiple-price auctions, the Treasury reveals the market-clearing yield and the lowest winning yield. For single-price auctions, the statistics include the market-clearing yield, the median winning yield, and the 95th-percentile winning yield (i.e., the lowest winning yield excluding the 5% tail of winning bids). Individual bids are never revealed. So although we do not observe each individual bid, the summary statistics allow one to observe the yield of the marginal bidder, as well as certain inframarginal bids.

3.1 Dominated bids

As explained above, since there is both a bidding grid in yield space and a second grid in price space, it is possible for a bid to be dominated when the maturity of a security is very short. In this section, I examine the frequency of dominated bids under both the multiple-price auction format and the single-price format. I distinguish between marginal bids and inframarginal bids to determine how a bidder's valuation affects his tendency to submit a dominated bid.

First consider multiple-price auctions. In the sample, there are 65 CMB auctions conducted under the multiple-price format. If bidders are perfectly rational, we should not observe any dominated bids. At the other extreme, if bidders only think in terms of yield, we should observe dominated bids in proportion to the number of potential bids on the bidding grid that would be dominated. The middle case is that bidders are boundedly rational and, using a heuristic of choosing bids in yield space, are more likely to submit dominated bids when the cost of doing so is low, but less likely to do so when the cost is high.

The frequency of dominated bids is reported in Panel A of Table 3. Among market-clearing bids, 43% are dominated. Among observed inframarginal bids, 14% are dominated. The existence of dominated bids, especially the high percentage among market-clearing bids is strong evidence that bidders do not fully optimize, but are subject to bounded rationality.[6]

Indeed, bidders do not think exclusively in yield space. Indeed, of all possible yields on the bidding grid for these bills, an average of 58% would be dominated, so if bidders completely disregard the rounding rule, approximately 58% of all observed bids should be dominated. In fact, fewer bids are dominated, indicating that at least some bidders are aware of this possibility and, at least sometimes, avoid dominated bids. This difference between the potential frequency of dominated bids and the observed frequency is highly statistically significant.

[6]One could argue that bidders want to lose the auction because they view it as negative NPV, and are only bidding to satisfy the requirement that primary dealers bid meaningfully. However, the high levels of oversubscription, and the fact that the observed bids are from those that actually win the auction, suggest that bidders are actively trying to win the auctions. Moreover there is substantial evidence that bidders in Treasury auctions earn positive rents on average. (See Cammack (1991), Goldreich (2007), Nyborg and Sundaresan (1996).)

Moreover, the difference in the frequency of dominated bids between marginal and inframarginal bids is statistically significant at the 1% level. This is suggestive of bounded rationality. Inframarginal bidders are those that choose to bid in a manner that has a high probability of winning, while at the same time paying the higher price of such a bid (because of the multiple-price format). Therefore, an inframarginal investor has incentive to choose a bid that simultaneously increases the probability of winning without raising the price excessively. Dominated bids are precisely those that an inframarginal bidder should avoid, as they lower the probability of winning without lowering the price. In comparison, marginal (market-clearing) bidders have a lower ex-ante probability of winning the auction, so their incentive to choose an undominated bid decreases accordingly. Thus, under the multiple-price format, bounded rationality implies that marginal bidders should be more likely to use the yield-space heuristic.

Panel B of Table 3 presents the results for 172 single-price auctions. Note that under this format, the observed inframarginal bids in the data include the median and 95th-percentile winning yields, rather than the lowest winning yield.

The results are superficially similar to those of the multiple-price auction. A large percentage of both marginal and inframarginal bids are dominated. The percentage of dominated bids among market-clearing bids and median winning bids is statistically significantly less than the 64 percent of all possible yields on the bidding grid that would be dominated. (In the case of the 95th percentile winning bids, this difference is only significant at the 10% level.)

The striking difference in contrast to the multiple-price auction is that under the single-price auction, inframarginal bids are *more* likely to be dominated than market-clearing bids. This difference is perfectly consistent with bounded rationality. Inframarginal bidders may be those with a high private valuation who knowingly submit a bid with a very high probability of winning, but since under the single-price format all winning bidders pay the same price, they have little incentive to carefully choose a bid from among those that are likely to win. However, marginal bidders are more likely to be those that only want to win if the price is sufficiently low, and are more likely to exert the effort to choose a bid carefully. Thus, marginal bidders are less likely to use a yield-space heuristic and submit dominated bids.

