THE VALUATION OF CORRELATION-DEPENDENT CREDIT DERIVATIVES USING A STRUCTURAL MODEL

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ABSTRACT

In 1976 Black and Cox proposed a structural model where an obligor defaults when the value of its assets hits a certain barrier. In 2001 Zhou showed how the model can be extended to two obligors whose assets are correlated. In this paper we show how the model can be extended to a large number of different obligors. The correlations between the assets of the obligors are determined by one or more factors. We examine the dynamics of credit spreads implied by the model and explore how the model prices tranches of collateralized debt obligations (CDOs). We compare the model with the widely used Gaussian copula model of survival time and test how well the model fits market data on the prices of CDO tranches. We consider three extensions of the model. The first reflects empirical research showing that default correlations are positively dependent on default rates. The second reflects empirical research showing that recovery rates are negatively dependent on default rates. The third reflects research showing that market prices are consistent with heavy tails for both the common factor and the idiosyncratic factor in a copula model.

The Valuation of Correlation-Dependent Credit Derivatives Using a Structural Model

The trading of products whose values depend on credit correlation has grown rapidly in recent years. The most popular credit-correlation product is a collateralized debt obligation (CDO). This is a structure designed to distribute the credit risk from an underlying portfolio to investors in much the same way that collateralized mortgage obligations distribute prepayment risk. In the cash CDO market, the underlying portfolio is formed from bonds or other credit-sensitive instruments. In the synthetic CDO market, it is formed from short positions in credit default swaps.

An important development in the synthetic CDO market has been the establishment of standard portfolios such as CDX NA IG and iTraxx Europe. CDX NA IG is a portfolio of 125 investment grade companies in the United States; iTraxx Europe is a portfolio of 125 European investment grade companies. Standard tranches for these portfolios have been defined and losses are allocated to tranches in order of reverse seniority.

To value a CDO tranche it is necessary to develop a model of the joint probability of default of the companies in the underlying portfolio. The first structural model of default for a single company was produced by Merton (1974). In this model a default occurs if the value of the assets of the company is below the face value of the debt at a particular future time. Black and Cox (1976) provide an important extension of Merton's model. Their model has a first passage time structure where a default takes place whenever the value of the assets of a company drops below a barrier level. Other extensions of the Merton's model are provided by Geske (1977), Kim, Ramaswamy and Sundaresan (1993), Leland (1994), Longstaff and Schwartz (1995), Leland and Toft (1996), and Zhou(2001a). All these papers looked at the default probability of only one issuer.

Zhou (2001b) and Hull and White (2001) were the first to incorporate default correlation between different issuers into the Black and Cox first passage time structural model. Zhou (2001b) finds a closed-form formula for the joint default probability of two issuers, but his results cannot easily be extended to more than two issuers. Hull and White (2001) show how many issuers can be handled, but their correlation model requires computationally time-consuming numerical procedures.

In this paper we provide a way to model default correlation using the Black-Cox structural model framework. Our approach can be used to price and calculate deltas for credit derivatives and is computationally feasible when there are a large number of different companies. We assume that the correlation between the asset values of different companies is driven by one or more common factors. We consider, first, a base case model, where the asset correlation and the recovery rate are both constant. We then extend the base case model in three ways. The first allows the asset correlation to be stochastic and correlated with default rates. This is motivated by empirical evidence that suggests that asset correlations are stochastic and increase when default rates are high (see Servigny and Renault (2002), Das, Freed, Geng and Kapadia (2004) and Ang and Chen (2002)). The second assumes that the recovery rate is stochastic and correlated with default rates. This is consistent with Altman et al (2005) and Cantor, Hamilton, and Ou (2002) who found that recovery rates are negatively dependent on default rates. The third assumes a mixture of processes for asset prices that leads to both the common factors and idiosyncratic factors having distributions with heavy tails. This model is in the spirit of the double *t* copula of Hull and White (2004) which was found to fit market prices well.

We implement the base-case model and the three extensions using Monte Carlo simulation and then investigate how well the model can explain empirical data on the market's prices for CDX NA IG and iTraxx Europe tranches. We also compare our model with the Gaussian copula model, a reduced form model of survival times that has become the standard market-model for pricing correlation-dependent credit derivatives. The Gaussian copula model was developed by Li (2000) and later extended Gregory and Laurent (2005).

This paper shows that the structural model is a computationally viable alternative to the Gaussian copula approach. Furthermore, it has two important advantages over that approach. First it is a dynamic model where the credit qualities of companies evolve through time. Second it has an economic rationale so that correlation parameters can be estimated empirically.

This paper is, to the best of our knowledge, the first study to test empirically the performance of structural models of default for explaining synthetic CDO tranche prices. The modeling approach we consider is a "bottom-up" approach where the individual defaults/credit losses are aggregated up to the portfolio losses which then determine the CDO tranche prices. An alternative model for explaining CDO tranche prices is provided by Longstaff and Rajan (2006). They use a "top down" framework similar to that employed by Duffie and Gârleanu (2001). They show that the evolution of losses on a portfolio can be explained by three independent jump processes. These processes give rise to losses of 0.4%, 6% and 30%. The first which is assumed to correspond to firm-specific default risk occurs on average once every 1.2 years; the second which is assumed to correspond to sector default risk occurs on average once every 41.5 years; the third which is assumed to correspond to economy-wide default risk occurs on average once every 763 years.

The rest of the paper is organized as follows. Section 1 discusses the market for CDOs. Section 2 describes the model. Section 3 compares the model to the survival-time copula model. Section 4 describes the data and the way we fit the model parameters to the data. Section 5 presents our empirical results. Conclusions are in Section 6.

1. The CDO Market

Synthetic CDOs are actively traded default-correlation-dependent credit derivatives. A synthetic CDO tranche is defined with reference to a (usually equally weighted) portfolio of credit exposures. The tranche has a notional principal, which we will denote by P. Two default loss levels for the portfolio, L_U and L_D, expressed as a fraction of the size of the portfolio, are specified with $L_U > L_D$. The seller of protection on the tranche is required to make payments to the buyer of protection for cumulative default losses on the reference portfolio that are in the range between L_D and L_U. If a loss equal to a proportion δ of the portfolio is incurred at a particular time and is in the range, the protection seller makes a payment of $P\delta/(L_U - L_D)$ at that time. The protection seller earns a rate of return, referred to as a spread. Initially the spread is earned on the whole notional principal, P. However, when cumulative losses on the portfolio are L and $L_U > L > L_D$, the spread is earned on $P(L_U – L)/(L_U - L_D)$. When losses exceed L_U, no spread is earned.

A synthetic CDO is created by selling a portfolio of credit default swaps in the market and then distributing the credit risk to investors. A key development in the synthetic CDO market involves what is known as single tranche trading. Standard portfolios and standard tranches are defined. One party to a contract agrees to buy protection on an individual tranche; the other party agrees to sell protection on the tranche. Cash flows are calculated in the same way as they would be if a synthetic CDO were constructed for the portfolio. However, in single tranche trading, the underlying portfolio of credits is never created. It is merely a reference portfolio used to calculate cash flows.

Two popular reference portfolios for single tranche trading are the CDX NA IG and the iTraxx Europe portfolios. In the case of CDX NA IG, the underlying portfolio consists of 125 equally weighted investment grade companies in the United States. In the case of iTraxx Europe the underlying portfolio is 125 equally weighted investment grade companies in Europe. The composition of both portfolios is reviewed and updated periodically.

Sample quotes for five- and ten-year tranches of the CDX NA IG and iTraxx Europe portfolios on August 25, 2004 are shown in Table 1. Consider first the CDX NA IG portfolio. The riskiest tranche that trades is known as the equity tranche. This is responsible for losses between 0% and 3% of the notional principal. The next tranche is known as the mezzanine tranche. This is responsible for losses between 3% and 7%. Other tranches are responsible for losses in higher ranges. To understand the quotes consider first the 15% to 30% tranche. The quote for five-year protection on August 25, 2004 was 12.43 basis points. This means that the buyer of protection would pay 0.1243% per year of the outstanding notional principal of the tranche and be compensated for any losses on the portfolio during the five years that are between 15% and 30% of the principal. (The relevant portfolio is the portfolio underlying the index at the time of the trade.) For 10-year protection on the 15 to 30% tranche, the spread is 0.495% per year.

The quotes for the other tranches are defined similarly to the quotes for the 15 to 30% tranche except that in the case of the equity tranche the quote is an upfront fee, expressed as a percent of the notional principal, that must be paid in addition to a spread of 500 basis points per year on the outstanding principal. The percent of the notional principal that buyers of the equity tranche had to pay on August 25, 2004 was 40.02% for five-year protection and 58.17% for ten-year protection. Table 2 provides statistics on tranche quotes for the period January 2 to December 21, 2004 indicating that the August 25, 2004 data is representative.

