Buyer Intermediation in Supplier Finance

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Abstract

Small suppliers often face challenges to obtain financing for their operations. Especially in developing economies, traditional financing methods can be very costly or unavailable for such suppliers. In order to reduce channel costs, in recent years large buyers started to implement their own financing methods that intermediate between suppliers and financing institutions. In this paper, we analyze the role and efficiency of buyer intermediation in supplier financing (BIF). Building a theoretical model, we show that without buyer intermediation, traditional supplier financing can be inefficient and can significantly reduce supply chain performance. Using data from a large Chinese online retailer, we demonstrate that BIF induces lower wholesale prices and higher order quantities. Through structural regression estimation, we demonstrate that the retailer overestimates the demand by 10-15%. We also show that the financed suppliers have cash positions of only about 56% of their operating costs, and that BIF improved channel profits by approximately 16.7%, yielding significant annual savings for the retailer.

Key Words: Operations-Finance Interface, Supply Chain Management, Contracting.

1 Introduction

When providing goods to a downstream retailer, suppliers often bear a long payment delay after delivery. Usually, this delay is contractually imposed by the buyer due to reasons that span from retailer financing to quality control and payments being contingent on lack of defects. As a consequence, suppliers, especially small ones, often find themselves in need of cash and financing in order to support their operations. However, it is often very difficult for small suppliers to obtain financing under favorable conditions. The situation is worse in developing economies such as China, due to lack of credit history, and in certain cases, lack of established financial mechanisms. As a result, suppliers often resort to very high interest loans, which increase the overall costs in the supply chain and reduce channel efficiency.

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Concerned about the rising supply chain costs and the difficulty, and sometimes inability, of their suppliers to receive financing, buyers are stepping up to help mitigate their suppliers' cash flow problems. Different companies take different approaches to the problem but in many cases, buyers employing these new schemes essentially act as intermediaries or underwriters, with a third-party bank lending money to the supplier. Usually, with their knowledge of historical transaction details and their past interactions with their suppliers, buyers have better information about the reliability of their own suppliers than banks do. This allows them to bridge the gap between a bank and the supplier, secure the loan back payment and price the supplier risk more efficiently. As a result, the bank can lend to the supplier at a better rate, the supplier can have his operations financed and the entire channel can operate more efficiently.

One example of a large buyer who provides financing intermediation to its suppliers is the Chinese online retailing giant Jingdong. Jingdong is currently China's largest online business-to-consumer retailer offering more than a million product selections of thousands of brands with an annual sales volume of approximately \$12 Billion. Jingdong's suppliers vary in size and many are small suppliers who are routinely in need of financing to continue their operations. To ease the supplier financing costs, Jingdong launched a supplier finance intermediation service in 2012. The financing scheme works as follows: Jingdong and a supplier agree on delivery of goods at a future date with Jingdong's payment to the supplier being due after a certain period, e.g., 45 days. However, often the supplier needs cash earlier than the payment due date to cover its operational costs and continue production. To provide this financing, Jingdong intermediates between the supplier and a third-party bank. In particular, the bank provides part of the payment, an amount that the supplier chooses, but charges a certain percentage of that amount, determined by Jingdong, to the supplier as "prepaid interest". In return, Jingdong guarantees the payment of the full amount to the bank at the due date. In doing so, Jingdong effectively underwrites the loan, assumes all the supplier risk and frees the bank of any concerns that a loan will not be paid back. This arrangement significantly reduces the cost of financing for the supplier as it reduces the riskiness of the loan for the bank. At the time scheduled for the payment, in addition to paying the bank its due, Jingdong pays the supplier the remaining amount for the product, provided that the product was defect free.

The buyer's pivotal position in supply chain finance is of special interest, as it sheds light on the question of how to best finance the supplier risk. The crux of the problem lies at how the supply chain risk should be distributed more efficiently to lower financing costs, keep the wholesale price low and avoid under-production. One important question is whether buyer intermediation in supplier financing is effective in reducing costs and increasing supply chain efficiency. Further, how does Buyer Intermediated Financing (BIF) compare to the traditional financing schemes that do not directly involve the buyer? In this paper, centered around these questions, we study the effectiveness of buyer intermediation in supplier finance.

To capture the economic gist behind the problem, we build a model of a two-layer supply chain with a potentially budget-constrained supplier. We first analyze the traditional supply chain financing namely, commercial loans such as those that Jingdong's suppliers were using before BIF was introduced. We then model BIF based on the financing scheme employed by Jingdong, compare its outcome and performance to the traditional financing scheme and demonstrate the efficiency gains BIF yields for the supply chain. Utilizing data obtained from Jingdong on its supplier finance program, we then test our model's conclusions, showing that as our model predicts, BIF reduces wholesale prices, reduces interest rates and increases order quantities. We further perform a structural estimation of our model and obtain estimates of unobservable model parameters. Finally, using the results of our model and calibrating, we estimate the efficiency gains by employing BIF compared to the traditional financing scheme to be approximately 16.7%, which translates into approximately \$7.3M savings for the current data set with projected total savings from employing the scheme reaching upwards of \$58M in 2013.

The rest of the paper is organized as follows. Section 2 reviews the previous literature. Section 3 lays out the model framework, describes the financing schemes we study, and provides the theoretical analysis and the comparison of the financing schemes. Section 4 presents our empirical analysis, including model parameter estimation, hypothesis development and tests, and efficiency analysis. Section 5 offers our concluding remarks. All proofs are given in the appendix.

2 Literature Review

Our work lies at the interface of operations and financial decisions. In the operations management literature, our work is closely related to three areas: supply chain finance, supply chain risk and operational hedging.

Supply chain finance literature studies how financial constraints and financing services influence supply chain performance. Xu and Birge (2004) provide one of the early studies in the literature that captures the decision of a capital constrained newsvendor. They show that firm value can be significantly improved by integrating financial and operational decisions. Klapper (2006) suggests that factoring allows a high risk supplier to transfer its risk to higher quality buyers, and factoring is more prevalent in countries with more mature credit information bureaus. Dada and Hu (2008) show that in a Stackelberg game setting, a capital constrained newsvendor would borrow from bank and order an amount that is less than what would be optimal if the cost of borrowing is not too high. Zhou and Groenevelt (2008) investigate the case when the supplier provide subsidies to a budget constrained retailer. Caldentey and Chen (2009), Kouvelis and Zhao (2011, 2012) and Jing et al. (2012) examine the interplay between a supplier, a budget constrained retailer and bank, demonstrating that when bank loans are competitively priced, retailers will prefer supplier financing to bank financing if an optimally structured trade credit contract is offered - but when the bank has market power in setting the interest rate, either form of financing can be preferable depending on the market parameters. Yang and Birge (2011) extend previous works by showing that even when bank financing and supplier financing can be used jointly, supplier financing is still preferred to bank financing. In addition, with the aid of a sample of firm-level data, they find that the financing pattern predicted by their model is used by a wide range of firms.

Alan and Gaur (2012) show that when bank is a profit maximizer, the collateral value of inventory is a function of the bank's belief regarding the firm's demand distribution. Luo and Shang (2013) explore the interaction of the inventory policy and trade-credit in multi-period setting, demonstrating that a simple myopic inventory policy based on a target stock level and the firm's working capital is optimal. Wu et al. (2014) explore the buyer backed supplier finance through a centralized two-stage stochastic programming model with exogenous wholesale price, finding that the buyer's guarantee in financing is necessary if the demand is large, supplier's capital is inadequate or the market finance interest rate is high. They also find that in this single decision-maker setting, the buyer can improve her payoff by guaranteeing the supplier's loan. Research on reverse factoring is closely related to our paper as it also studies how large retailer can collaborate with bank to help supplier to obtain cheap financing by reallocating the financial flow. Corsten (2010) notes that some suppliers resist changing from previous working practices to reverse factoring due to reasons such as supplier's being not accustomed to dealing with international bank that works with the retailer and complicated financial procedure required by implementing reverse factoring. Tanrisever et al. (2012) provide a theoretical treat showing how reverse factoring creates value for each party in the supply chain and how the value is affected by the spread in external financing cost, the working capital policy, the payment period extension and the risk free rate. The paper also explore the impact of reverse factoring on operations decision using make-to-order and make-to-stock models. In our paper, we differ from Tanrisever et al. (2012) by presenting a two-way decentralized game between the supplier, the buyer and the bank with endogenous wholesale pricing, and comparing the performance of two different financial schemes in this strategic setting with asymmetric information, as employed in practice by companies such as Jingdong. We further identify the conditions under which in equilibrium the buyer intermediated financing scheme can improve the buyer's payoff or reduce it. We further apply the theoretical findings to data from Jingdong to test the theory, estimate unobservable parameters and measure efficiency.

The second stream of research related to our paper concerns supply chain risk and supply chain default. Tomlin (2006) studies different types of disruption management strategies and shows that the nature of disruption such as length and uptime influences which strategy to choose. Babich (2006) and Babich et al. (2007) examine how correlated default affects the supplier competition and diversification within a supply chain. Dada et al. (2007) show that in a supply chain where there are both reliable and unreliable suppliers, low production cost, rather than reliability, is still the most important criteria for choosing suppliers. Lai et al. (2009) show that with financial constraints, a supply chain would maximize its efficiency by operating under a combination of consignment mode and pre-order mode. Swinney and Netessine (2009) demonstrate that in the presence of default risk, long-term contracts are preferred over short term contracts. Also, a dynamic contract whose contract price is tied to some index representing the production cost allows the buyer to coordinate the supply chain. Yang et al. (2009) examine how asymmetric information influences the value of risk mitigation strategy. They illustrate that information asymmetry will cause reliable suppliers to use back production option under default, and a manufacturer would be willing to pay for information when backup production cost is not very high. Babich (2010) presents conditions under which, when facing a risky supplier, a downstream firm could make ordering decisions independent of subsidy decisions. Dong and Tomlin (2012) examine how business interruption insurance, along with operational measures such as investing in inventory and using emergency sourcing, could help mitigate the disruption. They show that insurance and operations measures could be either substitutes or complements. We study a financing method that transfers the risk among supply chain members in a novel way, and show that by allocating risk away from the supplier and towards the buyer can improve supply chain performance.

The third stream of literature our paper is related to is on operational hedging. Boyabatlı and Toktay (2004) provide a comprehensive review of this branch of the literature. Gaur and Seshadri (2005) show how to construct optimal hedging transactions that minimize the profit variance and increase the expected utility for a risk averse decision maker. Ding et al. (2007) demonstrate that the firm's financial hedging strategy ties closely to the firm's operational strategy. Chod et al. (2010) examine the relationship between operational flexibility and financial hedging. Based on the value of financial hedging, they show that when product demands are positively (negatively) correlated, product flexibility and financial hedging tend to be complements (substitutes). Rao and Gutierrez (2010) propose a framework that can identify the combined effect of operations management practice and financial hedging, and find that coordination between the two functions can lower a firm's cash requirement and boost productivity. Hankins (2011) studies how firms manage risk by examining the relationship between financial and operational hedging. By using a sample of bank holding companies, he shows that the operational hedging can substitute for financial hedging.

Tomlin and Wang (2005) investigate the value of the combination of flexibility and dual sourcing in unreliable newsvendor networks. They demonstrate that the level of diversification and flexibility are sensitive to resource costs, reliabilities and downside risk tolerance. Wang et al. (2010) study whether,

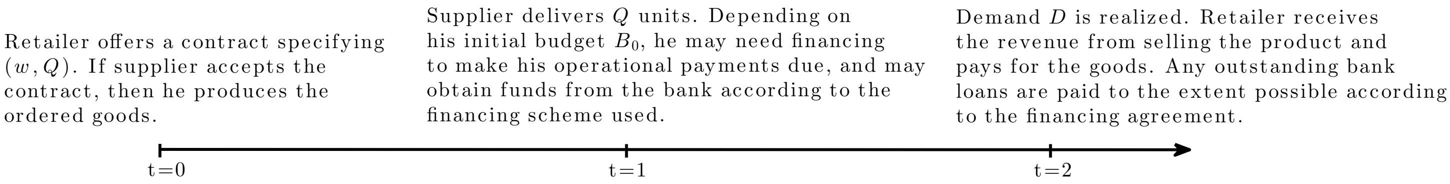


Figure 1: Sequence of events

dual sourcing or process improvement is preferable as a remedy to mitigate supply chain risk. They show that process improvement is more efficient than dual sourcing as the supply cost heterogeneity increases, and dual sourcing is preferable when supplier's reliability heterogeneity is high. Li and Debo (2009) examine the merits and drawbacks of sole sourcing and second sourcing under information asymmetry. They demonstrate that the benefits of second sourcing are influenced by the demand distribution and capacity costs. In addition, second sourcing is superior to sole sourcing when capacity cost is low, and second sourcing can lead to an overinvestment in initial capacity. In our paper, we theoretically and empirically demonstrate that when suppliers are cash constrained, in certain cases assuming more risk instead of trying to hedge it away can help boost a downstream buyer's profits.

3 Theory

In order to derive our hypotheses and provide the theoretical layout of our structural empirical analysis, we first present our game theoretical framework of the financing structures. We start with the general model description and then provide the details of the two financing models, namely the benchmark Commercial Loans and the Buyer Intermediated Financing model we are studying.

3.1 Model Description and Sequence of Events

Consider a two-layer supply chain with a large downstream *retailer* and an upstream *supplier*, who is potentially budget constrained. That is, the supplier's initial capital can be insufficient to produce what is ordered. The funds the supplier may need to finance its operations can be provided by a third-party *bank*. The retailer is large enough that, independent of the revenue from selling the product, she can always cover a loan she has committed to pay.[1]

[1]For the rest of the paper, for convenience in exposition, we will refer the retailer as "she", the supplier as "he", and the bank as "it".

There are three time periods in the model, indexed as $t = 0, 1$ and 2. At time $t = 0$, the retailer, who is at a dominant position within the supply chain, offers the supplier a contract, specified by the pair (w, Q), where w is the wholesale price, Q is the quantity ordered.[2] If the offer is accepted, supplier produces the goods to be delivered at $t = 1$ at a variable production cost is c_p. At time $t = 1$, supplier delivers the ordered goods, and needs to pay for the production costs due. If the supplier's cash position at $t = 1$ is insufficient to cover the production costs, he needs to borrow a certain amount from the bank. At $t = 2$, the consumer demand, D realizes. The consumer demand distribution has a c.d.f. $F(\cdot)$ and a p.d.f. $f(\cdot)$. The retailer delivers the product to the consumers and receives the corresponding revenue at a unit price p. For each unit of unmet demand, the retailer incurs a goodwill loss cost of c_g.