To summarize, the observation that bidders often submit dominated bids – especially under the multiple-price format is evidence of bounded rationality and the use of the yield-space heuristic. However, it is the pattern in the frequency of dominated bids – more frequent for marginal bids under the multiple-price format and more frequent for inframarginal bids under the single-price format – that provides even stronger and more nuanced evidence. Bidders use the yield-space heuristic and submit dominated bids more frequently when the benefits of optimization are low.

This distinction between the bidding strategies of inframarginal bidders across auction formats, that they are more cautious under the multiple-price format, is consistent with patterns in the observed spread of winning bids. The difference between the marginal bid and the most inframarginal observed bid is typically much wider under the single-price format than under the multiple-price format (even though the most inframarginal observed bids in single-price auctions excludes a 5% tail of winning bids). Although this could be interpreted as arising from more price uncertainty surrounding single-price auctions, in light of the evidence above it seems more likely that inframarginal bidders under the single-price format choose to bid more aggressively since their bids are unlikely to directly affect the price to be paid.

Furthermore, perhaps tangentially, the differences between marginal and inframarginal bidders are informative to the literature about how the market for Treasury securities should be modeled, and the nature of Treasury securities as common-value or private-value goods.

Treasury bills are often thought of as the quintessential common value good that is being purchased for resale. However, the results in this section suggest that bidders are not homogenous, and that inframarginal bidders are not simply those that happen to observe a high signal. Instead, they actively choose to submit a bid that has a higher probability of winning the auction. This would occur if, besides the obvious common-value component, Treasury securities also have a private value component. Bidders that have a high private value submit higher bids to ensure winning the auction and are more likely to be inframarginal.

While the evidence of bounded rationality inherent in dominated bids should be clear, it is also important to discuss economic significance. The pricing grid over the sample period is very small as a percentage of face value – only \$0.001 per \$100 of face value, but it is large when multiplied by more than \$40 billion of Treasury bills that were issued each week (and even larger amounts currently). However, it is not the dollar value of the tick size that is important here, since the phenomenon of dominated bids relates to allocation rather than price.

The relevant economic measure is the quantity of bills awarded to bids at the margin. Bidders who submit a dominated bid are reducing their probability of being awarded these securities. Although this quantity is not known, it can be roughly estimated based on the summary statistics revealed after each auction. On average, the difference between the market-clearing bid and the median winning bid is 1.5 basis points, or three ticks, for four-week Treasury bills. Conservatively assuming that the distribution of bids is uniform over this range, this corresponds to \$2.7 billion at the margin for a typical \$16 billion auction.[7] After accounting for rationing among bidders at the margin, there are some \$1.3 billion of securities that are allocated to bidders who submit the market-clearing bid. A bidder who

[7]Since the distribution of winning bids is right-skewed (as evidenced by the larger average difference between the median winning bid and the 95th percentile winning bid), the assumption of a uniform distribution is conservative and it is likely that the average quantity bid at the market-clearing bid exceeds \$2.7 billion.

submits a dominated bid reduces his probability of being allocated these securities.

3.2 Unevenly spaced bidding grid

The actions of bidders in the presence of the rounding rule leads to additional evidence that bidders use a heuristic of determining their bids in yield space.

The rounding rule in the yield-price conversion described above leads to an unevenly spaced bidding grid in price space. When prices are rounded to the nearest \$0.001, the grid step in price space (i.e., the price difference between two adjacent bids) is sometimes larger and sometimes smaller. For example, changing a bid by one basis in yield may correspond to a price change of \$0.001, but the next basis point change in yield may correspond to an additional \$0.002, or vice versa. (See Table 4 for an example of uneven grid steps for 13-week bills.) Dominated bids are simply an extreme form of this in which the size of a step is zero.

The average step size in price space corresponding to an incremental change in the bid yield is $\$0.001 \times (\delta \times ndays/36)$, where δ is the bidding increment in basis points and $ndays$ is the maturity of the bill in days. Only when the maturity of a bill is such that $\delta \times ndays/36$ is an integer are grid steps of constant size in price space. Otherwise there will be large steps and small steps. For thirteen-week Treasury bills (under a one basis point bidding grid), the step size averages very close to \$0.0025, so the step size almost always alternates between \$0.002 and \$0.003.

