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CVA AND WRONG WAY RISK

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ABSTRACT

This paper proposes a simple model for incorporating wrong-way and right-way risk into CVA (credit value adjustment) calculations. These are the calculations, involving Monte Carlo simulation, made by a dealer to determine the reduction in the value of its derivatives portfolio because of the possibility of a counterparty default. The model assumes a relationship between the hazard rate of the counterparty and variables whose values can be generated as part of the Monte Carlo simulation. Numerical results for portfolios of 25 instruments dependent on five underlying market variables are presented. The paper finds that wrong-way and right-way risk have a significant effect on the Greek letters of CVA as well as on CVA itself. It also finds that the percentage effect depends on the collateral arrangements.

CVA AND WRONG WAY RISK[1]

1. Introduction

It has for some time been standard practice for derivatives dealers to adjust the reported value of their derivatives transactions with a counterparty to reflect the possibility of losses being incurred because of a default by the counterparty. The adjustment is known as the credit value adjustment or CVA. The adjusted value of the derivative is the no-default value minus the CVA.

A derivatives dealer has one CVA for each counterparty. These CVAs are themselves derivatives and must be managed similarly to other derivatives. CVAs are particularly complex derivatives. In fact, the CVA for a counterparty is more complex, and more difficult to value, than any of the transactions between the dealer and the counterparty. This is because the CVA for a counterparty is, as we will see, contingent on the net value of the portfolio of derivatives outstanding with that counterparty.

Calculating CVAs is very computationally intensive. Gregory (2009) provides an excellent discussion of the issues. Statistics published about Lehman Brothers give a sense of the scale of the problem. Reuters (2008) reported that, at the time of its failure Lehman, had about 1.5 million derivatives transactions outstanding with 8,000 different counterparties. This means that it had to calculate 8,000 different CVAs and the number of derivative transactions on which each CVA was dependent averaged about 200.

Another measure, more controversial than CVA, is debit value adjustment or DVA.[2] The DVA is the CVA as seen from the point of view of the counterparty. It reflects the costs to the counterparty as a result of the possibility that the dealer might default. The possibility that it might default is in theory a benefit to the dealer since in some circumstances it may not have to honor its obligations. The reason why DVA is controversial is that it appears that there is no easy way a dealer can monetize DVA without actually defaulting. However, accounting standards

[1] We are grateful to Eduardo Canabarro, Jon Gregory, Dan Rosen, Ignacio Ruiz, Alexander Sokol, and Roger Stein for helpful comments on an earlier version of this paper.

[2] DVA is sometimes also called debt value adjustment.

require reported values to reflect DVA.[3] The book value of the derivatives outstanding with a counterparty is therefore calculated as their no-default value minus the CVA plus the DVA.[4]

It is interesting to note that when the credit spread of a derivatives dealer increases, the dealer's DVA increases. This in turn leads to an increase in the reported value of the derivatives on the books of the dealer and a corresponding increase in its profits. In the third quarter of 2011 the credit spreads of Wells Fargo, JPMorgan, Citigroup, Bank of America, and Morgan Stanley increased by 63, 81, 179, 266, and 328 basis points, respectively. As a result, these banks reported DVA gains that tended to swamp other income statement items. Not surprisingly, DVA gains and losses have now been excluded from the definition of common equity in determining regulatory capital. In this paper, we will focus on CVA, but many of the points we make are equally applicable to DVA.

Market variables that affect the no-default value of a dealer's outstanding transactions with a counterparty also affect the dealer's CVA for that counterparty. We will refer to these market variables as the "underlying market variables." In addition, CVA is affected by the counterparty's term structure of credit spreads (the "counterparty credit spreads"). CVA therefore gives rise to two types of exposures. One arises from potential movements in the underlying market variables; the other arises from potential movements in counterparty credit spreads.

In December 2010, the Basel Committee on Banking Supervision published a new regulatory framework for banks known as Basel III.[5] It requires a dealer's CVA risk arising from changes in counterparty credit spreads to be identified and included in the calculation of capital for market risk. However, the dealer's CVA risk arising from changes in the underlying market variables are not included in this calculation. Some dealers have developed their own sophisticated systems for managing both types of risk. These dealers feel that Basel III proposals are inadequate

[3] Both FAS157 and IAS39 require that, for instruments that are accounted for using mark-to-market accounting, the reported value should reflect both the CVA and DVA.
[4] There is in theory a dependence here in that if the dealer defaults subsequent defaults by the counterparty are irrelevant and vice versa. See Hull and White (2001) for a discussion of this point in the context of credit default swaps.
[5] See Basel Committee on Banking Supervision (2010)

because, if a dealer hedges against the underlying market variables, the hedging trades will lead to an increase, not a decrease, in its required capital.[6]

Since the crisis, governments have moved to require "standardized" over-the counter derivatives to be cleared through central clearing parties. This paper focuses on how counterparty credit risk is handled for those transactions that continue to be cleared bilaterally.[7] This includes all derivatives transactions classified as "non-standard." It also includes some categories of derivatives, such as forward foreign exchange contracts and transactions with end users, that are excluded from the central clearing legislation.

Transactions between a dealer and a counterparty are typically governed by an International Swaps and Derivatives Association (ISDA) Master Agreement. This specifies that all transactions between the two parties are to be netted and considered as a single transaction in the event that there is an early termination. The circumstances under which one side can send an early termination notice to the other side and the procedures that are then used are specified in the ISDA Master Agreement.

Collateralization has become an important feature of over-the-counter derivatives markets. An ISDA Master Agreement typically has a Credit Support Annex (CSA) which specifies the rules governing the collateral that has to be posted by the two sides. In particular, it specifies a variety of items including the threshold, the independent amount, the minimum transfer amount, haircuts that will apply to assets that are posted as collateral, etc. Suppose that the two sides are Party A and Party B, and Party B is required to post collateral. The threshold is the unsecured credit exposure to Party B that Party A is willing to bear. If the value of the derivatives portfolio to Party A is less than the threshold, no collateral is required from Party B. If the value of the derivatives portfolio to Party A is greater than the threshold, the required collateral is equal to the difference between the value and the threshold. The independent amount plays the same role as the initial margin in a futures contract and can be regarded as a negative threshold. A failure to

[6] See, for example, Pengelly (2011)

[7] How derivatives dealers will assess their credit exposure arising from transactions that are cleared through a central clearing party remains to be seen. The dealer is exposed to a) a default by any of the other clearing house members and b) a default by the clearing house itself.

post the required collateral by Party B is an "event of default" that unless corrected leads to the early termination of all outstanding transactions.