The index quote indicates the cost per company of buying protection against all companies in the underlying portfolio. The table shows that the cost of protection on the CDX NA IG portfolio on August 25, 2004 was 59.73 basis points for five years and 81 basis points for ten years. For iTraxx it was 37.79 basis points for five years and 51.25 for ten years. [1] The index is approximately equal to the average of the credit default swap spreads for the 125 companies.[2]

2. The Firm Model

We consider N_c different companies and define $V_i(t)$ as the value of the assets of company i at time t ($1 \le i \le N_c$). We assume the risk-neutral diffusion process followed by V_i is

$$dV_i = \mu_i V_i dt + \sigma_i V_i dz_i$$

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[1] The market considered the iTraxx IG portfolio to be less risky than the CDX NA IG portfolio on August 25, 2004. As mentioned in Kakodkar and Martin (2004), CDX NA IG had an average rating of BBB+ at the end of June 2004, while iTraxx Europe had an average rating of A−.

[2] The index is slightly lower than the average of the credit default swap spreads for the companies comprising the underlying portfolio. To understand the reason consider two companies, one with a spread of 1000 basis points and the other with a spread of 10 basis points. To buy protection on both companies would cost slightly less than 505 basis points per company. This is because the 1000 basis points is not expected to be paid for as long as the 10 basis points and should therefore carry less weight. Also, to facilitate trading the index is traded like a bond with a fixed coupon. The quoted spread is used to determine an upfront exchange of cash flow.

where z_i is a Wiener process and that μ_i and σ_i are constant[3].

We assume that company i defaults when V_i falls below a barrier H_i. This corresponds to the Black and Cox (1976) extension of Merton's (1974) model.

The probability of a default by a particular time T can be calculated analytically. In general, if a variable x follows an arithmetic Brownian motion:

$$dx = mdt + sdz$$

where m and s are constants and dz is a Wiener process and x is an amount K above a barrier at time zero, the probability of hitting the barrier by time T is

$$N\left(\frac{-K-mT}{s\sqrt{T}}\right)+\exp\left(\frac{-2Km}{s^2}\right)N\left(\frac{-K+mT}{s\sqrt{T}}\right) \qquad (1)$$

where N is the cumulative normal distribution function.[4] The probability density function of the time τ at which the barrier is hit is

$$\frac{1}{\sqrt{2\pi}}\frac{K}{s\tau^{3/2}}\exp\left(-\frac{(K+m\tau)^2}{2s^2\tau}\right)$$

The process assumed for V_i implies

$$d\ln V_i = \left(\mu_i - \sigma_i^2/2\right)dt + \sigma_i dX_i$$

.A default occurs when $\ln V_i = \ln H_i$. Assuming that the $V_i(0) > H_i$, the probability of a default between time 0 and T is given by substituting into equation (1): $K = \ln V_i(0) - \ln H_i$, $m = \mu_i - \sigma_i^2/2$, and $s = \sigma_i$. It is

$$N\left(\frac{\ln(H_i/V_i(0))-(\mu_i-\sigma_i^2/2)T}{\sigma_i\sqrt{T}}\right)+\exp\left(\frac{2\ln(H_i/V_i(0))(\mu_i-\sigma_i^2/2)}{\sigma_i^2}\right)N\left(\frac{\ln(H_i/V_i(0))+(\mu_i-\sigma_i^2/2)T}{\sigma_i\sqrt{T}}\right) \qquad (2)$$

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[3] When the model is fitted to market prices for credit derivatives the implied parameters are the risk-neutral values. In this case, for a non-dividend-paying company μ_i is the risk-free rate, r. For a company that pays a dividend that is a known percentage, q_i, of its assets $\mu_i = r - q_i$.

[4] See for example Harrison (1990)

This result can be used to find the probability of first hitting the barrier between times T_1 and T_2 in terms of the value of the company at time 0. The probability density function for the time to default, τ, is

$$\frac{1}{\sqrt{2\pi}}\frac{K}{\sigma_i\tau^{3/2}}\exp\left(-\frac{(\ln(V_i(0)/H_i)+(\mu_i-\sigma_i^2/2)\tau)^2}{2\sigma_i^2\tau}\right)$$

If we assume some recovery rate in the event of default, this can be used to determine the risk-neutral expected payoffs from a bond and its credit spread. A similar calculation can be used to determine a CDS spread.[5]

The four unobservable parameters $V_i(0)$, H_i , μ_i, and σ_i can be reduced to two by defining:

$$\beta_i=\frac{\ln H_i-\ln V_i(0)}{\sigma_i} \qquad \gamma_i=-\frac{(\mu_i-\sigma_i^2/2)}{\sigma_i}$$

Note that the structure of the model is the same if the barrier is linear (sloping up or down) rather than horizontal.. This is because the first passage time for the situation when the return on V_i is μ_i and the barrier starts at $H_i(0)$ sloping up at rate q_i per unit time is the same as that for the situation where the return on V_i is μ_i-q_i and the barrier is horizontal at $H_i(0)$.

2.1. Properties of the Model

Table 3 explores the process followed by the five-year credit spreads for a company in the structural model we have just presented. The table is based on the barrier for a representative company in the CDX NA IG index on August 25, 2004. On this date the five-year CDX NA IG index spread was 59.73 basis points and the 10 year spread was 81 basis points. The calibration involved an iterative search procedure for the two parameters β_i and γ_i. These two parameters enable the probability of default to be calculated in any time interval using equation (2). This in turn allows the CDS spread to be calculated. The two parameters were those for which the five- and ten-year CDS spreads equaled the five and ten-year indices, respectively.[6] The recovery rate was assumed to be 40%.

We find that the barrier parameters that match the CDX NA IG index levels on August 25, 2004 are $\beta = -3.89$ and $\gamma = -0.12$. These parameters are economically reasonable. If the asset volatility is 15% they correspond to

[5] See for example Hull and White (2003).

[6] We implicitly assume that index level is equal to the CDS spread for a representative company.

$$\frac{H}{V(0)} = 0.5583 \qquad \mu = 0.0289$$

The modified distance to default (MDD) measure in Table 3 is $-\beta$. This represents the number of standard deviations by which the value of the assets exceeds the barrier.[7] On August 25, 2004 the MDD for a representative company in the index was 3.89. The table is created by varying $V(0)$ while keeping the barrier the same.

The next two columns show the average drift per week and the standard deviation per week of the CDS spread. These were calculated by using equation (2) to estimate $\partial S/\partial t$, $\partial S/\partial X$, and $\partial^2 S/\partial X^2$ numerically and then using Ito's lemma, where S is the CDS spread.

The table shows that, as MDD declines (i.e., as the value of the assets of the company moves closer to the barrier), the CDS spread increases and both the drift and the standard deviation of the spread increases. This is as one would expect.

2.2 Asset Correlations

We now discuss how asset correlations are modeled. For convenience, we replace the state variable V_i by X_i where

$$X_i(t) = \frac{\ln V_i(t) - \ln V_i(0) - (\mu_i - \sigma_i^2/2)t}{\sigma_i}$$

The variable X_i follows a Wiener process. We assume that the process for X_i is

$$dX_i(t) = \alpha_i(t)dF(t) + \sqrt{1-\alpha_i(t)^2}\,dU_i(t) \qquad (3)$$

where F and U_i are Wiener processes with $F(0) = U_i(0) = 0$ and $dU_i(t)dF(t) = dU_i(t)dU_j(t)=0$, $j \neq i$. The Wiener processes X_i therefore have a common component F and an idiosyncratic component U_i. The

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[7] The distance to default measure in Kealhofer (2003) is a forward-looking measure. It is the number of standard deviations that the value of the assets must change to trigger a default at a particular future time $t+T$ or

$$\frac{\ln V_i(t) - \ln H_i + (\mu_i - \sigma_i^2/2)T}{\sigma\sqrt{T}}$$

Our modified distance to default measure is a simpler measure that is more appropriate to the results being presented.

variable α_i, which may be a function of time or stochastic, defines the weighting given to the two components.

F can be thought of as a factor defining the default environment. When $F(t)$ is low there is a tendency for the X_i's to be low and the rate at which defaults occur is relatively high. When $F(t)$ is high the reverse is true. One possible proxy for F is the Wiener process underlying the evolution of a well-diversified stock index such as the S&P 500. The parameter α_i defines how sensitive the default probability of company i is to the factor F. The correlation between the processes followed by the assets of companies i_1 and i_2 is $\alpha_{i_1}\alpha_{i_2}$.

2.3 Model Implementation

We implemented the model using Monte Carlo simulation. We consider points in time t_0, $t_1, \ldots, t_n$. (Typically $t_k - t_{k-1}$ is three months.) The continuous barrier is replaced by a discrete barrier that is defined only at these points. The barrier is chosen, using the numerical procedure explained in Hull and White (2001), so that the default probabilities in each interval (t_{k-1}, t_k) are the same as those for the continuous barrier given by equation (2).

