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The Valuation of Market-Leveraged Stock Units

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ABSTRACT

A market-leveraged stock unit (MSU) is a form of employee compensation in which the number of shares received on the vesting date depends on the stock price at that time. MSUs have been proposed as a way of overcoming some of the drawbacks of stock options and restricted stock units. In this paper, we show how MSUs can be valued and discuss their properties.

Market-Leveraged Stock Units

When companies granted stock options to their employees prior to 2005, only the intrinsic value of the option appeared as an expense on the income statement. This, not surprisingly, led to at-the-money options becoming a very popular form of compensation. Employees could be given something of value without the company appearing to bear any cost. Starting in 2005, this accounting anomaly was rectified. FAS 123 and IAS 2 now require the fair value of all forms of stock-based compensation to be estimated and charged to the income statement.[1] Accounting is now no longer the major consideration when compensation plans are being formulated and companies have started to consider alternatives to at-the-money stock options.

Many companies have started to use restricted stock units instead of options as a way of motivating employees. A restricted stock unit (RSU) gives the employee the right to receive one share of the company's stock at a future time (known as the vesting date) providing certain conditions are satisfied. Often the conditions are quite simple and are satisfied if the employee remains employed by the company. Occasionally, there are profit or other targets that must be met for vesting to occur.[2]

A variation on an RSU is what is known as a market-leveraged stock unit (MSU),[3] which was proposed by Cook and Neel (2009). In this, the number of shares received is equal to S_T/S_0 where S_0 is the stock price at time zero when the MSU is issued and S_T is the stock price at time T years (the vesting date). The number of shares received may be subject to a cap or a floor or both. Sometimes the number of shares received is specified as $\bar{S}_T/\bar{S}_0$ where $\bar{S}_0$ is defined as the average stock price during a period preceding the grant date and $\bar{S}_T$ is similarly defined as the average stock price during a period preceding the vesting date. Dividends during the life of an MSU reduce the terminal stock price and this reduces the payoff. In some cases, there may be some dividend protection with the number of units, and possibly the number of shares received for each unit, being adjusted.

[1] See Financial Accounting Standards Board (2004) and International Accounting Standards Board (2004).
[2] For valuation purposes it is usually assumed that the targets will be met.
[3] MSUs are also sometimes known as Performance Leveraged Stock Units, PSU.

The companies issuing MSUs include Newmont Mining, Xerox, and CarMax. The details of the issues can be obtained from their SEC filings. For example, Newmont has issued MSUs to senior executives at the beginning of each year starting in 2010. The MSUs vest after three years. One unit entitles the employee to receive $\bar{S}_T/\bar{S}_0$ shares where the averaging periods for both $\bar{S}_0$ and $\bar{S}_T$ is one quarter of a year. The floor and cap are 0.5 and 1.5. Suppose a Newmont employee is issued 1 million MSUs. If $\bar{S}_T/\bar{S}_0 \leq 0.5$, 500,000 shares are received in three years; if $0.5 < \bar{S}_T/\bar{S}_0 < 1.5$, 1,000,000 times $\bar{S}_T/\bar{S}_0$ shares are received; if $\bar{S}_T/\bar{S}_0 \geq 1.5$, 1,500,000 shares are received. The average Newmont stock price in the last quarter of 2009, $\bar{S}_0$, was 48.66 while the corresponding average in the last quarter of 2012, $\bar{S}_T$, was 49.20. As a result, an employee holding 1 million MSUs issued at the start of 2010 would receive 1.011 million Newmont shares in early 2013.

An MSU is designed to overcome one of the shortcomings of traditional stock option plans. If, for whatever reason, a stock option moves appreciably out of the money, the employee expects zero payoff and may not be incentivized to work hard. (This was an issue for many companies during the 2008 to 2009 period because of the steep decline in the stock market.) MSUs have the advantage over stock options that the payoff, although lower when the stock price declines, is always positive so that incentives are not completely lost. An MSU has the potentially attractive property that, for the range of stock prices for which the floor and cap have not become operative, the compensation is a convex function of the stock price. This means that there is a leverage element to an MSU. This distinguishes MSUs from RSUs.