In the calculation of CVA, it is usually assumed that the counterparty's probability of default is independent of the dealer's exposure to the counterparty. A situation where there is a positive dependence between the two, so that the probability of default by the counterparty tends to be high (low) when the dealer's exposure to the counterparty is high (low), is referred to as "wrong-way risk." A situation where there is negative dependence, so that the probability of default by the counterparty tends to be high (low) when the dealer's exposure to the counterparty is low (high) is referred to "right-way risk."

A subjective judgment of the amount of wrong-way or right-way risk in transactions with a counterparty requires a good knowledge of the counterparty's business, in particular the nature of the risks facing the business. It also requires knowledge of the transactions the counterparty has entered into with other dealers.[8] The latter is difficult to know precisely, but the extra transparency provided by post-crisis legislation may help.

One situation in which wrong-way risk tends to occur is when a counterparty is selling credit protection to the dealer. (AIG and monolines are obvious examples here.) This is because credit spreads are correlated. When credit spreads are high, the value of the protection to the dealer is high and as a result the dealer has a large exposure to its counterparty. At the same time, the credit spreads of the counterparty are also likely to be high indicating a relatively high probability of default for the counterparty. Similarly, right-way risk tends to occur when a counterparty is buying credit protection from the dealer.

A situation in which a counterparty is speculating by entering into many similar trades with one or more dealers is likely to lead to wrong-way risk for the dealer. This is because the counterparty's financial position and therefore its probability of default are likely to be affected adversely if the trades move against the counterparty. If the counterparty enters into transactions to partially hedge an existing exposure, there should in theory be right-way risk. This is because,

[8] A complication is that sometimes a company will trade with many different market participants in an attempt to conceal its true exposure.

when the transactions move against the counterparty, it will be benefitting from the unhedged portion of its exposure so that its probability of default will tend to be relatively low.[9]

This paper explains the way CVA is calculated and the advanced approach for calculating CVA risk capital under Basel III. The paper then proposes a model for incorporating wrong-way risk into CVA calculations. Other authors who have attempted to tackle the wrong-way risk problem include Cespedes *et al* (2010) and Sokol (2010). These authors propose an "exposure sampling" approach where the process followed by the exposure is approximated with a one-factor Markov process and a Gaussian (or other) copula is used to model the dependence between this process and the time to default.

Our model is simpler than the exposure sampling approach and involves a relatively small adjustment to the method used to calculate CVA when the usual assumption of no dependence between exposure and probability of default is made. We specify a relationship between the hazard rate of the counterparty and the value of variables whose values can be generated as part of the Monte Carlo simulation used to calculate CVA. Numerical results from the implementation of the model are presented.

2. Calculating CVA

Suppose that T is the longest-maturity derivative outstanding between a derivatives dealer and one of its counterparties. Define $v(t)$ as the value of a derivative that has a payoff equal to the dealer's net exposure to the counterparty at time t and R as the recovery rate (assumed to be constant). If we assume that the probability of a counterparty default at time t is independent of $v(t)$ then, as discussed by, for example, Hull and White (1995), Canabarro and Duffie (2003), and Picault (2005),

$$\text{CVA} = (1-R)\int_{t=0}^{T} q(t)\,v(t)\,dt \tag{1}$$

[9] An exception could be when the counterparty is liable to run into liquidity problems. Although the assets being hedged have increased in value, the counterparty might be unable to post collateral when required. An example here is Ashanti Goldfields in September 1999. See for example Hull (2010, page 395) for a description of what happened.

where $q(t)$ is the probability density function of the risk-neutral time to default for the counterparty.

The net exposure depends on whether collateral is posted. If no collateral is posted, the dealer'sexposure, $E_{NC}(t)$, at time t is

$$E_{NC}(t) = \max\left(w(t), 0\right) \tag{2}$$

where $w(t)$ is the value to the dealer of the portfolio of derivatives it has with the counterparty at time t.[10]

Suppose next that transactions are collateralized and the amount of collateral that is required to be posted by the counterparty with the dealer at time t is $C(t)$. We assume that a negative value of C(t) indicates that the dealer is required to post $-C(t)$ of collateral with the counterparty. If early termination happens as soon as the counterparty fails to post the required collateral, the collateralized exposure, $E_C(t)$, at time t is

$$E_C(t) = \max\left(w(t) - C(t),\, 0\right)$$

When only the counterparty is required to post collateral and there is a threshold, K, then

$$C(t) = \max\left(w(t) - K, 0\right) \tag{3}$$

In this situation an independent amount, I, can be treated as a negative threshold so that $C(t) = \max(w(t) + I, 0)$.

We define the "default unwind" date as the time when the dealer is able to either a) replace the transactions it has with the counterparty or b) unwind the hedges it has for those transactions. In practice, there is usually assumed to be a period of time prior to the default during which the counterparty ceases to post collateral and fails to return excess collateral. This period of time is referred to as the "cure period" or "margin period of risk." It can be thought of as having two

[10] We assume that collateral is posted continuously and that there is no minimum transfer amount. The procedures used can be modified to relax these assumptions.

components. The first is the period of time that elapses between the counterparty failing to post collateral or return excess collateral and the dealer declaring an early termination. This is the time during which attempts are being made to resolve disputes between the dealer and the counterparty about the value of the portfolio, whether the collateral demand is valid, and so on.[11] The second is the time that elapses between the dealer declaring an early termination and the default unwind date. This depends on market conditions, the nature of the portfolio, and the size of the portfolio. A total cure period of between 10 to 25 business days is commonly assumed.

