JOURNAL OF FINANCIAL AND QUANTITATIVE ANALYSIS Vol. 42, No. 2, June 2007, pp. 443–466

Underpricing in Discriminatory and Uniform-Price Treasury Auctions

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Abstract

This paper compares the newer uniform-price U.S. Treasury auctions to the traditional discriminatory mechanism and examines the extent to which the auction mechanisms are responsible for underpricing. Empirically, I find that even for the newer uniform-price auctions, the average price received by the Treasury is less than the price of the same securities in the concurrent secondary market although this underpricing is reduced by half relative to the older mechanism. From the summary statistics released by the Treasury, I calibrate common value auction models for the two mechanisms and predict the level of underpricing in each auction. I find that the observed magnitude of underpricing in the auctions is consistent with the model's predictions.

I. Introduction

Every year, the U.S. government auctions some three trillion dollars of Treasury securities to finance the public debt. Given the amounts involved, any mispricing implies a large wealth transfer. For instance, underpricing of one cent per $100 of face value results in an annual transfer of about $300 million from the government to auction participants.

A number of papers have documented underpricing in Treasury auctions under the older discriminatory auction mechanism. The mean winning yield in the auction, on average, exceeded the yield in the secondary and "when-issued" markets.[1] At least partially in response to this underpricing, the U.S. Treasury changed the auction format from the discriminatory auction, in which each winning bidder pays his own bid, to the uniform-price auction in which all winning

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I thank Julian Franks, Rick Green, Denis Gromb, Burton Hollifield, Ronen Israel, Kjell Nyborg, Uday Rajan, Matthew Rhodes-Kropf (the referee), Bryan Routledge, Kristian Rydqvist, Patrik Sandas, Chester Spatt, and seminar participants at Carnegie Mellon University, University of Iowa, London Business School, London School of Economics, Northwestern University, University of Pennsylvania, University of Pittsburgh, Rutgers University, Tel Aviv University, University of Texas at Dallas, University of Toronto, University of Washington, the European Finance Association Meetings, the American Finance Association Meetings, and the Treasury Auction Conference in Jerusalem for helpful discussions. Remaining errors are my own.

[1]The when-issued market is a forward market for the Treasury securities that are being auctioned with delivery to take place on the issue date.

bidders pay the same market-clearing price.[2] In contrast to the U.S., many government securities auctions worldwide still use the discriminatory mechanism.[3]

This paper makes two main contributions. First, I document that underpricing remains after the switch from discriminatory to uniform-price auctions although the amount of underpricing is reduced by about half. Second, I show that the *magnitude* of underpricing under each auction format is consistent with a multi-unit common values auction model with unit demand.

It is not surprising per se that there is underpricing under the discriminatory mechanism. Auction theory predicts that revenue in a common value auction depends on the mechanism. In particular, with risk-neutral investors the discriminatory auction is expected to result in underpricing.[4] However, the questions addressed in this paper are if underpricing has been reduced by the switch to the uniform-price mechanism, and if the magnitude of observed underpricing (under each mechanism) is consistent with auction theory.

Aside from the practical concern, the empirical question of whether underpricing has been reduced is of interest because of conflicting theoretical predictions. While traditional (unit demand) theory predicts less underpricing in uniform-price auctions, one prominent strand of the literature stresses that if bidders are allowed to submit entire demand curves, "collusive" equilibria can arise in uniform-price auctions and result in arbitrarily large underpricing.[5]

In fact, much of the existing theoretical Treasury auction literature comparing the two mechanisms, motivated by the policy debate, focuses on issues such as potential collusion, the interaction between the auction and secondary markets, and short squeezes. One motivation of this paper is to better understand the way in which the auction mechanism affects revenue in the absence of strategic behavior. Most importantly, by limiting myself to a relatively simple description of the auction, I can calibrate the model and predict the magnitude of underpricing for each individual auction and show that these other economic influences are unlikely to be needed to explain the data.

I model the auction in the traditional multi-unit common value auction framework. My innovation is to specify a model parameterization that predicts the magnitude of underpricing in each individual auction as a function of the dispersion of bidders' signals and the extent of competition in the auction. This specification allows the model to be calibrated with the statistics released after each auction, including the bid-to-cover ratio[6] and the dispersion of bids. By calibrating the model to the announced results of each individual auction, I am able to show that the magnitude of observed underpricing, both on average and in the cross section, is consistent with the model.

In the discriminatory auction model, the Treasury auctions a quantity of securities to a number of bidders, each of whom gets a noisy signal of the common

[2]Highlighting the uncertainty regarding the relative merits of the two auction formats, the U.S. Treasury reverted back to the discriminatory mechanism for its buyback program to repurchase outstanding 30-year bonds (see Hanke (2002)).

[3]See Bartolini and Cottarelli (1997).

[4]See Milgrom and Weber (2000) and Weber (1983) among others. For the single unit auction, the analogous comparison is between first- and second-price auctions (Milgrom and Weber (1982)).

[5]See Back and Zender (1993) who base their argument on Wilson (1979).

[6]The bid-to-cover is the ratio of aggregate bids tendered to the supply of securities offered.

value of the security. Each bidder bids for one unit at a price that maximizes his expected profit. The highest bidders pay their bids and are awarded the securities. If all bidders knew the true value of the securities, they would compete away all profits. Since there is uncertainty, there will be numerous winning prices so each bidder reduces his bid in an attempt to be among the lowest winning bidders. In equilibrium, all bidders reduce their bids—even beyond the winner's curse—and earn positive expected profits, i.e., underpricing occurs. The level of underpricing depends on the bid-to-cover ratio and the noise in bidders' signals.

In uniform-price auctions, all winning bidders pay the market-clearing price. In this case, lowering a bid beyond the winner's curse occurs to the extent that a bidder is likely to be marginal.

For the empirical analysis, I compare the auction results to the secondary market data from GovPX (which is considerably more comprehensive and accurate than the data used in the literature previously). I find that uniform-price auctions of notes and bonds have statistically significant underpricing averaging 0.32 basis points (corresponding to 1.3 cents per $100), which is economically close to the magnitude of underpricing predicted by the model. This is the first time that statistically significant underpricing has been documented since the U.S. Treasury switched to the uniform-price mechanism. I find that discriminatory auctions of notes and bonds result in underpricing averaging 0.59 basis points in yield space (corresponding to 3.5 cents per $100). This is, in fact, slightly below the underpricing predicted by the model on average. While underpricing remains after the change in the auction mechanism, underpricing in the uniform-price auctions is statistically significantly less than it was under the discriminatory mechanism.

I also measure the underpricing in each individual auction. Calibrating the model to the statistics revealed after each auction, I compare the underpricing in each auction to that predicted by the model. I find a strong correlation between predicted and observed underpricing.

The Treasury's use of the discriminatory auctions has raised controversy ever since Friedman (1960), (1991) argued that the Treasury could reduce the cost of financing the debt by switching to uniform-price auctions.[7] The recommendations of the Joint Report on the Government Securities Market (1992) prompted the Treasury's move to uniform-price auctions.

There has been little evidence of underpricing in uniform-price Treasury auctions until now. Nyborg and Sundaresan (1996), Malvey and Archibald (1998), and Reinhart and Belzer (1997) do not find statistically significant underpricing, most likely due to the limited number of auctions they consider. The earlier literature that documents underpricing in the older discriminatory Treasury auctions was limited by the quality of secondary market data available at the time. However, the estimate of the average underpricing in discriminatory auctions in this paper is similar to those of the studies that use interdealer broker data.[8]

[7] Apparently Friedman was referring to Simon's (1994b) study of the Treasury's experiment with uniform-price auctions in the 1970s when Friedman suggested that the Treasury could increase auction revenue by 0.75%. However, the results in Simon may be driven by an unusual outlier.

[8] Cammack (1991) and Spindt and Stolz (1992) use indicative quotes from the New York Federal Reserve. That data is biased, and as a result they overestimate the degree of underpricing. Bikhchandani, Edsparr, and Huang (2000), Simon (1994a), and Nyborg and Sundaresan (1996) are hampered by only having limited data from one interdealer broker.

The empirical literature comparing discriminatory and uniform-price auctions in other contexts generally favors uniform-price auctions. These papers include Umlauf's (1993) study of Mexican Treasury auctions (although Umlauf's results are affected by overt collusion in the Mexican auctions), Tenorio's (1993) analysis of foreign exchange auctions in Zambia, and Feldman and Reinhart's (1995) report on IMF gold auctions. In contrast, Hortacsu (2002) finds some evidence in favor of the discriminatory mechanism in Turkish treasury auctions.