MSUs have the property that, however badly a company performs, employees are always motivated to improve its performance. But, when the company does well, employees are motivated to work even harder. To quote Thomas G. Stemberg, Chair, Compensation Committee, CarMax Inc. “We see MSUs as being restricted stock units (RSUs) with leverage. They are a hybrid instrument with the characteristics of both options and RSUs, but without some of the disadvantages of each.” Exhibit 1 shows the payoffs from an MSU, an RSU, and an option.

This paper shows that MSUs can be valued analytically. It suggests a way in which the design of an MSU can be generalized to allow the leverage to be adjusted and shows that the resulting instrument can also be valued analytically. Numerical results illustrating the properties of MSUs are presented.

1. Valuation of Asset-or-Nothing Power Options

As a preliminary to valuing MSUs, we consider the valuation of an asset-or-nothing power option. This provides a payoff of S_T^α at time T when S_T, the value of the underlying stock price at time T, is greater than K. We define $G(S_t, K, \alpha, t, T)$ as the value of this option at time t when the stock price is S_t. (Note that, although α is usually a positive integer when power options are considered, the results we will present hold when α has any value, positive or negative or zero. We will make use of this in Section 3.)

Suppose that the stock price, S, follows geometric Brownian motion so that in a risk-neutral world:

$$dS = (r-q)S\,dt + \sigma S\,dz$$

where r is the risk-free rate, q is the dividend yield, σ is the volatility, and dz is a Wiener process. For ease of exposition, we assume that r, q, and σ are constant. (The analysis can be extended so that they are functions of time.)

When $\alpha = 1$, an asset-or-nothing power option becomes a regular asset-or-nothing option. As shown by Rubinstein and Reiner (1991), its value at time t is

$$G\left(S_t, K, 1, t, T\right) = e^{-q(T-t)} S_t N\left(d_1\right) \tag{1}$$

where

$$d_1 = \frac{\ln\left(S_t/K\right) + \left(r - q + \sigma^2/2\right)\left(T-t\right)}{\sigma\sqrt{T-t}}$$

and N is the cumulative distribution function for a standard normal variable. From Ito's lemma, the process followed by S^α in a risk-neutral world is

$$dS^\alpha = \left[\alpha\left(r - q - \sigma^2/2\right) + \alpha^2\sigma^2/2\right] S^\alpha dt + \alpha\sigma S^\alpha dz$$

This shows that S^α behaves like a stock where the volatility is $\sigma^* = \alpha\sigma$ and the dividend yield is

$$q^* = r - \alpha\left(r - q - \sigma^2/2\right) - \alpha^2\,\sigma^2/2$$

It follows that $G(S_t, K, \alpha, t, T)$ is given by equation (1) with S_t^α replacing S_t, K^α replacing K, q^* replacing q, and σ^* replacing σ:

$$G(S_t, K, \alpha, t, T) = S_t^\alpha \exp\left\{\left[\left(\alpha - 1\right)\left(r + \alpha\,\sigma^2/2\right) - \alpha q\right]\left(T - t\right)\right\} N\left(h\left(\alpha\right)\right) \tag{2}$$

where

$$h\left(\alpha\right) = \frac{\ln\left(S_t/K\right) + \left[r - q + \left(\alpha - 1/2\right)\sigma^2\right]\left(T - t\right)}{\sigma\sqrt{T - t}}$$

2. Valuation of MSUs

Consider first the situation where there is no averaging and there is no dividend protection. Define M_1 and M_2 as the floor and cap for S_T/S_0, respectively. The payoff from an MSU is

$$\begin{array}{lll} M_1 S_T & \text{if} & S_T < M_1 S_0 \\ (S_T/S_0) S_T & \text{if} & M_1 S_0 \le S_T \le M_2 S_0 \\ M_2 S_T & \text{if} & S_T > M_2 S_0 \end{array}$$