We can now refine the definition of $q(t)$. It is the probability distribution of the time to a default unwind date. Suppose that the assumed length of the cure period is c. If there is a default unwind at time t the collateral posted is $C(t - c)$ where as before a negative value indicates that the dealer posts collateral with the counterparty. The net exposure is

$$E_C(t) = \max\big(w(t) - C(t-c),\, 0\big) \tag{4}$$

Note that there can be an exposure when the value of the portfolio to the non-defaulting party, $w(t)$, is negative. This is because, if the portfolio has moved in the non-defaulting party's favor, the collateral posted at the beginning of the cure period by the non-defaulting party, $-C(t-c)$, may be more than the value of the portfolio at the end of the cure period to the defaulting party. The exposure arises from the defaulting party's assumed failure to return any excess collateral during the cure period.

The variable $v(t)$ in equation (1) is the value of a derivative that pays off $E_C(t)$ at time t. It can be calculated as the expected value of $E_C(t)$ in a risk-neutral world discounted at the risk-free rate.

DVA is calculated in the same way as CVA. To calculate DVA we let $v(t)$ be the value of a derivative that has a payoff equal to the counterparty's net exposure to the dealer at time t, and $q(t)$ be the probability density function of the dealer's time to a default unwind. With these altered definitions the right hand side of equation (1) gives the DVA. The counterparty's exposure to the dealer is calculated in a manner similar to how the dealer's exposure is calculated.

[11] The ISDA contract typically specifies that two days must be allowed for dispute resolution, but in practice it often takes much longer than this.

The variable $w(t)$ is the value of the portfolio of derivatives that the dealer has with the counterparty. This may consist of a large number of individual transactions. The variable $v(t)$ is the value of a derivative that pays off the net exposure $E_C(t)$ at time t. In CVA calculations, $v(t)$ is therefore a complex derivative contingent on the portfolio's value. As mentioned earlier, it is much more complex than any derivatives traded between the dealer and the counterparty. Netting tends to ensure that CVA is less than the sum of the dealer's exposures on the individual transactions constituting the portfolio.

3. Monte Carlo Simulation

In practice, CVA is almost invariably calculated using Monte Carlo simulation. To approximate the integral in equation (1) we can choose times t_i ($0 \le i \le n$) with $t_0 = 0$, $t_n = T$ and $t_0 < t_1 < t_2 < \ldots < t_n$ and set

$$\text{CVA} = (1-R)\sum_{i=1}^{n} q_i v_i \tag{5}$$

where q_i is the probability of a default unwind between times t_{i-1} and t_i and $v_i = v\left(t_i^*\right)$ with $t_i^* = 0.5\left(t_{i-1} + t_i\right)$.[12]

The q_i's are usually calculated from credit spreads. Sometimes a complete term structure of credit spreads for the counterparty can be observed in the market; sometimes it has to be estimated using credit spread data for other companies. If s_i is the credit spread for a maturity of t_i, a good estimate of the (risk-neutral) probability of no default between times 0 and t_i is $\exp\left[-s_i t_i/(1-R)\right]$.[13] It follows that

[12] Note that q_i is the unconditional risk-neutral probability of default between times t_{i-1} and t_i (as seen at time zero). It is not the probability of default conditional on no earlier default. A confusion concerning the use of conditional and unconditional default probabilities in calculating CVA was pointed out by Rebonato *et al* (2010).

[13] This is explained in Section 4.

$$q_i = \exp\left(-\frac{s_{i-1}t_{i-1}}{1-R}\right) - \exp\left(-\frac{s_i t_i}{1-R}\right) \quad (6)$$

To calculate the v_i's, the market variables affecting the no-default value of a dealer's derivatives with a counterparty are simulated between times 0 and T in a risk-neutral world.[14] One approach is to arrange the simulation so that the value of the dealer's portfolio with the counterparty is calculated at times $t_i^* - c$ and t_i^* $(1 \le i \le n)$. This means, on each simulation trial at each time $t_i^* - c$ $(1 \le i \le n)$, the collateral is determined using equation (3). Then at time t_i^* the value of the dealer's portfolio with the counterparty is determined and the net exposure of the dealer to the counterparty is calculated using equation (4). The variable v_i is estimated as the present value of the average of the calculated net exposures at time t_i^* $(1 \le i \le n)$.[15]

In practice, some approximations are usually made so that computations are feasible. The model used to value the portfolio during the Monte Carlo simulation may be simpler than either

a) the model used to simulate the underlying market variables, or

b) the model used by the dealer for marking to market its book.

Also, the risk-neutral measure for the v_i's may be different from that used for the q_i's.

The Monte Carlo simulation is computationally quite time consuming. Some dealers calculate CVA once a day, others less frequently. It is normal for the values of the market variables used in the simulation and the portfolio values to be stored. If a new transaction is contemplated with a counterparty, this stored data can then be used to quickly calculate the value of this transaction at each valuation time for each simulation trial. This enables the portfolio value at each valuation time for each simulation trial to be updated relatively quickly so that the impact of the new transaction on CVA is calculated.[16]

[14] When interest rates are stochastic a convenient numeraire is the value of a risk-free zero coupon bond maturing at the next valuation time. The Monte Carlo simulation can then be implemented by assuming that a) the returns on market variables between valuation dates and b) the discount rates between valuation dates equal the yield on the numeraire bond.

[15] Only a small amount of additional computation time is necessary to calculate DVA at the same time as CVA.

[16] Note that the impact of a new transaction on CVA can be positive or negative.

Dealers also often estimate "peak exposures" which are high percentiles (e.g., 97.5%) of the exposures at times $t_i^* \ (1 \le i \le n)$. The "maximum peak exposure" is the maximum of the peak exposures calculated at these times. An interesting point (usually ignored in practice) is that these estimates should in theory be made using the real-world measure to calculate exposures, not the risk-neutral measure.[17]

Using a delta/gamma approximation,[18] the impact on CVA of a small change Δs in all the s_i's is

$$\begin{aligned}\Delta(\text{CVA}) &= \sum_{i=1}^{n}\left[t_i \exp\left(-\frac{s_i t_i}{1-R}\right) - t_{i-1}\exp\left(-\frac{s_{i-1}t_{i-1}}{1-R}\right)\right]v_i \Delta s \\ &+ \frac{1}{2(1-R)}\sum_{i=1}^{n}\left[t_{i-1}^2 \exp\left(-\frac{s_{i-1}t_{i-1}}{1-R}\right) - t_i^2 \exp\left(-\frac{s_i t_i}{1-R}\right)\right]v_i(\Delta s)^2\end{aligned} \tag{7}$$