The equilibrium bidding strategy in the presence of an uneven grid is model dependent. For the purposes of this paper I do not impose any one model. Nevertheless, a bidder facing an uneven grid should consider the step size when evaluating potential bids. A bidder may optimally raise his yield by one tick if it corresponds to a substantially lower price, but not if it lowers the price by only a small amount. Of course, in equilibrium he will have to consider how other bidders also respond to the uneven grid.

However, if a bidder uses a yield-space heuristic, the distribution of bids in yield space will be unaffected by the uneven steps in the pricing grid.

Since the pricing grid alternates between \$0.002 and \$0.003 for the 13-week bill (for a one basis point tick size, as was the case from 1983 to 1997), it provides an opportunity to observe bidders' response to an uneven grid. A bid can either be before a "large" step or before a "small" step (i.e., so that an increase in yield of one basis point corresponds to a reduction of \$0.003 or \$0.002 in price space, respectively). Table 5 reports the distribution of bids between those before a large step and those before a small step for 760 13-week auctions conducted under the multiple-price format. I exclude the small number of bids that are in between two large steps or in between two small steps. If bidders rationally optimize, we are likely to observe different probabilities of bids before a large step and bids before a small step. However, if bidders think in yield space, they do not distinguish between large and small steps, and we should observe each with equal probability.

I find that almost exactly 50% of bids are before large steps and 50% before small steps. This holds for both marginal and inframarginal bids. This is consistent with bidders using a yield-space heuristic and not paying attention to the uneven grid in price space. Of course, without a well-defined alternative hypothesis, which is avoided here to keep the argument model independent, one cannot reject rationality. However, the tight confidence bands around 50% are suggestive of bidders thinking in yield-space.

4 Are dominated bids ever rational?

One of the central points of this paper is that the observed frequency of dominated bids is evidence of bounded rationality. In this section, in a fairly general setting, I show that dominated bids are not consistent with optimal bidding under the multiple-price auction format, and I show the conditions under which dominated bids are compatible or incompatible with optimization under the single-price format.

Consider an expected profit maximizing bidder who must choose a price for his bid. Normalize his bid quantity to one, and for simplicity, assume that this bid quantity is sufficiently small that it does not affect the market-clearing stop-out price.

Denote the bidding grid (in price space) of allowable prices as P_i, $i = 1, 2, 3...$, where P_i is (weakly) increasing in i. The probability, from the bidder's perspective, of P_i being the market-clearing price is denoted $\pi_i (> 0)$.

Because Treasury bills presumably have a common value component, the bidder's valuation of the security depends on the outcome of the auction. Denote V_i as the value of the security to the bidder if the market-clearing price is P_i. (The function $V_i (P_i)$ is likely to depend on the auction format.)

Denote the bid price submitted by the bidder as P_b. If this bid is above the market-clearing price (or more precisely, if $b > m$, where P_m is the market-clearing price), the bidder wins the auction and is awarded the security. Because in practice there is always rationing at the market-clearing price, assume that if the bid is equal to the market clearing price, (i.e., if $b = m$) the bidder is awarded half a security.

The price paid by a winning bidder depends on the auction format.

4.1 Multiple-price format

Under the multiple-price format, each winning bidder pays the price in his bid regardless of the market-clearing price. Thus, for a bid price P_b, the expected profit to a bidder is

$$Profit_b = \sum_{j=1}^{b-1} \pi_j (V_j - P_b) + \frac{1}{2} \pi_b (V_b - P_b) \tag{1}$$

For a bid P_b to be optimal, it is necessary that

$$\mathit{Profit}_b > 0 \tag{2}$$

$$\mathit{Profit}_b \geq \mathit{Profit}_{b+1} \tag{3}$$

$$\mathit{Profit}_b \geq \mathit{Profit}_{b-1} \tag{4}$$

Let us now consider the possibility of a bidder submitting a dominated bid P_b, where $P_b = P_{b+1}$.

Consider the expected profit (1). By assumption, V_j is increasing in j. So $(V_b - P_b)$ is similarly increasing in j. Since Profit_b is positive (from (2)) and the weights π_j are non-negative, the state with the highest per-unit profit must have positive profit, i.e.,

$$V_b - P_b > 0. \tag{5}$$

For the dominated bid to be optimal, $\mathit{Profit}_{b+1}-$ Profit_b must be negative. From (1), and using the definition of a dominated bid, $P_b = P_{b+1}$,

$$\mathit{Profit}_{b+1} - \mathit{Profit}_b = \frac{1}{2}\pi_b \left(V_b - P_b\right) + \frac{1}{2}\pi_{b+1} \left(V_{b+1} - P_b\right) \tag{6}$$

From (5), the first term is positive, and since V_i is an increasing function, the second term is also positive.