The MSU can be decomposed into a package of five derivatives

1. A long position in M_1 asset-or-nothing power options with $\alpha = 1$ and $K = 0$
2. A short position in M_1 asset-or-nothing power options with $\alpha = 1$ and $K = M_1 S_0$
3. A long position in $1/S_0$ asset-or-nothing power options with $\alpha = 2$ and $K = M_1 S_0$
4. A short position in $1/S_0$ asset-or-nothing power options with $\alpha = 2$ and $K = M_2 S_0$
5. A long position in M_2 asset-or-nothing power options with $\alpha = 1$ and $K = M_2 S_0$

The value of the MSU per unit issued is therefore

$$\begin{gathered} M_1 G(S_t, 0, 1, t, T) - M_1 G(S_t, M_1 S_0, 1, t, T) \\ + \frac{G(S_t, M_1 S_0, 2, t, T)}{S_0} - \frac{G(S_t, M_2 S_0, 2, t, T)}{S_0} + M_2 G(S_t, M_2 S_0, 1, t, T) \end{gathered} \tag{3}$$

For corporate reporting purposes the firm is interested in the fair market value of the MSUs at the inception date, $t = 0$. However, the holders of the MSU may be interested in the value of the MSU at other times.[4]

Exhibit 2 shows how the value of an MSU depends on M_1 and M_2 for an MSU when $S_t = S_0 = 100$, $r = 5\%$, $\sigma = 25\%$, $t = 0$, $T = 3$, and $q = 0$. ($M_2 = 50$ in effect corresponds to the situation where there is no cap to the number of options received.) When $M_1 = M_2 = 1$, the MSU is an RSU and its value is $S_0 e^{-qT}$. As M_2 increases the MSU becomes progressively more expensive

[4] For example, if the MSU holder is involved in a divorce the economic settlement may be related to the current value of the executive compensation package.

than an RSU. As M_1 decreases, the value of the MSU declines. In our example, the value of a three-year MSU can be as much as 28% greater than an RSU.

Exhibit 3 shows how the value of the MSU in Exhibit 2 depends on σ and T when $M_1 = 0.5$ and $M_2 = 1.5$. As expected the value of the MSU increases with both σ and T. This contrasts with an RSU which is independent of these parameters when q=0. The value of an MSU is less sensitive to σ and T than an at-the-money option. This is consistent with the observation that an MSU has properties that are intermediate between an option and an RSU.

The payment of dividends reduces the future stock price. This has a dual effect on the value of the MSU. First the reduced future stock price decreases the number of shares received, and second, it lowers the value of each share received. For example, suppose that the holder currently has one MSU with no cap or floor and that the stock pays a continuous dividend yield of q over the life of the MSU. This reduces the final stock price from S_T to $S_T\, e^{-qT}$ where S_T is the price that would have been observed had no dividends been paid. The number of shares issued is decreased from S_T / S_0 to $(S_T / S_0)\, e^{-qT}$ and the value of each share received is lowered from S_T to $S_T\, e^{-qT}$.

In some cases, a form of dividend protection is provided to the holder of the MSU in order to make the value of the MSU independent of the amount of dividends paid. In the case of the MSU with no caps or floors this is achieved by increasing the number of units issued from one to e^{qT} and the number of shares per unit from one to e^{qT}. Since we issue e^{qT} share per unit, if there is a floor at M_1 the floor level has to be adjust to $M_1\, e^{-qT}$ so that the total number of shares received when the final stock price is low is still M_1. This adjustment in combination with the increase in the number of units issued maintains the value of the floor option. The same adjustment must also be applied to any cap.

In summary, to provide complete dividend protection at time zero the following adjustments are necessary:

a) Increase the number of units issued by a multiplicative factor e^{qT}
b) Increase the number of shares received when each unit vests by a multiplicative factor e^{qT}
c) Multiply the cap and floor thresholds by e^{-qT}

The final payoff per original unit of MSU issued is then

$$
\begin{array}{lll}
M_1 e^{qT} S_T & \text{if} & S_T < M_1 e^{-qT} S_0 \\
\left(S_T / S_0\right) e^{2qT} S_T & \text{if} & M_1 e^{-qT} S_0 \le S_T \le M_2 e^{-qT} S_0 \\
M_2 e^{qT} S_T & \text{if} & S_T > M_2 e^{-qT} S_0
\end{array}
$$