This equation enables the dependence of CVA on counterparty credit spreads to be included in the bank's model for calculating market risk capital. Equations (5), (6), and (7) correspond to the equations used in the Basel III advanced approach for determining capital for CVA risk.[19]

Calculating the sensitivity of CVA to a small parallel shift in a counterparty's credit spread is straightforward using equation (7). Calculating the first and second partial derivatives of CVA with respect to the underlying market variables is liable to be more time consuming. Consider a market variable u with initial value u_0. It is necessary to calculate the effect on the paths sampled of changing u_0 to $u_0+\varepsilon$ and $u_0-\varepsilon$ for a small ε when all random number streams are kept the same. When the variable follows geometric Brownian motion this is not too difficult. A small percentage change at time zero leads to the same small percentage change at all future times on all simulation trials. (This is true both when the volatility is deterministic and when the volatility is stochastic.) For other variables such as those following mean reverting processes, the impact

[17] This nuance is usually ignored because the only difference between the real-world and risk-neutral world measures is in the drift rates of the variables. The vast majority of the peak exposure, particularly when short maturities are considered, is a result of volatility not drift. Further, in many cases it is not clear what the real-world drifts are or whether using real-world drifts would increase or decrease the peak exposure.

[18] The change in the value of a function can be approximated by using the first two terms of a Taylor series, the delta/gamma approximation: $\Delta f(x) = \delta \times \Delta x + 0.5\Gamma \times (\Delta x)^2$ where δ is df/dx and Γ is d^2f/dx^2.

[19] Charging capital for the CVA risk associated with credit spreads is likely to encourage dealers to hedge spread risk by buying credit default swaps. This might have the unintended consequence of increasing credit spreads.

of a change at time zero on the change at future times is liable to depend on the path followed by the market variable.

Suppose that v_i^+ and v_i^- are the values calculated for v_i when the initial value of the market variable is $u_0+\varepsilon$ and $u_0-\varepsilon$, respectively. From equation (5)

$$\frac{\partial \text{CVA}}{\partial u} = \frac{1-R}{2\varepsilon}\sum_{i=1}^{n} q_i(v_i^+ - v_i^-)$$

$$\frac{\partial^2 \text{CVA}}{\partial u^2} = \frac{1-R}{\varepsilon^2}\sum_{i=1}^{n} q_i(v_i^+ + v_i^- - 2v_i)$$

These equations enable CVA risks relating to the underlying market variables to be assessed and hedged. As already mentioned, under Basel III CVA exposures arising from the underlying market variables are not included in the calculation of market risk capital.

4. A Model for Wrong-Way and Right-Way Risk

The standard approach that we have described for calculating CVA assumes that $v(t)$ and $q(t)$ are independent. The situation where q is positively dependent on v is referred to as "wrong-way" risk. In this case, there is a tendency for a counterparty to default when the dealer's exposure is relatively high. The situation where q is negatively dependent on v is referred to as "right-way" risk. In this case, there is a tendency for a counterparty to default when the dealer's exposure is relatively low.

A simple way of dealing with wrong-way risk is to use what is termed the "alpha" multiplier to increase $v(t)$ in the version of the model in which $v(t)$ and $q(t)$ are assumed to be independent. The effect of this is to increase CVA by the alpha multiplier. The Basel II rules set alpha equal to 1.4 or allow banks to use their own models, with a floor for alpha of 1.2. This means that, at minimum, the CVA has to be 20% higher than that given by the model where $q(t)$ and $v(t)$ are independent. If a bank does not have its own model for wrong way risk it has to be 40% higher. Estimates of alpha reported by banks range from 1.07 to 1.10.

The usual approach to modeling wrong-way risk is to reflect it in the way the *v'*s are calculated. The alpha multiplier approach just mentioned is one example of this approach. Another method sometimes used is to set $v(t)$ equal to the present value of the exposure that is k standard deviations above the average exposure for some k.

We will use a different approach. Instead of changing the calculation of $v(t)$ we change the calculation of $q(t)$ so that $q(t)$ depends on the evolution of the variables in the Monte Carlo simulation used to calculate CVA up until time t. To do this, we have to introduce the concept of a hazard rate, a measure of the probability that a default will occur. If the hazard rate is h, the probability that a default will occur within any short period of time Δt, conditional on no earlier default, is $h\Delta t$. If the current time is zero, the probability that no default will occur before time t is $\exp(-ht)$. The hazard rate does not need to be constant. If the hazard rate changes deterministically over time, the probability that no default will occur before time t is $\exp(-\bar{h}t)$ where $\bar{h}$ is the average hazard rate between time zero and time t. If the hazard rate changes stochastically, the probability of no default between time zero and time t is the expected value of this.

Hazard rates are not directly observable, but credit spreads are. A very good approximation of the average (risk-neutral) hazard rate between time 0 and time t is $s(t)/(1-R)$ where $s(t)$ is the credit spread for a maturity of t.

Our approach is based on modeling a relationship between the hazard rate of a counterparty and a variable (or variables) that can be calculated in the Monte Carlo simulation and may affect the dealer's exposure to the counterparty. This relationship can be either deterministic or stochastic. There are three ways of proceeding:

1. Assume a relationship between the counterparty's hazard rate and a variable, x, that is closely related to the dealer's exposure to the counterparty. A reasonable approach here is to set x equal to w, the value of the dealer's portfolio with the counterparty. If there is no relationship between the hazard rate, $h(t)$, and $w(t)$ then there is no wrong way or right

way risk. A positive relationship is indicative of wrong way risk and a negative relationship is indicative of right-way risk.[20]

2. Assume a relationship between the counterparty's hazard rate and a variable, *x*, that a) affects the value of the counterparty's portfolio and b) has a big effect on the counterparty's health. This variable, because it affects the value of the counterparty's portfolio, is already part of the Monte Carlo simulation. The variable chosen for a gold producer might be the price of gold. For another company, it could be an exchange rate or interest rate.