The simulation is carried out by drawing a set of zero mean unit variance normally distributed random variables Δf_k and Δu_{ik} $(1 \le k \le n,\ 1 \le i \le N_c)$. These are used to sample four antithetic sets of paths for the X_i.

$$\begin{aligned} f_k^m &= f_{k-1}^m + (-1)^m \alpha_{ik} \Delta f_k \sqrt{\Delta t} & m &= 0,1 \\ u_{ik}^j &= u_{i,k-1}^j + (-1)^j \sqrt{1-\alpha_{ik}^2}\, \Delta u_{i,k} \sqrt{\Delta t} & j &= 0,1 \\ X_{ik}^{mj} &= f_k^m + u_{ik}^j & m, j &= 0,1 \end{aligned} \tag{4}$$

where X_{ik}^{mj} is the value of X_i at time t_k for a particular combination of m and j, and α_{ik} is the value of α_i between times t_{k-1} and t_k. The correlation parameters α_i are assumed to be constant in each interval $(t_{k-1},$ $t_k)$. Company i is assumed to default at the mid point of time interval (t_{k-1}, t_k) if the value of X_i is below the barrier for the first time at time t_k. The simulation is repeated 5,000 times generating 20,000 sets of paths.[8] For each set of paths the number of defaults, d_k, that occur in each time interval, (t_{k-1}, t_k), is determined.

If we assume that each CDS underlying a synthetic CDO has a notional principal of \$1 and the same recovery rate, R, the tranche attachment points, a_L and a_H, can be mapped into the number of losses, n_L

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[8] The use of antithetic paths does not reduce standard errors appreciably in this case, but it does cut down on the computational time spent sampling from normal distributions.

and n_H, where $n_L = a_L N_c/(1–R)$ and $n_H = a_H N_c/(1–R)$. The expression for the outstanding tranche notional after d defaults is

$$P(d, a_L, a_H) = \begin{cases} (a_H - a_L)M & d < m(n_L) \\ a_H M - j(1-R) & m(n_L) \le d < m(n_H) \\ 0 & d \ge m(n_H) \end{cases}$$

where $m(x)$ as the smallest integer greater than x.

The value of a contract given some default experience is the present value of the cash flows resulting from the defaults. The calculation of this present value involves three terms.

$$A = \sum_{k=1}^{n} (t_k - t_{k-1}) P(d_k, a_L, a_H) e^{-rt_i}$$

$$B = 0.5 \sum_{k=1}^{n} (t_k - t_{k-1}) \left\{ P(d_{k-1}, a_L, a_H) - P(d_k, a_L, a_H) \right\} e^{-r(t_i + t_{i-1})/2}$$

$$C = (1-R) \sum_{k=1}^{n} \left\{ P(d_{k-1}, a_L, a_H) - P(d_k, a_L, a_H) \right\} e^{-r(t_i + t_{i-1})/2}$$

where r is the risk-free rate of interest, A is the present value of the periodic payments at a rate of \$1 per year, B is the accrual payment that occurs when defaults reduce the notional principal, and C is the loss due to default. The value of the contract to the seller of protection per dollar of notional is $sA + sB – C$. The expected values of A, B and C, $\hat{A}$, $\hat{B}$ and $\hat{C}$, are estimated based on the simulations. The estimated breakeven spread is then $\hat{s} = \hat{C} / (\hat{A} + \hat{B})$. The standard error of the estimate is based on the sample variances and covariances of $\hat{A}$, $\hat{B}$, and $\hat{C}$.

2.4 Extension to Several Factors

The model can be extended so that there are several factors. Equation (3) then becomes

$$dX_i(t) = \sum_{j=1}^{n_F} \alpha_{ij}(t) dF_j(t) + \sqrt{1 - \sum_{j=1}^{n_F} \alpha_{ij}(t)^2} \, dU_i(t)$$

where n_F is the number of factors and α_{ij} is the jth factor loading for company i. The correlation between the processes followed by the assets of companies i_1 and i_2 is $\sum_{j=1}^{n_F} \alpha_{i_1 j} \alpha_{i_2 j}$.

3. Comparison with the Survival-Time Copula Model

As mentioned earlier, a popular way of modeling the joint default probability of many obligors is to use the factor-based Gaussian copula model of survival time that was suggested by Gregory and Laurent (2005). In the simplest form of the model, firm *i* has a random variable x_i associated with it. The x_i's are related through a single factor model

$$x_i = a_i M + Z_i\sqrt{1-a_i^2} \tag{5}$$

where $a_i^2 \le 1$ and *M* and the Z_i's are independent zero mean unit variance normally distributed random variables. The correlation between x_i and x_j is $a_i a_j$. This is referred to as the copula correlation.

The x_i is related to firm *i*'s time to default t_i by equating cumulative density functions so that

$$N(x_i) = Q_i(t_i)$$

where *N* is the cumulative normal distribution function and Q_i is the cumulative distribution function for t_i. In this model the factor *M* captures all the dependence between default events.

Conditional on a particular value of *M* the probability that firm *i* will survive until time *t* is

$$S_i\left(t\,|\,M\right) = 1 - N\left\{\frac{N^{-1}\left[Q_i\left(t\right)\right] - a_i M}{\sqrt{1-a_i^2}}\right\}$$

Conditional on *M*, survival times are independent. The present values of expected cash inflows and outflows conditional on *M* can be calculated and these can be integrated over the probability distribution of *M* to obtain their unconditional present values.

The survival time copula model has been extended in a number of ways by authors such as Andersen and Sidenius (2004), Giesecke (2003), and Hull and White (2004). One approach is to increase the number of factors. Another is to switch from the Gaussian copula to another copula such as the Student *t*, Clayton, Archimedian, Marshall-Olkin, or double *t*. A third approach is to assume a probability distribution for a_i that can be dependent on the factor *M*. These extensions are reviewed in Burtschell, Gregory, and Laurent (2005).

The survival time copula model is different from the structural model in that the realization of the common factor *M* happens at time zero and governs the default outcomes in all time periods. In other words the default environment is the same for the whole life of the model (usually five or ten years). The structural model we described in Section 2 is much richer because the realization of *F* at time *t* defines the default environment at that time. A bad default environment in one year can be followed by a good default environment in the next year.

In spite of these differences there are some reasons to suppose that the correlation environment generated by the Gaussian copula model is a reasonable approximation to that given by the structural model. Consider two companies i and j with cumulative probabilities of default by time t equal to $Q_i(t)$ and $Q_j(t)$. As before we define t_i and t_j as the times to default for the companies. Suppose that the Gaussian copula correlation is ρ_{ij}^*. The probability that both companies will default by time t is the joint probability that $t_i < t$ and $t_j < t$. Equivalently it is the probability that

$$x_i < N^{-1}(Q_i(t)) \text{ and } x_j < N^{-1}(Q_j(t))$$

where x_i and x_j are defined by equation (5). This is $B(Q_i(t), Q_j(t); \rho_{ij}^*)$ where $B(a, b; \rho)$ is the cumulative probability in a standardized bivariate normal distribution that the first variable will be less than a and the second variable will be less than b when the correlation between the two variables is ρ.

Under the structural model the joint probability that the companies will default by time t is the probability that the values of the assets of both companies have dropped below the barrier by time t. If we ignore the possibility of the value of a company's assets dropping below the barrier before time t and recovering to be above the barrier by time t, this equals the probability that the assets of both companies are below the barrier at time t. This is $B(Q_i(t), Q_j(t); \rho_{ij})$ where ρ_{ij} is the correlation between the asset price processes for i and j.

This shows that, if we make the simplifying assumption that once a company's assets are less than the barrier they remain less than the barrier, the Gaussian copula model and the structural model give the same joint probabilities of default by any time t when $\rho_{ij}^* = \rho_{ij}$. An extension of the argument shows that, when this simplifying assumption is made, they give the same joint default probabilities of the time to default for a set of N_c companies for any value of N_c. This in turn means that the models give rise to the same default correlation structure and the same pricing for CDO tranches.

To test the closeness of the results given by the two models we carry out three tests in the sections that follow. The results show that the models are very similar. However, it is dangerous to assume from this that the extensions to the basic Gaussian copula survival time model that have been proposed are reasonable approximations to the corresponding extensions to the basic structural model. For example making a_i dependent on M in equation (5) is clearly quite different from making α_i dependent on F in equation (3).

3.1 Pairwise Correlations

Zhou (2001b) provides exact analytic results for the joint probability of default of two companies under the Black and Cox (1976) structural model. As we have just explained, the joint probability of default in the Gaussian copula model can be calculated from the cumulative bivariate normal distribution function. In Tables 4 and 5 we compare the joint probabilities of default given by the Gaussian copula model with those given by the structural model when the same correlation parameter is used.

We fit the structural model to market data on August 25, 2004 as in Section 2.1. In Table 4 we investigate the effect of changing the MDD while keeping the barrier the same. In Table 5 we investigate the effect of changing the time horizon considered. In all cases the marginal default probabilities for the Gaussian copula model and the structural model are the same.