Thus $\mathit{Profit}_{b+1} - \mathit{Profit}_b > 0$, contradicting the optimality of the dominated bid.

Intuitively, the dominated bid P_b is suboptimal for the following reason. First, a bid of P_{b+1} does not change the profit when the market-clearing bid is below P_b. However, it awards more securities when the market-clearing bid is P_b or P_{b+1}. But since these are the states in which V_i is highest, and since $P_{b+1} = P_b$, then if it is worthwhile to bid P_b (i.e., if expected profits are positive), changing the bid to P_{b+1} can only increase expected profits.

Thus, observed dominated bids under the multiple-price format are always inconsistent with dealers acting optimally.

4.2 Single-price format

Under the single-price format, all winning bidders pay the market clearing price. For a bid price P_b, the expected profit to a bidder is

$$Profit_b = \sum_{j=1}^{b-1} \pi_j \left(V_j - P_j\right) + \frac{1}{2}\pi_b \left(V_b - P_b\right) \tag{7}$$

Unlike the case of the multiple-price format I show below that under certain conditions it is possible for a rational bidder to submit a dominated bid under the single-price format. This can occur because increasing a bid from P_{b-1} to P_b (which is dominated and equal to P_{b+1}) has two effects on the bidder's allocation. It results in a (rationed) allocation when the market clearing bid is P_b, and it also results in a full allocation (rather than rationing) when the market-clearing bid is P_{b-1}. Thus, a bidder may choose to submit a dominated bid P_b if the profit conditional on a market clearing price of P_b is negative, but the profit conditional on a market clearing price of P_{b-1} is positive and more than offsets the negative profit. Such a bidder will not increase his bid to the undominated P_{t+1} since that only increases his allocation in those states for which his profit is negative.

I now formalize the argument to show the conditions under which dominated bids are consistent with rationality and the conditions under which they are not.

As before, for a bid P_b to be optimal, it is necessary that

$$\mathit{Profit}_b > 0 \tag{8}$$

$$\mathit{Profit}_b \geq \mathit{Profit}_{b+1} \tag{9}$$

$$\mathit{Profit}_b \geq \mathit{Profit}_{b-1} \tag{10}$$

Substitute (7) into (9) and (10) to obtain

$$\frac{1}{2}\pi_{b+1}\left(V_{b+1} - P_{b+1}\right) + \frac{1}{2}\pi_b\left(V_b - P_b\right) \leq 0 \tag{11}$$

$$\frac{1}{2}\pi_b\left(V_b - P_b\right) + \frac{1}{2}\pi_{b-1}\left(V_{b-1} - P_{b-1}\right) \geq 0 \tag{12}$$

Noting that $(V_i - P_i)$ decreases in i, a bid P_b is optimal if both $\pi_{b+1}\left(V_{b+1} - P_{b+1}\right)$ is sufficiently negative and not fully offset by (a possibly positive) $\pi_b\left(V_b - P_b\right)$, and also $\pi_{b-1}\left(V_{b-1} - P_{b-1}\right)$ is sufficiently positive and not fully offset by (a possibly negative) $\pi_b\left(V_b - P_b\right)$.

For the purposes of clarity and to create a measure of the possibility that a bid will be optimal, let us add some structure and make some simplifying assumptions. Assume the following: $\pi_{b-1} = \pi_b = \pi_{b+1}$; the tick size in price space (other than for dominated bids) is a constant $\Delta = P_i - P_{i-1}$; and the increase in the bidder's valuation for an increase of δ in price (other than for dominated bids) is a constant $\alpha = V_i - V_{i-1}$, where $0 \leq \alpha < \Delta$.

With these assumptions, substituting into (11) and (12) results in

$$(\alpha - \Delta) + 2\left(V_b - P_b\right) \leq 0 \tag{13}$$

$$(\Delta - \alpha) + 2\left(V_b - P_b\right) \geq 0 \tag{14}$$

or,

$$\frac{-(\Delta - \alpha)}{2} \le (V_b - P_b) \le \frac{(\Delta - \alpha)}{2} \tag{15}$$

Under this condition, i.e., if the value of the security to the bidder is close to the bid price, P_b is optimal. The range of possible values for $V_b - P_b$ is of width $\Delta - \alpha$. (Note that if we allow α to be larger than Δ then there would be no finite optimal bid, as bidders would have upward sloping demand curves.)