3. Assume a relationship between the counterparty's hazard rate and a variable *x* that does not affect the value of the counterparty's portfolio, but potentially has a big affect on its health. Possible choices for *x* are the counterparty's five-year credit spread, its stock price, or the Moody's KMV distance to default for the counterparty.[21] Daily historical data must be used to estimate correlations between the variable chosen and the other variables in the Monte Carlo simulation so that the process assumed for the variable is appropriately modeled.

The relationship between the hazard rate *h* and the variable *x* has the form

$$h(t) = f\left(x(t)\right)$$

The function *f* can involve a noise term. The function chosen may depend on the nature of the variable *x* and the nature of the company. The function *f* must have the property that $h(t) \geq 0$ for all possible values of $x(t)$ and the noise term, if any. It must also have the property that for all times *t* the expected probability that there is no default before time *t* based on the random hazard rates equals the probability of no default before time *t* that is inferred from credit spreads at time zero. This means that the expected value of $\exp\left(-\bar{h}t\right)$ must equal $\exp\left(-st/(1-R)\right)$ where $\bar{h}$ is the average hazard rate between time zero and time *t* and expectations are taken over all possible

[20] An alternative we have experimented with is to set *x* equal to the exposure. However, this does not work as well as setting *x* equal to the value of the portfolio because the exposure at any particular time is zero for many of the simulation trials.

[21] Ruiz (2012) proposes a model where the hazard rate is assumed to depend on the equity price and shows how the model can be estimated empirically for Ford.

paths that x may follow. We explain how this condition is satisfied in the appendix. To allow the condition to be satisfied, the function f must involve a parameter which is a function of time.

The assumption of a relationship between hazard rates and other variables is likely to be more attractive to regulators than the exposure sampling method. This is because the model can be backtested fairly easily. Past hazard rates can be estimated from historical data on credit spreads. These can be combined with historical data on x to test whether the assumed relationship between h and x held in the past. If the assumed relationship is itself estimated from historical data, it can be tested out of sample.

An issue in the calculation of CVA is the existence of credit triggers in CSAs. A credit trigger can increase collateral requirements in the event of a downgrade. (An example here is AIG's downgrade to below AA on September 15, 2008.) Credit triggers can also give the dealer the option to terminate transactions early. If credit triggers are ignored, CVA estimates are liable to be too high. A side benefit of the model we propose is that it can be used to incorporate credit triggers. An approximate relationship between the counterparty's credit spreads (and therefore its hazard rate) and its credit rating can be estimated and adjustments to the CVA model to reflect the counterparty's credit rating can then be made.

In the rest of this paper we will illustrate the nature of wrong way risk by using the first of the approaches outlined above and set $x = w$, the value of the portfolio of derivatives. One function that we have found to be a robust choice in this case is

$$h(t) = \exp\left[a(t) + bw(t) + \sigma\varepsilon\right] \tag{8}$$

where b is a constant parameter, which measures the amount of right-way or wrong-way risk in the model, $a(t)$ is a function of time, σ is a constant measuring the amount of noise in the relationship and ε is normally distributed with zero mean and unit variance. In practice, we find that, unless σ is very large relative to *w*, the noise term makes little difference to the results so that the model can be safely assumed to be[22]

$$h(t) = \exp[a(t) + bw(t)] \tag{9}$$

In this model, small percentage changes in the hazard rate bear a simple relationship to small changes in *w*:

$$\frac{\Delta h}{h} = b\Delta w$$

The model is illustrated in Figure 1.

The relation between portfolio value and default probability can arise from the a correlation between *w* and the counterparty's credit spreads (e.g., because the counterparty is writing credit protection) or because the counterparty becomes more likely to default as $w(t)$ becomes high (e.g. because the counterparty is a hedge fund taking a big speculative position with the dealer in question and other dealers).

There are two approaches to estimating *b*. The first is to collect historical data on *w* and on credit spreads for the counterparty. The credit spreads can be converted into hazard rates and *b* can then be estimated. The disadvantage of this approach is that it assumes the factors influencing the counterparty's credit spreads in the past are the same as those that will do so in the future.

The other approach involves subjective judgement about the amount of right-way or wrong-way risk the counterparty has. It best illustrated with an example. Suppose that the current value of *w* is \$3 million and the counterparty's five-year credit spread is 300 basis points. Assuming a recovery rate of 40%, this means that the average five-year hazard rate is 5% per year. Also, suppose that it is estimated that, if *w* increased to \$20 million, the spread can be expected to rise to 600 basis points, corresponding to an average five-year hazard rate of 10%. Assuming the

[22] An alternative similar model is $h(t) = \ln[1 + \exp(a(t) + bw(t))]$. In this model $h(t)$ increases linearly with $w(t)$ when $x(t)$ becomes very large whereas in equation (8) it increases exponentially. In practice, we find very little difference between the two models.

term structure of hazard rates is flat,[23] when $w(0) = 3$, $h(0) = 5\%$ and when $w(0) = 20$, $h(0) = 10\%$. Solving a pair of simultaneous equations for $a(0)$ and b, we find that for the model in equation (9), b is 0.0408.

A key advantage of the approach we propose for incorporating wrong-way or right-way risk into CVA calculations is that it requires only a small change to the procedure for calculating CVA. This change is designed to determine $a(t)$ in a way that is consistent with credit spreads observed today and is explained in the appendix.

5. Numerical Results

In this section we consider the impact that both collateral arrangements and right-way or wrong-way risk have on CVA. There is a complex interplay between these factors and the results are often surprising.

We first illustrate the model in equation (9) by assuming that the dealer has a simple portfolio consisting of a single one-year forward foreign exchange transaction. We assume that the principal is \$100 million, the domestic and foreign risk free rates are 5%, the initial exchange rate is 1.0, the delivery exchange rate specified in the forward contract is also 1.0, and the volatility of the exchange rate is 15%. We suppose that the counterparty's credit spread (all maturities) is 125 basis points. Four different collateral arrangements are considered: no collateral, a threshold of \$10 million with a cure period of 15 days, a threshold of zero with a cure period of 15 days, and an independent amount of \$5 million with a cure period of 15 days. In all cases, it is assumed that only the counterparty is required to post collateral. Tables 1 to 4 report results for four possible cases: b is 0.03 and the dealer's position is long; b is 0.03 and the dealer's position is short; b is –0.03 and the dealer's position is long; and b is –0.03 and the dealer's position is short.