Table 5 shows that the joint probabilities given by the two models are very close. As MDD decreases, the probability of default increases and the absolute difference between the joint probabilities given by the two models increases. (This is consistent with the fact that companies far away from the barrier are likely to cross the barrier toward the end of the period making the Gaussian copula a better approximation than for companies close to the barrier.). However, the proportional difference decreases. As the correlation parameter increases, the difference (absolute and proportional) between the two models increases. Table 5 shows that as the time horizon is increased the absolute difference between the joint probabilities given by the two models increases while the percentage difference decreases. In all cases the Gaussian copula model overstates the joint probabilities.

3.2 CDO Tranche Pricing

Table 6 compares tranche prices for the structural model with those for the Gaussian copula survival time model. Again we use a structural model with parameters chosen to match the indices on August 25, 2004. As in the case of the pairwise correlation analysis, the marginal default probabilities for the two models are the same.

It can be seen that the models produce quite similar results. We measure the difference between the two models by calculating the mean absolute spread difference (MASD) across tranches. We obtained similar results using the 10-year CDX NA IG data and using iTraxx Europe data. It is clear that from these results and other tests we have carried out that the Gaussian copula survival time model is a good approximation to the basic structural model in a wide range of situations. When we increased spreads we found that two models do diverge, but only very slowly.

3.3 Deltas

It is sometimes assumed that an accurate calculation of deltas for a structural model is impossible. We have not found this to be the case. Our approach is to perturb the 5- and 10-year index spreads by x basis points, recalculate the barrier and the marginal default probabilities, and reprice the CDO tranches using the same sequence of random numbers. The delta is calculated as the change in the breakeven tranche spread divided by x. To test the robustness of our procedure we used values of x equal to −10, −5, −3, −2, −1, +1, +2, +3, +5, and +10.

Two alternative deltas can be calculated. For the first delta the perturbed barrier or marginal default probabilities applies to only one of the 125 firms in the portfolio underlying the CDO. The barrier or default probabilities for the remaining 124 firms remain unchanged. For the second delta the change is applied to all the firms in the portfolio.

By comparing the results for different values of x we found that the relationship between tranche spread and index spread is fairly linear, particularly in the case of the single-name delta. The results for the structural model using 20,000 trials (5,000 independent paths for F and U) are summarized in Table 7. Apart from confirming that our approach to calculating delta is fairly robust the table shows that the 125-name delta is approximately 125 times the one-name delta. This suggests an interesting approach to calculating a one-name delta when a homogeneous model is used. The common CDS spread is shifted by, say, 10 basis points and the resulting 125-name delta is divided by 125.

Table 8 compares the structural model delta with the delta calculated from the Gaussian copula model.[9] The standard error of the estimate was calculated by repeating the Monte Carlo simulation several hundred times. It can be seen that the Gaussian copula deltas are very close to the structural model deltas. This provides further evidence that the two models are very close to each other.

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[9] The latter required the bucketing approach to calculating the loss distribution explained by Andersen et al (2003) and Hull and White (2004).

4. Data

The data on CDX NA IG and iTraxx Europe tranches was provided by GFI, a large credit derivatives broker. It covers the period from January 2, 2004 to December 21, 2004 and consists of intraday bid and offer quotes as well as trades for CDO tranches. We computed the daily bid as the average of all intra-day bids. The daily offer quote was computed similarly. We used the daily mid-quote (the average of daily bid and offer quotes) as the daily observation in the sample.

Table 2 presents summary statistics on tranche spreads and bid-ask spreads for 5 year and 10 year index tranches. In general CDX NA IG and iTraxx Europe tranche spreads decreased over time. For example the mean 5 year CDX NA IG equity tranche spread for the first half a year (1/1/2004-30/6/2004) was 41.5%. It decreased to 37.9% in the second half. The spreads for the other five-year CDX NA IG tranches decreased from 332.3bp, 118.2bp, 51.7bp, and 13.8bp to 272.2bp, 105.1bp, 37.8bp and 11.6bp, respectively.[10]

The bid-ask spreads also fell over time, consistent with an increase in liquidity in this market. For example, in the case of CDX NA IG, the mean bid-ask spread for the five-year equity tranche for the first half a year was 2.4% and decreased to 1.5% in the second half. Similarly the mean bid-ask spreads for the 5Y CDX NA IG 3-7%, 7-10%, 10-15% and 15-30% tranches decreased from 43.6bp, 11.8bp, 13.1bp, 7.6bp to 9.6bp, 6.1bp, 6.3bp and 3.1 bp, respectively.

The data is incomplete in that not all the quotes shown in Table 1 are available for all days. In our analysis we considered only days where a complete set of bids and offers was available, that is, observations on all five tranches and the index for both five- and ten-year maturities. There were 51 such days for iTraxx Europe and 60 such days for CDX NA IG. We used the average of the daily bid and offer quotes in all our analyses.

In order to fit the structural model to a set of CDX NA IG or iTraxx Europe quotes on a particular day we must estimate the following: the risk-free zero curve, the asset correlation, the barrier slope, and the barrier level. We downloaded risk-free zero curves from Bloomberg. The applicable risk-free zero curve for CDX NA IG is the USD zero curve and the applicable zero curve for iTraxx Europe is the EUR zero curve. In the next section, we describe how the correlation parameter was chosen in the various tests we carried out.

[10] These numbers are not included in the table..

The barrier level and slope were chosen to be the same for all companies and consistent with the quoted five- and ten-year indices.[11] For a given barrier we can calculate the probability of default in any time interval using equation (2). This in turn allows us to calculate the CDS spread. The barrier level and slope were those for which the five- and ten-year CDS spreads equaled the five and ten-year indices, respectively.[12]

5. Empirical Analysis

We now test how well the structural model fits CDX NA IG market data on the days for which a complete set of data is available. The results for iTraxx Europe are similar.

5.1 Base Case

In our base case the correlation parameter, $\alpha_i(t)$ and recovery rate, R, are independent of the default environment parameter, $F(t)$. For the first test $\alpha_i(t)$ was the same for all i and t so that the pricing of the five-year equity tranche matched the market.[13] We then compared the model's pricing of the other five-year tranches and all ten-year tranches with the market. These results are referred to as Base Case (5) in our tables. As a second test, we assumed that $\alpha_i(t)$ is the same for all i and is a step function, having one value up to five years and another value beyond five years. The step function is chosen to match both the five-year and the ten-year equity tranches. The results comparing model and market pricing for other tranches for this test are referred to as Base Case (5, 10) in our tables.

The recovery rate, R, was assumed to be constant and 40% for all obligors. This is consistent with research in Varma and Cantor (2004).

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[11] This approach can be extended so that a non-linear barrier that is consistent with a complete term structure of default probabilities or CDS spreads is developed.

[12] Under our assumption that default probabilities are the same for all companies the CDS spread equals the index. The calculation of CDS spreads from default probabilities and recovery rates is described in, for example, Hull and White (2003). We assumed a recovery rate of 40%. This is consistent with Varma and Cantor (2004). Our definition of recovery rate is the standard one: value of cheapest to deliver bond as a percent of face value.

[13] We matched the equity tranche because the pricing of this tranche is highly sensitive to the value of the correlation parameter.

5.2. Stochastic Correlation

There is some empirical evidence to suggest that asset correlations are stochastic and increase when default rates are high. For example Servigny and Renault (2002), who look at historical data on defaults and ratings transitions to estimate default correlations, find that the correlations are higher in recessions than in expansion periods. Das, Freed, Geng and Kapadia (2004) employ a reduced form approach and compute the correlation between default intensities. They conclude that default correlations increase when default rates are high. Ang and Chen (2002) find that the correlation between equity returns is higher during a market downturn.

To test the impact of stochastic correlation we assumed that the correlation parameter α_{ik} applicable between times t_{k-1} and t_k is drawn from a beta distribution. The beta distribution was the same for all i. We used a Gaussian copula model to build in dependence between the value sampled for α_{ik} and the value sampled for F_{k-1}. The copula correlation was chosen as $-\sqrt{0.5}$.[14] This is achieved by making the value sampled for α_k the same fractile of a beta distribution as $\left(-\sqrt{0.5}F_{k-1}-\sqrt{0.5}\xi_{k-1}\right)$ is of a normal distribution, where the ξ 's have normal distributions with zero mean and unit variance rate and are independent of the F's and U's,

Our tests for the stochastic correlation model are similar to those for the base case model. For the first test, the mean of the beta distribution, $\bar{\alpha}$, is assumed to be the same for ten years and is chosen to match the 5-year equity tranche. The results from this test are referred to as "Stochastic Corr. (5)". For the second test $\bar{\alpha}$ is a step function, chosen to match both the five-year and the ten-year equity tranche. The results for the second test are referred to as "Stochastic Corr. (5, 10)".