Theoretical papers comparing the two Treasury auction procedures include Chari and Weber (1992) who appeal to single unit auction theory (as in Milgrom and Weber (1982)) to argue that uniform-price auctions are revenue superior, and Bikhchandani and Huang (1993) who focus on signaling. The model in this paper is in the same spirit, but explicitly considers the Treasury auction as a multi-unit sale without strategic behavior in order to clarify the relation between the mechanism and the extent of underpricing.

Much of the theoretical literature on Treasury auctions has been focused away from the basic auction theory and allows for strategic bidding. This includes discussions of how the auction market can interact with the secondary market. Chatterjea and Jarrow (1998) argue that discriminatory auctions are susceptible to manipulation. Bikhchandani and Huang (1989) argue that uniform-price auctions are more susceptible to overbidding as participants try to send signals to the secondary market. Viswanathan and Wang (2000) discuss how dealer inventories affect bidding strategies especially in discriminatory auctions. Another stream of the literature discusses the potential for implicit collusion in uniform-price auctions. This was first discussed in Wilson (1979), followed by Back and Zender (1993) and Wang and Zender (2002) and generalized by Ausubel and Cramton (2002) who show that with divisible goods the auction ranking is ambiguous. In an experimental study, Goswami, Noe, and Rebello (1996) find that collusive equilibria can occur. However, Kremer and Nyborg (2004) argue that these types of equilibria are not robust.

In this paper, I put aside the above issues by setting up the model so that individual bidders have little market power. Thus, one can more fully examine the auction mechanisms in a simpler setting. Most importantly, the setup allows for the quantitative predictions of underpricing in each auction, and shows that market power is not necessary to explain the observed levels of underpricing.

The paper proceeds as follows. Section II describes the data and presents the empirical results on average underpricing in discriminatory and uniform-price Treasury auctions. Section III presents the theoretical model. Section IV describes the methods used to infer the model variables from the publicly released auction results. Section V contains the empirical analysis comparing the theoretical predictions to the observed underpricing. Section VI concludes. All proofs are in Appendix A and a description of the Treasury auction procedure is in Appendix B.

II. Summary Statistics and Underpricing

A. Summary of Data

The data consists of the announced results for 283 Treasury note and bond auctions that took place between June 1991 and December 2000, and the corresponding secondary market prices at the times of the auctions. Traditionally, Treasury auctions were conducted using the discriminatory mechanism in which each winning bidder pays a price corresponding to his bid. In September 1992, as an experiment the Treasury began auctioning two- and five-year notes using the uniform-price mechanism in which all winning bidders pay the same market-clearing price. The uniform-price mechanism was extended to all maturities in August 1998.

In all, the data includes 105 discriminatory and 178 uniform-price auctions. Index linked securities are excluded as are reissues of existing securities. A small number of auctions is excluded when secondary market data is not available for the period immediately before the auction.

The auction result data are drawn from the announcements issued by the Treasury after each auction. For discriminatory auctions, the announced results include the highest winning (i.e., market-clearing) yield, the average winning yield, and the lowest yield bid in the auction.[9] For uniform-price auctions, the announced results include the highest winning yield, the median winning yield, and the 95th percentile winning bid in the auction. In all cases, the Treasury announces the total amount bid and awarded, as well as the extent of rationing to bidders exactly at the margin. Details of individual bids are never revealed.

Table 1 presents summary statistics of the auctions. The most commonly issued securities are the two- and five-year notes that were auctioned monthly over the time period. The auction size averages \$15.5 billion (face value) of securities offered, and the bid-to-cover ratio, i.e., the ratio of bids submitted to offering amount, averages 2.31. A commonly used measure of bid dispersion is the "tail," measured as the difference in yield space between the marginal winning bid and the average winning bid (or, in the case of uniform-price auctions, the median winning bid). The tail is substantially larger in uniform-price auctions than in discriminatory auctions (although the different definitions of tail make them not perfectly comparable). The wider dispersion of bids in uniform-price auctions may reflect weaker incentives for bidders to collect accurate information prior to participating in a uniform-price auction.

The secondary market data is from GovPX. The GovPX data is far more comprehensive than previously available data and includes all the quotes and trades from the interdealer market as supplied by all but one of the major interdealer brokers.[10] I consider the prices in the when-issued market, i.e., the forward market on the yet-to-be-issued securities, from a half hour before the 1:00 PM auction time until a half hour after the auction time. The when-issued market is

[9] Recall that the auction is conducted in yield space so the winning bids are those with the lowest yields.

[10] The missing interdealer broker is Cantor Fitzgerald. I obtain similar results using Cantor Fitzgerald's data for an earlier sample of discriminatory auctions.

TABLE 1

Auction Summary Statistics

Table 1 presents summary statistics of 283 Treasury auctions held between June 1991 and December 2000. In September 1992, the Treasury switched to the uniform-price mechanism for two- and five-year auctions and switched to the uniform-price mechanism for all auctions in August 1998. Auctions of index linked securities and reissues of seasoned securities are excluded. A small number of auctions are excluded because of a lack of secondary market data at the time of the auction. Maturity is the number of years in the life of the security although actual maturity may differ slightly from stated maturity. Supply is the face value of securities auctioned. Bid-to-cover is the ratio of the quantity of tenders to supply. Tail is the difference between the market-clearing yield and the mean or median winning yield (depending on the auction format).

	No. of Discriminatory	No. of Uniform	Average Maturity	Average Supply ($billion)	Average Bid-to-Cover	Average Tail (basis points)
All	105	178	5.7	15.5	2.31	1.94
By Auction Format:						
Discriminatory	105	—	9.1	14.4	2.34	1.01
Uniform price	—	178	3.7	16.1	2.29	2.49
By Maturity:						
2-year	15	94	2	18.1	2.31	2.01
3-year	28	0	3	19.7	2.21	0.93
5-year	15	78	5	12.8	2.42	2.31
7-year	8	0	7	10.2	2.07	2.00
10-year	23	5	10	13.5	2.09	2.00
30-year	16	1	30	11.4	2.29	1.06

TABLE 2

When-Issued Market Activity

Table 2 presents measures of market activity in the when-issued market around the time of Treasury auctions. Panel A presents the statistics for the when-issued market near the time of discriminatory auctions and Panel B near the time of uniform-price auctions. The auction is held at 1:00 PM and the results are announced approximately 30 minutes later.

Time of Day	No. of Auctions	No. of Trades per Auction	Quantity Traded per Auction ($million)
Panel A. When-Issued Market Activity—Discriminatory Auctions			
12:50 PM–1:00 PM	105	10.0	250
12:30 PM–1:00 PM	105	26.1	606
1:00 PM–1:30 PM	105	22.6	213
Panel B. When-Issued Market Activity—Uniform-Price Auctions			
12:50 PM–1:00 PM	178	12.3	359
12:30 PM–1:00 PM	178	31.9	875
1:00 PM–1:30 PM	176	23.0	288

very active with an average of about $1 billion of the auctioned security trading in the hour around the auction. Table 2 presents statistics of secondary market activity at the time of the auction.

B. Average Underpricing

Underpricing is measured as the auction yield minus the yield on the same securities in the when-issued market. For discriminatory auctions, the auction yield is defined as the quantity-weighted average yield of the winning bids.[11] For

[11]Because yield is locally linear in price, this measure is almost exactly the same as the yield corresponding to the average winning price. For the empirical purposes of this paper, measuring underpricing in yield space reduces the heteroskedasticity in price space that would otherwise result from differences in duration across the securities.

uniform-price auctions, the auction yield is defined as the market-clearing yield. The when-issued yield is taken as the quantity-weighted average transaction yield in the 10 minutes prior to the 1:00 PM auction deadline.[12] As a second, more conservative measure, I also calculate underpricing relative to the average quoted bid in the when-issued market. For robustness, I also measure underpricing in the half hour before the auction, as well as the half hour after the auction. Table 3 presents the average underpricing in discriminatory and uniform-price auctions.

