Simulation Optimization via Robust Optimization

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To understand the performance of stochastic systems, the traditional approach consists of positing a probability distribution over the uncertain parameters and evaluate the distribution associated with a certain performance measure. When the system dynamics are complex, common practice is to resort to Monte Carlo simulation models to evaluate the system's performance. Difficulties arising in this setting include the choice of probability distributions to posit around the uncertain parameters, given the presence of correlations and limited information, as well as computational challenges, especially for heavy-tailed systems which require a high number of samples to obtain meaningful simulation outputs.

A step further is to optimally design stochastic systems by finding the best design or input parameters with the goal of optimizing a given performance measure. Simulation optimization is commonly used to that end, by taking the output of a simulation model, then feeding it to an optimization strategy which provides a feedback on the progress of the search for the optimal solution, henceforth guiding the next input to the simulation model. Simulation optimization methods include gradient based search methods, such as perturbation analysis (e.g., see this approach applied to inventory systems by Glasserman and Tayur [1994, 1995] and Fu [1994]), and stochastic optimization, among others. However, generating the gradient for search methods, on the one hand, performing Monte Carlo approximations and generating scenarios for the stochastic optimization framework, on the other hand, can be challenging.

Proposal

We propose a robust optimization approach to simulating and optimizing stochastic systems characterized by linear dynamics, based on a robust optimization framework. Specifically,

(1) Instead of positing some joint probability distribution over the uncertain parameters affecting the system's performance, we model uncertainty via polyhedral sets inspired by the limit laws of probability. The size of each uncertainty set is characterized by a single variability parameter, which controls the degree of conservatism of the model. We further assume that the variability parameter follows some limiting distribution (a derivative of the normal distribution for light-tailed systems and the stable distribution for heavy-tailed systems).

(2) We formulate the worst case performance analysis as an optimization problem over the parametrized uncertainty sets and obtain a worst case output as a function of the variability parameters. To analyze the average case performance of a stochastic system, we propose to take the average of the worst case outputs over the variability parameters. Furthermore, we cast the problem of finding the optimal design inputs of a stochastic system that optimize its average performance as a robust optimization problem.

This framework allows a *significant reduction in the dimension of uncertainty*, yielding tractable analyses. Furthermore, the distribution of the variability parameters is deduced from the limiting distributions, hence bypassing the challenge of fitting probability distributions or generating scenarios to model the uncertainty. The approach also demonstrates *the use of robust optimization for evaluating average performance*, as opposed to being merely restricted to a worst case approach.

Results

We illustrate the tractability and accuracy of our approach by (a) simulating the transient behavior of multi-server queueing systems, (b) determining optimal base-stock levels in supply chains, and (c) designing a optimal portfolio minimizing conditional value-at risk (CVaR).

Transient Queues. Queueing systems in call centers and cloud computing are often subject to heavy-tailed arrivals and experience substantially long transient regime. The probabilistic analysis of even simple transient queues remains intractable. We assume that the interarrival and service times satisfy uncertainty sets characterized by variability parameters. We formulate the worst case performance analysis as a one-dimensional nonlinear optimization problem, which yields closed form expressions as a function of the variability parameters. The resulting average case performance reduces to a single-dimensional integral. Besides significant tractability, the proposed methodology yields accurate results, with errors within 10% relative to simulation for light and heavy tailed multi-server queueing systems.

Inventory Systems. The problem of finding optimal base-stock levels in multi-echelon supply chain networks is challenging given the multi-dimensional state space of possible demand realizations. In our framework, we assume demand realizations at the sink nodes satisfy parameterized uncertainty sets. For given base-stock levels, we take a worst case approach and determine the highest total cost incurred, subject to the demand satisfying the parameterized uncertainty set. We then cast the problem of finding the optimal base-stock levels that minimize the expected total cost as a robust optimization problem, specifically one that minimizes the average worst case total cost. Our approach generates optimal base-stock levels within reasonable run times, using a Benders-like decomposition inspired by Bienstock and Özbay [2008], with optimal base-stock levels and expected costs that are within 8% of those obtained via simulation optimization.

Portfolio Optimization. Multi-period optimal portfolio selection is a challenging problem given the intrinsic correlation among different assets within the portfolio. While practitioners generally assume a geometric Brownian motion for price dynamics, empirical studies have shown that market prices differ from the ascribed model prices. We propose to model uncertainty in correlated returns via parametrized uncertainty sets. More specifically, we deduce the covariance matrix from bootstrapping over available data on stock returns, and then express price returns as a function of independent variables using the Cholesky decomposition. We constrain the independent variables to follow parametrized uncertainty sets inspired by the central limit law and obtain an optimal portfolio which minimizes the CVaR of the worst case, given our uncertainty set assumptions. Our computations suggest that our methodology captures the $\alpha-$CVaR of the total returns with errors within 12% compared to simulations, for $\alpha = 0.5, \ldots, 0.99$.

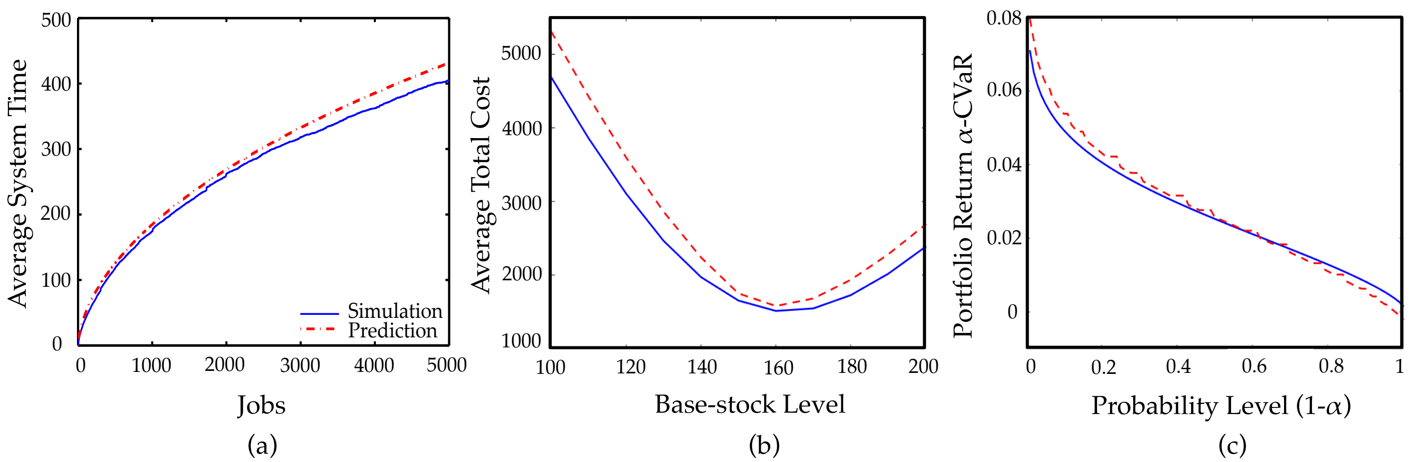

Figure 1: Simulated (solid blue line) versus predicted values (dotted red line). Panel (a) depicts the average system time in a single-server queue with Pareto distributed primitives with a tail coefficient of 1.6. Panel (b) depicts the average total cost of holding and backlogging stock in a single installation over 24 time periods. The optimal base-stock level is 160 as inferred by simulation and prediction, with an error of 4% with the corresponding expected cost relative to simulation. Panel (c) depicts the α-CVaR of the return for a given five-asset portfolio over a one month horizon of 22 trading days.

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