5.3 Stochastic Recovery Rate

Altman et al. (2005) and Cantor, Hamilton, and Ou (2002, p.19) show that recovery rates are negatively dependent on default rates. To test the impact of this we assumed that the recovery rate, R_k, applicable between times t_{k-1} and t_k has a beta distribution and used a Gaussian copula model to build in a dependence between it and F_{k-1}. In this case the copula correlation was chosen to be $+\sqrt{0.5}$.[15] As before

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[14] A negative correlation between the α's and the F's corresponds to a positive correlation between α and the default rates.

[15] A positive correlation between the recovery rate and the F's corresponds to a negative correlation between recovery rates and default rates.

this means that value sampled for α_k is the same fractile of a beta distribution as $\left(-\sqrt{0.5}F_{k-1}-\sqrt{0.5}\xi_{k-1}\right)$ is of a normal distribution, where the ξ's have normal distributions with zero mean and unit variance rate and are independent of the *F*'s and *U*'s,

The mean of the recovery rate distribution, $\overline{R}$, was chosen so that the expected recovery given default is 0.4. This does not mean that $\overline{R}=0.4$. In practice a higher value of $\overline{R}$ (about 0.47) was necessary because the recovery rates tend to be observed when the factor *F* is low.

5.4 Stochastic Volatility

Hull and White (2004) and Burtschell, Gregory and Laurent (2005) show that a double-*t* copula model where both the common factor and the idiosyncratic factor have a *t*-distribution with 4 degrees of freedom fits the market prices well. These findings suggest that market prices are consistent with heavy tails for the common factor and the idiosyncratic factor.

To investigate this in the context of the structural framework, we consider a third extension to the base case model where both the common factor *F* and the idiosyncratic factor *U* have stochastic volatilities. Specifically we assume that each of the factors is drawn from a mixture of Wiener processes with different variance rates. *F* is assumed to be drawn with probability p_{F1} from a process with variance rate $V_{F,1}$ and probability $(1-p_{F,1})$ from a process with variance rate $V_{F,2}$. The expected variance of *F* is 1. *U* is treated similarly. The parameters we chose are $V_{F,1}$=1.5, $V_{F,2}$=0.5, $p_{F,1}$=0.5; $V_{U,1}$ =1.5, $V_{U,2}$=0.5, $p_{U,1}$=0.5.

5.5 The Results

Our results are shown in Tables 9 to 12. Table 9 compares the model prices from the base case model and the stochastic correlation model with the market quotes for CDX NA IG. Table 10 compares model prices from the base case model and the stochastic recovery rate model with the market quotes for CDX NA IG. Table 11 compares model prices from the base case model and the stochastic volatility model with the market quotes for CDX NA IG.[16] The overall fit of the models is examined in Table 12.

Table 9 shows that Base Case (5), where the correlation parameter is constant and fits the market price of the five-year equity tranche, greatly overprices the mezzanine tranche and greatly underprices the most senior tranches. Stochastic Corr. (5), where the mean correlation is constant and fits the market price of the five-year equity tranche, brings pricing closer to that in the market for both these tranches. Base Case (5) sometimes does a better job at pricing intermediate tranches than Stochastic Corr. (5) (for example,

[16] Similar tests to those in Tables 9, 10 and 11 were carried out for iTraxx. The results were similar.

the 7% to 10% tranche) but the improvement is relatively small. The results from comparing Base Case (5,10) where the correlation is a step function with Stochastic Corr. (5, 10) are similar.

Table 10 shows the results from the base case and the stochastic recovery models are fairly close. This is a little surprising. A possible explanation is that the positive effect on the fit of the model caused by the negative correlation of recovery rate and default rate is counteracted by the negative effect on the fit of the lower default correlation necessary to match the equity tranche

From Table 11 we see that Stoch Vol (5) does significantly better than Base Case (5) and Stoch Vol (5,10) does significantly better than Base Case (5,10) for almost all tranches. The one exception is that Base Case (5) outperforms Stoch Vol (5) for the 10-year equity tranche. But the extent of the outperformance is relatively small. .

Table 12 compares the overall fit of the models by computing the mean absolute pricing errors and the mean absolute percentage pricing errors. The mean absolute pricing error is the average of absolute differences between the model prices and market prices. The mean absolute percentage pricing error is the average of absolute pricing errors divided by the market prices. The statistical significance of the mean difference between the pricing errors from the two models is tested using both a parametric *t*-test and a non-parametric bootstrap test. All three (5) and (5,10) extensions of the Base Case model provide an improvement over the Base Case model at the 1% level. However, the economic significance of the results for the stochastic correlation model and the stochastic volatility model is much greater than that for the stochastic recovery rate model.

The stochastic correlation model does a better job than the stochastic volatility model when mean absolute pricing errors are used but slightly worse when mean absolute percentage pricing errors are used. This is because the stochastic volatility model tends to be closer in absolute terms on senior tranches where the quote is low and less close on junior tranches where the quote is high.

6. Conclusions

We have proposed a structural model similar to that in Black and Cox (1976) for valuing correlation-dependent credit derivatives. The correlation between the assets of different companies is determined by one or more common factors.

The basic Gaussian copula survival time model provides a good approximation to our base-case structural model when the correlation parameter in the model is set equal to the asset correlation in the structural model. Even for relatively large credit spreads the prices given by the two models are fairly close. The advantage of the structural model is that it provides a way of simultaneously modeling credit rating changes and defaults. Also it is possible to make extensions to the model while maintaining its economic integrity as a structural model.

The basic structural model does not provide a good fit to the market prices of CDO tranches. When the correlation parameter is chosen to match the price of the equity tranche, the mezzanine tranche is overpriced and the other tranches (particularly the most senior tranche) are underpriced.

The basic structural model assumes that asset correlations are constant. Empirical evidence suggests that asset correlations are positively related to default rates. We therefore tested whether an extension of the structural model that incorporates this positive relationship fits market prices better than the basic structural model. We found that there is a significantly better fit at the 1% level. Our results suggest that further research quantifying the relationship between asset correlations and default rates could lead to better pricing models. Another model where the common factor and the idiosyncratic factor in the structural model have heavy tails produced approximately the same improvement over the basic structural model as the stochastic correlation model.

There is a growing body of empirical evidence to show that recovery rates are negatively correlated with default rates. Surprisingly, an extension of the basic structural model to incorporate this phenomenon produces only a marginally better fit to market prices. Although statistically significant this is not economically significant.

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Table 1

Quotes for CDX NA IG and iTraxx Europe Tranches on August 25, 2004. The quote for the 0% to 3% tranche is the upfront payment as a percentage of the notional principal that is paid in addition to 500 basis points per year. Quotes for all other tranches and the index are in basis points per year. Source: GFI

	CDX NA IG Tranches					
	0% to 3%	3% to 7%	7% to 10%	10% to 15%	15% to 30%	Index
5-year Quotes	40.02	295.71	120.50	43.00	12.43	59.73
10-year Quotes	58.17	632.00	301.00	154.00	49.50	81.00

	iTraxx Europe Tranches					
	0% to 3%	3% to 6%	6% to 9%	9% to 12%	12% to 22%	Index
5-year Quotes	24.10	127.50	54.00	32.50	18.00	37.79
10-year Quotes	43.80	350.17	167.17	97.67	54.33	51.25

Table 2

Descriptive Statistics on Daily Mid-Quotes and Bid-Ask Spreads for CDX NA IG and iTraxx Europe tranches between January 2 and December 21, 2004. The daily bid for each tranche is computed as the average of all intra-day bids for that tranche. The daily offer quote was computed similarly. The mid-quote is the average of daily bid and daily ask. The bid-ask spread is defined as the difference between the daily ask and daily bid, while the percentage bid-ask spread is the ratio of the bid-ask spread and the mid-quote. Panels A present the time-series means and standard deviations of daily tranche mid-quotes. Panels B and C present the same statistics for bid-ask spreads and percentage bid-ask spreads.