TABLE 3

Average Underpricing

Table 3 presents the average underpricing in Treasury auctions. Underpricing is measured in basis points and is the average auction yield (defined as the average winning bid in discriminatory auctions and defined as the market-clearing yield in uniform-price auctions) minus the average when-issued yield close to the 1:00 PM auction time. The three rows refer to different time periods over which the when-issued yield is averaged. The when-issued yield is taken first as a quantity-weighted average of trade prices in the when-issued market and second as an average of bid prices quoted in the when-issued market. Reissues and index linked securities are excluded. ***, **, and * indicate one-tailed significance at the 1%, 5%, and 10% levels, respectively.

Time of Day	No. of Auctions	Underpricing (relative to when-issued transactions)	Standard Error	Underpricing (relative to when-issued bid quote)	Standard Error
Panel A. Underpricing in Discriminatory Auctions					
12:50 PM–1:00 PM	105	0.59***	0.07	0.50***	0.07
12:30 PM–1:00 PM	105	0.61***	0.08	0.51***	0.08
1:00 PM–1:30 PM	105	0.86***	0.10	0.67***	0.10
Panel B. Underpricing in Uniform-Price Auctions					
12:50 PM–1:00 PM	178	0.32***	0.12	0.20**	0.12
12:30 PM–1:00 PM	178	0.40***	0.12	0.29***	0.12
1:00 PM–1:30 PM	176	0.32***	0.10	0.15*	0.10

Time of Day	Underpricing in Discriminatory Auctions (relative to when-issued transactions)	Underpricing in Uniform-Price Auctions (relative to when-issued transactions)	Difference	Standard Error
Panel C. Difference in Underpricing between Discriminatory and Uniform-Price Auctions				
12:50 PM–1:00 PM	0.59	0.32	0.27**	0.14
12:30 PM–1:00 PM	0.61	0.40	0.22*	0.15
1:00 PM–1:30 PM	0.86	0.32	0.53***	0.15

In the discriminatory auctions, the average underpricing relative to when-issued transactions is 0.59 basis points. This is statistically significant and consistent with previous studies on the older auction mechanism. With more that $3 trillion of government debt held by the public, this implies about $200 million of extra debt servicing per year.

One of the basic policy questions is whether the use of uniform-price auctions reduces or even eliminates underpricing. In uniform-price auctions, I find a statistically significant 0.32 basis points of underpricing on average. This is the first time that statistically significant underpricing has been documented for the newer mechanism. However, I also find that the difference in underpricing between the two methods (0.27 basis points) is statistically significant. Thus, one

[12]Measuring the when-issued yield using equally-weighted transaction yields does not change any results in this paper. Using midquotes rather than transactions also makes very little difference.

can conclude that the switch to the new uniform-price format has reduced underpricing although a lower level of underpricing still persists.

The average underpricing statistics quoted above are relative to transaction prices in the when-issued market. Almost identical estimates result when comparing the auction yield to the midpoint of the bid-ask spread in the when-issued market. While transaction prices and midquotes are often used as proxies for the "true" value of a security, one could argue that the relevant comparison might be between the auction yield and either the bid yield or the ask yield quoted in the when-issued market.

Since the auction is a market for dealers purchasing Treasury securities, the comparison might be to the quoted ask, the price at which the security can be bought in the when-issued market. Such a comparison could be interpreted as asking how much better the government could do if it were to replace the market makers and sell directly to the buyers of securities. This comparison is not perfect because purchases in the auction are for much larger quantities than available in the when-issued market. When underpricing is measured relative to the ask, higher estimates of underpricing result.

Perhaps the most relevant (and more conservative) measure of underpricing is relative to the quoted bid in the when-issued market. This could be interpreted as how much worse the government does than sellers in the when-issued market. It could also be interpreted as the hypothetical profit of short selling in the when-issued market and covering the short position in the auction (as is often done by primary dealers in practice). Again, this is an imperfect comparison because of differences in quantities in the two markets. Underpricing relative to the quoted bid averages 0.50 basis points and 0.20 basis points for discriminatory and uniform-price auctions, respectively, and both are statistically significant (Table 3).[13]

For the remainder of the paper, I use transaction yields as the basis for measuring underpricing.

C. Determinants of Underpricing

At this point, it is natural to ask what are the determinants of underpricing. In Table 4, I report the results of regressions of underpricing on a number of characteristics. Regression 1 corresponds to the results already reported above: there is underpricing in uniform-price auctions (as captured by the intercept), but there is more underpricing in discriminatory auctions.

However, the comparison between the auction price and the when-issued price could also depend on characteristics in the pre-auction when-issued market. Most importantly, the change in the auction format from discriminatory to uniform price was associated with changes in the when-issued market—most notably volume as shown in Table 2—so it is necessary to control for when-issued market characteristics. In regressions 2 to 5, I include measures from the when-issued market to see how they affect underpricing. I find that the bid-ask spread and volatility are both positively related to underpricing. This suggests that un-

[13]In Panel C of Table 3, I do not report the difference in underpricing across the two mechanisms using bid yields because the results are almost identical to those reported for transaction yields.

TABLE 4

Regressions of Underpricing on Auction and When-Issued Market Characteristics

The dependent variable in these regressions is underpricing. Underpricing is measured in basis points and is the average auction yield (defined as the average winning yield in discriminatory auctions and defined as the market-clearing yield in uniform-price auctions) minus the average when-issued transaction yield in the 10-minute period before each auction. Discrim is a dummy variable set equal to one if the auction is discriminatory and set to zero for uniform-price auctions. Bid-ask spread is the quoted bid-ask spread (in basis points) in the when-issued market averaged over the 30-minute period prior to each auction. Volatility is the standard deviation of yields in the when-issued market, based on transactions in the 30-minute period prior to each auction. Log(volume) is the logarithm of transaction volume in the when-issued market (in units of millions of dollars) in the 30-minute period prior to each auction. Bid-to-cover is the ratio of the quantity of tenders to supply in each auction. Tail is the difference between the market-clearing yield and the mean or median winning yield (depending on whether the auction format is discriminatory or uniform-price). Regression (10) only includes the time period when both auction formats are used. Regression (11) is limited to two- and five-year note auctions. Regression (12) is run in price space in dollars per $100 of face value instead of yield space. To control for heteroskedasticity, the regressions are weighted by the inverse of the standard error in the when-issued yield average. *t*-statistics are in parentheses.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	Limited Time (10)	2- & 5-Year (11)	Price Space (12)
Intercept	0.20	−0.16	−0.01	1.28	1.06	3.23	−1.51	−0.36	0.35	2.65	1.69	0.034
	(2.2)	(−1.0)	(0.1)	(3.0)	(1.4)	(8.2)	(−11.8)	(−0.9)	(0.5)	(2.3)	(2.2)	(1.08)
Discrim	0.36	0.32	0.40	0.28	0.31	0.48	1.44	1.38	1.37	0.91	1.47	0.051
	(2.6)	(2.3)	(2.9)	(2.0)	(2.2)	(3.8)	(11.8)	(11.2)	(10.6)	(5.1)	(7.3)	(8.05)
Bid-ask spread		1.58			0.26				−0.80	−0.34	−1.05	−1.968
		(2.7)			(0.3)				(−1.2)	(−0.35)	(−1.3)	(−6.94)
Volatility			0.20		0.25				−0.02	−0.04	0.04	0.177
			(2.0)		(2.3)				(−0.2)	(−0.39)	(0.4)	(1.99)
Log(volume)				−0.17	−0.18				−0.09	−0.23	−0.18	−0.002
				(−2.6)	(−1.9)				(−1.3)	(−1.8)	(−2.1)	(−0.43)
Bid-to-cover						−1.33		−0.43	−0.42	−0.88	−0.73	−0.023
						(−7.9)		(−2.8)	(−2.7)	(−3.6)	(−4.1)	(−2.84)
Tail							0.73	0.66	0.68	0.59	0.68	0.568
							(15.6)	(12.7)	(12.3)	(8.2)	(10.4)	(10.78)
Adj. R^2	2.0%	4.1%	3.0%	4.0%	5.7%	19.6%	47.8%	49.0%	48.9%	49.2%	50.4%	46.8%
N	280	280	280	280	280	280	280	280	280	196	199	280

derpricing is related to uncertainty about the value of the security. There is a negative relation between (the logarithm of) pre-auction volume and underpricing. This could be interpreted that pre-auction trade reduces uncertainty. An alternative interpretation is that volume is associated with short positions held by primary dealers, and such dealers would be more aggressive in the auction. Nevertheless, in all regressions, the auction mechanism dummy remains statistically and economically significant, suggesting that the reduction in underpricing under the uniform-price auction is robust to variations in the when-issued market.