The results illustrate that wrong-way and right-way risks have a material effect on the deltas and gammas of CVA as well as on CVA itself. This is true for both the deltas and gammas with

[23] Given the imprecision of any attempt to quantify wrong-way risk, this is a reasonable assumption.

respect to the exchange rate and the deltas and gammas with respect to the credit spread. The magnitude of the effect is difficult to predict. Indeed, in some cases, even the direction of effect can be difficult to predict. As shown by the tables, sometimes the direction of the effect depends on the collateral arrangements.

In general, the impact of wrong-way and right-way risk on CVA depends in a complex way on CVA itself and the collateral arrangements. To illustrate this for more realistic portfolios, we randomly generated 250 portfolios. Each portfolio consists of 25 options on one of five different assets. The asset prices are assumed to follow geometric Brownian motion with pairwise correlations of 0.36. As before, only the counterparty is required to post collateral. Each option has the following properties:

(a) It is equally likely to be long or short
(b) It is equally likely to be a call or a put
(c) The underlying is equally likely to any one of the five assets
(d) All maturities between one and five years are equally likely
(e) All strike prices within 30% of the current asset price are equally likely
(f) The underlying principal is $25 million.

The assets do not provide any income. They have an initial price of $25 and a volatility of 25%. The risk-free rate is 5%, the credit spread of the counterparty (all maturities) is 125 basis points, and the recovery rate is 40%.

Figure 2a provides a scatter plot of the relationship between the dollar change in CVA caused increasing b from 0 to 0.01 and the CVA for b=0 for the 250 portfolios when there is no collateralization. It can be seen that the change in CVA tends to increase as CVA increases. The reason is that as CVA increases there is a tendency for both the mean and standard deviation of the value of the portfolio at future times to increase. The gap between the exposures on high-$w(t)$ paths and low-$w(t)$ paths increases. Wrong-way risk causes the hazard rate to increase dramatically on the high-$w(t)$ paths and decrease (modestly) on the low-$w(t)$ paths.

The case illustrated in Figure 2a is roughly consistent with an alpha of 1.4, the regulatory requirement. The increase in CVA as a result of wrong-way risk is on average about 30% to 40%

of the CVA calculated when there is no wrong-way risk. However, the impact of b=0.01 for the portfolios considered does depend on the collateral arrangements and the assumption that wrong-way risk on average tends to increase CVA by a certain percentage is not always valid.. This is illustrated by the next chart, Figure 2b, where there is a non-zero threshold.

In Figure 2b, the threshold is \$10 million and the cure period is 15 days. The average relationship between the change in CVA and CVA is no longer monotonic. The change first increases and then decreases. To understand the reason for this, we ignore the cure period and focus on the threshold. With zero threshold, the impact of the threshold is to restrict the net exposure to less than \$10 million. For low $w(t)$-paths this restriction has little effect. However, as $w(t)$ increases, the net exposure is increasingly impacted by the \$10 million restriction. The gap between the exposure on high-$w(t)$ paths and low-$w(t)$ paths is much less than in the no collateral case. The highest theoretical CVA is achieved when the value of $w(t)$ is certain to be above \$10 million at all times. In this case, the net exposure is the same on all $w(t)$ paths and wrong-way risk has no effect. This explains the pattern observed in Figure 2b. The cure period does have an effect on the results, but its effect is less than the effect of the threshold.

Figure 2c provides a plot similar to Figures 2a and 2b for the situation where the threshold is zero and the cure period is 15 days. In this case the net exposure is entirely as a result of the cure period. When $w(t)$ is positive, the net exposure increases as the standard deviation of the change in $w(t)$ during the cure period increases. High standard deviations of $w(t)$ tend to be associated with high $w(t)$-path. High $w(t)$-paths are in turn associated with increases in the hazard rate when there is wrong-way risk. This leads to the pattern shown in Figure 2c.

Figure 2d provides a plot for the situation where there is an independent amount equal to \$5 million and a cure period of 15 days. As in the case of Figure 2c the impact of the cure period on the exposure at time t for a particular $w(t)$ depends on the standard deviation of the change in $w(t)$ during the cure period. Indeed, for positive exposures to be generated, the standard deviation has to be sufficiently high that there is a reasonable chance of a \$5 million increase in $w(t)$ during the cure period. The reasons for the pattern observed are similar to those for Figure 2c.

The impact of right-way risk can be examined by changing b from 0 to −0.01 instead of from 0 to +0.01. The results are similar to those shown in Figure 2, except that the change in CVA is negative instead of positive. We have carried out other experiments involving portfolios of interest rate swaps and other derivatives. The results are similar to those in Figures 2.

6. Conclusions

We have proposed a simple model for handling wrong-way/right-way risk in the calculation of CVA. The model involves estimating a relationship between the counterparty's hazard rate and other relevant variables. The model can be implemented by making a small change to the usual method for calculating CVA and can incorporate credit triggers.

Tests of the model show that wrong-way and right-way risk have a significant effect on the Greek letters of CVA as well as on CVA itself. Because CVA is such a complex derivative, it is difficult to estimate these effects without a model. Indeed, even the sign of the effect can be counterintuitive.

Tests involving the random generation of portfolios indicate that when there is no collateral or when collateral is posted with zero threshold or when collateral is posted with an independent amount, the dollar impact of wrong-way risk on CVA tends to increase as CVA increases. The situation where the threshold is materially positive is more interesting. As CVA increases, the impact of wrong way risk tends to first increase and then decrease.

Models involving a relationship between hazard rates and other observable variables have a number of advantages over the more complex models that have been proposed in the literature to date. They are simpler, computationally faster, and easier to backtest. As indicated, a number of different observable variables can be chosen. Further research is needed to determine variables which work best and to determine the appropriate functional form for the relationship.

