When Graph Theory Meets Dynamic Pricing

Maxime Cohen, Swati Gupta, Jeremy J Kalas, Georgia Perakis

When Graph Theory Meets Dynamic Pricing

1 Introduction

Dynamically pricing an item through sales and promotions plays a significant role in determining the profits of any commodity-based industry. Sales can dramatically increase product visibility, result in increased store traffic, and sometimes cause customers to switch their brands. A study by A.C. Nielsen during January-July 2004 estimated that 12-25% of the sales in five big European countries were made during promotions. Promotions not only help retailers meet their sales targets, but also contribute to a significant percentage of the total profits. Using a demand model estimated from real data for a brand of coffee, it has been observed that the promotions for this brand of coffee achieved a profit gain of 3% compared to using only the regular price (i.e., no promotions). This improvement in the profits *is* of the order of the profit margins for retail companies such as super-markets. A paper published by the Community Development Financial Institutions (CDFI) Fund reported that the average profit margin for the supermarket industry was 1.9% in 2010 and Yahoo! Finance data concluded that the average net profit margin for publicly traded US-based grocery stores for 2012 was also close to 2010's 1.9% average. These studies provide ample evidence that manipulating promotions gives the retailers a good handle on increasing their sales and their profits. However, the current process of planning promotions is still fairly manual and does not account for the effect of a sale-item on other items in the same category. The natural question is, *can one do better using optimization models that cater to various business requirements?*

In this work, we study the problem of promotion planning for a retailer who needs to decide when to lower an item's price over a time horizon and to what amount, with the goal to maximize the total profits. In practice, retailers need to obey several business rules, for example a limited number of price changes. Further, one might expect that lowered prices would always result in increased demand. However a rather interesting consequence of promotions is that the customers tend to buy a sale-item in bulk and *stockpile* it resulting in a much lower demand (even lower than the baseline demand) in subsequent time periods. To capture this behavior, we consider general demand functions that depend on several past prices to capture the stockpiling behavior of the customers. The goal is to propose a scalable method for this problem such that the retailers can easily test several *what-if* scenarios.

Summary of Contributions We study a multi-period, multi-item *Promotion Optimization Problem* (POP) for a retailer. One seeks to maximize the total profits by choosing optimal prices that satisfy several business requirements. The strength of our work lies in the graphical model reformulation we introduce which can handle naturally various complexities of the POP while providing access to a whole range of ideas from combinatorial optimization. Using our graphical model, we specifically show how to capture the *end-of-horizon* effects, i.e. the interaction of prices at the end of the time horizon with the next planning period. The complexity of our method depends linearly on the number of time periods and the length of the price ladder of each item but is exponential in the consumer memory of the model and the number of items. We show that the POP is in fact NP-hard for large memory, and thus, one cannot hope for a polynomial time algorithm for problems with large memory (which is often the case) for general demand functions. Consequently, we approximate our demand model using the well-known *reference price* model and give a fully polynomial-time approximation scheme for the promotion optimization problem through the reference price model. We extend the reference price model to handle cross-item effects using the notion of a single "virtual" competitor item. We finally report promising performance of these methods on real-world data sets, in collaboration with the Oracle Retail Science group, where we achieve 4-6% improvement in profits.

Literature Review [1] and [2] contain extensive reviews on sales promotion and dynamic pricing respectively. One of the main challenging issues in modeling our problem is related to how one should incorporate the dependence of the demand on past prices while modeling various business rules that arise in practice. Moreover, for a problem with sensitive profit margins, it is important to explore exact methods and good approximations when one cannot hope to find exact methods. In addition, our model can be applied to any objective and demand function and includes several important business requirements.

2 Model and Contributions

Given a set of items and a finite planning horizon, the promotion optimization problem aims to decide for each item when a promotion must happen and to what extent, such that the total profit is maximized. The prices for the items are assumed to come from a discrete set, for example the prices must be rounded up to a cent. Our demand functions depend on a constant number (referred to as the *memory* of the model) of past prices of each item and are allowed to vary with time as well.

We reformulate the problem in the form of a layered graph. For each time period, the nodes in the corresponding layer contain the price in that time period as well as the memory of some past prices. Edges in the graph exist only between nodes in consecutive layers such that the information of the past prices in each node is consistent. The cost of the edges denotes the profit that is obtained for using the prices in the incident nodes, and thus the problem of maximizing the profits reduces to finding the maximum cost path from the first layer (i.e. the first time period) to the last layer (i.e. the last time period) in the graph. Given this graph, one can enforce various business rules by either deleting some edges (i.e. for markdown prices) or by adding more information in each node of the graph (i.e. to limit the number of promotions, or enforce minimum time between promotions to protect brand image). Since the planning horizon is finite, optimization models often ignore the effect of prices in the end of horizon time periods. We capture these *end-of-horizon* effects by assuming similar prices in the next planning period. Another important constraint that arises in the fashion industry is that of available inventory. Using our graphical model, we can model inventory constraints as a *flow* problem with only a few binary variables! These ideas extend to multiple items naturally by introducing additional memory in the model, however the size of the graph increases exponentially with the number of items.

We prove that the POP is NP-hard by giving a polynomial reduction of the traveling salesman problem to POP. Subsequently, we incorporate large memory through an approximation using the well-studied reference price model. In our graphical model, this approximation behaves as if memory is two, and thus gives good performance. Finally, to capture cross-item effects, we extend the reference price model to account for the price of a "virtual" item that captures the effects of the remaining items. This research has been conducted in collaboration with industry practitioners from the Oracle Retail Science group. We report that we can achieve a projected 4-6% improvement in profits using our methods on available data sets.

References

[1] Robert C Blattberg and Scott A Neslin. *Sales promotion: Concepts, methods, and strategies*. Prentice Hall Englewood Cliffs, NJ, 1990.

[2] Kalyan T Talluri and Garrett J Van Ryzin. *The theory and practice of revenue management*, volume 68. Springer Science & Business Media, 2006.