The supplier can be one of two types: low product defect with probability $\pi_l \in (0, 1)$ or high product defect with probability $1 - \pi_l$.[3] If the supplier is the low product defect type, his product is defective with probability $a_l \in (0, 1)$ and when he is the high product defect type, his product is defective with probability $a_h > a_l$. Define μ_a, as the ex-ante product defect probability for a given supplier, i.e., $\mu_a = \pi_l a_l + (1 - \pi_l) a_h$. After the product reaches the customers at time $t = 2$, it is revealed whether it is defective or good. If the product is defective, customers return the product to the retailer and the retailer returns it to the supplier. The retailer gets full refund for the defective products but it costs her c_e for each item returned from customers for processing. A customer may also return the product with probability a_n even when a certain product is not defective. Finally, the retailer sends all unsold products to the supplier for a full refund, w.[4] We assume that the expected revenue from selling the product exceeds the expected losses from exchange costs, i.e., $(1 - a_l)((1 - a_n)p - a_n c_e) - a_l c_e > 0$, since otherwise the product would not be a viable one for the supply chain with negative expected proceeds from each unit sold. The bank does not know the type of the supplier and hence the true defect rate of the product. The retailer on the other hand, has the history of the transactions with the supplier and knows the type of the supplier and the true defect rate. Figure 1 summarizes the general outline of the sequence of events.

We next present the details of the two financing schemes we study individually. For simplicity in exposition, we will focus on the case where the retailer is low product defect type (i.e., the probability of defect for his product is a_l).[5]

[2] This reflects the contract procedure for many large buyers, including Jingdong. Such retailers tend to have significant bargaining power, especially over their small suppliers and set the wholesale price and quantity for the contracts they engage in. Naturally, they set the wholesale price high enough to make sure the supplier still agrees to the contract. Please see problem formulations we give below in Section 3.2 for more details.

[3] This implies a separation between two supplier types and may have an impact on their creditworthiness. As an example, Jingdong separates its suppliers into two broad groups. The first group (labeled categories A,B, and C suppliers by Jingdong) are considered "higher quality" suppliers who are more dependable. The second group (labeled D and E) are considered lower quality suppliers who are perceived by Jingdong as risky. Jingdong chooses to offer supplier finance services only to those suppliers in categories A,B, and C, i.e., higher quality suppliers with low expected problem rates.

[4] This is a common return policy employed by many large retailers, including Jingdong.

[5] This reflects Jingdong's practice of underwriting financing of only "higher quality" suppliers as we mentioned above.

3.2 Financing Alternatives

3.2.1 Commercial Loan

We start with the case where the supplier borrows from a bank without the retailer being an intermediary. This is a traditional commercial loan scenario, which we denote by the subscript $_{cl}$. The time line follows the general outline: The retailer makes the contract offer (w, Q) at $t = 0$, and the supplier produces and delivers the goods at $t = 1$. The consumer demand and the retailer revenues materialize at $t = 2$ and if the product is not defective, the retailer makes the payment in full to the supplier. If the products are defective, the retailer returns all the products to the supplier and the supplier does not get paid. In the mean time, in order to finance its production, the supplier may need to borrow a loan L at time $t = 1$ from the bank, payable due at $t = 2$. At the time the loan payment is due, if the supplier has not defaulted, he pays the bank the loan principal plus interest to the extent possible depending on his cash position after obtaining the retailer's payment at $t = 2$. The interest rate for the bank loan is determined competitively. Denote the risk-free rate by r_f and the bank's interest rate by r_{cl}. Also denote the retailer's and the supplier's expected profits as Π_r and Π_s, respectively.

The supplier has to make sure that he borrows enough to cover his production cost. The retailer, on the other hand, has to ensure that the supplier's terminal cash position will be no less than what he would otherwise obtain by investing his money on the risk free asset.

Given, this setting, first, B_2, the supplier's cash position at $t = 2$ before paying the loan can be written as

$$B_2 = \begin{cases} (B_0(1+r_f) + L - c_pQ)(1+r_f) + w(Q - \mathbb{E}[(Q-D)^+]) & \text{if the product is not defective}\,; \\ (B_0(1+r_f) + L - c_pQ)(1+r_f) & \text{if the product is defective}\,. \end{cases} \tag{1}$$

Then the supplier's problem for determining the amount to borrow from the bank can be written as

$$\begin{aligned} \max_{l\geq 0}\Pi_s(Q,w,l) = \quad & \max_{l\geq 0}\{(B_0(1+r_f) + l - c_pQ)(1+r_f) \\ & \qquad +(1-a_l)(wQ - l(1+r_{cl}) - w\mathbb{E}[(Q-D)^+]) \\ & \qquad -a_l \min\{l(1+r_{cl}), (B_0(1+r_f) + l - c_pQ)(1+r_f)\}\} \\ \text{s.t.} \quad & B_0(1+r_f) + l - c_pQ \geq 0, \\ & l(1+r_f) = (1-\mu_a)l(1+r_{cl}) \\ & \qquad\qquad +\mu_a \min\{l(1+r_{cl}), (B_0(1+r_f) + l - c_pQ)(1+r_f)\}. \end{aligned} \tag{2}$$

Hence the retailer's problem when the supplier financing is obtained through a commercial loan can be written as

$$
\begin{aligned}
\max_{Q,w\geq 0} \Pi_r(Q,w) = \quad & \max_{Q,w\geq 0} \{\mathbb{E}[(1-a_l)(((1-a_n)p - a_n c_e)\min(D,Q) \\
& \qquad -c_g(D-Q)^+ + w(Q-D)^+ - wQ) \\
& \qquad +a_l(-c_e \min(D,Q) - c_g(D-Q)^+)]\} \\
\text{s.t.} \quad & \Pi_s(Q,w,L) \geq B_0(1+r_f)^2, \\
& \text{where } L \text{ solves the supplier's optimization problem for } (Q,w) \\
& \text{as given in (2).}
\end{aligned} \tag{3}
$$

3.2.2 Buyer Intermediated Financing

Traditional commercial loans, though being able to provide the suppliers with some liquidity, still make the supplier to face the bank as the borrower. This may be especially a problem for small suppliers or new businesses who have little or no credit history. In Buyer Intermediated Financing (BIF), however, a larger retailer can help small suppliers get better financing by intermediating between them and the bank and effectively underwriting the supplier's loan back payment. We denote the BIF financing case with the subscript $_{bi}$.

The timeline of the BIF approach is as follows: after the supplier and the retailer agree on the contract at $t = 0$ and the products are delivered at $t = 1$, the supplier can obtain a loan from the buyer to cover his due payments *arranged* by the retailer: The retailer works together with the supplier and the bank to get a loan at a discount $\delta_{bi} \in (0, 1)$ set by the retailer herself. In return, the retailer commits to paying back the loan. In particular, for $L \leq wQ$, the supplier obtains $L(1 - \delta_{bi})$ at $t = 1$ and the retailer agrees to pay L back to the bank at $t = 2$. At that time, if the product is not defective, the retailer pays the supplier the remainder of the account $wQ - L$. If the product is defective, however, the retailer just pays the loan due amount L back to the bank and does not pay the remaining due to the supplier as all the products are returned to the supplier.

Supplier's problem in this case can be written as

$$
\begin{aligned}
\max_{0\leq l\leq wQ} \Pi_s(Q,w,\delta_{bi},l) = \quad & \max_{0\leq l\leq wQ} \{(B_0(1+r_f) + l(1-\delta_{bi}) - c_p Q)(1+r_f) \\
& \qquad +(1-a_l)(wQ - l - w\mathbb{E}[(Q-D)^+])\} \\
\text{s.t.} \quad & B_0(1+r_f) + l(1-\delta_{bi}) - c_p Q \geq 0,
\end{aligned} \tag{4}
$$

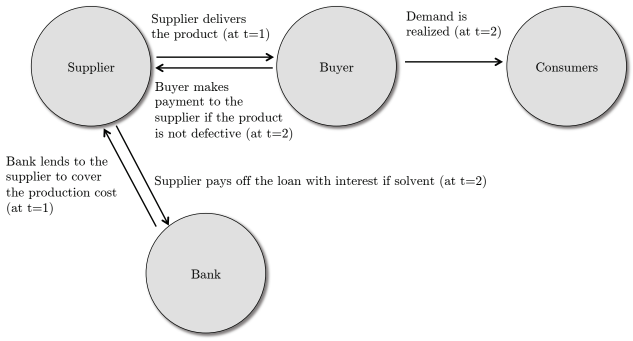
Table 1: Model Notation

w:	Unit wholesale price for the product
Q:	Retailer's order quantity
D:	Consumer demand
F, f:	c.d.f. and p.d.f. for D
p:	Unit retail price for the product
c_p:	Supplier's unit production cost
c_g:	Retailer's unit goodwill loss cost
c_e:	Retailer's processing and shipping cost for returned product
r_f:	Risk-free interest rate for each period
δ:	Bank's discount rate on the loan in the cases of factoring and BIF
B_0:	Supplier's initial cash position at t=0.
B_2:	Supplier's cash position before paying the bank loan at t=2
L:	The loan amount
Π_r:	Retailer's expected net profit at time t=2
Π_s:	Supplier's expected net profit at time t=2
a_l:	The defect probability of the supplier's product given low defect rate
a_h:	The defect probability of the supplier's product given high defect rate
a_n:	The return rate of non-defective products by the consumers
π_l:	Ex-ante probability of the supplier's product having a low defect rate
μ_a:	Ex-ante defect rate of the supplier's product Probability

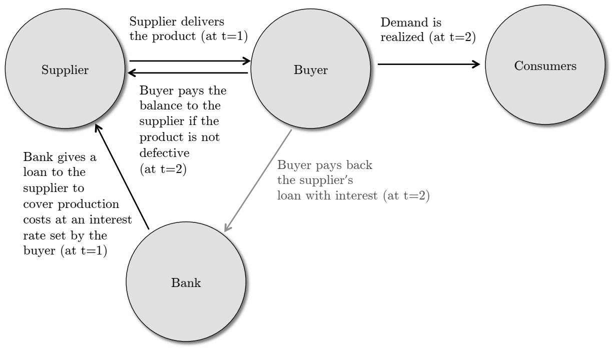
and the retailer's problem under a BIF policy is

$$\begin{aligned}\max_{Q,w\geq 0,\,\delta_{bi}\in(0,1)} \Pi_r(Q,w,\delta_{bi}) = \quad & \max_{Q,w\geq 0}\big\{\mathbb{E}[(1-a_l)(((1-a_n)p-a_nc_e)\min(D,Q)\\ &\qquad -c_g(D-Q)^+ + w(Q-D)^+ - wQ)\\ &\qquad +a_l(-c_e\min(D,Q)-c_g(D-Q)^+ - L)]\big\}\\ \text{s.t.}\quad & \Pi_s(Q,w,\delta_{bi},L)\geq B_0(1+r_f)^2,\\ & (1+r_f)(1-\delta_{bi})\leq 1\\ & \text{and where } L \text{ solves the supplier's optimization problem}\\ & \text{for } (Q,w,\delta_{bi}) \text{ as given in (4).}\end{aligned} \tag{5}$$

Figure 2 depicts a comparison of the two financing schemes. As shown in panel (a), when financing through a commercial loan, the loan transaction is fully between the supplier and the bank. Any risk of non-payment of the loan back to the bank is carried by the bank. For the Buyer Intermediated Financing scheme, on the other hand, the buyer sets the interest rate on the loan the supplier is receiving at $t = 1$, and as shown in panel (b), and commits to pay back the loan with interest at $t = 2$ to the bank. Table 1 summarizes our model notation.



(a) Commercial Loan



(c) Buyer Intermediated Financing

Figure 2: Comparison of the two financing schemes. Panel (a) shows the transaction structure for financing through a Commercial Loan and panel (b) through Buyer Intermediated Financing.

3.2.3 The First Best Solution

Lastly, before we present the equilibrium solution for each financing scheme we study, we present the formulation of the supply chain ideal benchmark, i.e., first-base case, where the supply chain is integrated and the decisions are centralized. In this case, the supplier's budget constraint as well as the participation and incentive compatibility constraints no longer enter the formulation. The first best problem, which we denote by the subscript $_{fb}$, then can be formulated as

$$\begin{aligned}\max_{Q\geq 0}\Pi_{fb} = \max_{Q\geq 0}\{&(1-a_l)\mathbb{E}[((1-a_n)p - a_n c_e)\min(D,Q) - c_g(D-Q)^+ - c_p Q(1+r_f)] \\ &+ a_l\mathbb{E}[-c_e\min(D,Q) - c_g(D-Q)^+ - c_p Q(1+r_f)] + B_0(1+r_f)^2\}.\end{aligned} \tag{6}$$

Solving (6), the optimal order quantity for the first best case can be found as

$$Q_{fb}^* = F^{-1}\left(1 - \frac{c_p(1+r_f)}{(1-a_l)((1-a_n)p - a_n c_e) - a_l c_e + c_g}\right). \tag{7}$$

Plugging (7) back in (6), we can obtain the first-best channel profit Π_{fb}^*. Throughout the rest of the paper, we will be using the first-best quantity Q_{fb}^* and surplus Π_{fb}^* as our benchmarks for quantity and surplus under full supply chain efficiency.

3.3 Equilibrium Analysis

In this section, we provide the equilibrium solutions and comparisons of the three financing methods we described above in Section 3.1. We start with the equilibrium outcome for the case when the supplier's initial budget position is relatively high.