The value of the MSU at time t per original unit issued is then

$$
M_1 e^{qT} G\left(S_t, 0, 1, t, T\right) - M_1 e^{qT} \left(S_t, M_1 e^{-qT} S_0, 1, t, T\right)
$$
$$
+ e^{2qT} \frac{G\left(S_t, M_1 e^{-qT} S_0, 2, t, T\right)}{S_0} - e^{2qT} \frac{G\left(S_t, M_2 e^{-qT} S_0, 2, t, T\right)}{S_0} + M_2 e^{qT} G\left(S_t, M_2 e^{-qT} S_0, 1, t, T\right)
$$

This is independent of q when t = 0, but not at later times.

In some cases the initial and final stock prices used in the MSU calculation are average stock prices. If there is averaging the number of shares received is calculated as $\bar{S}_T / \bar{S}_0$ where averages are taken over a length of time τ years. Averaging the initial stock price reduces both the incentive and the ability of management to game their compensation by choosing a ‘good’ date on which to issue the MSU. Averaging the final stock price protects the MSU holder from the effects of potential short term down-spikes in the stock price at maturity. With averaging, the payoff from the option is

$$
\begin{array}{lll}
M_1 S_T & \text{if} & \bar{S}_T < M_1 \bar{S}_0 \\
\left(\bar{S}_T / \bar{S}_0\right) S_T & \text{if} & M_1 \bar{S}_0 \le \bar{S}_T \le M_2 \bar{S}_0 \\
M_2 S_T & \text{if} & \bar{S}_T > M_2 \bar{S}_0
\end{array}
\tag{4}
$$

The averaging feature makes the valuation of the option more complicated. One approach is to use Monte Carlo simulation. Providing the final averaging period is not too long, we find that averaging can be handled to a good approximation by shortening the maturity of the contract by half of the averaging period to $T^* = T - \tau / 2$ and setting both S_T and $\bar{S}_T$ equal to S_{T^*}.[5] When there is no dividend protection, the value of the MSU is then

$$M_1 G\left(S_t, 0, 1, t, T^*\right) - M_1 G\left(S_t, M_1 \bar{S}_0, 1, t, T^*\right)$$
$$+ \frac{G\left(S_t, M_1 \bar{S}_0, 2, t, T^*\right)}{\bar{S}_0} - \frac{G\left(S_t, M_2 \bar{S}_0, 2, t, T^*\right)}{\bar{S}_0} + M_2 G\left(S_t, M_2 \bar{S}_0, 1, t, T^*\right)$$

A more sophisticated analytic approximation can be obtained using the results produced for average strike options by authors such as Levy (1992) and Bouaziz (1994). These results show how the joint distribution of $\ln(S_T)$ and $\ln(\bar{S}_T)$ can be approximated as a bivariate normal distribution. From this, the joint distribution of $\ln(\bar{S}_T^{\alpha} S_T)$ and $\ln(\bar{S}_T)$ can be obtained as a bivariate normal distribution. Define $X = \bar{S}_T^{\alpha} S_T$ and $Y = \bar{S}_T$. The value of a derivative that pays off X when $Y > K$ is

$$\int_{-\infty}^{+\infty} \int_{K}^{+\infty} X g(X|Y) f(Y) dY dX$$

where f anf g are the probability densities of Y and $X|Y$.

This can be evaluated in a straightforward way by reversing the order of integration and produces a result for the averaging case that corresponds to the result in equation (2). This enables the value of the MSU for the averaging case to be calculated analogously to the way it is calculated for the non-averaging case. However, we emphasize that, for the averaging periods encountered in practice, the simple approach of shortening the maturity works well.