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Appendix

Calibration of the Model

To match survival probabilities in the model in equation (8), we discretize the model so that

$$h_{ij} = \exp\left[\, a\left(t_i^*\right) + b w_{ij} + \sigma \varepsilon_{ij} \,\right]$$

where h_{ij} and w_{ij} are the values of $h\left(t_i^*\right)$ and $w\left(t_i^*\right)$ on the jth simulation trial and ε_{ij} is a random sample from a standard normal distribution. We require

$$\frac{1}{m}\sum_{j=1}^{m}\left[\exp\left(-\sum_{i=1}^{k} h_{ij}\Delta t\right)\right] = \exp\left(-\frac{s_k t_k}{1-R}\right) \quad \text{for} \quad 1 \le k \le n$$

where m is the number of simulation trials, and s_k is the credit spread for a maturity of t_k.

The values of $a\left(t_k^*\right)$ for $1 \le k \le n$ are determined sequentially so that the average survival probability, across all simulations, up to time t_k^* equals the survival probability calculated from the term structure of credit spreads. First, k is set equal to 1 and an iterative search is used to determine $a\left(t_1^*\right)$ from the w_{1j}. This determines the h_{1j}. Second, k is set equal to 2 and an iterative search is carried out to determine $a\left(t_2^*\right)$ from the w_{2j} values and the h_{1j}. This determines the h_{2j}; and so on.

For example, suppose we are using the model in equation (8) with $\sigma = 0$ and that we simulate three paths (m=3). The credit spread for all maturities is 1%, the recovery rate is zero, the time step in the simulation is 0.5 years, and the value of b is 0.01. The values of w at the first step of each path are 100, 200 and 300. We choose the first value of a in order to satisfy

$$\frac{1}{3}\left[e^{-\exp(a+0.01\times 100)\times 0.5} + e^{-\exp(a+0.01\times 200)\times 0.5} + e^{-\exp(a+0.01\times 300)\times 0.5}\right] = \exp\left(-\frac{.01\times 0.5}{1-0}\right)$$

The solution to this is $a\left(t_1^*\right)$ = –6.9128 and the three hazard rates are h_{11}=0.00270, h_{12}=0.00735, and h_{13}=0.01998. The corresponding values of w at the second step of each path are 100, 300 and 400. We choose the second value of a in order to satisfy

$$\frac{1}{3}\left[e^{-[.00270+\exp(a+1)]\times 0.5}+e^{-[.00735+\exp(a+3)]\times 0.5}+e^{-[.01998+\exp(a+4)]\times 0.5}\right]=\exp\left(-\frac{.01\times 1.0}{1-0}\right)$$

The solution to this is $a\left(t_2^*\right)$ = –7.8509 and the three hazard rates are h_{21}=0.00106, h_{22}=0.00782, and h_{23}=0.02126.

Table 1: Impact of wrong-way risk on CVA for a long forward contract to buy 100 million units of a foreign currency in one year. The current exchange rate is 1.0, the domestic and foreign risk-free interest rates are both 5%, and the volatility of the exchange rate is 15%. The credit spread is 125 basis points for all maturities and the recovery rate is 40%. *K* is the threshold in $ millions, *c* is the cure period in days, and *b* is the parameter in equation (9).

	No Collateral	$K = 10$ $c = 15$	$K = 0$ $c = 15$	$K = –5$ $c = 15$
CVA ($ millions) for b = 0	0.048	0.036	0.011	0.002
Impact of b = 0.03 per $mm on:				
CVA	54.8%	41.7%	37.3%	53.5%
Delta wrt Exch Rate	32.0%	15.6%	12.8%	39.3%
Gamma wrt Exch Rate	2.6%	–25.4%	17.7%	–0.7%
Delta wrt Spread	53.8%	41.2%	36.8%	52.8%
Gamma wrt Spread	181.8%	124.3%	122.8%	184.3%

Table 2: Impact of wrong-way risk on CVA for a short forward contract to sell 100 million units of a foreign currency in one year. The current exchange rate is 1.0, the domestic and foreign risk-free interest rates are both 5%, and the volatility of the exchange rate is 15%. The credit spread is 125 basis points for all maturities and the recovery rate is 40%. *K* is the threshold in $ millions, *c* is the cure period in days, and *b* is the parameter in equation (9).

	No Collateral	$K = 10$ $c = 15$	$K = 0$ $c = 15$	$K = –5$ $c = 15$
CVA ($ millions) for b = 0	0.048	0.039	0.011	0.001
Impact of b = 0.03 per $mm on:				
CVA	40.5%	34.0%	27.6%	28.9%
Delta wrt Exch Rate	16.2%	7.7%	–1.9%	–341.9%
Gamma wrt Exch Rate	–7.0%	–21.4%	16.4%	26.5%
Delta wrt Spread	40.0%	33.7%	27.4%	28.8%
Gamma wrt Spread	114.8%	91.0%	77.0%	70.7%

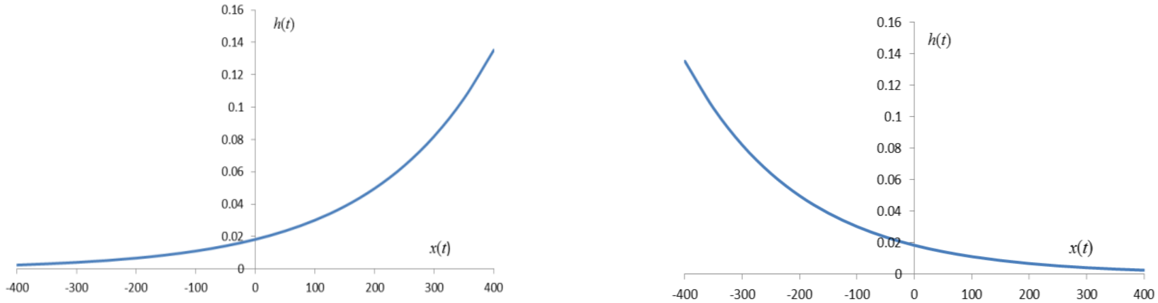
Table 3: Impact of right-way risk on CVA for a long forward contract to buy 100 million units of a foreign currency in one year. The current exchange rate is 1.0, the domestic and foreign risk-free interest rates are both 5%, and the volatility of the exchange rate is 15%. The credit spread is 125 basis points for all maturities and the recovery rate is 40%. *K* is the threshold in $ millions, *c* is the cure period in days, and *b* is the parameter in equation (9).