Proposition 1 *Define*

$$\bar{Q}_{cl} = F^{-1}\left(1 - \frac{c_p(1-a_l)(1+r_f)}{(1-\mu_a)((1-a_l)((1-a_n)p - a_n c_e) - a_l c_e + c_g)}\right), \tag{8}$$

$$\bar{Q}_{bi} = F^{-1}\left(1 - \frac{c_p(1+r_f)}{(1-a_l)((1-a_l)((1-a_n)p - a_n c_e) - a_l c_e + c_g)}\right). \tag{9}$$

For $\rho \in \{cl, bi\}$, $Q_{fb}^* > \bar{Q}_\rho$. In each financing method ρ, if $B_0 \geq c_p \bar{Q}_\rho/(1+r_f)$ then the supplier obtains no financing in equilibrium. Further, (i) if $B_0 \geq c_p Q_{fb}^*/(1+r_f)$, then the equilibrium quantity $Q_\rho^* = Q_{fb}^*$, (ii) otherwise $Q_\rho^* = B_0(1+r_f)/c_p$. In each case, the equilibrium wholesale price is

$$w_\rho^* = \frac{c_p Q_\rho^*(1+r_f)}{(1-a_l)(Q_\rho^* - \mathbb{E}[(Q_\rho^* - D)^+])}. \tag{10}$$

Proposition 1 states that, when the supplier's budget level is sufficiently high, in all three of the financing schemes we study, he will not need to borrow to produce, and instead he will pay for his operations using his own funds. Further, as stated in part (i), if the supplier's budget is sufficiently high to produce the first-best quantity, he produces at that level using his own funds. The financing costs that are paid to a third party as well as the decentralized decision of supplier choosing his loan amount disappears and the supply chain gets coordinated. As indicated in part (ii), if the supplier's budget is insufficient to produce the first-best quantity but still not too low, in the optimal contract, he will still not borrow and will use his own funds. However, in this case, the retailer sets the wholesale price that makes supplier break even and orders a quantity less than the first-best (notice that in this case $Q_\rho^* = B_0(1+r_f)/c_p < Q_{fb}^*$). Consequently the equilibrium outcome will be strictly worse than the first-best outcome. That is, for an intermediate supplier budget band, the supply chain will operate without financing from a third-party entity but without full efficiency.

On the other hand, when the supplier's budget is not sufficiently high, he will need to obtain financing. We study this case under each financing scheme next. Note that henceforth, we will be using the quantity thresholds defined in (8)-(9) in our notation. We start with the case of financing through a commercial loan, which is presented in the following proposition.

Proposition 2 *For commercial loan financing, if $B_0 < c_p\bar{Q}_{cl}/(1+r_f)$, then in equilibrium*

$$
\begin{aligned}
r_{cl} &= \frac{1+r_f}{1-\mu_a} - 1, && (11)\\
Q^*_{cl} &= \bar{Q}_{cl}, && (12)\\
w^*_{cl} &= \frac{c_p Q^*_{cl}\frac{(1+r_f)}{1-\mu_a} - B_0(1+r_f)^2(\frac{1}{1-\mu_a} - \frac{1}{1-a_l})}{Q^*_{cl} - \mathbb{E}[(Q^*_{cl} - D)^+]}, && (13)\\
L^*_{cl} &= c_p Q^*_{cl} - B_0(1+r_f)\,. && (14)
\end{aligned}
$$

Proposition (2), states that when the supplier's budget is sufficiently low, it becomes optimal for the retailer to offer a contract that induces the supplier to borrow to support his production. Given the loan amount requested by the supplier, the bank will set the interest or the discount rate competitively. In particular, the commercial loan interest rate will depend on the loan amount. Note that projecting the bank's interest and discount rate setting behavior, the supplier will borrow the exact amount needed to cover his production costs at $t = 1$.

Next, we present the equilibrium outcome for the Buyer Intermediated Financing (BIF) scheme.

Proposition 3 *For buyer intermediated financing, if $B_0 < c_p\bar{Q}_{bi}/(1+r_f)$, then in equilibrium, the supplier borrows up to the level to cover his production costs. Further,*

$$
\begin{aligned}
\delta_{bi} &= 1 - \frac{1-a_l}{1+r_f}, && (15)\\
Q^*_{bi} &= \bar{Q}_{bi}, && (16)\\
w^*_{bi} &= \frac{c_p Q^*_{bi}(1+r_f)}{(1-a_l)(Q^*_{bi} - \mathbb{E}[(Q^*_{bi} - D)^+])}, && (17)\\
L^*_{bi} &= (c_p Q^*_{bi} - B_0(1+r_f))\,\frac{1+r_f}{1-a_l}\,. && (18)
\end{aligned}
$$

Note that the relationship between an interest rate r and the corresponding discount rate δ is $1 + r = 1/(1-\delta)$. Therefore, the corresponding interest rate for the buyer intermediated financing scheme is $r_{bi} = 1/(1-\delta_{bi}) - 1$. Hence, by Proposition 3, $r_{bi} = 1/(1-\delta_{bi}) - 1 = (1+r_f)/(1-a_l) - 1 > r_f$. That is, unlike the commercial loan, under the BIF structure the bank makes strictly positive profits. Yet, this is still optimal for the retailer who sets the rate in its three-way contract with the bank and the supplier, because the high interest rate restrains the supplier from borrowing too much, which keeps the retailer's costs low. As a result, in equilibrium, the supplier will again borrow only up to the amount he needs to cover for his production costs.

We can now compare the outcomes of the two financing schemes in several performance and efficiency aspects. The following proposition presents the comparison.

Proposition 4

(i) Buyer intermediated financing has a lower interest rate than the commercial loan, i.e., $r_f < r_{bi} < r_{cl}$.

(ii) There exist $\underline{\kappa}, \bar{\kappa} > 0$ *such that given that the supplier borrows a positive loan amount,*

(a) if $\mu_a < a_l + \underline{\kappa}$, *BIF induces a lower wholesale price, i.e.,* $w_{bi}^* < w_{cl}^*$;

(b) if $\mu_a > \bar{\kappa}$, *then BIF induces a higher wholesale price, i.e.,* $w_{bi}^* \geq w_{cl}^*$.

(iii) If $\mu_a \leq 1 - (1 - a_l)^2$, *then* $Q_{cl}^* \geq Q_{bi}^*$ *and* $\Pi_{cl}^* \geq \Pi_{bi}^*$, *otherwise* $Q_{bi}^* \geq Q_{cl}^*$ *and* $\Pi_{bi}^* \geq \Pi_{cl}^*$.

Figure 3 demonstrates the comparison of the outcomes of the traditional commercial loan and the BIF cases. As part (i) of Proposition 4 states, buyer intermediated financing (BIF) can reduce the effective interest rate, i.e., reduce the costliness of the loan. In fact, as can be seen from panel (a) of Figure 3, commercial loan interest rate increases sharply as the supplier's ex-ante defect rate (μ_a) increases. As can again be seen in panel (a) and as stated in part (iii) of Proposition 4, when the ex-ante defect rate (μ_a) is sufficiently low, the order quantity under the commercial loan is higher than that under BIF. This is because, even though BIF reduces the interest rate the supplier faces, it makes the retailer assume *increased risk*, since the retailer commits to cover the supplier's loan payment even when the product is defective and the supplier is unable to pay back the loan. This added risk reduces the retailer's incentives to order under BIF. However, as μ_a becomes larger, because of the rising commercial loan interest rate, the order quantity under the commercial loan sharply plunges and the order quantity under BIF can significantly be higher.

The wholesale price with BIF, on the other hand, can be lower or higher than the commercial loan. As stated in part (ii)(a), if μ_a is not too high, then BIF reduces the wholesale price by reducing the cost of borrowing. However, when the ex-ante riskiness of the loan rises the commercial loan interest rate increases and the order quantity decreases significantly. Then the supplier needs to borrow little, which means that the higher bank interest rate under commercial loan does not inflate the supplier compensation much and the wholesale price can be lower with a commercial loan than BIF as stated in part (ii)(b). However, increased μ_a means sharply decreasing profits for the supply chain for the commercial loan case and the

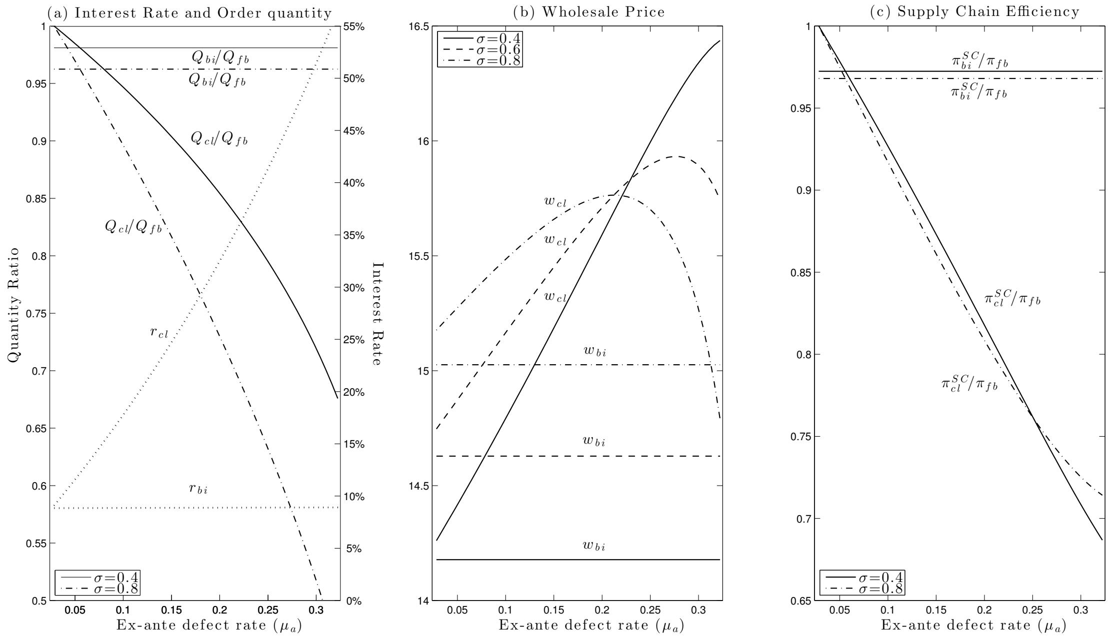


Figure 3: Comparison of the equilibrium outcomes for financing through Commercial Loans and BIF. Panel (a) illustrates the order quantity ratio with respect to first best for each financing scheme, panel (b) illustrates wholesale prices, and panel (c) shows the supply chain efficiency for varying ex-ante product defect rate (μ_a). For all panels, the parameter values are a_l=0.03, c_p=12, p=20, c_g=1, c_e=0.2, a_n=0.015, B_0=5, r_f=0.06, and the demand follows a log normal distribution with log of mean 0.5.

channel performance becomes significantly higher with BIF for high supplier ex-ante risk levels, as stated in part (iii) of Proposition 4. The efficiency gains with BIF can quickly reach 20% or more as the ex-ante defect rate increases as can be seen in panel (c) of Figure 3.

We will be using our theoretical results from this section in the rest of the paper in our empirical analysis in order to derive hypotheses, to make structural parameter estimations, and to perform efficiency and savings calculations.

4 Empirical Analysis

Utilizing the theoretical foundations we have laid out in Section 3, we now present our empirical results. Our data comes from Jingdong, the largest Chinese online retailer (JD.com). Jingdong employs a buyer intermediated financing scheme, which it launched in the end of 2012, and the service was adopted by a wide range of suppliers during the year of 2013. To this date the company has helped its suppliers get a combined 1 Billion Chinese Yuan (approximately $167 Million) financing.[6] In this section, we test our theory by using data from this initiative. First, we provide a parametric estimation of product demand

[6]For the rest of the paper, all currency figures will be given in Chinese Yuan unless indicated otherwise.

distributions. Using the derived demand distributions, we present a structural estimation of our theoretical model, obtaining estimates for goodwill costs, average interest rate for the suppliers under commercial loans in 2012 and retailer's demand forecast errors. Then we test four hypotheses on the outcome of BIF based on our theoretical results in Section 3 and our structural parameter estimation. Finally, again using our structural estimation we estimate Jingdong's supply chain efficiency gains from employing the Buyer Intermediated Financing scheme.

4.1 Data description

The data consists of more than 60000 SKUs that are sourced from 186 different suppliers of Jingdong. Among these suppliers, 143 of them have used the buyer intermediated finance service provided by the retailer (the *BIF* group), and the remaining 43 were randomly selected among suppliers that did not use financing or used other sources of financing (the *control* group). The data, which is collected from January 1, 2012 to December 31, 2013, contains information on both the procurement and retail sides. The procurement data includes product name, wholesale price, annual order quantity, as well as the supplier identification. The retail data includes the retail price and the annual realized demand. In addition to the procurement and retail data, we also have data from the finance service, which contains each supplier's reliability rating evaluated by the retailer. Furthermore, for those suppliers who have used the supplier finance program, the data includes account receivables and the amount borrowed through the supplier finance service.

4.1.1 Demand Estimation

We start by estimating the demand pattern for each industry. We use a log-log demand estimation model

$$\log(D_{ij}) = a_j + b_j \log(p_{ij}) + \epsilon_{ij}, \tag{19}$$

where D_{ij} is the demand for SKU i in industry segment j, p_{ij} is the price for that SKU, a_j is the industry-specific fixed-effect for demand, b_j is the industry-specific price elasticity, and ϵ_{ij} is the corresponding error term. Demand-Price scatter plots for two example industries (Electronic Products and Cosmetics are demonstrated as examples in panles (a) and (b) of Figure 4.

