Two-Stage Portfolio Optimization with Higher-Order Conditional Measures of Risk

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We describe a study of application of novel risk modeling and optimization techniques to daily portfolio management. In the first part of the study, we develop and compare specialized methods for scenario generation and scenario tree construction. In the second part, we construct a two-stage stochastic programming problem with conditional measures of risk, which is used to re-balance the portfolio on a rolling horizon basis, with transaction costs included in the model. In the third part, we present an extensive simulation study on real-world data of several versions of the methodology.

The main objective of this paper is to evaluate the usefulness of several risk modeling and optimization techniques for daily stock portfolio optimization. The portfolio problem is to find an "optimal" way to distribute the initial capital among the available assets.

In a pioneering study, Markowitz argued that portfolio performance can be measured by using two scalar characteristics: the mean of the portfolio return, and the variance of the return, which characterizes its riskiness. We can then minimize the variance for a fixed value of the mean, or maximize the mean, while keeping the variance bounded. Since then, numerous theoretical and practical studies evaluated the usefulness of the mean–variance approach in portfolio optimization.

Further improvement was made by considering more general mean–risk models, with different measures of variability. By considering consistency with stochastic dominance, a family of mean–semideviation models were introduced. These models are particularly useful for portfolio management.

In the last decade, axiomatic models of risk have been studied extensively, in particular, *coherent risk measures*, introduced by Artzner et al. We will use coherent risk measures in this study.

In this study, two-stage portfolio models with higher-order conditional coherent risk measures are studied. First, adequate number of scenarios are generated to model the probabilistic information on random data. Next, scenario trees are constructed by different methods, and the best one is chosen based on the distance between the probability distribution on the scenario tree and the empirical distribution on the raw scenarios. It is found that the two-step forward clustering method is most efficient in terms of the CPU time, because it passes over the data just once. However, that multi-facility location clustering method is the most accurate, in terms of the Monge-Kantorovich metric.

Next, conditional mean-semideviation risk functions of first-order and higher-orders (Order 2 and 3) are used to formulate the risk-averse two-stage portfolio problem on the trees generated from the multi-facility location clustering method. The problems are solved by a generic risk averse multicut algorithm for any higher-order risk function.

In order to obtain the empirical results, a simulation analysis was carried out. Each day, the preceding chosen days of data were used to calibrate a multivariate GARCH model. The model was then used to generate large amount of scenarios for the following two days. The multi-facility location clustering method was used to construct a scenario tree for the next two days. Then the tree model with conditional measures of risk was solved, the investments were re-balanced, and the method continued. On the next day, new return data were available, new scenario trees were generated, new models solved, etc.

The simulation study had two objectives. First, we compared the two-stage portfolio model with the static model. Results showed that two-stage portfolio model performs better than the static model for both mean-semideviation risk functions of order 1 and 2. This was partly due to the reduced volume of trades, which resulted in significantly lower transaction costs, but also to a better portfolio composition.

Secondly, we compared two-stage portfolio models with the mean-semideviation risk measures of first-order and higher-orders (Order 2 and 3) with static minimum variance model. In each case we used fixed risk aversion constant and bid-ask spread transaction costs. The two-stage portfolio optimization problem was solved with the risk-averse multi-cut method, implemented in MATLAB with the CPLEX solver. We observed that there is no significant difference between two-stage portfolio problem with mean-semideviation risk measure of second-order and third-order. Results showed that using the second-order methods leads to significant improvements in cumulative wealth trajectories compared to first order and Markowitz's minimum variance models.

The results show that the portfolio allocations for mean-semideviation models of first-order and higher-orders are similar. However, portfolios with the mean-semideviation of second-order and third-order perform better compared to mean-semideviation of first-order, minimum variance model, and the Dow Jones index. All two-stage models outperform the static model.