Rather than using measures taken from the pre-auction when-issued markets, regressions 6 to 8 use auction statistics as explanatory variables. These auction statistics are the bid-to-cover ratio and the tail. Bid-to-cover, the ratio of aggregate bids to supply, captures the extent of competition in the auction, and the tail is a measure of bid dispersion. (Recall that the tail is defined differently for discriminatory and uniform-price auctions so the results should be interpreted cautiously.) I find strong results for these variables as measured by *t*-statistics. In comparison to the previous results, the adjusted R^2 is markedly higher—especially once the tail is introduced.

The strong negative relation between the bid-to-cover ratio and underpricing suggests that more competition in the auction reduces underpricing. The strongest relation, however, is a positive one between the tail and underpricing. The coefficient is both economically and statistically very significant. Most striking is an adjusted R^2 that jumps to almost 50%.

The tail can have different interpretations. A wide tail could result from disagreement among bidders about the value of the securities. Alternatively, a wide tail could also result from individual bidders submitting demand curves over a wide range of prices. The latter would suggest a role for risk aversion or implicit collusion (as in Back and Zender (1993) in the case of uniform-price auctions). However, in the model in the next section I stress the role of noisy signals, and in Section IV I show that the noise in the signals can be inferred from the dispersion of bids as measured by the tail. Under this interpretation, the regression results show more underpricing when there is more uncertainty about the value of the securities.

Regression 9 shows that when all the explanatory variables are included together, the auction statistics (bid-to-cover and tail) completely subsume the when-issued market statistics (bid-ask spread, volatility, and volume). Note that in all regressions that include the auction statistics the coefficient on the auction mechanism dummy is much stronger than when only when-issued statistics are included.

Because the earliest of the data includes only discriminatory auctions and the end of the dataset only includes uniform-price auctions, I repeat the full regression using only data from the shorter time period during which both auction methods were used (regression 10). The results are robust over this subset of data.

Since the securities in this study vary by maturity, I also repeat the full regression over the entire time period limited to just the most frequently issued securities: two- and five-year notes (regression 11). Additionally, in unreported regressions, I use a variety of controls for maturity. The results are robust.

Finally, regression 12 repeats the analysis in price space and shows that the main results are qualitatively similar.

Overall, the results show that underpricing depends on the auction mechanism, the extent of competition in the auction, and uncertainty in the auction. These are exactly the economic forces that theory would suggest should affect auction prices.

In the next section, I present a theoretical model of discriminatory and uniform-price Treasury auctions to illustrate how the auction mechanism, the competition in the auction, and uncertainty all interact to produce underpricing. Most importantly, the model allows me to go beyond the regression estimation in this section. When calibrated using the bid-to-cover ratio and the tail, the model gives quantitative predictions of the magnitude of underpricing in each auction.

III. The Auction Model

In this section, I model the discriminatory and uniform-price auctions in the spirit of the traditional multi-unit common value auction literature in which the auction mechanism affects the degree of underpricing in equilibrium. That the auction mechanism can affect underpricing in common value auctions is well known (see, for example, Milgrom and Weber (1982) for the single unit case and

Milgrom and Weber (2000) and Weber (1983) for the multi-unit case).[14] However, this model is designed to be taken to data and calibrated using the announced results of each Treasury auction. It can then be used to predict the magnitude of underpricing in each auction, thus capturing the heterogeneity in underpricing across auctions.

A. Discriminatory Auction Model

I model the discriminatory Treasury auction as an auction of M securities to N risk-neutral bidders, each with unit demand. The fraction of all bids that are accepted is thus $\alpha = M/N$, $0 < \alpha < 1$.

Each bidder receives a noisy signal of the true common value of the security, V, which can be interpreted as the resale price of the securities in the secondary market. Each bidder's signal of the common value has two noise components, one common and one idiosyncratic. Priors are diffuse. Bidder i receives a signal,

$$S_i = V + \sigma_0\tilde{\epsilon}_0 + \sigma_1\tilde{\epsilon}_i, \tag{1}$$

where $\tilde{\epsilon}_0$ and $\tilde{\epsilon}_i$ are drawn independently from distributions with zero mean and unit variance. The common noise term $\sigma_0\tilde{\epsilon}_0$ captures uncertainty that is common to all bidders and does not play an important role in the model. Assume that the idiosyncratic noise, $\tilde{\epsilon}_i$, is drawn i.i.d. from a standard normal distribution, and denote the cumulative distribution of the standard normal as $F(\cdot)$ and the density as $f(\cdot)$. This idiosyncratic noise term captures the differences of opinion among bidders about the common value, perhaps due to private information about order flow.[15]

The signal that each bidder receives is the realized common component, $\overline{V} \equiv V + \sigma_0\epsilon_0$, plus the i.i.d. idiosyncratic noise, $\sigma_1\tilde{\epsilon}_i$. Given the diffuse prior, since S_i is the only information available to bidder i and since all the signals have the same accuracy, bidder i has no way of knowing whether his signal is a high or low draw. In fact, from bidder i's perspective, $\overline{V}$ is random with mean S_i and variance σ_1^2.

The discriminatory auction proceeds as follows. Simultaneously, each bidder submits a bid $B_i(S_i)$.[16] The seller awards the securities to the M bidders (i.e., the fraction α of bidders) that submit the highest bids. Each winning bidder pays his submitted bid. Define the market-clearing stop-out bid, B_s, as the highest losing bid.

Bidder i's problem is to choose a bid function $B_i(S_i)$ that maximizes his expected profits. He solves:

$$\max_{B_i} \text{Prob}(B_i > B_s|S_i)E(V - B_i|S_i, B_i > B_s), \tag{2}$$

[14] Other papers that discuss discriminatory and uniform-price auctions in a similar multi-unit setting include Milgrom (1981), Bikhchandani and Huang (1989), Chari and Weber (1992), Pesendorfer and Swinkels (1997), Ausubel and Cramton (2002), and Parlour, Prasnikar, and Rajan (2004), among others. See also Milgrom (1989), McAfee and McMillan (1987), and Wilson (1990) for more general discussions of auctions.

[15] Hortacsu and Sareen (2005) provide evidence that bidders respond to private information in order flow from customers in Government of Canada securities auctions.

[16] In this model, bids are submitted as prices. In actual Treasury auctions, bids are submitted as yields.

i.e., he maximizes the probability of winning the auction times the expected profit conditional on winning.

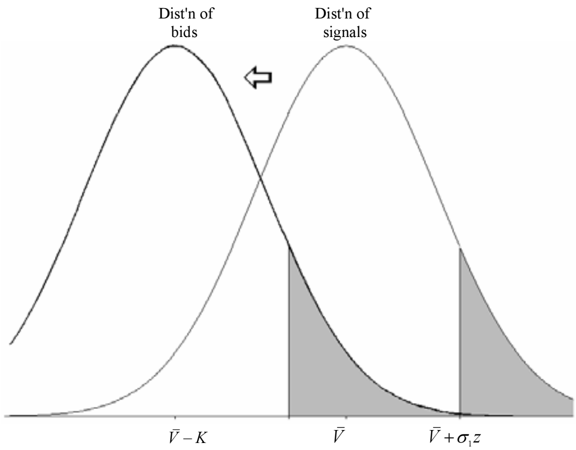
I limit the potential equilibria to those that are "efficient," defined as those in which the bidders with the highest signals win the auction.[17] Efficiency necessarily implies symmetry—that all bidders have the same bid function $B_i(S_i) = B(S_i)$. In other words, each bidder uses his information in exactly the same way as all other bidders. Efficiency also requires pure strategies.

Define the random variable $\widetilde{Z}_m^n$ as the mth order statistic from n draws from the standard normal distribution. Thus, $\sigma_1 \widetilde{Z}_M^N$ is the bias in the Mth highest signal. The winning bidders will be those with signals at or above $\bar{V} + \sigma_1 \widetilde{Z}_M^N$. The distribution of signals is illustrated in Figure 1.

FIGURE 1

Signal and Bid Distributions

Figure 1 illustrates the distribution of signals and bids in the model. The distribution of signals is centered around $\bar{V}$. In equilibrium, the distribution of bids is the same as the distribution of signals shifted to the left by a fixed amount K. The fraction α of bidders with the highest bids win the auction. The winning bids (and the corresponding signals above $\bar{V} + \sigma_1 z$) are shaded.