Since the demand data only reveals the realized sales, censored in the sense that unmet demand is lost and unobserved, the estimation needs to be appropriately adjusted. To this end, we employ an expectation-maximization regression method to account for the unobserved component of demand (see, e.g., Dempster et al. 1977, Aitkin and Wilson 1980). There are two steps in the estimation:

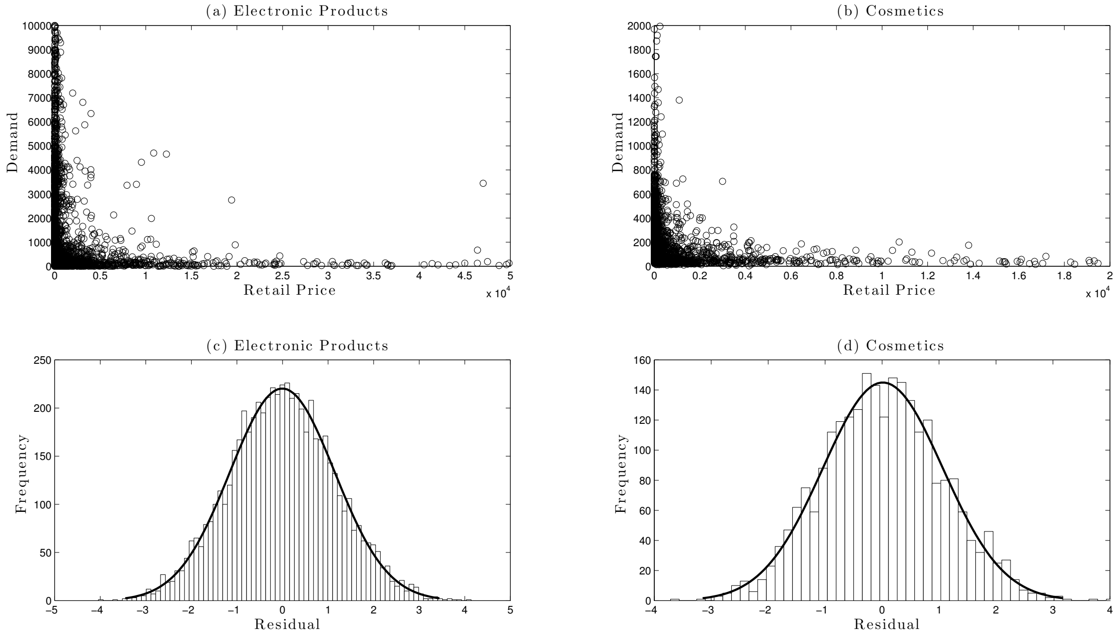


Figure 4: Panels (a) and (b) demonstrate the scatter plots for demand and price for Electronic Products and Cosmetics. Panels (c) and (d) exhibit the distributions for the residuals of the demand estimation regression specified in (19).

1- At iteration $t = k$, compute $Q(\theta; \theta^{(k)})$ where

$$Q(\theta; \theta^{(k)}) = \mathbb{E}_{\theta^{(k)}} [\mathcal{L}(\theta; y)|y_{obs}] . \tag{20}$$

2- At iteration $t = k + 1$, find $\theta^{(k+1)}$ s.t.

$$\theta^{(k+1)} = \arg\max_{\theta} Q(\theta; \theta^{(k)}) . \tag{21}$$

In (20) and (21), θ represents the vector of parameters to estimate. As demand for industry segment j is modeled as in the regression equation (19), the parameters to be updated can be expressed by $\theta = (a_j, b_j, \sigma_j^2)$, where σ_j^2 is the demand variance for industry j. $\mathcal{L}(\theta; y)$ is the log-likelihood function of the uncensored demand data y. Again two examples of distributions for industry residuals again (Electronic Products and Cosmetics) are given in panels (c) and (d) of Figure 4. As can be seen in Table 2, in 14 of the 15 industry segments Pearson Chi-square normality tests on the residuals from regression specified by (19) indicate that demand follows a log-normal distribution. Thus assuming normality for the log-residuals, for

Table 2: Results for Pearson chi-square Normality Test

	chi-square	p-value	Sample size
Electronic products	71.6080	0.2138	6049
Clothing	66.5776	0.6266	8101
Household appliances	27.0102	0.5712	982
Staple goods	40.2386	0.5485	2288
Baby and Pregnancy products	27.7972	0.5288	1006
Sporting goods	38.2971	0.4104	1750
Cosmetics	37.2061	0.7199	2504
Auto parts	49.2258	0.1049	1891
Computer accessories	32.1572	0.1231	649
Wine	19.1979	0.5724	475
Computer hardware	16.04	0.0661	75
Coffee	19.791	0.1005	177
Ceramics	5.5634	0.7827	71
Home improvement	47.6126	0.0005	413
Pet supplies	16.2603	0.0616	73

each industry j, we have

$$\mathcal{L}(a_j, b_j, \sigma_j^2; \log(D_{ij})) = -\frac{n_j}{2}\log(2\pi\sigma_j^2) - \frac{1}{2}\sum_{i=1}^{n_j}\frac{(\log(D_{ij}) - (a_j + b_j\log(p_{ij})))^2}{\sigma_j^2}. \tag{22}$$

Without loss of generality, let n_j be the sample size for industry segment j, in which the first m_j demand entries are not binding with the respective order quantity and the rest $n_j - m_j$ demand entries are binding. Further, let D_{ij} denote the real demand, and let D_{obs} denote the observed binding demand documented in the data.

Combining (21) and (22), the parameters at iteration $k+1$ can be updated as

$$a_j^{(k+1)} = \frac{h_{j1}^k \sum_{i=1}^{n_j}(\log(p_{ij}))^2 - h_{j2}^k \sum_{i=1}^{n_j}\log(p_{ij})}{n_j \sum_{i=1}^{n_j}(\log(p_{ij}))^2 - \left(\sum_{i=1}^{n_j}\log(p_{ij})\right)^2}, \tag{23}$$

$$b_j^{(k+1)} = \frac{n_j h_{j2}^k - h_{j1}^k \sum_{i=1}^{n_j}\log(p_{ij})}{n_j \sum_{i=1}^{n_j}(\log(p_{ij}))^2 - \left(\sum_{i=1}^{n_j}\log(p_{ij})\right)^2}, \tag{24}$$

and

$$\sigma_j^{2(k+1)} = \frac{\sum_{i=1}^{m_j}(\log(D_{ij}))^2 + \mathbb{E}[\sum_{i=m_j+1}^{n_j}(\log(D_{ij}))^2|\log(D_{obs}), a_j^{(k)}, b_j^{(k)}, \sigma_j^{2(k)}]}{n_j} - \left(\frac{\sum_{i=1}^{m_j}(\log D_{ij}) + \mathbb{E}[\sum_{i=m_j+1}^{n_j}\log(D_{ij})|\log(D_{obs}), a_j^{(k)}, b_j^{(k)}, \sigma_j^{2(k)}]}{n_j}\right)^2, \quad (25)$$

where

$$h_{j1}^k = \sum_{i=1}^{m_j}\log(D_{ij}) + \sum_{i=m_j+1}^{n_j}\mathbb{E}[\log(D_{ij})|\log(D_{obs}), a_j^{(k)}, b_j^{(k)}, \sigma_j^{2(k)}], \quad (26)$$

and

$$h_{j2}^k = \sum_{i=1}^{m_j}\log(D_{ij})\log(p_{ij}) + \sum_{i=m_j+1}^{n_j}\log(p_{ij})\mathbb{E}[\log(D_{ij})|\log(D_{obs}), a_j^{(k)}, b_j^{(k)}, \sigma_j^{2(k)}]. \quad (27)$$

A more detailed derivation of the distribution parameter update equations are given in Appendix B. The estimation results are presented in Table 3. Furthermore, the estimated parameters a_j^*, b_j^* together with $\sigma_j^{2^*}$ rendered by the regression iterations at convergence allow us to calculate estimated demand distributions. The mean of the logarithm of uncensored demand for each SKU is calculated as $\mu_{ij}^* = a_j^* + b_j^* log(p_{ij})$. The variance for each SKU, on the other hand, can be approximated by $\sigma_{ij}^{2^*} = \sigma_j^{2^*}(\mu_{ij}/(\sum_{s=1}^{n_j}\mu_{sj}))$. The logarithm of the uncensored demand therefore follows $\log(D_{ij}) \sim \mathcal{N}(\mu_{ij}^*, \sigma_{ij}^{2^*})$.

4.1.2 Structural Estimation of the Parameters

In order to estimate the performance of the supply chain through our theoretical analysis presented in Section 3, we need to estimate values of several parameters that are not directly provided in the data set. To start with, a_l, a_n, c_e and r_{bi} have to be calibrated. First, information obtained from Jingdong's financial disclosures reveals that the shipping fee c_e is on average 8 Yuan per item.[7] The information obtained from Supplier Finance Division of Jingdong annual interest rate for the Buyer Intermediated Financing (r_{bi}) is 9% for all suppliers who are qualified for using the supplier financing service, which are the low product defect rate suppliers as assessed by the company. In addition, the risk-free rate can be obtained through public information from The People's Bank of China and is 6% for the period encompassed by the data. Then since by equation (15), $(1 + r_f)/(1 - a_l) = 1 + r_{bi}$, we obtain our estimate for a_l as 0.0283. Moreover Jingdong's average consumer product return rate, including defect and non-defect cases, is 4.2%.[8] Since the total return rate is $a_l + (1 - a_l)a_n$ and $a_l = 0.0283$, the estimate for a_n can then be calculated as 0.0141.

With the above parameters obtained, we move to derive the suppliers' budget levels and production

[7]Presentation of Jingdong CEO Qiangdong Liu at Zhong Guan Cun 100, Beijing, China, March 27 2014.
[8]Interview with Jingdong's CMO Yan Lan on November 15 2013.

Table 3: Regression Outcome for Demand Distribution Estimation

	Estimate	Std. Error	t value	Pr(> |t|)
Industry specific fixed-effects:				
Auto parts	6.7561***	0.2055	32.88	0.0000
Baby and Pregnancy products	7.7163***	0.4155	18.57	0.0000
Ceramics	2.3015	1.6255	1.42	0.1568
Clothing	4.5735***	0.1181	38.73	0.0000
Coffee	2.4489**	0.8620	2.84	0.0045
Computer accessories	6.8503***	0.2711	25.27	0.0000
Computer hardware	8.7985***	1.9407	4.53	0.0000
Cosmetics	8.7901***	0.2499	35.17	0.0000
Electronic products	6.0549***	0.0935	64.78	0.0000
Home improvement	6.4238***	0.4769	13.47	0.0000
Household appliances	5.0736***	0.2703	18.77	0.0000
Pet supplies	4.7388**	2.2760	2.08	0.0373
Sporting goods	2.9811***	0.2414	12.35	0.0000
Staple goods	8.4921***	0.1712	49.60	0.0000
Wine	6.8432***	0.4101	16.69	0.0000
Price elasticity of demand:				
Auto parts	-0.5063***	0.0435	-11.65	0.0000
Baby and Pregnancy products	-0.3230***	0.1100	-2.93	0.0033
Ceramics	1.0063**	0.4053	2.48	0.0130
Clothing	0.2282***	0.0469	4.87	0.0000
Coffee	0.8800***	0.2149	4.10	0.0000
Computer accessories	0.1905***	0.0686	2.77	0.0055
Computer hardware	0.0111	0.2786	0.04	0.9681
Cosmetics	-0.4318***	0.0662	-6.52	0.0000
Electronic products	0.1630***	0.0465	3.50	0.0005
Home improvement	0.0841	0.1035	0.81	0.4165
Household appliances	0.3740***	0.0689	5.42	0.0000
Pet supplies	0.3550	0.4074	0.87	0.3835
Sporting goods	0.5154***	0.0621	8.30	0.0000
Staple goods	-0.2513***	0.0558	-4.50	0.0000
Wine	0.0298	0.0912	0.33	0.7441
Number of Obs:	18956			
Adjusted R-squared:	0.7924			
p<0.01 ***, p<0.05 **, p<0.1 *				

costs. For a given supplier k in the industry segment j, the estimate for budget $B_{0(kj)}$ of companies in BIF group in 2013 can be derived by combining equations (17) and (18) and taking the sum of the binding individual rationality constraint for the buyer intermediated finance scheme over all applicable SKU's. That is,

$$B_{0(kj)} = \frac{1}{(1+r_f)^2} \sum_{i=1}^{M_{kj}} \left\{ (w_{ikj} Q_{ikj} - w_{ikj} \mathbb{E}[(Q_{ikj} - D_{ikj})^+])(1-a_l) \right\} - L_{kj} \frac{(1-a_l)}{(1+r_f)^2}, \tag{28}$$

where for supplier k in industry segment j, M_{kj} is the number of SKU's, L_{kj}, the loan obtained by supplier as documented in our finance data set, and the expectation is computed using the corresponding demand distribution for each i derived in Section 4.1.1.

By Proposition 3, a supplier will borrow no more than what is needed to cover his production cost. Thus, we can provide an estimate for an SKU's unit production cost using his budget constraint. From (18), assuming proportional allocation of loan over the budget for each SKU i for a given supplier k in industry segment j, we have

$$c_{p(ikj)} = \frac{L_{kj}(1-a_l) + B_{0(kj)}(1+r_f)^2}{Q_{ikj}(1+r_f)} \cdot \frac{w_{ikj}Q_{ikj}}{\sum_{s=1}^{M_{kj}} w_{skj}Q_{skj}}. \tag{29}$$

Lastly, assuming that suppliers financed their operations through commercial loans in 2012, the goodwill loss c_g and the average interest rate for commercial loan r_{cl} can be jointly structurally estimated by combining (8) and (9). The estimation results are derived from the following equation:

$$\begin{aligned} \min_{\xi_j^{(12)},\xi_j^{(13)},c_g,r_{cl}} & \left\{ \sum_{j=1}^{N}\sum_{i=1}^{M_j} \left(Q_{ij}^{(12)} - F^{-1}_{(\mu_{ij}^{(12)}(1+\xi_j^{(12)}),\sigma_{ij}^{(12)}(1+\xi_j^{(12)}))} \left(1 - \frac{c_{p(ij)}^{(12)}(1+r_{cl})}{(1-a_l)((1-a_n)p - a_n c_e) - a_l c_e + c_g} \right) \right)^2 \right. \\ & \left. + \sum_{j=1}^{N}\sum_{i=1}^{M_j} \left(Q_{ij}^{(13)} - F^{-1}_{(\mu_{ij}^{(13)}(1+\xi_j^{(13)}),\sigma_{ij}^{(13)}(1+\xi_j^{(13)}))} \left(1 - \frac{c_{p(ij)}^{(13)}(1+r_f)/(1-a_l)}{(1-a_l)((1-a_n)p - a_n c_e) - a_l c_e + c_g} \right) \right)^2 \right\}, \end{aligned} \tag{30}$$

where the superscript indicates which year the data is from, M_j is the number of SKUs in industry j, N is the number of industry segments and is equal to 15. $\{\xi_j\}$'s are defined as the industry based demand forecast errors that take different values in 2012 and 2013. Since Jingdong makes the ordering decision based on its forecasted demand instead of the actual demand, we explicitly model and estimate the forecast errors so that the estimation results for c_g and r_{cl} will not be affected by the disparity in the perception of demand distribution. In addition, $c_{p(ij)}^{(13)}$ is the production cost in 2013 and is derived from (29), the unit production cost for each SKU i, j in 2012, $c_{p(ij)}^{(12)}$, is adjusted from the corresponding estimated production cost in 2013 $c_{p(ij)}^{(13)}$, only by adjusting for inflation.[9] The estimation results are given in Table 4.