Dividend protection when there is averaging can be handled similarly to the way it is handled for the non-averaging case. Increasing the number of units received by e^{qT} increases the value of the instrument by e^{qT} (not e^{qT^*}). Increasing the number of shares received on vesting by e^{qT} also has

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[5] To see why the approximation works well, suppose the averaging takes place over n days. A payoff of S_T at time T is equivalent to payoffs of S_T / n at on each of the n days. These payoffs are in turn is approximately the same as receiving $\bar{S}_T$ at the midpoint of the n days

this effect.   However, perfect dividend protection is not realized because of the impact of averaging. A simple way of achieving approximate dividend protection is to

a) Increase the number of units issued by a multiplicative factor e^{qT^*}
b) Increase the number of shares received when each unit vests by a multiplicative factor e^{qT^*}
c) Multiply the cap and floor thresholds by e^{-qT^*}

3. **Generalization**

An MSU can be generalized so that the number of shares received is $\left(S_T/A\right)^{\alpha-1}$ for some constants A and α. (For the standard MSU, α is two.) The payoff from the generalized MSU is

$$\begin{array}{lll} M_1 S_T & \text{if} & S_T < M_1^{1/(\alpha-1)} A \\ \left(S_T/A\right)^{\alpha-1} S_T & \text{if} & M_1^{1/(\alpha-1)} A \le S_T \le M_2^{1/(\alpha-1)} A \\ M_2 S_T & \text{if} & S_T > M_2^{1/(\alpha-1)} A \end{array}$$

The cap and floor for the number of shares received is M_1 and M_2, as before. The value of the generalized MSU then becomes:

$$M_1 G\left(S_t, 0, 1, t, T\right) - M_1 G\left(S_t, M_1^{1/(\alpha-1)} A, 1, t, T\right)$$
$$+\frac{G\left(S_t, M_1^{1/(\alpha-1)} A, \alpha, t, T\right)}{A^\alpha} - \frac{G\left(S_t, M_2^{1/(\alpha-1)} A, \alpha, t, T\right)}{A^\alpha} + M_2 G\left(S_t, M_2^{1/(\alpha-1)} A, 1, t, T\right)$$

The parameter α enables the amount of leverage to be adjusted. If less leverage than the basic MSU is required then α can be set to a value less than two. If more leverage is required a value of α greater than two can be used. As α tends to one, the MSU becomes an RSU.

When there is no cap or floor and $\alpha = 2$ the parameter A plays a simple scaling role. Doubling A cuts the value of the MSU in half. When there is a cap and a floor increasing A makes it more likely that the floor applies while decreasing A makes it more likely that the cap will apply.

Exhibit 4 shows how the value of the generalized MSU considered earlier depends on α and M_2 when $S_0 = A = 100$, $r = 5\%$, $t = 0$, $T = 3$, $q = 0$, $\sigma = 25\%$, and $M_1 = 0.5$. (The value has very little dependence on M_2.) For $1 < \alpha < 2$, the MSU is less expensive than a regular MSU. Interestingly, the value of the MSU does not change much as α increases above 2 when $M_2 = 1.5$. But for higher values of M_2 the benefits of the increased leverage are substantial.

Dividend protection and averaging can be handled analogously to the basic MSU case discussed in the previous section by increasing the number of units received and/or increasing the number of shares received per unit.

Other generalizations are possible where the number of shares is a polynomial in S_T/A.

4. Conclusions

MSUs are an interesting alternative to options and RSUs. They have the leverage characteristics of options while providing incentives that are similar to those of RSUs. This paper has shown that MSUs can be valued analytically. The early exercise of an MSU is usually not permitted. This has two potential advantages. First, all executives granted a particular MSU receive the same payoff from that MSU. They do not have to spend time worrying about when to exercise. Second, the calculation of the MSU value for reporting purposes is less subjective than the valuation of options because no assumptions about the early exercise behavior of employees are necessary.

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Exhibit 1

Payoff from a) an MSU where M_1=0.5, M_2=1.5, and S_0=100, b) an RSU, and c) and an option with strike price 100.

