The Financial Holding Cost of Inventory

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1. Introduction

In a nutshell, stochastic inventory management models deal with the optimal inventory decisions by balancing two costs, the underage cost (including the backlogging or lost sale cost) when inventory is lower than the realized demand, and the overage cost when inventory is higher than the realized demand. On inventory holding cost, Zipkin (2000) points out that "the holding cost h typically includes two major components. The first comprises all direct costs associated with inventory itself, including costs for physical handling, insurance, refrigeration, and warehouse rental. The second component is a financing cost αc, where α is an interest rate, reflecting the fact that holding inventory ties up capital." While physical component of holding cost is tangible and relatively easy to estimate, the financial component is intangible, and hence more difficult to grasp conceptually or empirically. This difficulty is further complicated as in practice, firms rarely face a simple constant interest rate. Instead, they may face various sources of financing and different frictions in the financial market. Therefore, a natural question to ask is: in the presence of such frictions in the financial market, what should be a proper financial holding cost of inventory? Or equivalently, how does a firm's inventory decision depends on its financial situation?

To answer this question, we build a model that captures the firm's financing decision and cash dynamics explicitly, and then fully characterize the firm's jointly optimal decisions on inventory, cash, and (dividend) distribution. We show that a firm's optimal inventory level is closely linked to its cash policy and the form of capital market imperfection. Therefore, using a constant financial holding cost to approximate the impact of capital market imperfections on the inventory decisions could lead to significantly sub-optimal decisions.

Our paper is closely related to the booming literature of OM-Finance interface, especially those examining firm's operational inventory policy under financial constraints, such as Buzacott and Zhang (2004), Li et al. (2013), Luo and Shang (2013) and Gong et al. (2014). Our paper enriches this literature by showing that under realistic assumptions on capital market imperfection, a firm's optimal inventory and cash targets should be jointly determined by the firm's existing inventory and cash positions.

2. The Model

We develop a finite-horizon periodic-review model in which the firm jointly optimize the inventory and financing decisions to maximize the discounted expected value for shareholders. On the operational side, similar to the classic inventory models, the firm faces stochastic demand, unit holding cost for excess inventory, and unit backlogging cost for unsatisfied demand. Note that the unit holding cost modeled here only includes the physical component, which the financial implication of holding additional inventory will be modeled explicitly as follows.

On the financial side, we model three forms of capital market frictions: first, costly equity issuance (flotation cost); second, agency cost of free cash (in reduced form); and third, a credit limit linked to the firm's physical assets, especially inventory. The assumptions we adopt to model these market imperfections are widely used in the recent finance literature (Hennessy and Whited 2005, Bolton et al. 2013).

3. Main Results

Using the above model, we first show that in the absence of equity issuance cost, the firm does not hold any cash, and its optimal inventory level is characterized by the classic critical fractile formulation, with the shareholder's discount rate, as an approximation of the firm's cost of capital, influencing not only the overage cost, and hence can be seen as a component of inventory holding cost, but also the underage cost.

As equity issuance becomes costly, this capital market imperfection creates a precautionary demand for holding cash in the firm. We fully characterize the firm's optimal cash and inventory policy, which follows a state-dependent base-stock structure. Specifically, the firm's jointly optimal inventory and cash target level depend on the firm's existing inventory and cash positions according to the following three regions.

1. When the firm's inventory position or cash position is sufficiently low, the firm obtains external equity from shareholders to bring the cash and inventory up to certain targets. In this region, the firm does not keep more cash than absolutely needed to maintain liquidity, and its inventory decision follows a simple state-independent base-stock policy.

2. When the firm's cash position is high and inventory position is not too low, after keeping the cash and inventory levels at certain targets, the firm distributes additional cash to shareholders.

3. When the firm's cash and inventory positions are in between the above two regions, the firm neither raises equity nor distributes. It simply re-allocate its total asset between cash and inventory by matching the marginal benefits of the two items as close as possible.

Numerical results show that under realistic equity issuance costs, the optimal inventory target levels between Region 1 and 3 above can be significant. Therefore, the implied financial holding

costs are also significantly different, and using a single financial holding cost term regardless of the firm's inventory and cash position can lead to significant losses. In addition, we find that while the optimal inventory target level is non-decreasing in the cash position, the cash target level may first increase and then decrease in the inventory position.

As an extension, we construct managerially intuitive heuristic algorithms to approximate the optimal joint policy. One such heuristics is to incorporate the firm's financial status into a simple parameter as the financial holding cost and use classic inventory models to characterize the inventory policy.

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