Some Specially Structured Assemble-to-Order Systems

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Abstract

Assemble-to-order systems are important in practice but challenging computationally. This paper combines some notions from combinatorial optimization, namely polymatroids and discrete convexity, to ease the computational burden significantly, for certain specially structured models. We point out that polymatroids have a concrete, intuitive interpretation in this context.

Extended Abstract

This paper considers a one-period *assemble-to-order* (or *ATO*) *system* with stochastic demand. There are several *components* and several *products*. Demands occur only for products, while the system keeps inventory only of components. To make a unit of each product requires a certain amount of each component. The time to assemble a product from components is negligible. A product is assembled only in response to demand. The components must be acquired, however, before demand is realized. The overall problem can be viewed as a stochastic program with recourse. See Song and Zipkin [3] for a review of the research literature and applications up to the early 2000's. We mention several more recent works in the full paper.

Once demand is realized, the available components must be allocated among the products. This problem can be represented as an integer linear program (ILP). For certain special product-component structures, under a certain condition on the component inventories, the feasible set has a special form called a *polymatroid*. One implication of this form is that the ILP is easy to solve; its solution can be written almost in closed form.

We then prove that, assuming only that the acquisition cost is increasing, the component inventories *should* satisfy a related but weaker condition. (More precisely, this is a necessary condition for optimality in the larger problem of component acquisition.) In certain special but important cases, this weaker condition is in fact equivalent to the polymatroid condition. In any case, we assume the polymatroid condition from then on.

Next, we show that the expected cost of the ILP has a property related to *$L^\natural$-convexity.* This notion, developed by Murota [1], [2] and others, is part of a broader topic called discrete convexity. It has proven quite useful in understanding and solving other operational problems. (The last section below mentions some of these.) Now, the cost here is not quite $\mathrm{L}^\natural$-convex; it has a property we call *cover-$L^\natural$-convexity.* We argue that this property is nearly as good as $\mathrm{L}^\natural$-convexity for practical purposes. (Caveat: The argument is not quite complete.) Provided the acquisition cost also is well behaved, the overall problem should be fairly tractable, using the powerful algorithms developed in the discrete-convexity literature.

The polymatroid condition may seem abstract. We point out, however, that it and its consequences have rather appealing, concrete meanings in the context of ATO systems.

The full paper is organized as follows: Section 2 reviews relevant material on polymatroids and discrete convexity, derives a few small but useful results about polymatroids, and introduces the notion of cover-$\mathrm{L}^\natural$-convexity. Section 3 presents the formulation of the ATO model. Section 4 contains the main results concerning the relations of the ATO model with polymatroids and cover-$\mathrm{L}^\natural$-convexity. Section 5 extends the model and results to two multi-period systems, one with backorders and one with lost sales (but zero leadtimes). Finally, Section 6 points out some directions for further study, while suggesting some intuitive interpretations.

References

[1] Murota, K. (2003). *Discrete Convex Analysis.* SIAM, Philadelphia, PA.

[2] Murota, K. (2009). Recent developments in discrete convex analysis. In W. Cook, L. Lovász, and J. Vygen (eds.), *Research Trends in Combinatorial Optimization* (pp. 219-260). Springer, Berlin-Heidelberg.

[3] Song, J. and P. Zipkin (2003). Supply-chain operations: Assemble-to-order systems. In A. de Kok and S. Graves (eds.), *Handbooks in OR/MS 30: Supply Chain Management* (Chapter 11). Elsevier, Amsterdam.