Multi-Period Stock Allocation Via Robust Optimization

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One important instance of the central warehouse, multi-retailer problem is the following. The central warehouse is replenished once in each cycle of a lengthy time horizon. Each cycle consists of multiple periods. For example, a cycle could be a week in duration and a period could be a day in length. The question then is how much inventory should be allocated to each retailer so as to minimize costs. Suppose that there are two types of costs, holding and backorder costs. Furthermore, suppose the holding cost rate is the same at the central warehouse as it is at each of the retailers. In this case, there is no economic incentive to hold inventory at the central warehouse. The only motive for holding inventory at the central warehouse is to reduce the risk of inventory imbalance. Holding inventory back at the central warehouse is a form of risk pooling. Suppose we make the so-called Clark- Scarf balance assumption. Then, when holding costs are equal among all locations, and the balance assumption holds, it is easy to show that all inventory received at the central warehouse will be immediately allocated to the retailers. This is the policy first employed for this situation by Eppen and Schrage.

Suppose, on the other hand, we do not make the balance assumption. To study this problem for the environment we have stated, we use a robust optimization approach. Robust optimization has been proposed as a tractable optimization approach for stochastic planning problems that are too large to be solved by dynamic programming. Proponents of robust optimization make several claims for the approach:

1. Robust optimization models lead to solutions whose expected cost is close to the minimum expected cost in cases where the optimum can be computed, especially when compared to the error possible from selecting probability distribution models incorrectly;

2. Robust optimization models do not suffer the curse of dimensionality which plagues expected cost optimization models; in many cases, the dimension of the robust optimization formulation is no larger than the size of a corresponding deterministic model.
3. Robust optimization models in special cases lead to closed form solutions which provide analytical insight into the underlying problems.

Our approach is similar to other robust optimization applications. We divide the decisions between inventory target variables and actual stock allocation decisions. The former are chosen before demands are realized and the latter are consequences of the policy variables and the actual demands. It is an adaptive policy approach and we use Benders' decomposition to solve the overall problem. The Benders' subproblem is a bilinear program but the simplicity of our formulation allows us to recast it as a mixed-integer linear program.

Our primary contribution to modeling inventory systems using robust optimization is to include the phenomenon of risk pooling across retailers. In particular, we develop a computationally tractable approach, via robust optimization, to recommend stock allocations from a central warehouse to multiple retailers over multiple time periods when imbalance is a distinct possibility. Recall that we assume that holding costs are the same at all locations in order to focus solely on the risk-pooling motive for holding stock centrally, distinct from economic motivations. We also assume that demand distributions are characterized by their means and variances and that demands are independent across retailers and across time. We find that the claimed benefits of robust optimization, with some qualification, do materialize for this problem. In particular,

1. Stock allocation decisions recommended by this approach capture a large fraction of the risk-pooling benefit that is possible in an expected value sense;
2. The approach is capable of quickly solving problems far beyond the reach of dynamic programming methods; however, to adequately capture the risk-pooling effect, the model size grows with the square of the number of retailers; and
3. A closed form solution is possible for a relaxed version of a two-period, identical-retailer stock allocation problem; this solution provides analytical insight not seen before in the study of this problem.

That is, all the benefits of robust optimization emerge except that the problem size is not proportional to the size of a corresponding deterministic model.

In the course of evaluating the performance of the robust allocation approach, we consider a variety of situations in which risk pooling, and the balance assumption, may or may not play a role in system performance. This study reinforces some conclusions from earlier studies, but also points to new managerial insights. For example, previous work has shown the balance assumption to be violated to a significant extent when demand among retailers is unbalanced, even for moderate coefficients of variation. Our results point to a different conclusion, and this can be traced to our contention that coefficients of variation are inversely correlated with demand rates, as we have observed in several practical instances.