	No Collateral	$K = 10$ $c = 15$	$K = 0$ $c = 15$	$K = –5$ $c = 15$
CVA ($ millions) for b = 0	0.048	0.036	0.011	0.002
Impact of b = –0.03 per $mm on:				
CVA	–37.5%	–32.7%	–29.1%	–35.7%
Delta wrt Exch Rate	–26.7%	–18.8%	–14.8%	–28.9%
Gamma wrt Exch Rate	–8.2%	11.7%	–16.0%	6.2%
Delta wrt Spread	–37.2%	–32.5%	–28.9%	–35.6%
Gamma wrt Spread	–79.2%	–74.5%	–72.1%	–77.3%

Table 4: Impact of right-way risk on CVA for a short forward contract to buy 100 million units of a foreign currency in one year. The current exchange rate is 1.0, the domestic and foreign risk-free interest rates are both 5%, and the volatility of the exchange rate is 15%. The credit spread is 125 basis points for all maturities and the recovery rate is 40%. *K* is the threshold in $ millions, *c* is the cure period in days, and *b* is the parameter in equation (9).

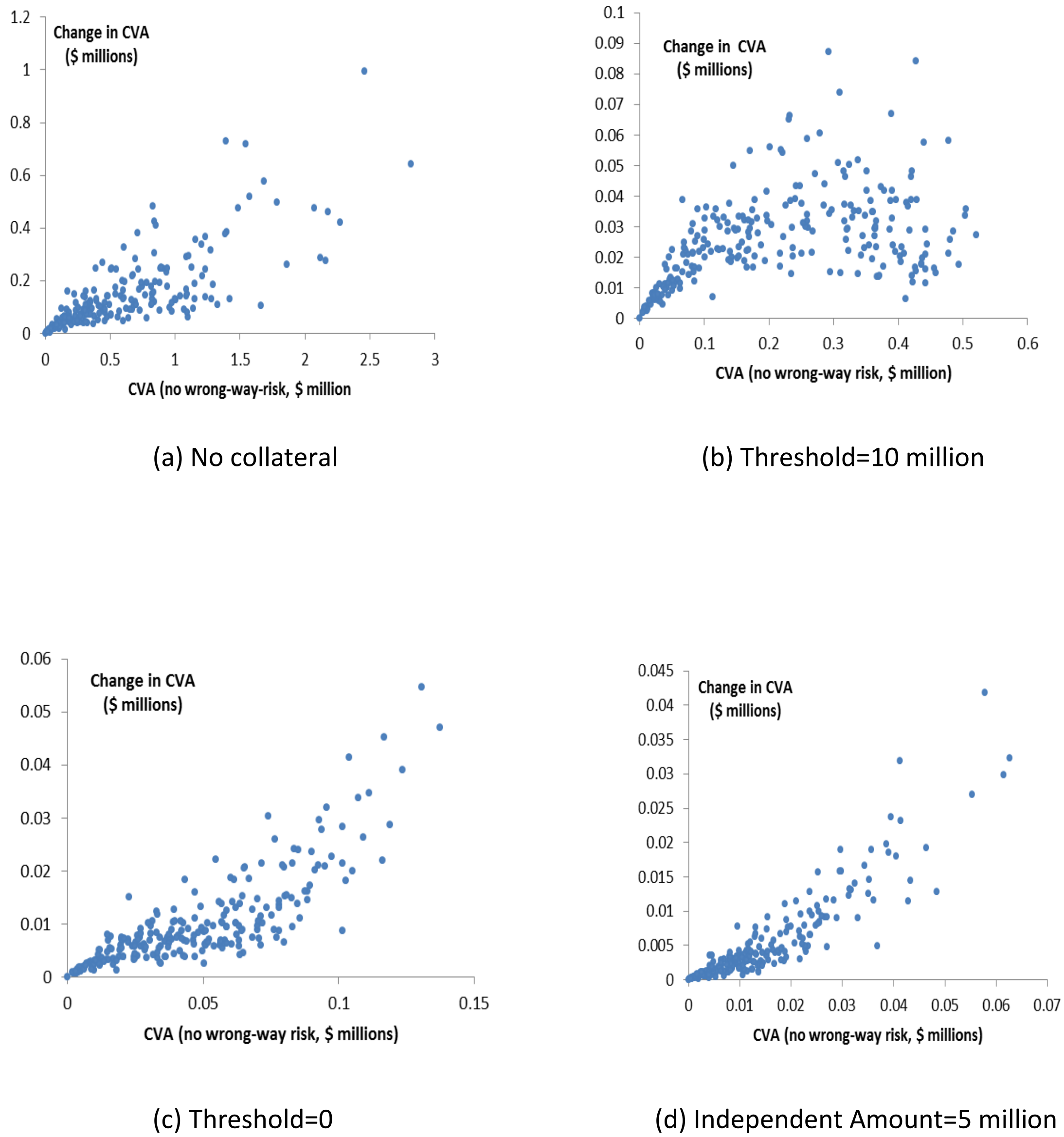
	No Collateral	$K = 10$ $c = 15$	$K = 0$ $c = 15$	$K = –5$ $c = 15$
CVA ($ millions) for b = 0	0.048	0.039	0.011	0.001
Impact of b = –0.03 per $mm on:				
CVA	–33.9%	–30.8%	–25.9%	–26.9%
Delta wrt Exch Rate	–19.3%	–13.6%	–4.9%	209.1%
Gamma wrt Exch Rate	0.9%	14.4%	–16.7%	–37.5%
Delta wrt Spread	–33.6%	–30.6%	–25.7%	–26.7%
Gamma wrt Spread	–78.8%	–75.5%	–71.3%	–69.0%

Figure 1: The model in equation (9) when $a(t) = -4$.



$b = 0.005$ $b = -0.005$

Figure 2: Impact of wrong-way risk for 250 portfolios of options when there is no collateral. The horizontal axis shows CVA when *b*=0. The vertical axis shows the change in CVA when *b* is increased from 0 to 0.01 per million.



(a) No collateral

(b) Threshold=10 million

(c) Threshold=0

(d) Independent Amount=5 million