# Selling Passes to Strategic Customers Jue Wang, Yuri Levin, Mikhail Nediak

### SELLING PASSES TO STRATEGIC CUSTOMERS

#### 1. INTRODUCTION

Passes are prepaid packages of multiple units of goods or services, typically with flexible delivery or consumption dates. They can take a variety of forms, such as commuter passes in transportation (e.g., flight passes in the airlines), capped quota in telecommunication, or memberships in resorts, health, and beauty clubs.

In the airlines, a "flight pass" is a prepaid package of finite number of electronic flight credits that can be used for travel during a specific period. The pass permits its holder to obtain a seat on a specified number of flights at a fixed price. Flight passes are typically sold through the same booking systems as regular tickets. Although individual tickets are priced dynamically, most passes are sold at fixed prices. This inconsistency gives rise to the question: how can we dynamically price the pass and individual tickets in a consistent and integrated manner? To answer this question, airlines must consider customer choice behavior between passes and individual tickets, and account for the credit utilization process and repurchases behaviours. This paper addresses the following research questions: Should the passes be priced dynamically? How far is the fixed price from optimality? What are the benefits of passes (if any) and how do strategic behavior and capacity affect their benefits?

We focus on strategic customers who are inherently forward-looking in both purchase and utilization of passes. That is, they compare all foreseeable future prices and make the purchase that maximizes their long-term utilities. After purchasing the pass, the customers become pass holders and strategize on utilizing the finite number of credits on the pass. For example, high future prices may slow down the credit utilization because pass holders may curb their consumption to avoid renewing passes at high prices. Further, the credits utilization rate is generally decreasing in the number of remaining credits because smaller quantities are often perceived as more valuable. This non-stationary credit utilization process is an integral part of customer behavior and should be considered while making price decisions.

#### 2. Model description

We formulate the problem as a fluid model. We consider a monopoly with a finite capacity of items to be sold over a finite horizon. Each item can be sold either individually or through the pass. Each pass initially has a finite number of credits but will expire at the end of the horizon. After spend all credits on the pass, the customer can purchase the pass again (renewal) or buy individual tickets.

Along with the pricing policy and the pass expiration date, one of the most important drivers of customer choice is the number of remaining credits. We assume that the dependence on the number of remaining credits is essentially captured by the utility of these credits. A customer without the pass can choose one of three actions: purchase regular ticket, purchase the pass, and no-purchase. The choice probability is characterized by the nested logit model. A pass holder focuses on utilizing the pass credits and chooses between using a credit and wait. The binary choice probabilities are captured by the logit model.

The objective is to determine the open-loop dynamic regular price and the price of the pass to maximize the revenues from the available capacity, subject to dynamics that include the remaining capacity, the breakdown of customers according to the number of remaining credits on their passes, and customer perceptions of future utility.

We solve the problem in the framework of continuous-time deterministic optimal control. Using the Pontryagin's maximum principle, we derive the necessary conditions for the optimal pricing policy and examine the structures of price, demand and revenue.

## 3. Summary of Results

**Turnpike property.** We find that the optimal pricing policy has the "turnpike" property: the optimal price trajectories stay close to a steady state (turnpike) most of the time except at the beginning and the end of the horizon, provided that the sales horizon is sufficiently long (to the extend that pass holders can spend all credits before the expiration date). Further, the revenue loss from a fixed-price policy does not increase with the sales horizon, indicating that its relative sub-optimality decreases as the sales horizon increases. This suggests that dynamic pricing of passes cannot significantly improve the revenue. After introducing the pass, even the regular price for individual sales need not be dynamic.

**Price & revenue structure.** We also study the impact of strategic behavior and capacity on the turnpike revenue rate. We find that passes offer quantity discounts and encourage consumption as long as customers are not fully strategic, which is consistent with practice and intuition. What is less intuitive is that the pass credit generates a higher turnpike revenue rate than a regular sale when customers are not fully strategic *and* the capacity is limited, in which case the capacity scarcity also makes an appreciable contribution to the revenue.

**Capitalization on strategic behavior.** Perhaps the most significant advantage of the pass is that it allows the issuer to *benefit from strategic behavior*. In prior results, strategic customer behavior typically decreases the revenue. However, this trend can be reversed in the presence of passes: more strategic customers generate higher revenues.

Passes capitalize on strategic behavior by offering a quantity discount contingent on advance purchase. Since passes require the customers to prepay for their future consumption, they are less appealing to myopic customers. Therefore, the quantity discount offered by passes is more likely to be appreciated by strategic customers who carefully plan future consumption. If the quantity discount is sufficiently attractive, the customers need not involve in strategic waiting. As a result, the pass can encourage advance purchase made by strategic customers. It has been discussed in the literature that the firm can utilize rationing strategies to create a sense of urgency in order to induce early purchase. Unlike the negative incentive of shortage risk, passes provide a new mechanism to capitalize on strategic behavior by offering a quantity discount combined with advance purchase.