As shown in the following proposition, in equilibrium each bidder bids below his signal S_i, despite S_i being an unconditionally unbiased estimate of V. The amount of this "shading" depends on the ratio of supply M to demand N, and the amount of idiosyncratic noise in the signals σ_1.

Proposition 1. In the discriminatory auction, it is an equilibrium for all bidders to bid

$$
\begin{aligned}
(3) \qquad\qquad B(S_i) &= S_i - K(\sigma, M, N), \\
\text{where} \qquad K(\sigma, M, N) &= \sigma_1 \left[\frac{\alpha + E[\tilde{Z}_M^{N-1} f(\tilde{Z}_M^{N-1})]}{E[f(\tilde{Z}_M^{N-1})]} \right].
\end{aligned}
$$

[17]The term "efficiency" is analogous to the same term in the context of private values auctions. In the private values literature, it means that the bidder with the highest valuation wins the auction. In the context of common values, that definition is inappropriate.

The expected underpricing per unit sold in the auction is

$$(4) \qquad E(\pi) \quad = \quad \sigma_1 \left[\frac{\alpha + E[\tilde{Z}_M^{N-1} f(\tilde{Z}_M^{N-1})]}{E[f(\tilde{Z}_M^{N-1})]} - \frac{E[f(\tilde{Z}_M^{N-1})]}{\alpha} \right].$$

In the limit, as the number of bidders and the number of securities gets large (i.e., as $M, N \to \infty$, keeping $M/N{=}\alpha$) the shading simplifies to $K{=}\sigma_1(\alpha/(f(z)) + z)$, and underpricing simplifies to $E(\pi){=}\sigma_1(\alpha/(f(z)){+}z{-}(f(z))/\alpha)$, where z is the value above which the standard normal distribution has mass α, i.e., $F(z) = 1 - \alpha$.

Since all bidders shade their bids by exactly the same amount, the distribution of bids is simply the distribution of signals shifted to the left by the fixed amount K, i.e., with a mean of $\overline{V} - K$ instead of $\overline{V}$ (see Figure 1). The winning bids belong to the fraction α of bidders with the highest signals. It is not surprising that they all shade by the same amount, as the only information each bidder has is S_i, they all have the same signal noise, and there is no metric against which to measure the relative accuracy of their signals. (If the assumption of diffuse priors is relaxed, bidders with high signals would shade more than bidders with low signals. Diffuse priors allow for the simpler equilibrium presented in Proposition 1, which can be more easily used to calibrate the model to the observed Treasury auction data.)

The equilibrium can be understood by considering the bidders' first-order condition. The first-order condition that results in equation (3) can be written as

$$(5) \qquad \alpha - E\left(\frac{f\left(\widetilde{Z}_M^{N-1}\right)}{\sigma_1} \left[K - \sigma_1 \widetilde{Z}_M^{N-1}\right] \right) \quad = \quad 0.$$

By symmetry, the equilibrium probability of winning the auction is α for each bidder. On the one hand, decreasing a bid by one unit will have a marginal benefit of α. On the other hand, the probability density of being the marginal bidder is $f(\widetilde{Z}_M^{N-1})/\sigma_1$, and by decreasing his bid, a bidder risks losing the expected profits earned by the bidder on the margin, $K - \sigma_1 \widetilde{Z}_M^{N-1}$. (This is the expected profit to the bidder on the margin since his signal is biased by $\sigma_1 \widetilde{Z}_M^{N-1}$, but he shades his bid by K.) The optimal bid equates the marginal benefit of reducing a bid, α, with the marginal cost, $(f(\widetilde{Z}_M^{N-1})/\sigma_1)[K - \sigma_1 \widetilde{Z}_M^{N-1}]$.

The expected profit to bidders is the shading, K, minus the *winner's curse*. The idea of the winner's curse is that while each signal is an unbiased estimate of the true value, the winners are only those that received a high signal. So although a signal is an unconditionally unbiased estimate of the true value, it is biased upward conditional upon winning the auction. The winner's curse for a given α is defined as the bias in the signal conditional on winning,

$$(6) \qquad WC_\alpha \quad \equiv \quad E(V|S_i) - E(V|S_i, B(S_i) > B_s) \quad = \quad E\left(\frac{f\left(\widetilde{Z}_M^{N-1}\right)}{\alpha} \right).$$

Although rational bidders are aware of this and should be expected to shade their bids to compensate for the winner's curse, it is not the winner's curse that determines the extent of shading. The intuition is as follows. If a bidder shades his bid only to compensate for the winner's curse, he is bidding his conditional expectation and expected profits are zero. However, all the winning bidders pay different prices. Since a given bidder is unlikely to be the marginal bidder (i.e., the one with the lowest winning bid), he can lower his bid further and still have a positive probability of being awarded securities. In those states in which this lower bid still wins, positive expected profits are earned. If his bid is much lower than the conditional expectation, then the marginal cost of any further decrease is large as he risks losing substantial profits. This trade-off determines the extent of bid shading.

As shown in Proposition 1, shading and underpricing depend on the dispersion of signals, σ_1 (which is also the measure of bid dispersion in equilibrium) and by the ratio of securities supplied to the quantity of bids tendered, $M/N = \alpha$. As bids become more dispersed, it is less likely that a bid is marginal and reducing a bid further is unlikely to result in losing the auction, so in equilibrium bids are lower and underpricing is higher.

In the extreme case when $\sigma_1 = 0$, there is no disagreement and everyone bids $\overline{V}$, competing away all expected profits. This is closely related to the linkage principle in single-unit common value auction theory (Milgrom and Weber (1982)) that revelation of information prior to the auction leads to greater revenue for the seller.

Expected underpricing is an increasing function of α (see Figure 2). When α is large, there is little competition and the marginal benefit of lowering a bid is high. So shading is high even though the winner's curse is small resulting in high underpricing. However, for small α the winner's curse is large. As a result, underpricing decreases and approaches 0 as α approaches 0. This result is intuitively appealing as a low ratio of supply to aggregate bids tendered can be interpreted as a high level of competition that reduces profits for bidders.

B. Uniform-Price Auction Model

So far I have considered discriminatory auctions. In uniform-price auctions, all winning bidders pay the market-clearing stop-out bid, B_s^u, which I model as the $(M + 1)$st highest bid (i.e., the highest losing bid). I model this uniform-price auction in the same way as the discriminatory auction except that bidder i's problem is

$$\max_{B_i^u} \text{Prob}\left(B_i^u > B_s^u | S_i\right) E\left(V - B_s^u | S_i, B_i^u > B_s^u\right). \tag{7}$$

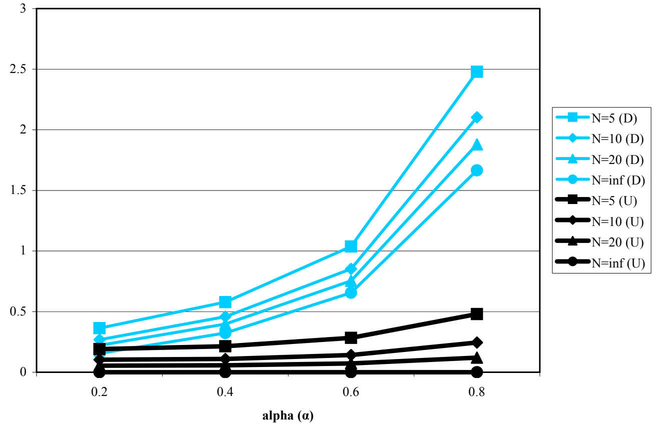
As before, the bidder maximizes the probability of winning the auction times the expected profit conditional on winning, except now this profit is $V - B_s^u$, the difference between the true value and the stop-out bid. (The superscript u indicates bids submitted under the uniform-price auction format.)

In a uniform-price auction, the bias in the signal conditional on winning the auction is not the important bias. More important is the bias in the signal of the $(M+1)$st bidder, $E(S_i - V|S_i, B^u(S_i) = B_s^u) = \sigma_1 \widetilde{Z}_{M+1}^N$.

FIGURE 2

Theoretical Underpricing (discriminatory and uniform-price auctions)

Figure 2 displays the theoretical underpricing for the two auction formats for different numbers of bidders. α is the ratio of securities offered to number of bidders. The theoretical underpricing in the figure is per unit of σ_1, the standard deviation of noise in the individual signals. This figure linearly interpolates between marked points. Underpricing is lower when there is a lower ratio of securities to bidders (α). Underpricing decreases with the number of bidders. Discriminatory auctions have more underpricing.