Now consider the possibility of a dominated bid P_b. In such case $P_b - P_{b-1} = \Delta$, and $P_{b+1} - P_b = 0$. We also have to address the value function V_i. Below the dominated bid, $V_b - V_{b-1} = \alpha$ as before, but $V_{b+1} - V_b$ is ambiguous. An increase in the market-clearing bid from P_b to P_{b+1} signifies increased demand, so the value of the security surely increases. However, since this doesn't correspond to an increase in the actual price of the market-clearing bid, the increase in value may be less than α. Denote the value increase as $\beta\alpha = V_{b+1} - V_b$, where $0 < \beta < 1$. In other words, β represents the incremental increase in the value of the security as a proportion of the "normal" incremental value increase.

It follows that a dominated bid, P_b, is optimal if

$$\frac{-(\Delta - \alpha)}{2} \le (V_b - P_b) \le \frac{(0 - \beta\alpha)}{2} \tag{16}$$

The width of the range of possible values of $V_b - P_b$ which allow dominated bids to be optimal is

$$\frac{\Delta - (1 + \beta)\,\alpha}{2}$$

which is much narrower than the width $\Delta - \alpha$ that allows a bid to be optimal in the absence of dominated bids.

In particular, if $\Delta < (1 + \beta)\,\alpha$, then dominated bids are never optimal. In words, for an increase in the market-clearing bid from P_{b-1} to P_{b+1}, if the increase in the bill's value to the bidder exceeds a single price tick Δ, then dominated bids are incompatible with rationality. If the inequality is reversed, then dominated bids would be observed, albeit with less frequency than undominated bids.

So while the existence of dominated bids under the single-price format is not *per se* proof of behavioral bias, the frequency of the dominated bids, the differences between marginal and inframarginal bids, and the existence of dominated bids under the multiple-price format, is evidence of behavioral bias and is consistent with bounded rationality.

5 Conclusion

Complementing the literature on bounded rationality in individual investors, I have shown that even large sophisticated institutions are subject to bounded rationality. In auctions for U.S. Treasury bills, very large primary bond dealers regularly submit dominated bids which reduce their chance of winning the auction while still requiring the same price to be paid conditional on winning. When considering the allocation of bills to primary dealers, this is a very large effect.

Most importantly, when comparing single-price and multiple-price auctions, and when comparing marginal bids and inframarginal bids, I find that dealers are less likely to be subject to this bias when the cost of suboptimal bidding is higher.

Additionally, dealer disregard uneven steps in the pricing grid when bidding for the regular 13-week Treasury bills.

These results are consistent with boundedly rational dealers. When faced with a bidding problem framed in yield space, they use a yield-space heuristic when choosing their bids.

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Table 1: Short-Term Treasury Bill Auction Summary Statistics

This table summarizes the auction characteristics for Treasury bills, including cash management bills (CMBs), four-week bills, and longer term (13-week, 26-week and 52-week) bills from April 1983 to December 2003. (Four-week bills were first auctioned in July 2001. Fifty-two week bills were discontinued in February 2001.) The statistics for CMBs are also reported excluding those with a maturity long enough to preclude the possibility of dominated bids. Under the multiple-price format, each winning bidder pays a price corresponding to his submitted bid. Under the single-price format, all winning bidders pay a price corresponding to the yield of the market-clearing bid. *Tick size* refers to the grid in yield space. *Maturity* is the number of days in the life of the security. *Bid-to-cover* is the ratio of the quantity of tenders to supply. *Bid spread* is the difference between the market clearing yield and the lowest winning yield (or, under the single-price format, the difference between the market clearing yield and the 95th percentile winning yield).