As shown in Table 4, the interest rate on loan is on average 20.44%, which indicates that commercial loans tend to be expensive financial tools for small suppliers. Moreover, we can observe from the table that Jingdong overestimates the demand on average in each industry segment in both 2012 and 2013 with the possible exception of Staple Goods in 2013 (which is not significant at the 10% level). Our estimation indicates that Jingdong overestimates the demand by an average of 10.16% in 2012, compared to 15.34% in 2013. The increase in overestimation of the demand may have resulted from Jingdong's lower

[9] The inflation rate in China During 2012-2013 is 2.6% (NBSC 2014), thus $c_{p(ij)}^{(12)} = c_{p(ij)}^{(13)}/1.026$.

Table 4: Results for the NLS Regression for Parameter Estimation

	Estimate	Std. Error		
c_g	14.7647***	1.1326		
r_{cl}	0.2044***	0.0209		
Industry Segment Forecasting Error (ξ_j):				
	2012		2013	
Electronic products	0.0465***	0.0083	0.0617***	0.0039
Clothing	0.0861***	0.0124	0.1167***	0.0128
Household appliances	0.1550**	0.0837	0.0758	0.0562
Staple goods	0.1021***	0.0136	0.3208***	0.0227
Baby and Pregnancy products	0.0756***	0.0263	0.0867***	0.0284
Sporting goods	0.0690***	0.0155	0.1322***	0.0189
Cosmetics	0.0873***	0.0263	0.1057***	0.0265
Auto parts	0.1712	0.3435	0.1072***	0.0333
Computer accessories	0.0665***	0.0065	0.0838***	0.0004
Wine	0.1261***	0.0142	0.1504***	0.0104
Computer hardware	0.1812*	0.1028	0.6201***	0.1577
Coffee	0.1103***	0.0140	0.1094***	0.0128
Ceramics	0.0327***	0.0125	0.1394***	0.0147
Home improvement	0.1184***	0.0113	0.1340***	0.0070
Pet supplies	0.0879***	0.0313	0.0529**	0.0227
Number of Observations:	18956			

p<0.01 ***, p<0.05 **, p<0.1 *

than expected growth rate in 2013. The demand overestimation 3.27% (for Ceramics in 2012) to 62% (Computer Hardware in 2013) but mostly vary around the 10% range. Utilizing our estimation results from Table 4, we will next test hypotheses derived from our theory in Section 3, and perform efficiency analysis.

4.2 Tests on Borrowing, Wholesale Prices, and Order Quantities

In this section, based on the results from our theoretical analysis from Section 3 we present and test hypotheses on supplier borrowing behavior, the effect of BIF on wholesale prices and the order quantities.

Our first hypothesis is on supplier borrowing and draws from one of our conclusions from Proposition 3, which states that in equilibrium, suppliers will only borrow what is needed to cover their production costs, even though their credit limits are higher. Notice that if in the BIF contract design, it is in the retailer's interest to set a low interest rate for the loan since the retailer herself will be paying the loan back to the bank. However, if the retailer sets an interest rate that is too low, the supplier may have an opportunity to borrow more than he needs for production just to boost his expected profit since the retailer commits to pay the supplier's loan regardless of a product defect. In such a case, a supplier would borrow all the way up to his credit limit, which can increase total procurement costs and reduce retailer profits. Proposition

3 implies that in equilibrium, the retailer will not set the interest rate too low and as a consequence the supplier will *not* borrow up to their credit limits. Formally stated the hypothesis to test is:

Hypothesis 1 *When using BIF, suppliers borrow less than their available credit limits.*

As we have shown in part (i) of Proposition 4, if a supplier with a low product defect rate chooses to use commercial loan to finance his production, the interest rate on the loan should be higher compared to the case where the supplier uses BIF.

Hypothesis 2 *The use of BIF induces lower interest.*

As a consequence of higher interest rates under a commercial loan, the supplier's total costs increase and in order to cover for these increasing costs, the retailer has to offer a higher wholesale price. As we have found in part (ii) of Proposition 4 and as demonstrated in Figure 3, for moderate supplier ex-ante defect rate, with lower interest rates BIF can induce lower wholesale prices, and we can state the following hypothesis:

Hypothesis 3

(a) The use of BIF induces lower wholesale prices.

(b) The use of BIF induces lower wholesale-to-retail price ratios.

Finally, again as we discussed in Section 3, decreased wholesale prices lead to increased order quantities. Therefore, as we have shown in part (iii) of Proposition 4, for suppliers that are financed through BIF, order quantities tend to increase. Hence we have the following hypothesis:

Hypothesis 4 *The use of BIF induces higher order quantities.*

To test Hypothesis 1, for the 143 suppliers who have used the financing provided by the retailer, we first compute the ratio of the amount borrowed by each supplier over his total account payable. Define M_{kj} as the the number of SKU's supplier k in industry segment j has. Then the loan ratio for this supplier is $L_{kj}/\sum_{i=1}^{M_k} w_{ikj}Q_{ikj}$. We then conduct paired t-test on the the difference of the ratio from 1. The test result shows that the mean of the ratio is 0.337, with a t-value of -33.41 and is significant at the 0.01% level. Therefore, Hypothesis 1 is supported.

In order to test Hypothesis 2, we compare the interest rate on the commercial loan used by the suppliers in 2012 to the interest rate charged in BIF using a t-test. From our structural estimation of our model in Section 4.1.2, the average interest rate for commercial loans r_{cl}, is found to be 20.44% with standard deviation 2.09% (Table 4). The BIF interest rate, $r_{bi} = \frac{1+r_f}{1-\mu_a} - 1$, is known to be a flat rate 9%. The t-test shows that commercial loan induces a higher interest rate with a t-value of 494.507 and is significant at 0.01% level. Hence, Hypothesis 2 is supported.

Hypothesis 3(a) can be tested by conducting a t-test on the ratio of 2013 wholesale price over 2012 wholesale price for both the control group and the BIF group. The results from the t-test indicates that the average wholesale price drop in the BIF group is 3.1%, compared to a 1.7% drop in control group. Moreover, the t-test shows that the difference in wholesale price renders a t-value of -4.062, and the result is significant at the 0.01% level. Thus, the data indicates that the relative 1.4% drop between the two groups is due to the use of the buyer intermediated financing scheme, and hence Hypothesis 3(a) is supported. For Hypothesis 3(b), to take into account the effect of the change of overall retail price in the market, we perform t-test on the ratio of 2013 wholesale-to-retail price ratio, defined as w_{ikj}/p_{ikj}, over 2012 wholesale-to-retail price ratio, for both the BIF and the control groups. The results from the t-test indicates a 1.88% increase in the wholesale-to-retail price ratio from 2012 to 2013 for BIF group, in contrast with a 3.63% increase for control group, with a t-value of -3.943, and the difference is significant at 0.01% level. Hence, Hypothesis 3(b) is also supported.

Finally, to test Hypothesis 4, for each SKU in BIF and control group, we calculate the ratio of order quantity over the mean of untruncated demand with forecast error included, that is

$$Q_{ikj}/e^{\mu^*_{ij}(1+\xi_j)+(\sigma^*_{ij}(1+\xi_j))^2/2}. \tag{31}$$

This is because with forecast errors, the logarithm of demand follows

$$\log(D_{ij}) \sim \mathcal{N}(\mu^*_{ij}(1+\xi_j), (\sigma^*_{ij}(1+\xi_j))^2), \tag{32}$$

and the demand, which follows a log-normal distribution, has mean $e^{\mu^*_{ij}(1+\xi_j)+(\sigma^*_{ij}(1+\xi_j))^2/2}$. This operation allows us to see to which percentage will Jingdong fulfill the demand with and without BIF when demand forecast is known. Hence, Hypothesis 4 can be tested by implementing a t-test on the ratio of 2013 order quantity to mean demand ratio, over 2012 order quantity to mean demand ratio for both BIF and non-BIF groups. The results from the t-test shows that the non-BIF group has a ratio of 0.733, and BIF group's ratio is 0.871, indicating that employing BIF induced a relative 13% increase in the order quantity, with a t-value 2.661, and is significant at the 1% level. Therefore Hypothesis 4 is supported.

We can further test Hypothesis 4 as follows. For each SKU i in industry segment j,

$$\log(Q_{ij}) = \alpha + \beta_0 \log(\mu_{ij}(1+\xi_j)) + \beta_1 D_{13} + \beta_2 \cdot BIF_i + \beta_3 \cdot D_{13} \cdot BIF_i + \epsilon_{ij}. \tag{33}$$

In equation (33), D_{13} is the year indicator for 2013 and BIF is the indicator for whether the supplier used Jingdong's financing. Hence, β_0 captures the effect of demand forecast on order quantity, β_1 shows the impact of the year 2013 over 2012 on order quantity, and β_2 seizes the difference in firm characteristics between BIF group and control group. Finally, β_3, the coefficient of the interaction variable indicating a buyer financed SKU order, isolates the impact on order quantity brought by the implementation of BIF scheme. The regression results are given in Table 5. As can be seen in the table, after controlling for the effects of annual change in demand and the wholesale price fluctuations, the use of buyer intermediated finance leads to a significant increase in the order quantity (coefficient of BIF is 0.3411 which is significant at the 0.01% level). This again supports Hypothesis 4.

Table 5: Regression Estimation for the test of Hypothesis 4

	Estimate	Std. Error	t value	Pr(> |t|)
α	1.3655***	0.0290	47.09	0.0000
β_0	0.7611***	0.0027	285.70	0.0000
β_1	-1.1731***	0.0370	-31.75	0.0000
β_2	0.1656***	0.0277	5.98	0.0000
β_3	0.3411***	0.0398	8.57	0.0000
Number of Observations:	8810			
Adjusted R-squared:	0.9051			

p<0.01 ***, p<0.05 **, p<0.1 *

4.3 Empirical Efficiency Analysis

In this section, we present efficiency analysis based on the results from both our theoretical analysis and the data. By combining our theoretical results with empirical findings, we are able to estimate elements that are not directly contained in the data set, such as suppliers' budget and the production costs for each product. Further, we substitute our estimation from the structural regressions back for calibration and make direct profit comparisons between different financing scenarios to calculate efficiency gains.

4.3.1 Efficiency Comparisons

In this section, we conduct efficiency comparisons between commercial loan and BIF. We start by analyzing the budget of the supplier under each financing scheme. With the assumption that the production cost in 2012 differs from the production cost in 2013 only by the inflation factor 2.6%, by (13), we can estimate the 2012 budget for supplier k in industry segment j as

$$B_{0(kj)} = \sum_{i=1}^{M_{kj}} \frac{c_{p(ikj)} Q_{ijk} \frac{(1+r_f)}{(1-\mu_a)} - w_{ijk}(Q_{ijk} - \mathbb{E}[(Q_{ijk} - D)^+])}{(1+r_f)^2 (\frac{1}{1-\mu_a} - \frac{1}{1-a_l})}, \tag{34}$$

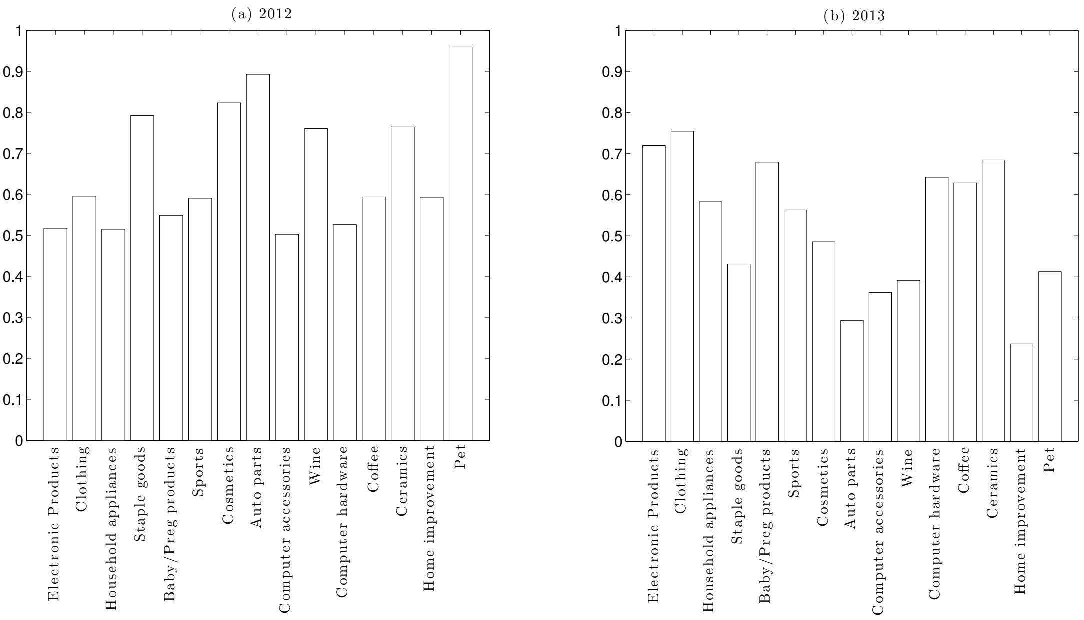


Figure 5: Industry segment breakdown of estimated supplier budget ($B_0(1 + r_f)$) as a percentage of the total production cost (c_pQ) in (a) 2012 and (b) 2013.

where M_{kj} is the number of SKU's for supplier k in industry segment j, and $(1 + r_f)/(1 - \mu_a)$ is 0.2044, as presented in 4.

The industry segment breakdown of the suppliers' estimated initial budget as a percentage of their total production cost is shown in Figure 5. Our estimates show that in 2012, suppliers's budget level is on average 66.7% of their production cost, and in 2013, this number drops to 55.6%. In addition, Figure 5(b) shows that 7 out of 15 industries have on average less than 50% of the cash needed to support their production, indicating that the suppliers in need of financing require very significant support, and for many of them close to double their cash positions is needed just to cover their production costs.