Proposition 2. In the uniform-price auction, it is an equilibrium for all bidders to bid

$$B(S_i) \quad = \quad S_i - \sigma_1 \left[\frac{E[\tilde{Z}_M^{N-1} f(\tilde{Z}_M^{N-1})]}{E[f(\tilde{Z}_M^{N-1})]} \right]. \tag{8}$$

The expected underpricing per unit sold in the auction is

$$E(\pi) \quad = \quad \sigma_1 \left[\frac{E[\tilde{Z}_M^{N-1} f(\tilde{Z}_M^{N-1})]}{E[f(\tilde{Z}_M^{N-1})]} - E(\tilde{Z}_{M+1}^N) \right]. \tag{9}$$

The main cause of the underpricing is that the important bias in the signals is $\sigma_1 \widetilde{Z}_{M+1}^N$, the bias of the $(M+1)$st highest bidder, but when choosing the amount of shading, the relevant bias is $\sigma_1 \widetilde{Z}_M^{N-1}$ (which, in expectation, is more than $\sigma_1 \widetilde{Z}_{M+1}^N$). This is because when bidder i decides how much to shade his bid, he considers the distribution of the other $N-1$ bids and realizes that he will win the auction if his bid is higher than the Mth of those other $N-1$ bids. Therefore, the shading overcompensates for the bias in the signal of the marginal bidder. Note that in the limit as $M, N \to \infty$ (keeping $M/N = \alpha$), $\widetilde{Z}_M^{N-1}$, $\widetilde{Z}_{M+1}^N \to z$, resulting in zero expected underpricing.

Figure 2 also plots the expected underpricing in the uniform-price auction. Given σ_1, underpricing increases in α. Underpricing decreases as the number of bidders and the supply of securities rises (holding α constant).

For a given α and σ_1, there is less underpricing in the uniform-price auction than the discriminatory auction (consistent with the more general results for multi-unit auctions in Weber (1983) and Milgrom and Weber (2000)). However, the revenue rankings remain ambiguous if α or σ_1 depends on the auction format.

Indeed, I have already seen empirically that the dispersion of bids is much wider in uniform-price auctions, indicating a larger σ_1 under that format. (See below for the relation between bid dispersion and signal dispersion.)[18]

The most important question, however, is if the expected underpricing predicted by this model (primarily as a function of α and σ_1) is consistent with the empirically observed underpricing in Treasury auctions.

IV. Calibration from Auction Results

Section III shows theoretically that underpricing should occur under both auction formats. The strength of the model is that it gives quantitative predictions for the magnitude of underpricing in each individual auction as a function of σ_1 and α (given the number of bidders N). To test these predictions, I infer α and σ_1 from the announced results of each auction.

As mentioned above, the Treasury releases summary statistics shortly after each auction. The statistics include: quantity tendered, quantity accepted, low, high, and average (or median) accepted bids, as well as a number of other statistics. In the context of the model, the variables α and σ_1 can be inferred from these statistics.

First, for each auction, α is simply taken as (*Quantity Accepted/Quantity Tendered*).[19]

Second, σ_1, being the standard deviation of the bidders' signals, is not directly observable; however, it can be inferred from the distribution of bids. Recall from the theory that the distribution of bids in equilibrium is simply the distribution of signals shifted to the left by K, so σ_1 is also the standard deviation of bids. I can infer the standard deviation of bids from the statistics released after the auction.

In discriminatory auctions, the *tail*, the difference between the average winning bid and the stop-out bid, is announced. In uniform-price auctions, the difference between the *median* winning bid and the stop-out bid (which I denote as $Tail^{med}$) is announced. Given α, and assuming that the signals are drawn from a normal distribution, the tail is sufficient to infer σ_1. Similarly, $Tail^{med}$ can be used to infer σ_1. The relations are:[20]

$$(10) \qquad Tail \equiv B_{AvgWinning} - B_s = \frac{1}{\alpha}\int_{B_s}^{\infty} Bg(B)dB - B_s = \frac{\sigma_1 f(z)}{\alpha} - \sigma_1 z,$$

$$\text{or} \qquad \sigma_1 = \frac{\alpha \cdot Tail}{f(z) - \alpha z}.$$

$$(11) \qquad Tail^{med} \equiv B_{MedWinning} - B_s = \sigma_1 z_{\alpha/2} - \sigma_1 z,$$

$$\text{or} \qquad \sigma_1 = \frac{Tail^{med}}{z_{\alpha/2} - z},$$

[18]Moreover, even holding α and σ_1 fixed, it is possible to construct distributions of $\tilde{\epsilon}_i$ for which the rankings reverse.

[19]α can either be taken with or without noncompetitive bids included. I include the noncompetitive bids, but the empirical results are not very sensitive to this choice.

[20]For simplicity, I use the case of $M, N \rightarrow \infty$ to make the inference.

where $g(B)$ is the density of the bids. Recall that z is such that $1 - F(z) = \alpha$. $z_{\alpha/2}$ is defined analogously such that $1 - F(z_{\alpha/2}) = \alpha/2$.

I infer the values of α and σ_1 for each auction in the dataset.[21] Over the entire dataset, α averages 0.45 in discriminatory auctions and 0.44 in uniform-price auctions, while σ_1 averages 1.7 basis points in discriminatory auctions and 4.2 basis points in uniform-price auctions. The values of α and σ_1 in each auction lead to quantitative prediction of underpricing for each individual auction.[22]

V. Comparison of Theoretical and Empirical Underpricing

In Section II, I show that underpricing exists under both the discriminatory and the uniform-price auction formats although more so under the former. In this section, I first test whether the model explains the average observed underpricing for each auction format, and then I test whether the model predicts the observed cross section of underpricing in the auctions.

As described in Section II, I measure the observed underpricing as the difference between the auction yield (i.e., the average winning yield in discriminatory auctions or the market-clearing yield in uniform-price auctions) and the yield of the same securities—with the same delivery date—in the forward when-issued market. The when-issued yield used for this comparison is the average transaction yield over the 10-minute period prior to the 1:00 PM auction.

Table 5, Panel A compares the observed and theoretical underpricing for the discriminatory auctions. While the average observed underpricing is statistically different from the average theoretical underpricing (but only marginally so under the infinite number of bidders assumption), they are of similar economic magnitude—0.59 basis points of observed underpricing compared to 0.73 basis points of predicted underpricing with infinite bidders. The similar magnitude is striking considering that the theoretical prediction is not based on any in-sample estimation. In fact, the theoretical underpricing is higher than the observed underpricing, even when no bidder has any market power (i.e., when there are an infinite number of bidders). This suggests that the market power arguments proposed in the literature are not necessary to explain the level of observed underpricing. When allowing for a small amount of market power (by assuming fewer bidders), the theoretical underpricing is further above the observed underpricing although still of the same order of magnitude. Rather than wondering about the existence of underpricing, I might ask why there is not more underpricing.

Under the uniform-price format (Panel B of Table 5), the existence of statistically significant underpricing immediately rejects the implications of the model under the infinite number of bidders assumption. However, with 20 bidders assumed the average theoretical underpricing is economically similar to (and not

[21] Since the inferred α cannot, in general, be written as an integer number of securities divided by the assumed number of bidders, I linearly interpolate the theoretical underpricing when necessary.

[22] I make a small adjustment to the tail to correct for a wide bidding grid in the early part of the sample. In brief, I use the reported rationing at the margin to estimate what the stop-out bid would have been absent the discrete grid. If most bids at the margin are accepted, then I infer that the stop-out price would have been slightly lower. If few bids at the margin are accepted, then I infer that the stop-out price would have been higher. This adjustment does not have a strong effect on the results.

TABLE 5

Actual versus Theoretical Underpricing

Table 5 compares the underpricing observed in discriminatory auctions and the theoretical underpricing based on the model and parameters inferred from the auction results. Observed underpricing is the difference between the average winning yield in the auction and the quantity-weighted average trade yield in the when-issued market in the 10-minute period prior to the auction. The theoretical underpricing is based on the model (using various assumptions on the number of bidders). The model parameter values are inferred for each auction based on the bid-to-cover ratio and the auction tail.