CDX NA IG Tranches

	5Y Tranches					10Y Tranches				
	0%-3%	3%-7%	7%-10%	10%-15%	15%-30%	0%-3%	3%-7%	7%-10%	10%-15%	15%-30%
Panel A: Mid Quotes										
Mean	39.6	301.4	111.4	44.4	12.7	57.0	597.8	270.7	145.2	43.4
Std. Dev.	3.6	57.9	24.1	10.7	2.2	2.3	70.7	62.1	36.1	8.1
No. Obs.	214	214	215	207	210	84	101	107	106	86
Panel B: Bid-Ask Spreads										
Mean	1.9	26.1	8.8	9.5	5.2	3.7	37.1	29.0	20.9	9.6
Std. Dev.	1.0	24.8	5.0	5.0	2.8	1.4	16.7	14.5	6.4	4.1
Panel C: Percentage Bid-Ask Spreads										
Mean	4.8%	8.4%	8.3%	21.7%	41.0%	6.5%	6.2%	10.7%	15.0%	22.5%
Std. Dev.	2.5%	7.8%	5.5%	10.2%	20.5%	2.4%	2.8%	5.1%	5.2%	9.1%

iTraxx Europe Tranches

	5Y Tranches					10Y Tranches				
	0%-3%	3%-6%	6%-9%	9%-12%	12%-22%	0%-3%	3%-6%	6%-9%	9%-12%	12%-22%
Panel A: Mid Quotes										
Mean	25.4	148.5	56.9	36.6	17.4	45.1	376.7	165.5	95.4	50.2
Std. Dev.	3.3	28.8	16.0	9.1	3.3	4.0	37.8	26.5	17.1	9.8
No. Obs.	131	132	131	130	131	68	63	64	61	62
Panel B: Bid-Ask Spreads										
Mean	1.4	7.4	5.8	6.0	2.9	3.3	37.5	21.1	20.4	17.6
Std. Dev.	0.8	4.6	2.6	1.6	1.4	1.0	7.3	8.6	5.1	2.7
Panel C: Percentage Bid-Ask Spreads										
Mean	5.4%	4.9%	10.2%	16.5%	16.2%	7.3%	10.0%	12.6%	21.4%	36.5%
Std. Dev.	3.0%	2.3%	3.2%	3.4%	5.2%	2.3%	2.2%	4.9%	4.5%	9.4%

Table 3

Five-year credit spread dynamics given by the structural model for a representative CDX NA IG company. The model parameters (barrier level (β) and slope (γ)) for the representative company are calibrated to the five-year and ten-year CDX NA IG index levels on August 25, 2004. The Modified Distance to Default (MDD) represents the number of standard deviations by which the asset value exceeds the default barrier ($MDD = -\beta = (\ln V(0) - \ln H)/\sigma$). The table is created by varying *MDD* (i.e. the barrier level) and keeping the barrier slope the same. The CDS spread, drift and standard deviation are all expressed in basis points.

Modified Distance to Default (MDD)	5Y CDS Spread	Drift of 5Y Credit Spread (per week)	Std Dev of 5Y credit spread (per week)
2.50	246.11	3.2	33.0
3.00	150.66	2.1	20.9
3.50	90.12	1.4	13.2
4.00	52.21	1.0	8.2
4.50	29.14	0.6	4.9
5.00	15.61	0.4	2.8

Table 4

This table compares the five-year joint default probability of two correlated issuers implied by the structural model with that implied by the Gaussian copula model. The comparison is performed for different levels of issuers' riskiness (as represented by the Modified Distance to Default) and different levels of correlation (ρ) between the issuers' asset values. The structural model parameters are fitted to August 25th data on CDX NA IG indices. The five-year joint default probabilities from the structural model (presented in Panel A) are computed using the analytical formulas in Zhou(2001). Panel B presents the joint five-year default probabilities of two-issuers as implied by the Gaussian copula model. These are calculated from the cumulative bivariate normal distribution function under the assumptions that the marginal default probabilities of the two issuers are matched to those in the structural model and the copula correlation is equal to the asset returns correlation in the structural model.

Panel A: Structural Model

	Modified Distance to Default (MDD)						
Correlation (ρ)	2	2.5	3	3.5	4	4.5	5
0.1	15.11%	7.97%	3.91%	1.78%	0.75%	0.29%	0.10%
0.2	16.48%	9.05%	4.65%	2.23%	1.00%	0.42%	0.16%
0.3	17.91%	10.19%	5.46%	2.74%	1.29%	0.57%	0.24%
0.4	19.42%	11.41%	6.34%	3.32%	1.64%	0.76%	0.33%
0.5	21.02%	12.72%	7.31%	3.98%	2.05%	1.00%	0.46%

Panel B: Gaussian Copula Model

	MDD						
Correlation (ρ)	2	2.5	3	3.5	4	4.5	5
0.1	15.21%	8.03%	3.95%	1.80%	0.76%	0.29%	0.11%
0.2	16.67%	9.17%	4.72%	2.27%	1.02%	0.42%	0.16%
0.3	18.18%	10.37%	5.56%	2.80%	1.32%	0.58%	0.24%
0.4	19.75%	11.63%	6.47%	3.40%	1.68%	0.78%	0.34%
0.5	21.41%	12.98%	7.47%	4.07%	2.10%	1.02%	0.47%

Table 5

This table compares the joint default probability of two correlated issuers implied by the structural model with that implied by the Gaussian copula model. The comparison is performed for different time horizons and different levels of correlation (ρ) between the issuers' asset values. The structural model parameters are fitted to August 25th data on CDX NA IG indices. The joint default probabilities from the structural model (presented in Panel A) are computed using the analytical formulas in Zhou(2001). Panel B presents the joint default probabilities of two-issuers as implied by the Gaussian copula model. These are calculated from the cumulative bivariate normal distribution function under the assumptions that the marginal default probabilities of the two issuers are matched to those in the structural model and the copula correlation is equal to the asset returns correlation.

Panel A: Structural Model

	Maturity (in years)						
Correlation	1	3	5	7	10	15	20
0.1	0.00%	0.10%	0.92%	2.52%	5.65%	11.16%	16.17%
0.2	0.00%	0.16%	1.21%	3.09%	6.56%	12.41%	17.58%
0.3	0.00%	0.23%	1.55%	3.72%	7.53%	13.71%	19.03%
0.4	0.00%	0.32%	1.94%	4.42%	8.58%	15.09%	20.56%
0.5	0.00%	0.45%	2.40%	5.21%	9.73%	16.56%	22.18%

Panel B: Gaussian Copula Model

	Maturity (in years)						
Correlation	1	3	5	7	10	15	20
0.1	0.00%	0.10%	0.93%	2.55%	5.70%	11.24%	16.28%
0.2	0.00%	0.16%	1.23%	3.14%	6.65%	12.56%	17.78%
0.3	0.00%	0.23%	1.58%	3.79%	7.67%	13.93%	19.31%
0.4	0.00%	0.33%	1.98%	4.52%	8.76%	15.36%	20.91%
0.5	0.00%	0.46%	2.46%	5.33%	9.94%	16.89%	22.59%

Table 6

Comparison of the 5-year CDX NA IG tranche prices computed from the structural model with the prices from the Gaussian Copula model. The 0% to 3% tranche is quoted as the percentage upfront payment required on the assumption that subsequent payments will be 500 basis points per year. The other tranches are quoted as basis points per year. The numbers in parentheses are standard errors of the tranche price estimates from the structural model.

5-YEAR DJ CDX NA IG										
Structural Model Prices										
Correlation	Tranches									
	0-3%		3-7%		7-10%		10-15%		15-30%	
0	59.2%	(0.10%)	231.7	(1.8)	0.4	(0.1)	0.0	(0.0)	0.0	(0.0)
0.1	46.2%	(0.16%)	376.0	(3.3)	66.9	(1.5)	11.0	(0.6)	0.3	(0.1)
0.2	37.1%	(0.19%)	407.5	(3.7)	135.1	(2.3)	45.8	(1.3)	5.1	(0.3)
0.3	29.4%	(0.20%)	407.4	(3.9)	175.8	(2.7)	81.4	(1.8)	16.2	(0.7)
0.4	22.5%	(0.21%)	393.1	(3.9)	198.9	(2.9)	109.7	(2.1)	31.2	(1.0)
0.5	16.1%	(0.21%)	369.6	(3.9)	210.8	(3.0)	130.2	(2.4)	47.8	(1.3)

Gaussian Copula Model Prices					
Correlation	Tranches				
	0-3%	3-7%	7-10%	10-15%	15-30%
0	59.5%	231.9	0.5	0.0	0.0
0.1	45.8%	383.4	70.9	12.0	0.4
0.2	36.5%	414.8	140.8	48.7	5.5
0.3	28.7%	413.0	182.1	84.8	17.2
0.4	21.7%	396.5	205.5	114.3	32.3
0.5	15.4%	370.9	218.3	132.4	50.1

Differences: Structural-Gaussian Copula					
Correlation	Tranches				
	0-3%	3-7%	7-10%	10-15%	15-30%
0	-0.3%	-0.2	-0.1	0.0	0.0
0.1	0.4%	-7.4	-4.0	-1.0	0.0
0.2	0.6%	-7.3	-5.8	-2.8	-0.4
0.3	0.7%	-5.7	-6.3	-3.4	-1.1
0.4	0.8%	-3.4	-6.6	-4.6	-1.1
0.5	0.7%	-1.3	-7.5	-2.2	-2.3

Table 7

Deltas for the breakeven spreads of CDX NA IG tranches calculated from a Monte Carlo implementation of the structural model on August 25, 2004. Delta is calculated by shifting the CDS spread up and down by x basis points and calculating the values $V(+x)$ and $V(–x)$. Delta is $[V(+x)–V(–x)] / 2x$. The table shows the average deltas calculated for different values of x. In the first part of the table the shift is applied to just one company. In the second part it is applied to all 125 companies. The third part of the table shows the ratio of the 125-name delta to the 1-name delta