Further, by using the estimated SKU based demand distributions, we can also estimate a supplier's expected losses from unsold product returns for that SKU, $w_{ikj}\mathbb{E}[(Q_{ikj} - D_{ikj})^+]$. Utilizing these estimates, Figure 6 demonstrates the retailer's gross margin ($(p-w)/w$)and the suppliers' expected losses from unsold product returns as a percentage of total production cost ($w\mathbb{E}[(Q - D)^+]/wQ$). The average gross margin for the retailer is 12.7%, and Figure 6 shows that the margin, which ranges from less than 1% to 20%, varies to a great extent among industry segments. The overall average of estimated expected supplier losses from returned products is 25.1% varying between approximately 11%-45% among the industry segments.

With B_0, c_p, c_g, $\{\xi_j\}$, a_l, a_n, r_{cl} and SKU demand distribution estimates in hand, we can calculate the optimal order quantities and the expected profit for the first-best solution, as well as the expected supply

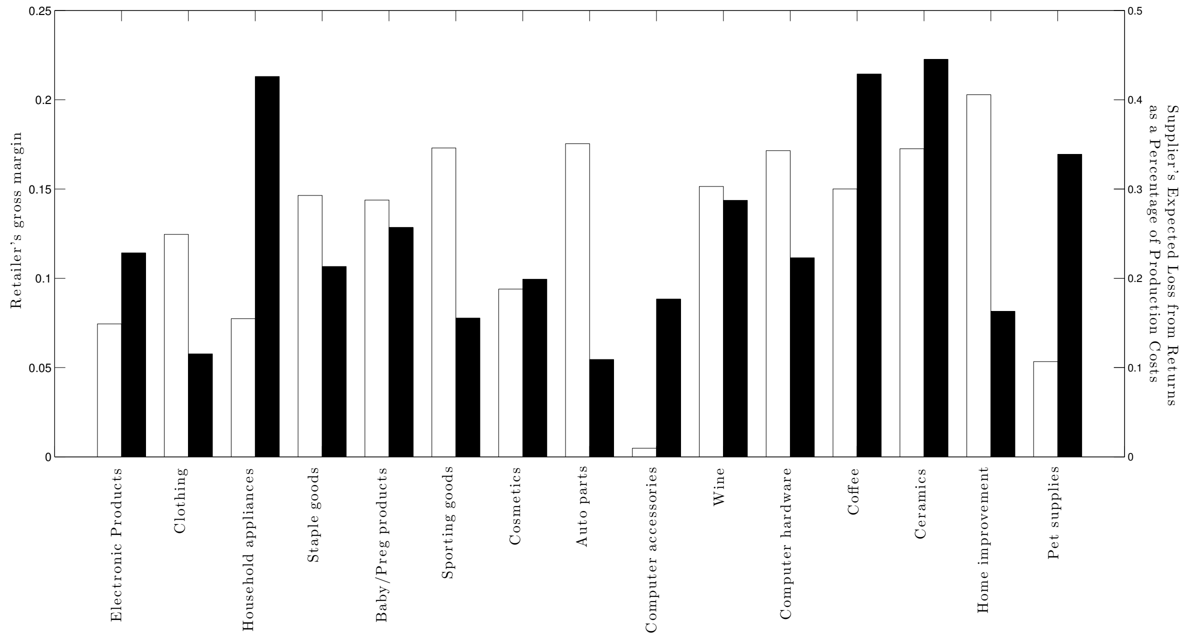


Figure 6: Industry segment breakdown of retailer's gross margin ($(p-w)/w$), light colored bars, measured in left y-axis), and the suppliers' expected losses from product returns as a percentage of the total production cost ($w\mathbb{E}[(Q-D)^+]/wQ$), dark colored bars, measured in right y-axis).

chain profit under Buyer Intermediated Financing. Moreover, we can also calculate the estimated wholesale price, order quantity and the expected supply chain profit for each SKU using Propositions 2 through a counterfactual analysis *had* the suppliers financed through commercial loans. We can then measure the efficiency of each scheme as well as the efficiency gains obtained by employing BIF using the first-best outcome as a benchmark for varying μ_a values for both with and without forecasting errors cases.

Table 6 presents these results. In particular, it exhibits the ratio of the mean of the optimal order quantity in each scheme as a fraction of the order quantity from the first best solution, the mean of optimal wholesale price under each scheme, and the ratio of expected profit for each scheme over the first-best total profit. As can be seen from Table 6, affected by the forecast error, retailer in general orders more than the first best order quantity under both financing schemes. With the estimated r_{cl}, the quantity increase by employing BIF over commercial loan ranges from 3.8% to 16.7% with forecast error and from 3.4% to 12.8% without forecast error. Profit gains on the other hand vary from from 1.2% to 15.2% with forecast error and from 2.0% to 12.5% without forecast error. The empirical efficiency comparison of order quantity, wholesale prices and profits as μ_a varies are further illustrated in Figure 7. As can be seen in panel (a), the retailer orders more than the first best order quantity due to forecast error. In panel (b) the presence of forecast error leads to higher wholesale price and a more rapid growth for wholesale price of commercial loan as μ_a increases. BIF appears to be less sensitive to forecast error than commercial loan,

Table 6: Empirical Efficiency Analysis

With Forecasting Error						
	Commercial Loan					BIF
	(r_{cl}=0.12)	(r_{cl}=0.18)	(r_{cl}=0.22)	(r_{cl}=0.26)	(r_{cl}=0.30)	
Order Quantity	1.0666	1.0348	1.0046	0.9761	0.9490	1.1091
Wholesale Price	221.0383	222.1179	223.2270	224.3419	225.5796	209.3470
Total Profit	0.7239	0.7034	0.6868	0.6714	0.6575	0.8099
Without Forecasting Error						
	Commercial Loan					BIF
	(r_{cl}=0.12)	(r_{cl}=0.18)	(r_{cl}=0.22)	(r_{cl}=0.26)	(r_{cl}=0.30)	
Order Quantity	0.9440	0.9186	0.8946	0.8720	0.8506	0.9782
Wholesale Price	186.6618	187.0444	187.4575	187.9133	188.3631	175.7583
Total Profit	0.9341	0.9033	0.8760	0.8509	0.8280	0.9539

highlighting another advantage of BIF scheme as demonstrated in panel (c).

According to the NLS estimation, the interest rate for loans for Jingdong's suppliers is 20.44%. Substituting this estimate in the supply chain profit expressions for commercial loan with our above calibration, we find that under commercial loan financing and with Jingdong's demand forecast, supply chain profits are 69.42% of the first-best compared to 80.99% for the BIF as calculated from the data. This indicates that the employment of BIF resulted in approximately 16.7% net profit increase for the supply chain. From the sales data set, the average gross margin for Jingdong can be calculated as approximately 12.77% and the total revenue for the 143 companies that used BIF can be calculated as approximately \$400M. Therefore, we can conclude that employing BIF resulted in approximately \$7.3M savings for Jingdong for the companies in our sample data set. Note that our data covers only about 1 Billion Yuan total financed through BIF by Jingdong whereas the total amount financed by the company in 2013 was approximately 8 Billion Yuan. Therefore, the company's projected total savings from BIF in 2013 reaches upwards of \$58M for 2013.

5 Concluding Remarks

In this paper we studied a novel financing scheme that is recently employed by large buyers to provide accessible financing to small suppliers who are facing budget constraints. We first built a game theoretical model to analyze traditional supplier financing schemes such as commercial loans and factoring, and compared the efficiency of Buyer Intermediated Financing to the efficiency of these traditional schemes. We theoretically demonstrated that BIF reduces the loan interest rate and the wholesale price, and can increase the order quantities significantly. As a result, the supply chain efficiency can improve substantially with BIF.

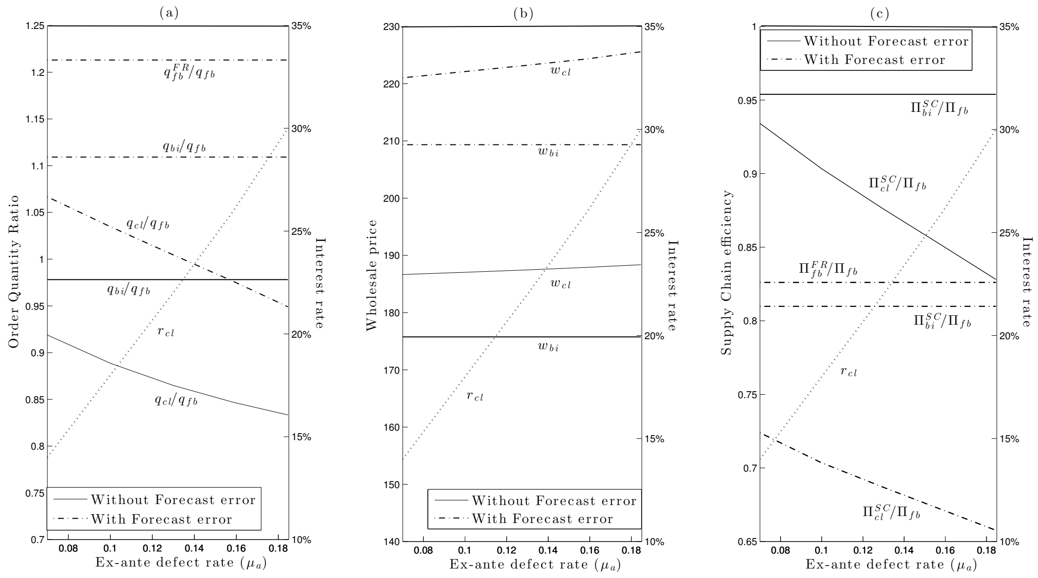


Figure 7: Empirical estimates for quantity ratio(Q/Q_{fb}, panel (a)), wholesale price (w, panel (b)), and Supply Chain Efficiency (Π^{SC}/Π_{fb}, panel (c)) for commercial loan and BIF.

We then use data from Chinese online retailer Jingdong to test the predictions of our theory as well as measuring efficiency gains and savings resulting from buyer intermediation in financing. Our empirical analysis verifies our theoretical prediction that the suppliers who use the buyer intermediated financing service will not use up their credit limits – in fact they use only about 33.7% of their available credit. The data analysis also provides evidence for our theoretical predictions that BIF significantly reduces the wholesale price and can substantially increase order quantities. Further, we perform a structural estimation of the model to predict the model parameters that are not directly unobservable. We estimate demand distributions, the retailer's unit goodwill loss, and average value of channel presence for suppliers in each industry segment. Using our estimations to calibrate our model, we then perform an empirical efficiency analysis. We estimate that the cash positions for the suppliers who need the financing service can cover only about 55.6% of their production costs. We also estimate the supply chain efficiency gains through BIF can reach up to 16.7% over the commercial loan, resulting in significant potential annual savings.

Our study provides theory and evidence on the efficiency of the Buyer Intermediated Supplier Financing schemes that are gaining increased usage in supply chains, especially in emerging economies. This innovative approach in easing suppliers' budget constraints can help not only improving supply chain efficiency significantly, but also help many small suppliers to gain their footing in the industry and grow their business, ultimately helping the development of economies, trade growth and value generation around the globe. The insights obtained from our study and future follow up studies can contribute to the understanding of these useful financing schemes and to their evolution and progress in practice.

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Online Supplement for
Buyer Intermediation in Supplier Finance

A Proofs of Propositions

Proof of Proposition 1: We will present the proof only for the commercial loan case. The proofs for factoring and the buyer intermediated financing schemes for this proposition will be similar and hence be omitted.

First, by (7) and (12),

$$
\begin{aligned}
Q^*_{fb} = F^{-1}\left(1 - \frac{c_p(1+r_f)}{(1-a_l)((1-a_n)p - a_n c_e) - a_l c_e + c_g}\right) \\
> F^{-1}\left(1 - \frac{c_p(1-a_l)(1+r_f)}{(1-\mu_a)((1-a_l)((1-a_n)p - a_n c_e) - a_l c_e + c_g)}\right) = \bar{Q}_{cl}\,, \quad \text{(A.1)}
\end{aligned}
$$

as stated in the proposition, since $a_l < \mu_a$. Now, given the supplier's loan amount request l, the bank competitively sets the interest rate $r_c l$ that solves

$$
l(1+r_f) = (1-\mu_a)l(1+r_{cl}) + \mu_a \min\{B_2^d, l(1+r_{cl})\}\,, \tag{A.2}
$$

where, by (1) $B_2^d = (B_0(1+r_f)+l-c_pQ)(1+r_f)$ is the supplier's cash position at $t=2$ in case the product is defective. Notice that since $\min\{B_2^d, l(1+r_{cl})\} \leq l(1+r_{cl})$, then by (A.2), $1+r_{cl} \geq 1+r_f$, where the equality holds only when $\min\{B_2^d, l(1+r_{cl})\} = l(1+r_{cl}) = l(1+r_f)$. Further, r_{cl} is monotonically non-increasing in B_2^d. Hence, solving (A.2) we obtain

$$
r_{cl} = \begin{cases} r_f, & \text{if } B_2^d \geq \frac{l(1+r_f)}{1-\pi_d} \\ \frac{l(1+r_f) - \mu_a(B_0(1+r_f)+l-c_pQ)(1+r_f)}{l(1-\mu_a)} - 1, & \text{if } B_2^d < \frac{l(1+r_f)}{1-\pi_d}\,. \end{cases} \tag{A.3}
$$

Plugging (A.3) into the supplier's objective function given in (2), we then have

$$
\frac{d\Pi_s(Q,w,l)}{dl} = \begin{cases} 0, & \text{if } B_2^d \geq \frac{l(1+r_f)}{1-\pi_d} \\ 0, & \text{if } B_2^d < \frac{l(1+r_f)}{1-\pi_d}\,. \end{cases} \tag{A.4}
$$

Hence, we have $d\Pi_s/dl = 0$ for all cases. This implies that given w and Q, supplier will be indifferent between borrowing and not borrowing. That is, every value for l is an equilibrium. However in reality, due to the existence of commission fee, we assume supplier will borrow exactly what is needed to cover his production costs, i.e., $l = c_pQ - B_0(1+r_f)$ if $B_0(1+r_f) \leq c_pQ$. Now, first suppose $B_0(1+r_f) > c_pQ$, i.e.,

$l = 0$. Then the supplier's participation constraint in (3) becomes

$$(B_0(1+r_f) - c_p Q)(1+r_f) + (1-a_l)(wQ - w\mathbb{E}[(Q-D)^+]) \geq B_0(1+r_f)^2 . \tag{A.5}$$