No. of Bidders	Observed Underpricing (basis points)	Theoretical Underpricing (basis points)	Difference	*p*-Value of Difference (two-tailed)	Correlation	*p*-Value of Correlation (two-tailed)
Panel A. Discriminatory Auctions (N = 105)						
∞		0.73	−0.14	0.055	0.57	0.000
20	0.59	0.88	−0.29	0.000	0.58	0.000
10		1.01	−0.42	0.000	0.57	0.000
5		1.25	−0.66	0.000	0.57	0.000
Panel B. Uniform-Price Auctions (N = 178)						
20	0.32	0.26	0.06	0.567	0.75	0.000
10		0.50	−0.18	0.087	0.75	0.000
5		0.98	−0.66	0.000	0.79	0.000

statistically different from) the average observed underpricing. With 10 or five bidders, the average observed underpricing is below the theoretical prediction.

I now turn to the model's prediction for underpricing in the cross section, i.e., I examine how successful it is in explaining underpricing auction by auction. Table 5 reports the cross-sectional correlation between the theoretical and observed underpricing. In all cases, the correlation is positive (0.57 to 0.58 for discriminatory auctions and 0.75 to 0.79 for uniform-price auctions) and highly significant.

Table 6 reports the results of regressions of observed underpricing on theoretical underpricing. A positive slope coefficient would indicate a correlation between the theoretical and observed underpricing. If the model were a perfect fit, I would expect an intercept of zero and a slope coefficient of one.

For the discriminatory auction, I find a positive and statistically significant slope coefficient (ranging from 0.33 to 0.49) reflecting the correlation between the theoretical and observed underpricing. The adjusted R^2 is approximately 0.32. However, the slope coefficient is also significantly less than one and the intercept is statistically significantly greater than zero, indicating that there are effects that are not captured by the model. Errors in measuring the theoretical underpricing can also explain the slope of less than one.

For the uniform-price auction, the regression again shows a positive and significant slope indicating support for the model. However, the estimated slope coefficient is well above one (ranging from 2.43 to 9.27) suggesting that underpricing is far more sensitive to the auction statistics used to calibrate the model than the model predicts. The adjusted R^2 is 0.57.

In regressions that have both the theoretical underpricing and the explanatory variables from Table 4 (including the components of theoretical underpricing—the tail and bid-to-cover ratio) as explanatory variables, the F-statistic shows that the theoretical underpricing adds explanatory power under the discriminatory auc-

TABLE 6

Regression of Observed Underpricing on Theoretical Underpricing

Table 6 reports the results for the following regression: *Observed underpricing* = α + β(*theoretical underpricing*) + ε. Underpricing is measured in basis points. Standard errors of estimates are in parentheses.

No. of Bidders	N	α	β	Adj. R^2
Panel A. Discriminatory Auctions				
∞	105	0.23 (0.08)	0.49 (0.07)	0.31
20	105	0.19 (0.08)	0.46 (0.06)	0.32
10	105	0.19 (0.08)	0.40 (0.06)	0.32
5	105	0.18 (0.08)	0.33 (0.05)	0.32
Panel B. Uniform-Price Auctions				
20	178	−2.06 (0.18)	9.27 (0.61)	0.57
10	178	−2.06 (0.18)	4.77 (0.31)	0.57
5	178	−2.06 (0.17)	2.43 (0.16)	0.57

tion, but not the uniform-price auction.[23] This further supports the model for the discriminatory auction and suggests at least some level of misspecification for the uniform-price auction.

In summary, the average underpricing is similar to that predicted by the model for both the discriminatory and the uniform-price auctions. Moreover, the underpricing predicted by the model for each individual auction is highly correlated with the observed underpricing in the cross section. However, the regressions show that particularly for the uniform-price auctions, the model does not fully explain the cross-sectional variation in underpricing.

VI. Implications and Conclusion

The price obtained by the U.S. Treasury in securities auctions is less than that observed in the concurrent secondary market. I find that this underpricing occurs under both the older discriminatory auctions and the newer uniform-price auctions although to a lesser extent in uniform-price auctions.

This paper explores the role of the auction mechanism in underpricing. The model, in which each bidder has little market power, predicts underpricing that depends on the auction mechanism, the dispersion of signals observed by bidders, and the ratio of securities offered to bids tendered.

The model is designed so that the summary statistics revealed after each auction can be used to predict the amount of underpricing in that auction. I find that the average level of underpricing predicted by the model is of similar magnitude to the observed underpricing for each auction format. Additionally, the observed underpricing is highly correlated with the theoretical underpricing over the cross section of auctions.

[23]These regression results are available from the author.

I show that underpricing increases in the dispersion of signals. A policy implication is that if information that reduces uncertainty about the value of the security can be revealed, underpricing can be reduced. This supports the rationale for the existence of the when-issued market prior to the auction as a path to reducing price uncertainty.

However, the main policy implication of this paper is that the use of the uniform-price mechanism enhances Treasury auction revenue. The theoretical arguments in the literature cautioning that uniform-price auctions may lead to lower revenue (e.g., through collusive-type equilibria) do not appear to be a problem in practice in this market.

Appendix A: Proofs

Proof of Proposition 1. Since there are N bidders, bidder i wins the auction if his signal is higher than the Mth highest signal out of the other $N-1$ bidders.

Bidder i's maximization problem is

$$\max_{B_i} \operatorname{Prob}(B_i > B_s|S_i)E(V - B_i|S_i, B_i > B_s).$$

Defining bidder i's shading as K_i (i.e., so that $B_i = S_i - K_i$), defining the stop-out bidder's shading as K, and recalling that $\widetilde{Z}_b^a$ is defined as the bth order statistic from a draws from the distribution $f(.)$, the bidder's problem can be written as

$$\begin{aligned}\max{}_{K_i} \quad & \operatorname{Prob}\left(S_i - K_i > S_i - \sigma_1\widetilde{\epsilon}_i + \sigma_1\widetilde{Z}_M^{N-1} - K\right) \times \\ & E\left(S_i - \sigma_0\widetilde{\epsilon}_0 - \sigma_1\widetilde{\epsilon}_i - (S_i - K_i)\,|S_i - K_i > S_i - \sigma_1\widetilde{\epsilon}_i + \sigma_1\widetilde{Z}_M^{N-1} - K\right)\end{aligned}$$

or

$$\max_{K_i} \operatorname{Prob}\left(\widetilde{\epsilon}_i > \widetilde{Z}_M^{N-1} + \frac{K_i - K}{\sigma_1}\right) E\left(K_i - \sigma_1\widetilde{\epsilon}_i|\widetilde{\epsilon}_i > \widetilde{Z}_M^{N-1} + \frac{K_i - K}{\sigma_1}\right).$$

The first-order condition is:

$$E\left(\frac{\partial}{\partial K_i}\int_{\widetilde{Z}_M^{N-1}+\frac{K_i-K}{\sigma_1}}^{\infty}(K_i - \sigma_1\epsilon_i)f(\epsilon_i)d\epsilon_i\right) = 0$$

$$E\left[\int_{\widetilde{Z}_M^{N-1}+\frac{K_i-K}{\sigma_1}}^{\infty} f(\epsilon_i)d\epsilon_i - \frac{1}{\sigma_1}\left(K_i - \sigma_1\left(\widetilde{Z}_M^{N-1} + \frac{K_i - K}{\sigma_1}\right)\right)f\left(\widetilde{Z}_M^{N-1} + \frac{K_i - K}{\sigma_1}\right)\right] = 0.$$

Setting $K_i = K$,

$$E\left(\int_{\widetilde{Z}_M^{N-1}}^{\infty} f(\epsilon_i)d\epsilon_i - \frac{1}{\sigma_1}(K - \sigma_1\widetilde{Z}_M^{N-1})f\left(\widetilde{Z}_M^{N-1}\right)\right) = 0.$$

The integral $\int_{\widetilde{Z}_M^{N-1}}^{\infty} f(\epsilon_i)d\epsilon_i$ is the probability of a draw being *above* the top M among $N-1$, i.e., the probability of being *within* the top M out of N. In expectation, this is (by symmetry) $M/N = \alpha$.

$$\alpha - \frac{1}{\sigma_1}E\left(\left(K - \sigma_1\widetilde{Z}_M^{N-1}\right)f\left(\widetilde{Z}_M^{N-1}\right)\right) = 0$$

$$\alpha - \frac{K}{\sigma_1}E\left(f\left(\widetilde{Z}_M^{N-1}\right)\right) + E\left(\widetilde{Z}_M^{N-1}f\left(\widetilde{Z}_M^{N-1}\right)\right) = 0$$

$$K = \sigma_1 \left(\frac{\alpha + E\left(\widetilde{Z}_M^{N-1} f\left(\widetilde{Z}_M^{N-1}\right)\right)}{E\left(f\left(\widetilde{Z}_M^{N-1}\right)\right)} \right).$$