Deltas Calculated by Perturbing the Spread for One Name.					
Tranche	$x = 10$	$x = 5$	$x = 3$	$x = 2$	$x = 1$
0% to 3%	0.00470	0.00470	0.00470	0.00470	0.00470
3% to 7%	0.08002	0.08003	0.08003	0.08003	0.08003
7% to 10%	0.03679	0.03677	0.03677	0.03677	0.03677
10% to 15%	0.01543	0.01541	0.01541	0.01541	0.01541
15% to 30%	0.00221	0.00220	0.00220	0.00220	0.00220
Delta Calculated by Perturbing the Spread for All 125 names					
Tranche	$x = 10$	$x = 5$	$x = 3$	$x = 2$	$x = 1$
0% to 3%	0.59081	0.58877	0.58830	0.58817	0.58808
3% to 7%	9.97683	9.99772	10.00149	10.00285	10.00366
7% to 10%	4.58313	4.59289	4.59452	4.59512	4.59548
10% to 15%	1.92200	1.92506	1.92560	1.92582	1.92595
15% to 30%	0.27608	0.27513	0.27491	0.27485	0.27481
Ratio of 125-name Delta to 1-name Delta					
Tranche	$x = 10$	$x = 5$	$x = 3$	$x = 2$	$x = 1$
0% to 3%	125.7	125.2	125.1	125.0	125.0
3% to 7%	124.7	124.9	125.0	125.0	125.0
7% to 10%	124.6	124.9	125.0	125.0	125.0
10% to 15%	124.5	124.9	125.0	125.0	125.0
15% to 30%	125.2	125.0	125.0	125.0	125.0

Table 8

Comparison of single name deltas calculated from the Gaussian copula model with those calculated using the structural model for CDX NA IG tranches on August 25, 2004. Delta is calculated by shifting the CDS spread up and down by *x* basis points and calculating the values *V*(+*x*) and V(–*x*). Delta is [*V*(+*x*)– V(–*x*)] / 2*x*. The table shows the average deltas calculated for different values of *x*. Standard errors are shown for the structural model. The *t*-statistic shows the difference between the deltas divided by the standard error.

	Gaussian Copula Model Deltas				
Tranche	$x = 10$	$x = 5$	$x = 3$	$x = 2$	$x = 1$
0% to 3%	0.00470	0.00470	0.00470	0.00470	0.00470
3% to 7%	0.08002	0.08003	0.08003	0.08003	0.08003
7% to 10%	0.03679	0.03677	0.03677	0.03677	0.03677
10% to 15%	0.01543	0.01541	0.01541	0.01541	0.01541
15% to 30%	0.00221	0.00220	0.00220	0.00220	0.00220
	Structural Model Deltas				
Tranche	$x = 10$	$x = 5$	$x = 3$	$x = 2$	$x = 1$
0% to 3%	0.00479	0.00477	0.00478	0.00473	0.00475
3% to 7%	0.08088	0.08005	0.08070	0.08070	0.08087
7% to 10%	0.03682	0.03668	0.03709	0.03675	0.03645
10% to 15%	0.01466	0.01467	0.01508	0.01540	0.01571
15% to 30%	0.00203	0.00206	0.00214	0.00216	0.00224
	Standard Error of Structural Model Deltas				
Tranche	$x = 10$	$x = 5$	$x = 3$	$x = 2$	$x = 1$
0% to 3%	0.00003	0.00004	0.00005	0.00006	0.00010
3% to 7%	0.00050	0.00070	0.00089	0.00107	0.00146
7% to 10%	0.00036	0.00052	0.00071	0.00088	0.00113
10% to 15%	0.00017	0.00027	0.00034	0.00043	0.00064
15% to 30%	0.00004	0.00006	0.00008	0.00009	0.00014
	t-statistic				
Tranche	$x = 10$	$x = 5$	$x = 3$	$x = 2$	$x = 1$
0% to 3%	2.72	1.52	1.40	0.43	0.48
3% to 7%	1.72	0.03	0.75	0.63	0.57
7% to 10%	0.08	-0.19	0.45	-0.02	-0.28
10% to 15%	-4.55	-2.79	-0.99	-0.01	0.48
15% to 30%	-4.27	-2.50	-0.77	-0.36	0.31

Table 9

Comparison of the model prices with the market quotes for the 5-year and 10-year CDX NA IG index tranches. The market quotes are the average quotes for the 60 days in 2004 for which a complete set of data is available. This table shows the results from two models: the base case model ("Base Case") and the structural model with stochastic correlation ("Stochastic Corr."). For each model two cases are considered: in the first case the (mean) correlation is the same for all ten years and is chosen to match the 5-year equity tranche ("Base Case(5)", "Stochastic Corr.(5)"); in the second case the (mean) correlation is a step function and is chosen to match both the 5-year equity tranche and the 10-year equity tranche ("Base Case(5,10)", "Stochastic Corr.(5,10)"). The reported model prices are sample averages of the daily prices. The numbers in parentheses are sample averages of the standard errors from Monte Carlo simulations. The mean absolute pricing error is the average of absolute differences between the model prices and market prices. The mean absolute percentage pricing error is the average of absolute pricing errors divided by the market prices. "***,**,*" denotes superior fit of one model at the 1%, 5%, and 10% levels.

Panel A: Market Quotes												
			5-year CDX					10-year CDX				
			0-3%	3-7%	7-10%	10-15%	15-30%	0-3%	3-7%	7-10%	10-15%	15-30%
Average Market Quotes			38.56%	283.54	110.75	39.39	11.95	57.37%	599.33	273.18	140.17	44.56
Panel B: Model Prices												
	Corr.	Corr.	5-year CDX					10-year CDX				
Model	(0-5Y)	(5-10Y)	0-3%	3-7%	7-10%	10-15%	15-30%	0-3%	3-7%	7-10%	10-15%	15-30%
Base Case(5)	0.156	0.156	38.56%	371.72	98.53	26.38	1.89	57.59%	774.2	368.66	164.68	25.89
				(3.41)	(1.90)	(0.92)	(0.17)	(0.13%)	(3.51)	(2.56)	(1.67)	(0.52)
Stochastic Corr.(5)	0.083	0.083	38.56%	251.23	89.93	49.55	19.1	58.73%	702	247.52	114.34	47.33
				(2.85)	(1.95)	(1.44)	(0.84)	(0.11%)	(3.00)	(2.12)	(1.52)	(0.95)
Base Case(5,10)	0.156	0.170	38.56%	371.72	98.53	26.38	1.89	57.37%	770.8	368.4	165.89	26.64
				(3.41)	(1.90)	(0.92)	(0.17)		(3.51)	(2.56)	(1.68)	(0.54)
Stochastic Corr.(5,10)	0.083	0.230	38.56%	251.23	89.93	49.55	19.1	57.37%	668.36	250.08	123.05	51.92
				(2.85)	(1.95)	(1.44)	(0.84)		(3.03)	(2.16)	(1.58)	(0.99)
Panel C: Mean Absolute Pricing Errors												
Base Case(5)				88.18	12.44***	13.02	10.05	0.85%***	174.87	95.47	24.51	18.67
Stochastic Corr.(5)				32.31***	20.82	10.15***	7.16***	1.49%	102.68***	26.31***	25.83	4.01***
Base Case(5,10)				88.18	12.44***	13.02	10.05		171.47	95.22	25.72	17.99
Stochastic Corr.(5,10)				32.31***	20.82	10.15***	7.16***		69.03***	24.10***	19.93**	7.53***
Panel D: Mean Absolute Percentage Errors												
Base Case(5)				31.77%	12.42%***	35.02%	84.89%	1.49%***	29.41%	36.10%	18.70%	42.73%
Stochastic Corr.(5)				11.33%***	18.93%	25.67%***	58.94%***	2.63%	17.25%***	9.48%***	17.24%	9.37%***
Base Case(5,10)				31.77%	12.42%***	35.02%	84.89%		28.65%	35.91%	20.08%	40.40%
Stochastic Corr.(5,10)				11.33%***	18.93%	25.67%***	58.94%***		11.39%***	8.48%***	12.87%***	18.14%***

Table 10

Comparison of the model prices with the market quotes for the 5-year and 10-year CDX NA IG index tranches. The market quotes are the average quotes for the 60 days in 2004 for which a complete set of data is available. This table shows the results from two models: the base case model ("Base Case") and the structural model with stochastic volatility ("Stoch. Vol."). For each model two cases are considered: in the first case the correlation is the same for all ten years is chosen to match the 5-year equity tranche ("Base Case (5)", "Stochastic RR(5)"); in the second case the correlation is a step function and is chosen to match both the 5-year and the 10-year equity tranches ("Base Case (5,10)", "Stochastic RR(5,10)"). The reported model prices are sample averages of the daily prices. The numbers in parenthesis are sample averages of the standard errors from Monte Carlo simulations. The mean absolute pricing error is the average of absolute differences between the model prices and market prices. The mean absolute percentage pricing error is the average of absolute pricing errors divided by the market prices. "***,**,*" denotes superior fit of one model at the 1%, 5%, and 10% levels.