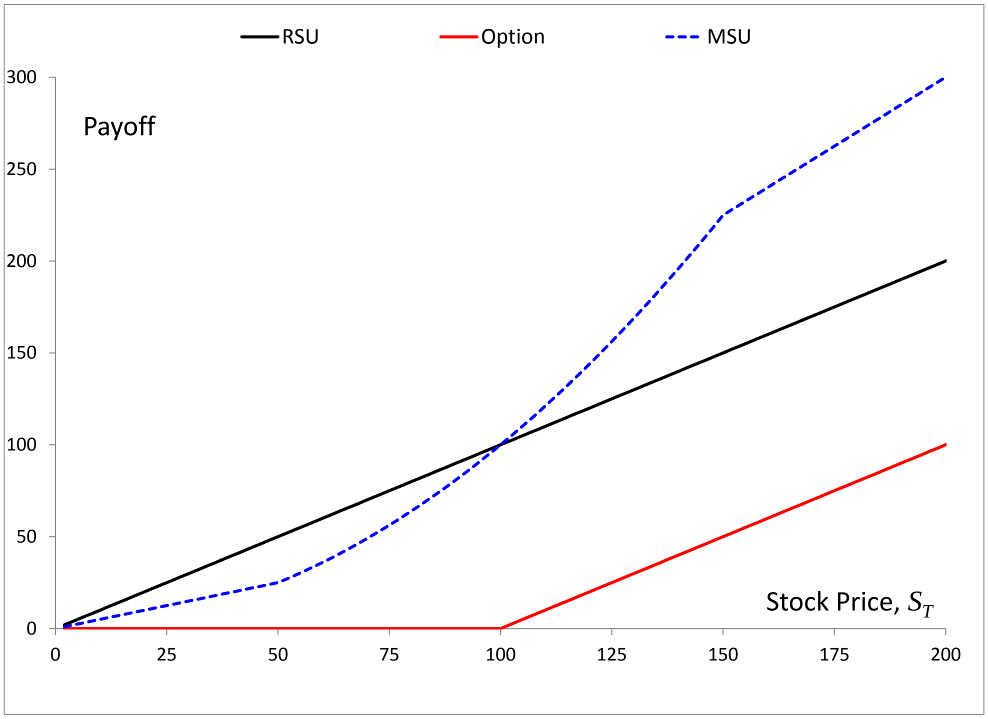

Exhibit 2

Value of an MSU for different M_1 and M_2 when
S_t = S_0 = 100, r = 2%, q = 0, σ = 25%, and t = 0, T = 3.

		M_1				
		0.00	0.25	0.50	0.75	1.00
M_2	1.0	91.45	91.45	91.63	93.63	100.00
	1.5	113.67	113.67	113.85	115.84	122.22
	2.0	122.70	122.70	122.88	124.88	131.25
	3.0	127.31	127.31	127.49	129.48	135.85
	5.0	128.06	128.06	128.24	130.23	136.61
	50	128.08	128.08	128.26	130.26	136.63

Exhibit 3

Value of an MSU for different σ and T when
$S_t = S_0 = 100$, $r = 2\%$, $q = 0$, $t = 0$, $M_1 = 0.5$ and $M_2 = 1.5$.

		σ				
		15%	25%	35%	45%	55%
	0.5	102.15	104.05	106.11	107.89	109.41
	1.0	104.29	107.13	109.42	111.30	113.04
T	2.0	108.10	111.12	113.39	115.54	117.71
(yrs)	3.0	111.15	113.85	116.17	118.59	121.10
	5.0	115.74	117.81	120.31	123.16	126.13
	10	123.18	124.35	127.21	130.61	134.02

Exhibit 4

Value of an MSU for different α and M_2 when
$S_0 = A = 100$, $r = 2\%$, $q = 0$, $\sigma = 25\%$, $t = 0$, $T = 3$ and $M_1 = 0.5$.

		M_2				
		1.50	2.00	3.00	5.00	50.00
	1.00	100.00	100.00	100.00	100.00	100.00
	1.25	104.53	104.53	104.53	104.53	104.53
	1.50	109.56	110.52	110.55	110.55	110.55
	1.75	112.51	117.23	118.29	118.33	118.33
α	2.00	113.85	122.88	127.49	128.24	128.26
	2.50	114.70	130.17	145.35	153.91	156.41
	3.00	114.84	134.21	158.75	181.15	199.78
	6.00	114.51	141.46	188.87	266.84	908.42