Thus, it is an equilibrium, for each bidder i, to bid

$$B_i = S_i - K = S_i - \sigma_1 \left(\frac{\alpha + E\left(\widetilde{Z}_M^{N-1} f\left(\widetilde{Z}_M^{N-1}\right)\right)}{E\left(f\left(\widetilde{Z}_M^{N-1}\right)\right)} \right).$$

The amount of expected underpricing is equal to this shading, K, minus the bias in the winning bids (i.e., the winner's curse). The winner's curse is the bias in the signal conditional upon being above the top Mth highest signals out of the other $N-1$ bidders.

$$\begin{aligned} WC &= E\left(S_i - V \mid \widetilde{\epsilon}_i > \widetilde{Z}_M^{N-1}\right) \\ WC &= \sigma_1 E\left(\widetilde{\epsilon}_i \mid \widetilde{\epsilon}_i > \widetilde{Z}_M^{N-1}\right). \end{aligned}$$

Since $\widetilde{\epsilon}_i$ is drawn from the normal distribution, the winner's curse is

$$WC = \sigma_1 \frac{E\left(f\left(\widetilde{Z}_M^{N-1}\right)\right)}{\alpha}.$$

Thus, the expected underpricing is

$$E(\pi) = K - WC = \sigma_1 \left(\frac{\alpha + E\left(\widetilde{Z}_M^{N-1} f\left(\widetilde{Z}_M^{N-1}\right)\right)}{E\left(f\left(\widetilde{Z}_M^{N-1}\right)\right)} - \frac{E\left(f\left(\widetilde{Z}_M^{N-1}\right)\right)}{\alpha} \right).$$

Proof of Proposition 2. In the uniform-price auction, the winning bidders all pay the $M+1$st highest bid. To win, a given bid must be higher than the Mth highest of the other $N-1$ bidders, and the winning bidder then pays the Mth highest of the those $N-1$ bids.

The bidder's maximization problem is:

$$\max_{B_i^u} \text{Prob}(B_i^u > B_s^u | S_i) E(V - B_s^u | S_i, B_i^u > B_s^u)$$

$$\begin{aligned} &\max_{K_i^u} \text{Prob}\left(S_i - K_i^u > S_i - \sigma_1\widetilde{\epsilon}_i + \sigma_1 \widetilde{Z}_M^{N-1} - K^u\right) \times \\ &E\left(S_i - \sigma_0\widetilde{\epsilon}_0 - \sigma_1\widetilde{\epsilon}_i - \left(S_i - \sigma_1\widetilde{\epsilon}_i + \sigma_1\widetilde{Z}_M^{N-1} - K^u\right) | S_i - K_i^u \right. > \\ &\qquad \left. S_i - \sigma_1\widetilde{\epsilon}_i + \sigma_1\widetilde{Z}_M^{N-1} - K^u\right) \end{aligned}$$

$$\max_{K_i^u} \text{Prob}\left(\widetilde{\epsilon}_i > \widetilde{Z}_M^{N-1} + \frac{K_i^u - K^u}{\sigma_1}\right) E\left(K^u - \sigma_1 \widetilde{Z}_M^{N-1} | \widetilde{\epsilon}_i > \widetilde{Z}_M^{N-1} + \frac{K_i^u - K^u}{\sigma_1}\right).$$

The first-order condition is:

$$\begin{aligned} E\left(\frac{\partial}{\partial K_i^u} \int_{\widetilde{Z}_M^{N-1} + \frac{K_i^u - K^u}{\sigma_1}}^{\infty} \left(K^u - \sigma_1 \widetilde{Z}_M^{N-1}\right) f(\epsilon_i) d\epsilon_i \right) &= 0 \\ E\left(-\frac{1}{\sigma_1}\left(K^u - \sigma_1 \widetilde{Z}_M^{N-1}\right) f\left(\widetilde{Z}_M^{N-1} + \frac{K_i^u - K^u}{\sigma_1}\right)\right) &= 0. \end{aligned}$$

Setting $K_i = K$,

$$\begin{aligned} E\left(\left(K^u - \sigma_1 \widetilde{Z}_M^{N-1}\right) f\left(\widetilde{Z}_M^{N-1}\right)\right) &= 0 \\ K^u = \frac{\sigma_1 E\left(\widetilde{Z}_M^{N-1} f\left(\widetilde{Z}_M^{N-1}\right)\right)}{E\left(f\left(\widetilde{Z}_M^{N-1}\right)\right)} &= 0. \end{aligned}$$

Thus, it is an equilibrium for each bidder i to bid

$$B_i^u = S_i - K^u = S_i - \sigma_1 \left(\frac{E\left(\widetilde{Z}_M^{N-1} f\left(\widetilde{Z}_M^{N-1}\right)\right)}{E\left(f\left(\widetilde{Z}_M^{N-1}\right)\right)} \right).$$

The amount of expected underpricing is equal to this shading, K^u, minus the bias in the $M+1$st highest signals out of all N signals, $\sigma_1 E(\widetilde{Z}_{M+1}^N)$. Thus, the expected underpricing is

$$E(\pi) = K^u - WC^u = \sigma_1 \left(\frac{E\left(\widetilde{Z}_M^{N-1} f\left(\widetilde{Z}_M^{N-1}\right)\right)}{E\left(f\left(\widetilde{Z}_M^{N-1}\right)\right)} - E\left(\widetilde{Z}_{M+1}^N\right) \right).$$

Appendix B: Treasury Auction Procedures[24]

The U.S. Treasury issues marketable securities as the primary method of financing the public debt. The securities are issued with different maturities. Securities with an initial maturity of less than a year are called *bills*. Those with an initial maturity from two to 10 years are *notes*. Long-term 30-year securities are called *bonds*.

Notes and bonds are semi-annual coupon-bearing instruments with the coupon rate normally set so that the initial price will be just below par. Bills, on the other hand, do not have coupons, but rather are sold at a discount to the face value.

The issue schedule for Treasury notes changes from time to time, but the most frequently and regularly auctioned notes are the two-year securities that are auctioned toward the end of each month. Three-year, five-year, and 10-year notes are issued less frequently. Occasionally, an auctioned security has exactly the same maturity and coupon as an existing older security. In that case, the new security is a reissue of the older security. In the primary market (i.e., the auction) and in the pre-auction when-issued market, the notes are quoted in yields. In the secondary market, prices are quoted.

Prior to the auction, the approximate total face value of securities is announced.[25] Auction participants include those that bid competitively and noncompetitively. Noncompetitive bidders submit a quantity order and are guaranteed awards at the average yield of the winning competitive bids. Noncompetitive bids are limited to face values of $1 million for bills and $5 million for notes and bonds. The remaining securities are available for competitive bidders.

The vast majority of bids are submitted by the fewer than 30 *primary dealers* authorized to deal directly with the Federal Reserve. Competitive bids are limited so that no one entity can control more than 35% of a single issue. Bidders may not submit both competitive and noncompetitive bids in the same auction.

The competitive portion of the auction proceeds as follows. Bidders submit (possibly multiple) sealed bids as yield-quantity pairs prior to the auction. Bids are submitted on a half basis point grid for bills and a tenth of a basis point grid for notes and bonds.[26]

[24] More information about the Treasury auction and the when-issued markets can be found in Sundaresan (1997). The definitive rules of the Treasury auction are in the Federal Register (2004).

[25] Recent auctions have been on the order of $20 billion.

[26] In the earlier part of the data used in this study, bids were submitted in whole basis points.

Securities are awarded to those that bid the lowest yield, and further awards are made successively at higher rates until the supply is exhausted. Bids at the highest accepted rate are pro-rated as necessary. For coupon-bearing securities, the coupon is set at the time of the auction as the average winning yield rounded down to the nearest eighth of a percent.

Under the old discriminatory mechanism, each of the winning bidders would pay a price corresponding to the yield in his own bid. Thus, not all winning bidders would pay the same price.

The uniform-price mechanism began as an experiment for the two- and five-year notes in 1992 and was expanded to all securities in 1998. With this mechanism, all winning bidders pay the price corresponding to the highest winning yield.

Auctions are usually held at 1:00 PM and the results are announced shortly afterward.

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