			5-year CDX					10-year CDX				
Panel A: Market Quotes												
			0-3%	3-7%	7-10%	10-15%	15-30%	0-3%	3-7%	7-10%	10-15%	15-30%
Average Market Quotes			38.56%	283.54	110.75	39.39	11.95	57.37%	599.33	273.18	140.17	44.56
Panel B: Model Prices												
Model	Corr. (0-5Y)	Corr. (5-10Y)	0-3%	3-7%	7-10%	10-15%	15-30%	0-3%	3-7%	7-10%	10-15%	15-30%
Base Case(5)	0.156	0.156	38.56%	371.72	98.53	26.38	1.89	57.59%	774.2	368.66	164.68	25.89
				(3.41)	(1.90)	(0.92)	(0.17)	(0.13%)	(3.51)	(2.56)	(1.67)	(0.52)
Stochastic RR(5)	0.118	0.118	38.56%	377.35	108.77	33.41	3.42	56.42%	744.32	368.88	181.33	39.23
				(3.46)	(2.02)	(1.06)	(0.26)	(0.14%)	(3.76)	(2.76)	(1.90)	(0.75)
Base Case (5,10)	0.156	0.17	38.56%	371.72	98.53	26.38	1.89	57.37%	770.8	368.4	165.89	26.64
				(3.41)	(1.90)	(0.92)	(0.17)		(3.51)	(2.56)	(1.68)	(0.54)
Stochastic RR(5,10)	0.118	0.052	38.56%	377.35	108.77	33.41	3.42	57.37%	750.65	361.59	169.91	32.86
				(3.46)	(2.02)	(1.06)	(0.26)		(3.54)	(2.58)	(1.74)	(0.64)
Panel C: Mean Absolute Pricing Errors												
Base Case (5)				88.18***	12.44	13.02	10.05	0.85%**	174.87	95.47	24.51***	18.67
Stochastic RR (5)				93.8	10.05**	7.30***	8.53***	1.09%	145.00***	95.69	41.15	6.22***
Base Case (5,10)				88.18***	12.44	13.02	10.05		171.47	95.22	25.72***	17.99
Stochastic RR (5,10)				93.8	10.05**	7.30***	8.53***		151.32***	88.40***	29.74	12.21***
Panel D: Mean Absolute Percentage Errors												
Base Case (5)				31.77%***	12.42%	35.02%	84.89%	1.49%**	29.41%	36.1%*	18.7%***	42.73%
Stochastic RR (5)				33.86%	9.41%***	20.1%***	72.48%***	1.88%	24.43%***	36.41%	31.38%	14.66%***
Base Case (5,10)				31.77%***	12.42%	35.02%	84.89%		28.65%	35.91%	20.08%***	40.40%
Stochastic RR (5,10)				33.86%	9.41%***	20.1%***	72.48%***		25.42%***	33.68%***	23.45%	27.68%***

Table 11

Comparison of the model prices with the market quotes for the 5-year and 10-year CDX NA IG tranches. The market quotes are the average quotes for the 60 days in 2004 for which a complete set of data is available. This table shows the results from two models: the base case model ("Base Case") and the structural model with stochastic volatility ("Stoch. Vol."). For each model two cases are considered: in the first case the correlation is the same for all ten years is chosen to match the 5-year equity tranche ("Base Case (5)", "Stoch. Vol(5)"); in the second case the correlation is a step function and is chosen to match both the 5-year equity tranche and the 10-year equity tranche ("Base Case (5,10)", "Stoch. Vol(5,10)"). The reported model prices are sample averages of the daily prices. The numbers in parenthesis are sample averages of the standard errors from Monte Carlo simulations. The mean absolute pricing error is the average of absolute differences between the model prices and market prices. The mean absolute percentage pricing error is the average of absolute pricing errors divided by the market prices. "***,**,*" denotes superior fit of one model at the 1%, 5%, and 10% levels.

Panel A: Market Quotes

	5-year CDX					10-year CDX				
	0-3%	3-7%	7-10%	10-15%	15-30%	0-3%	3-7%	7-10%	10-15%	15-30%
Average Market Quotes	38.56%	283.54	110.75	39.39	11.95	57.37%	599.33	273.19	140.17	44.56

Panel B: Model Prices

Model	Corr.	Corr.	5-year CDX					10-year CDX				
	(0-5Y)	(5-10Y)	0-3%	3-7%	7-10%	10-15%	15-30%	0-3%	3-7%	7-10%	10-15%	15-30%
Base Case(5)	0.156	0.156	38.56%	371.72	98.53	26.38	1.89	57.59	774.20	368.66	164.68	25.89
				(3.41)	(1.9)	(0.92)	(0.17)	(0.13)	(3.51)	(2.56)	(1.67)	(0.52)
Stoch. Vol.(5)	0.215	0.215	38.56%	329.87	98.08	38.73	7.40	57.73	730.19	329.09	154.01	38.77
				(3.23)	(1.96)	(1.20)	(0.44)	(0.12)	(3.36)	(2.45)	(1.68)	(0.76)
Base Case(5,10)	0.156	0.170	38.56%	371.72	98.53	26.38	1.89	57.37%	770.80	368.40	165.89	26.64
				(3.41)	(1.9)	(0.92)	(0.17)		(3.51)	(2.56)	(1.68)	(0.54)
Stoch. Vol. (5,10)	0.215	0.239	38.56%	329.87	98.08	38.73	7.40	57.37%	722.92	328.27	155.59	40.27
				(3.23)	(1.96)	(1.20)	(0.44)		(3.35)	(2.45)	(1.69)	(0.78)

Panel C: Mean Absolute Pricing Errors

	3-7%	7-10%	10-15%	15-30%	0-3%	3-7%	7-10%	10-15%	15-30%
Base Case (5)	88.18	12.44	13.02	10.05	0.85%***	174.87	95.47	24.51	18.67
Stoch. Vol (5)	46.33***	12.66	4.73***	4.55***	0.97%	130.87***	55.91***	13.88***	6.06***
Base Case (5, 10)	88.18	12.44	13.02	10.05		171.47	95.22	25.72	17.99
Stoch. Vol (5,10)	46.33***	12.66	4.73***	4.55***		123.59***	55.08***	16.76***	6.35***

Panel D: Mean Absolute Percentage Errors

	3-7%	7-10%	10-15%	15-30%	0-3%	3-7%	7-10%	10-15%	15-30%
Base Case (5)	31.77%	12.42%	35.02%	84.89%	1.49%***	29.41%	36.10%	18.70%	42.73%
Stoch Vol. (5)	16.93%***	12.11%	12.19%***	39.79%***	1.70%	22.07%***	21.53%***	11.48%***	14.18%***
Base Case (5, 10)	31.77%	12.42%	35.02%	84.89%		28.65%	35.91%	20.08%	40.40%
Stoch. Vol (5,10)	16.93%***	12.11%	12.19%***	39.79%***		20.58%***	21.04%***	13.91%***	14.38%***

Table 12

Overall fit of models to the market data. This table shows the results from four models: the base case model ("Base Case"), the structural model with stochastic correlation ("Stochastic Corr."), the structural model with stochastic recovery ("Stochastic RR") and the structural model with stochastic volatility ("Stoch. Vol"),. Column "(5)" corresponds to the first case where the correlation is the same for all ten years and is chosen to match the 5-year equity tranche, while column "(5,10)" corresponds to the second case in which the correlation is a step function and is chosen to match both the 5-year equity tranche and the 10-year equity tranche. The mean absolute pricing error is the average of absolute differences between the model prices and market prices. The mean absolute percentage pricing error is the average of absolute pricing errors divided by the market prices. "***" denotes statistically significant difference from zero at the 1% level.

CDX NA IG				
Model	Mean Absolute Pricing Errors		Mean Absolute Percentage Pricing Errors	
	(5)	(5,10)	(5)	(5,10)
Base Case	54.65	54.26	36.38%	36.14%
Stochastic Correlation	28.66	23.88	21.03%	20.72%
Difference(Stochastic Corr. - BaseCase)	-25.99***	-30.38***	-15.35%***	-15.42%***
Base Case	54.65	54.26	36.38%	36.14%
Stochastic Recovery	50.97	50.17	30.34%	30.76%
Difference(Stochastic Rec.-BaseCase)	-3.68***	-4.09***	-6.04%***	-5.38%***
Base Case	54.65	54.26	36.38%	36.14%
Stochastic Volatility	34.37	33.76	18.78%	18.86%
Difference(Stochastic Vol.-BaseCase)	-20.28***	-20.50***	-17.60%***	-17.28%***