Notice that the left hand side of (A.5) is increasing in w. However, again from (3), we have

$$\frac{\partial \Pi_r}{\partial w} = (1-a_l)(\mathbb{E}(Q-D)^+ - Q) < 0, \tag{A.6}$$

which means that the retailer's profit is decreasing in w. Therefore, for any given $Q \geq 0$, in the optimal solution (A.5) must be binding. Thus, solving for w and plugging in $\Pi_r(Q,w)$, the retailer's profit function on $B_0(1+r_f) > c_p Q$ then is

$$\begin{aligned} \Pi_r(Q) = \Pi_r^1(Q) \;\triangleq\; & (1-a_l)\mathbb{E}\left[((1-a_n)p - a_n c_e)\min(D,Q) - c_g(D-Q)^+\right] \\ & + a_l \mathbb{E}[-c_e \min(D,Q) - c_g(D-Q)^+] - c_p Q(1+r_f) . \end{aligned} \tag{A.7}$$

Also note that,

$$\frac{d^2\Pi_r^1(Q)}{dQ^2} = -((1-a_l)((1-a_n)p - a_n c_e) - a_l c_e + c_g) f(Q) < 0, \tag{A.8}$$

since $(1-a_l)((1-a_n)p - a_n c_e) - a_l c_e > 0$. Therefore $\Pi_r^1(Q)$ is concave and, by solving the first order condition, is maximized at

$$Q = Q^*_{fb} = F^{-1}\left(1 - \frac{c_p(1+r_f)}{(1-a_l)((1-a_n)p - a_n c_e) - a_l c_e + c_g}\right) . \tag{A.9}$$

Next, suppose $B_0(1+r_f) < c_p Q$, then the supplier will borrow $L = c_p Q - B_0(1+r_f)$ and his participation constraint in (3) will be

$$(1-a_l)(wQ - L - w\mathbb{E}[(Q-D)^+]) \geq B_0(1+r_f)^2 . \tag{A.10}$$

Once again, since the retailer's objective function is decreasing in w, (A.10) is binding. Therefore solving for w and plugging it in Π_r, the retailer's profit function on $B_0(1+r_f) < c_p Q$ is

$$\begin{aligned} \Pi_r(Q) = \Pi_r^2(Q) \;\triangleq\; & (1-a_l)\mathbb{E}[((1-a_n)p - a_n c_e)\min(D,Q) - c_g(D-Q)^+)] \\ & + a_l \mathbb{E}[-c_e \min(D,Q) - c_g(D-Q)^+] \\ & -(c_p Q - B_0(1+r_f))\frac{(1+r_f)(1-a_l)}{(1-\mu_a)} - B_0(1+r_f)^2 . \end{aligned} \tag{A.11}$$

Further, $d^2\Pi_r^2(Q)/dQ^2$ is also as given in (A.8) and is negative. That is $\Pi_r^2(Q)$ is also concave and by solving its first order condition, is maximized at

$$Q = \bar{Q}_{cl} \triangleq F^{-1}\left(1 - \frac{c_p(1+r_f)(1-a_l)}{(1-\mu_a)((1-a_l)((1-a_n)p - a_n c_e) - a_l c_e + c_g)}\right) . \tag{A.12}$$

In addition, comparing the two optimizers, since $a_l < \mu_a$, we have $\bar{Q}_{cl} < Q^*_{fb}$. Finally also note that

$$\Pi^1_r\left(\frac{B_0(1+r_f)}{c_p}\right) = \Pi^2_r\left(\frac{B_0(1+r_f)}{c_p}\right) = B_0(1+r_f)^2. \tag{A.13}$$

Since $B_0(1+r_f)/c_p > Q^*_{fb}$, $\bar{Q}_{cl} < B_0(1+r_f)/c_p$, and since Π^2_r is concave, Π^2_r is then decreasing on $Q > B_0(1+r_f)/c_p$. Further, again since $B_0(1+r_f)/c_p > Q^*_{fb}$, the maximizer Q^*_{fb} of $\Pi^1_r(Q)$ is in $0 \leq Q < B_0(1+r_f)/c_p$. Hence $\Pi_r(Q^*_{fb}) = \Pi^1_r(Q^*_{fb}) > \Pi^1_r(B_0(1+r_f)/c_p) = \Pi^2_r(B_0(1+r_f)/c_p) > \Pi^2_2(Q) = \Pi_r(Q)$ for any $Q > B_0(1+r_f)/c_p$. Therefore, we can conclude that the retailer's optimal order quantity is Q^*_{fb} with the wholesale price given in (13). This proves part (i).

For part (ii), notice that when $\bar{Q}_{cl} < B_0(1+r_f)/c_p < Q^*_{fb}$, since $\Pi^1_r(Q)$ is concave and maximized at Q^*_{fb}, $\Pi^1(Q)$ is increasing on $0 \leq Q \leq B_0(1+r_f)/c_p$, and attains its maximum on this interval at $Q = B_0(1+r_f)/c_p$. On the other hand, since $\Pi^2_r(Q)$ is also concave and is maximized at $\bar{Q}_{cl}$, it is decreasing on $Q \geq B_0(1+r_f)/c_p$. Since $\Pi^1_r(B_0(1+r_f)/c_p) = \Pi^2_r(B_0(1+r_f)/c_p)$, and since $\Pi_r(Q) = \Pi^1_r(Q)$ for $0 \leq Q \leq B_0(1+r_f)/c_p$ and $\Pi_r(Q) = \Pi^2_r(Q)$, $Q > B_0(1+r_f)/c_p$ it follows that $\Pi_r(Q)$ is maximized at $Q^*_{cl} = B_0(1+r_f)/c_p$. Plugging this value into the supplier's binding participation constraint as described in the proof of Proposition 2, we again find that w^*_{cl} satisfies (13). ■

Proof of Proposition 2: Suppose $B_0(1+r_f)/c_p < \bar{Q}_{cl}$. Then, using the notation of the proof of Proposition 1, since $\bar{Q}_{cl} < Q^*_{fb}$, and by concavity of Π^1_r, $\Pi^1_r(Q)$ is increasing on $0 \leq Q \leq B_0(1+r_f)/c_p$ and attains its maximum at $Q = B_0(1+r_f)/c_p$. On the other hand, $\Pi^2(Q)$ has its global maximizer $\bar{Q}_{cl}$ in $B_0(1+r_f)/c_p > \bar{Q}_{cl}$. Again since, Π^2_r is concave and $\Pi^1_r(B_0(1+r_f)/c_p) = \Pi^2_r(B_0(1+r_f)/c_p)$, this means that $\Pi^2_r(Q)$ is increasing on $0 \leq Q \leq \bar{Q}_{cl}$ and decreasing for $Q \geq \bar{Q}_{cl}$, i.e., $\Pi^2_r(Q)$ is maximized at $Q^*_{cl} = \bar{Q}_{cl}$. Once again, plugging Q^*_{cl} into into the supplier's binding participation constraint, we obtain w^*_{cl} as given in (13). Further, as we have shown in the proof of Proposition 1, the supplier's budget constraint is binding in (2), i.e., $B_0(1+r_f)+l-c_pQ = 0$. Therefore by plugging in $Q^*_{cl} = \bar{Q}_{cl}$ we obtain $L = c_p\bar{Q}_{cl} - B_0(1+r_f) > 0$ as given in (14). Finally, plugging (14) into (A.3) for $L > 0$, we obtain r_{cl} as given in (11). This completes the proof. ■

Proof of Proposition 3: First, on $Q < B_0(1+r_f)/c_p$, the retailer's objective function will again be $\Pi_r(Q) = \Pi^1_r(Q)$ where Π^1_r is as given in the proof of Proposition 1. For $Q \geq B_0(1+r_f)/c_p$, taking the total derivative of the retailer's objective function in (5) with respect to δ_{bi}, we have

$$\frac{d\Pi_r}{d\delta_{bi}} = \frac{\partial\Pi_r}{\partial\delta_{bi}} + \frac{\partial\Pi_r}{\partial\delta_L} \cdot \frac{dL}{d\delta_{bi}}\,. \tag{A.14}$$

Notice that the first term in (A.14) is zero. Further $\partial\Pi_r/\partial\delta_L < 0$. Now, from (4),

$$\frac{\partial\Pi_s}{\partial l} = (1+r_f)(1-\delta_{bi}) - (1-a_l)\,, \tag{A.15}$$

and the supplier's profit is non-increasing in l if and only if $\delta_{bi} \geq 1 - (1-a_l)/(1+r_f)$. So for any δ_{bi}

for which this condition is not satisfied, supplier will have incentives to obtain a loan $L \to \infty$, which will be suboptimal for the retailer. Further, notice that for $\delta_{bi} \geq 1 - (1 - a_l)/(1 + r_f)$, the supplier will borrow to exactly cover his production costs, meaning $L = (c_p Q - B_0(1 + r_f))/(1 - \delta_i)$, and hence $dL/d\delta_{bi} > 0$. Consequently, by (A.14), on $\delta_{bi} \geq 1 - (1 - a_l)/(1 + r_f)$, the retailer will minimize δ_{bi}, which means $\delta_{bi} = 1 - (1 - a_l)/(1 + r_f)$. Further, this discount factor satisfies the bank's participation constraint, as

$$\delta_{bi}^* = 1 - \frac{1 - a_l}{1 + r_f} \geq \frac{1}{1 + r_f} . \tag{A.16}$$

Hence, for δ_{bi}^*, the supplier will choose $L = (c_p Q - B_0(1 + r_f))(1 + r_f)/(1 - a_l)$, and plugging it in the supplier's participation constraint, we have

$$wQ - w\mathbb{E}[(Q - D)]^+ - L \geq B_0 \frac{(1 + r_f)^2}{1 - a_l} . \tag{A.17}$$

As the left hand side of (A.17) is increasing in w, and the retailer's objective function in (5) is decreasing in w, (A.17) will bind in optimality. Notice that with $w > c_p$, if $(1 + r_f)/(1 - \mu_a) \leq w/c_p$ then $L \leq wQ$ for all Q, and if $(1 + r_f)/(1 - \mu_a) > w/c_p$, then $L \leq wQ$ for all $0 \leq Q < c_p(1 + r_f)/w$, which is non-empty. Therefore, there is always a contract offer the retailer can make that is feasible for the supplier's problem. Solving for w and plugging in the retailer's objective in (5) we obtain

$$\begin{aligned} \Pi_r(Q) = \Pi_r^4(Q) \quad \triangleq \quad & (1 - a_l)\mathbb{E}[((1 - a_n)p - a_n c_e) \min(D, Q) - c_g(D - Q)^+] && \text{(A.18)} \\ & + a_l \mathbb{E}[-c_e \min(D, Q) - c_g(D - Q)^] && \text{(A.19)} \\ & -(c_p Q - B_0(1 + r_f)) \frac{(1 + r_f)}{(1 - a_l)} - B_0(1 + r_f)^2 , && \text{(A.20)} \end{aligned}$$

on $Q > B_0(1 + r_f)/c_p$, which is again concave in Q, and has a unique maximum at

$$Q_{bi}^* = \bar{Q}_{bi} = F^{-1}\left(1 - \frac{c_p(1 + r_f)}{(1 - a_l)((1 - a_l)((1 - a_n)p - a_n c_e) - a_l c_e + c_g)}\right) . \tag{A.21}$$

Finally, again notice that as in the proofs of Propositions (1), $\Pi_r^1(B_0(1 + r_f)/c_p) = \Pi_r^4(B_0(1 + r_f)/c_p)$. Given these, the rest of the proof proceeds in the similar fashion as in the proofs of Proposition 2 and is skipped. L_{bi}^* can be obtained by plugging Q_{bi}^* in the supplier's binding budget constraint in (4), and w_{bi}^* can again be obtained by plugging Q_{bi}^* and L_{bi}^* in (A.17). ■

Proof of Proposition 4: To see part (i), since $1 + r_{bi} = 1/(1 - \delta_{bi})$, by (15) we have

$$1 + r_{cl} = \frac{1 + r_f}{1 - \mu_a}, \quad \text{and} \quad 1 + r_{bi} = \frac{1 + r_f}{1 - a_l}. \tag{A.22}$$

Since $0 < a_l < \mu_a$, we have $r_f < r_{bi} < r_{cl}$.

For part (ii)(a), denoting $\mu_a = a_l + \delta$, plugging in (12), and by (12),

$$Q_{cl}^* = F^{-1}\left(1 - \frac{c_p(1+r_f)}{(1-a_l)((1-a_n)p - a_n c_e) - a_l c_e + c_g}\right) + O(\delta) = Q_{fb}^* + O(\delta)\,. \tag{A.23}$$

Plugging (A.23) in (13), we have

$$w_{cl}^* = \frac{c_p Q_{fb}^*(1+r_f)}{(1-a_l)(Q_{fb}^* - \mathbb{E}[(Q_{fb}^* - D)^+])} + O(\delta)\,. \tag{A.24}$$

Now, define

$$g(Q) = \frac{c_p Q(1+r_f)}{(1-a_l)(Q - \mathbb{E}[(Q-D)^+])}. \tag{A.25}$$

Now

$$\frac{dg(Q)}{dQ} = \frac{c_p(1+r_f)\int_0^Q Df(D)dD + v(1-F(Q))}{(1-a_l)(Q - \mathbb{E}[(Q-D)^+])^2} > 0, \tag{A.26}$$

i.e., $g(Q)$ is increasing in Q. Further, by Propositions 1 and 3, $Q_{fb}^* > Q_{bi}^*$. Since $w_{bi}^* = g(Q_{bi}^*)$, it then follows that there exists $\bar{d} > 0$ such that if $\mu_a < a_l + \bar{d}$, $w_{cl}^* > w_{bi}^*$. This proves part (ii)(a).

