Why Has Black−Scholes−Merton Been So Successful?

John Hull
Joseph L Rotman School of Management
University of Toronto

After 50 years, the model developed by Black and Scholes (1973) and Merton (1973) still plays a key role in the way traders manage option portfolios. The model has also received a great deal of attention from researchers over the last 50 years. For example, in Merigo et al (2016), the Black-Scholes paper was listed as the tenth most cited paper in business and economics of all time.

Why has the Black−Scholes−Merton model (BSM) been so successful? It is not because the model is a really good description of the way options are priced by the market. BSM assumes a constant volatility when, in practice for most assets, volatility fluctuates with periods of high volatility and periods of low volatility. Furthermore, the underlying asset price sometimes exhibits jumps, so that the assumption that movements are always continuous is at best an approximation.

The reason for the success of BSM is the fact that it has only one important unobservable variable. As is well known, the inputs to BSM for valuing European options on an investment asset that provides no income are:

- Underlying asset price
- Strike price
- Risk-free rate
- Time to maturity
- Volatility

The underlying asset price, strike price and time to maturity are all known.[1] Theoretically the risk-free rate should be the instantaneous risk-free rate. The model requires this to be constant during the life of the option. Fortunately the sensitivity of the option price to the risk-free interest rate is generally fairly small and a reasonable approach is to substitute into the formula the risk-free rate that has the same maturity as the option. If dividends or other income are provided by the underlying asset, it appears to be necessary to introduce one or more extra unobservable variables defining income during the life of

[1] There is an issue as to whether the time to maturity should be measured in trading days or calendar days when the model is applied. This can make a difference, particularly for short maturity options. Research such as Roll (1984) shows that volatility is much greater when the market is open than when it is closed. As a result, most traders prefer to measure time in trading days with 252 days per year.

the asset. However, the market's expectation about the present value of this income is defined by forward or futures contracts. Indeed, as shown by Black (1976), defining the price of a European option in terms of futures prices avoids the need to worry about income altogether.

The option price is a monotonically increasing function of volatility. This means that there is a one-to-one correspondence between the price and volatility. BSM allows option prices to be computed unambiguously from volatilities and volatilities to be implied unambiguously from option prices. Implied volatilities based on BSM have become a key communication tool. Rather than quoting the price of a particular option, traders would much rather quote its implied volatility with the understanding that BSM will be used to convert the implied volatility to a price. The reason they like to do this is that, as the price of the underlying asset moves, the dollar option price will nearly always move by much more than the implied volatility. As a result, an implied volatility quote is more attractive than a price quote because it is likely to be good for a longer period of time.

What all this means is that traders have to have access to the BSM model so that they can move from implied volatilities to prices, and vice versa. They may use much more sophisticated models than BSM to try and understand the market, but they use BSM implied volatilities to communicate prices.

Imagine what would have happened if Black, Scholes and Merton had come up with a different model in 1973. For example, because volatilities are often negatively correlated with asset prices, they might have proposed the following constant elasticity of variance model for asset prices:

$$dS = rS\,dt + \sigma\sqrt{S}\,dz$$

where S is the asset price, r is the instantaneous risk-free rate, σ is a constant determining price uncertainty and dz is a Wiener process. They would then have produced a model where the option price depends on cumulative non-central chi square distributions rather than cumulative normal distributions.[2] It is conceivable that this would have become the market standard. If that had happened, there is no reason to suppose that option prices in the market would be any different from those observed, but the implied σ's used as communication tools would be quite different.

The key point here is that a model with one unobservable parameter, even if the model is only a rough approximation to reality, is likely to be very popular with practitioners. The unobservable parameter can

[2] The constant elasticity of variance model was considered by Cox and Ross (1976), with a publication date three years later than BSM.

be a useful communication tool. Unfortunately there are few other models in economics and finance which have been constructed with just one unobservable parameter. With two unobservable parameters, the set of parameters corresponding to a particular output must be represented by a curve in two-dimensions; with three parameters, they are represented by a two-dimensional surface in three dimensions; and so on.

The assumptions underlying BSM have been used for derivatives other than European options. For example, many exchange-traded options are American-style and can be valued numerically using BSM assumptions with binomial trees, finite difference methods, or other approaches. Implied volatilities can be estimated and used to price over-the-counter options, many of which are European.

The widespread use of volatility surfaces is an example of the success of BSM. If the BSM assumptions were a perfect reflection of the pricing of options, the volatility surface would be flat, i.e., all implied volatilities would be the same. In fact many different volatility surface shapes are observed in practice.

A natural approach for trying to beat the market is to develop a better model than geometric Brownian motion (the model assumed by BSM) for how the prices of assets behave. Authors such as Hull and White (1987), Heston (1993), and Hagan et al (2002) allow volatility to be stochastic and produced some analytic results. Although the asset price has to follow a Markov process, volatility does not. This has led researchers such as Gatheral et al (2018) to develop what are known as rough volatility models where the process for volatility is non-Markov so that changes in one time period are influenced by changes in previous time periods

Other researchers have produced models where there are jumps in the asset price. Examples are Merton (1976) and Madan et al (1998). Yet another line of research, initiated by Dupire (1994), Derman and Kani (1994), and Rubinstein (1994), has involved developing one-factor models where volatility is a function of the asset price and time so that the volatility surface is perfectly matched at time zero.

In the last few years machine learning has had an increasing impact on pricing and hedging options. As described in for example Hull (2021), methods have been developed to generate synthetic paths for a variable that are indistinguishable from those observed historically. This is an alternative to specifying a stochastic model algebraically. Two popular approaches involve the use of a variational autoencoders (VAE) or a generative adversarial network (GAN).

The properties observed in practice for the behavior of asset prices include:

- fat tails (relative to a normal distribution) for daily changes
- volatility clustering (periods of high volatility and periods of low volatility)
- the leverage effect (negative correlation between the asset price and volatility)
- no significant autocorrelation in returns
- positive autocorrelation in squared or absolute returns that decreases as the number of lags increase

It is difficult to develop an algebraic model with all these properties, but generative models such as VAE and GAN can potentially incorporate them.

Generative models can produce as many paths as required. This is convenient because Monte Carlo simulation can be used to value European options and path-dependent options. Other approaches such as those suggested by Longstaff and Schwartz (2001) can be used in conjunction with Monte Carlo simulation for American options.

Pricing options and other derivatives is of course only half the battle. Traders need to develop good hedging tools. Traditionally, the approach of practitioners has been to substitute the implied volatility into the BSM formulas for Greek letters such as delta, gamma, and vega. Bates (2005) provides a way of calculating delta and gamma that takes volatility smiles into account. More recently, researchers have used a machine learning tool known as reinforcement learning to develop hedging strategies. This can be used in conjunction with the generation of synthetic data to potentially develop improvements to current hedging practice.

Does this mean we are finally leaving BSM behind? The answer is No! Researchers such as Cont and Vuletić (2022) and Francois et al (2023) have very recently developed generative models for the dynamics of volatility surfaces. (A volatilty surface is of course a BSM construct.) In theory, the stochastic evolution of the volatility surface can be derived from a knowledge of the stochastic evolution of the underlying asset. However, the volatility surface is forward looking and contains more information about the market's perception of potential future asset price movements than the past history of asset prices.

What is the key takeaway for researchers from all this. Perhaps it is the following. If you want to develop a model that people are still going to be using in 50 years, include just one unobservable variable on the right hand side!

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