	Date range	# of auctions	Auction format	Tick size (basis points)	Avg auction size ($ billion)	Avg maturity (days)	Average bid-to-cover	Bid spread (basis points)
CMBs	5/1983 - 8/1998	116	Multiple Price	1.0	11.6	44.3	4.3	4.6
	exclud. > 35 days	65			12.4	15.2	4.3	6.0
	11/1998 - 3/2002	33	Single Price	1.0	24.5	26.2	2.4	5.9
	exclud. > 35 days	24			24.1	14.7	2.5	10.1
	4/2002 - 12/2003	22	Single Price	0.5	16.6	8.7	3.2	4.5
	exclud. > 71 days	22			16.6	8.7	3.2	4.5
4-Week Bills	7/2001-12/2003	126	Single Price	0.5	16.3	28.0	2.3	5.0
Longer-Term Bills	4/1983-11/1997	1710	Multiple Price	1.0	9.5	161.8	3.4	3.1
	11/1997-10/1998	114	Multiple Price	0.5	7.4	160.4	3.3	2.2
	11/1998-12/2003	592	Single Price	0.5	12.0	145.0	2.0	6.0

Table 2: Example of Possible Dominated Bids

This table displays a portion of the bidding grid for a 20-day cash management bill to illustrate the possibility of dominated bids. In this example, bids are submitted in one basis point increments. Prices are rounded to the nearest $.001 per $100 face value. Bids marked with "D" are "dominated".

Rate	Price (unrounded)	Price (rounded)
3.02%	99.83222	99.832
3.03%	99.83167	99.832 D
3.04%	99.83111	99.831
3.05%	99.83056	99.831 D
3.06%	99.83000	99.830
3.07%	99.82944	99.829
3.08%	99.82889	99.829 D
3.09%	99.82833	99.828

Table 3: Frequency of Dominated Bids

This table displays the frequency of dominated bids among observed bids for short-term Treasury bill auctions under both auction formats. Cash management bill (CMB) auctions are included only when the maturity is short enough for the possibility of dominated bids. Under the multiple-price format, the market-clearing yield and the lowest winning yield are observed. Under the single-price format, the observed bids include the market-clearing yield (i.e., highest winning yield), the median winning yield, and the 95th-percentile winning yield (i.e., the lowest winning yield excluding the 5% tail of winning bids).

Panel A: Multiple-price auctions (cash management bills)

	Number of dominated bids	Percentage of observed bids that are dominated	Percentage of bids on the bidding grid that would be dominated (assuming random bidding)	P-value of difference
Market clearing yield	28	43%	58%	1.08%
Lowest winning yield	9	14%	58%	0.00%
# of auctions = 65				

Panel B: Single-price auctions (cash management bills and four-week bills)

	Number of dominated bids	Percentage of observed bids that are dominated	Percentage of bids on the bidding grid that would be dominated (assuming random bidding)	P-value of difference
Market clearing yield	75	44%	64%	0.00%
Median winning yield	92	53%	64%	0.47%
95th percentile winning yield	100	58%	64%	9.51%
# of auctions = 172				

Table 4: Example of Uneven Grid Steps (13-Week Treasury Bill)

This table displays a portion of the bidding grid for 91-day Treasury bills to illustrate the uneven grid steps in price space. In this illustration, the bidding grid is one basis point in yield space. Prices are rounded to the nearest $.001 per $100 face value. *Step size* is the price difference between two adjacent bids.

Rate	Price		Step size
4.00%	98.989		
		}	0.003
4.01%	98.986		
		}	0.002
4.02%	98.984		
		}	0.003
4.03%	98.981		
		}	0.002
4.04%	98.979		
		}	0.003
4.05%	98.976		
		}	0.002
4.06%	98.974		

Table 5: Bidding and Uneven Grid Steps

Under a one basis point bidding grid (as was the case from 1983 to 1997), the pricing grid for 91-day Treasury bills alternates between \$0.003 and \$0.002 with few exceptions. A *bid before a "large" step* is an observed bid such that an increase in one basis point in yield would correspond to \$0.003 decrease in price, i.e., $P(r) - P(r+\delta) = .003$, where $P(r)$ is the price corresponding to a yield bid of r, and δ is the one basis point bidding increment. A *bid before a "small" step* is an observed bid such that an increase in one basis point in yield would correspond to \$0.002 decrease in price, i.e., $P(r) - P(r+\delta) = .002$. Occasionally, bids are in between two large steps or in between two small steps and are thus excluded from the data. The auctions were conducted under the multiple-price format, and the observed bids are the market clearing yield and the lowest winning yield.

	Bids before a "large" step	Bids before a "small" step	H_0: p = 50%	
			χ_1^2	P-value
Market clearing yield	353 (49.2%)	365 (50.8%)	0.201	65.4%
Lowest winning yield	358 (50.7%)	348 (49.3%)	0.142	70.7%
# of auctions = 760				