For part (ii)(b), denote $(c_p(1-a_l)(1+r_f))/((1-a_l)((1-a_n)p - a_n c_e) - a_l c_e + c_g)$ by K_0. Note that when $K_0 \geq 1$, the supplier will choose not to produce by eq.(12). For $K_0 < 1$, let $\mu_a = 1 - K_0 - \delta$, and plug it in (12). With a non-empty lower of the demand distribution support $f(F^{-1}(0)) > 0$, we have

$$Q_{cl}^* = O(\delta). \tag{A.27}$$

Now, since

$$Q_{cl}^* - \mathbb{E}[(Q_{cl}^* - D)^+] = Q_{cl}^*(1 - F(Q_{cl}^*)) + \int_0^{Q_{cl}^*} Df(D)dD = Q_{cl}^*(1 - F(Q_{cl}^*)) + K_1(Q_{cl})Q_{cl}^*\ , \tag{A.28}$$

where the second equality holds because of mean value theorem, and $K_1(Q_{cl}) \leq F(Q_{cl})$, the expression for w_{cl}, after plugging (A.27) into (13), becomes

$$w_{cl}^* = \frac{c_p O(\delta)\frac{(1+r_f)}{(1-a_l)} - B_0(1+r_f)^2(\frac{1}{1-\mu_a} - \frac{1}{1-a_l})}{O(\delta)(1-O(\delta))) + K_1(O(\delta))O(\delta)}\,. \tag{A.29}$$

When $B_0(1+r_f) \geq c_p\bar{Q}_{bi}$, by Proposition 1, w_{cl}=w_{bi}. When $c_p\bar{Q}_{bi} \geq B_0(1+r_f) \geq c_p\bar{Q}_{cl}$, since A.25 is decreasing with Q and $Q_{bi}^* > Q_{cl}^*$ by proposition 1, 2 and 3, We have $w_{cl} < w_{bi}$. Finally, when $B_0(1+r_f) < c_p Q_{cl}^*$, from (A.29), we obtain

$$w_{cl}^* = \frac{c_p(1+r_f)}{1-a_l} - \frac{B_0(1+r_f)^2(\frac{1}{1-\mu_a} - \frac{1}{1-a_l})}{O(\delta)(1-O(\delta))) + K_1(O(\delta))O(\delta)}\,. \tag{A.30}$$

Further, by (17), we have

$$w_{bi}^* = \frac{c_p Q_{bi}^*(1+r_f)}{(1-a_l)(Q_{bi}^* - \mathbb{E}[(Q_{bi}^* - D)^+])} > \frac{c_p(1+r_f)}{1-a_l}. \tag{A.31}$$

Therefore,

$$w_{bi}^* - w_{cl}^* > \frac{B_0(1+r_f)^2(\frac{1}{1-\mu_a} - \frac{1}{1-a_l})}{O(\delta)(1-O(\delta)) + K_1(O(\delta))O(\delta)}, \tag{A.32}$$

which is positive since $\mu_a > a_l$. Thus, there exists $\bar{d} > 0$ such that if $1 - \mu_a < K_0 + \bar{d}$, i.e., when $\mu_a > 1 - K_0 - \bar{d} = \kappa > 0$, we have $w_{cl}^* \leq w_{bi}^*$.

Finally, to see part (iii), we start by comparing the equilibrium order quantities. By (8) and (9), $\bar{Q}_{cl} \geq \bar{Q}_{bi}$ if and only if

$$\begin{aligned} F^{-1}\left(1 - \frac{c_p(1-a_l)(1+r_f)}{(1-\mu_a)((1-a_l)((1-a_n)p - a_n c_e) - a_l c_e + c_g)}\right) \\ \geq F^{-1}\left(1 - \frac{c_p(1+r_f)}{(1-a_l)((1-a_l)((1-a_n)p - a_n c_e) - a_l c_e + c_g)}\right). \end{aligned} \tag{A.33}$$

Since F^{-1} is monotonically non-decreasing, (A.33) is satisfied if and only if

$$\frac{1-a_l}{(1-\mu_a)} \leq \frac{1}{1-a_l}. \tag{A.34}$$

Now, (A.34) is satisfied if and only if $\mu_a \leq 1-(1-a_l)^2$. Therefore, if $\mu_a \leq 1-(1-a_l)^2$, then $\bar{Q}_{cl} \geq \bar{Q}_{bi}$. Otherwise, if $1-(1-a_l)^2 \leq \mu_a$, then $\bar{Q}_{bi} \geq \bar{Q}_{cl}$ follows.

When $\bar{Q}_{cl} \geq \bar{Q}_{bi}$, if $B_0 < c_p\bar{Q}_{bi}/(1+r_f)$, then $Q_{bi}^* = \bar{Q}_{bi}$ and $Q_{cl}^* = \bar{Q}_{cl}$. As a result, $Q_{cl}^* \geq Q_{bi}^*$. If $c_p\bar{Q}_{bi}/(1+r_f) \leq B_0 < c_p\bar{Q}_{cl}/(1+r_f)$, then $Q_{bi}^* = B_0(1+r_f)/c_p < \bar{Q}_{cl}$ and $Q_{cl}^* = \bar{Q}_{cl}$. Thus, $Q_{cl}^* \geq Q_{bi}^*$ still holds. Moreover, if $c_p\bar{Q}_{cl}/(1+r_f) \leq B_0$, $Q_{bi}^* = Q_{cl}^* = B(1+r_f)/c_p$, and $Q_{cl}^* \geq Q_{bi}^*$ remains valid. Therefore, the quantity comparisons stated in part (iii) hold for all budget levels. The proofs for the case where $\bar{Q}_{cl} < \bar{Q}_{bi}$ follows the similar logic.

To show the profit comparisons, when $\bar{Q}_{cl} \geq \bar{Q}_{bi}$, for $B_0 < c_p\bar{Q}_{bi}/(1+r_f)$, by plugging in (11)-(14) into the objective in (3) and plugging in (15)-(18) into the objective in (5), we find that

$$\Pi_{cl}^* - \Pi_{bi}^* = (c_p Q - B_0(1+r_f))(1+r_f)\left(\frac{(1-a_l)}{(1-\mu_a)} - \frac{1}{1-a_l}\right). \tag{A.35}$$

It follows that $\Pi_{cl}^* \geq \Pi_{bi}^*$ if and only if (A.34) is satisfied. The rest of the proof for the case $B_0 < c_p\bar{Q}_{bi}/(1+r_f)$ proceeds in an identical manner as presented above for the quantity comparisons.

When $c_p\bar{Q}_{bi}/(1+r_f) \leq B_0 < c_p\bar{Q}_{cl}/(1+r_f)$, retailer's profit under BIF is $\Pi_{bi}^* = \Pi_r^1(B_0(1+r_f)/c_p)$, where $\Pi_r^1(\cdot)$ is defined in the proof of Proposition 1. The retailer's profit under commercial loan on the other hand is $\Pi_{cl}^* = \Pi_r^2(Q_{cl}^*)$, where $\Pi_r^2(\cdot)$ is again defined in the proof of Proposition 1. As $\Pi_r^2(\cdot)$ achieves global maximum at Q_{cl}^*, we have $\Pi_r^2(Q_{cl}^*) > \Pi_r^2(B_0(1+r_f)/c_p)$. Moreover, given the fact that

$\Pi_r^1(B_0(1+r_f)/c_p) = \Pi_r^2(B_0(1+r_f)/c_p)$, we again conclude that $\Pi_{cl}^* \geq \Pi_{bi}^*$. The proof for the cases in which $B_0 \geq c_p\bar{Q}_{cl}/(1+r_f)$ follows the same procedure as for the case when $c_p\bar{Q}_{bi}/(1+r_f) \leq B_0 < c_p\bar{Q}_{cl}/(1+r_f)$, and the proof for $\Pi_{cl}^* < \Pi_{bi}^*$ can be derived in a similar way. ■

B Derivations of the Distribution Parameter Updates in Demand Estimation

The proof for the distribution parameter update formulas for the demand estimation given in (23)-(25) involves deriving the conditional expectation updating rules for $a_j^{(k)}$, $b_j^{(k)}$ and $\sigma_j^{2^{(k)}}$. We start by writing the joint probability density for $\log(D_{ij})$. Assuming each SKU demand follows a normal distribution, we have

$$f(a_j, b_j, \sigma_j^2; \log(D_{ij})) = (\frac{1}{2\pi\sigma_j^2})^{n_j/2} e^{-\frac{1}{2}\sum_{i=1}^{n_j}\frac{(\log(D_{ij})-(a_j+b_j\log(p_{ij})))^2}{\sigma_j^2}}. \tag{B.1}$$

The maximization step requires the formulation for log-likelihood of the log-normal demand data, which is

$$\mathcal{L}(a_j, b_j, \sigma_j^2; \log(D_{ij})) = -\frac{n_j}{2}\log(2\pi\sigma_j^2) - \frac{1}{2}\sum_{i=1}^{n_j}\frac{(\log(D_{ij}) - (a_j + b_j\log(p_{ij})))^2}{\sigma_j^2}. \tag{B.2}$$

Expanding (B.2), we have

$$\mathcal{L}(a_j, b_j, \sigma_j^2; \log(D_{ij})) = -\frac{n_j}{2}log(\sigma_j^2) - \sum_{i=1}^{n_j}\frac{(\log(D_{ij}))^2}{2\sigma_j^2} + \sum_{i=1}^{n_j}\frac{\log(D_{ij})(a_j + b_j\log(p_{ij}))}{\sigma_j^2} - \sum_{i=1}^{n_j}\frac{(a_j + b_j\log(p_{ij}))^2}{2\sigma_j^2} + c_0, \tag{B.3}$$

where c_0 is a constant. We now choose $a_j^{(k+1)}$, $b_j^{(k+1)}$ and $\sigma_j^{2^{(k+1)}}$ maximize the expected log utility at iteration k. Since

$$\frac{\partial^2\mathbb{E}[\mathcal{L}(a_j, b_j, \sigma_j^2; \log(D_{ij}))|\log(D_{obs})), a_j^{(k)}, b_j^{(k)}, \sigma_j^{2^{(k)}}]}{\partial a_j^2} = -\frac{n_j}{\sigma_j^2} < 0, \tag{B.4}$$

and

$$\frac{\partial^2\mathbb{E}[\mathcal{L}(a_j, b_j, \sigma_j^2; \log(D_{ij}))|\log(D_{obs})), a_j^{(k)}, b_j^{(k)}, \sigma_j^{2^{(k)}}]}{\partial b_j^2} = -\sum_{i=1}^{n_j}\frac{(\log(p_{ij}))^2}{\sigma_j^2} < 0, \tag{B.5}$$

the expected log likelihood function is concave in a_j and b_j, and hence the first order conditions are sufficient for optimality. From (B.3),

$$\partial\mathbb{E}[\mathcal{L}(a_j, b_j, \sigma_j^2; \log(D_{ij}))|\log(D_{obs})), a_j^{(k)}, b_j^{(k)}, \sigma_j^{2^{(k)}}]/\partial a_j = 0, \tag{B.6}$$

from which we have

$$n_j a_j^{(k+1)} + b_j^{(k+1)} \sum_{i=1}^{n_j} log(p_{ij}) = \sum_{i=1}^{m_j} \log(D_{ij}) + \mathbb{E}[\sum_{i=m_j+1}^{n_j} \log(D_{ij})| \log(D_{obs}), a_j^{(k)}, b_j^{(k)}, \sigma_j^{2^{(k)}}]. \tag{B.7}$$

Similarly, from

$$\partial \mathbb{E}[\mathcal{L}(a_j, b_j, \sigma_j^2; \log(D_{ij}))| \log(D_{obs})), a_j^{(k)}, b_j^{(k)}, \sigma_j^{2^{(k)}}]/\partial b_j = 0, \tag{B.8}$$

we have

$$\begin{aligned} a_j^{(k+1)} \sum_{i=1}^{n_j} \log(p_{ij}) + b_j^{(k+1)} \sum_{i=1}^{n_j} (\log(p_{ij}))^2 = \sum_{i=1}^{m_j} \log(D_{ij}) \log(p_{ij}) \\ + \mathbb{E}[\sum_{i=m_j+1}^{n_j} \log(D_{ij}) \log(p_{ij})| \log(D_{obs}), a_j^{(k)}, b_j^{(k)}, \sigma_j^{2^{(k)}}]. \end{aligned} \tag{B.9}$$

By jointly solving (B.7) and (B.9) for a_i and b_i, we obtain

$$a_j^{(k+1)} = \frac{h_{j1}^k \sum\limits_{i=1}^{n_j} (\log(p_{ij}))^2 - h_{j2}^k \sum\limits_{i=1}^{n_j} \log(p_{ij})}{n_j \sum\limits_{i=1}^{n_j} (\log(p_{ij}))^2 - \left(\sum\limits_{i=1}^{n_j} \log(p_{ij})\right)^2}, \tag{B.10}$$

and

$$b_j^{(k+1)} = \frac{n_j h_{j2}^k - h_{j1}^k \sum\limits_{i=1}^{n_j} \log(p_{ij})}{n_j \sum\limits_{i=1}^{n_j} (\log(p_{ij}))^2 - \left(\sum\limits_{i=1}^{n_j} \log(p_{ij})\right)^2}, \tag{B.11}$$

where h_{j1}^k, and h_{j2}^k are as given in (26) and (27). $\sigma_j^{2^{(k+1)}}$ can be derived by a simpler approach. Since the normal distribution falls into the exponential family, the conditional expectations of the moments can be directly substituted for the moments that occur in the expressions obtained for the complete-data maximum likelihood estimators to perform the next iteration. That is, we can replace the sample moments in $\hat{\sigma}^2 = \frac{\sum_{i=1}^{n_j} (\log(D_{ij}))^2}{n_j} - \left(\frac{\sum_{i=1}^{n_j} \log(D_{ij})}{n_j}\right)^2$ by their conditional expectations and obtain $\sigma_j^{2^{(k+1)}}$. Thus, we also have

$$\begin{aligned} \sigma_j^{2^{(k+1)}} = \frac{\sum_{i=1}^{m_j} (\log(D_{ij}))^2 + \mathbb{E}[\sum_{i=m_j+1}^{n_j} (\log(D_{ij}))^2 | \log(D_{obs}), a_j^{(k)}, b_j^{(k)}, \sigma_j^{2^{(k)}}]}{n_j} \\ - \left(\frac{\sum_{j=i}^{m_j} (\log D_{ij}) + \mathbb{E}[\sum_{i=m_j+1}^{n_j} \log(D_{ij}) | \log(D_{obs}), a_j^{(k)}, b_j^{(k)}, \sigma_j^{2^{(k)}}]}{n_j}\right)^2. \end{aligned} \tag{B.12}$$

■